# 1 Introduction

## 1.1 Life Cycle of a Theoretical Physicist

1. Write down a *Lagrangian density* $L$. This is a polynomial in *fields* $\psi$ and their derivatives. For example
$$L[\psi] = \partial_\mu \psi \partial^\mu \psi - m^2 \psi^2 + \lambda \psi^4$$

2. Write down the *Feynman path integral*. Roughly speaking this is
$$\int e^{i \int L[\psi]} \mathcal{D}\psi$$
The value of this integral can be used to compute "cross sections" for various processes.

3. Calculate the Feynman path integral by expanding as a formal power series in the "coupling constant" $\lambda$.
$$a_0 + a_1 \lambda + a_2 \lambda + \cdots$$
The $a_i$ are finite sums over *Feynman diagrams*. Feynman diagrams are a graphical shorthand for finite dimensional integrals.

4. Work out the integrals and add everything up.

5. Realise that the finite dimensional integrals do not converge.

6. Regularise the integrals by introducing a "cutoff" $\epsilon$ (there is usually an infinite dimensional space of possible regularisations). For example
$$\int_{\mathbb{R}} \frac{1}{x^2} dx \longrightarrow \int_{|x|>\epsilon} \frac{1}{x^2} dx$$

7. Now we have the series
$$a_0(\epsilon) + a_1(\epsilon)\lambda + \cdots$$
**Amazing Idea:** Make $\lambda$, $m$ and other parameters of the Lagrangian depend on $\epsilon$ in such a way that terms of the series are independent of $\epsilon$.

8. Realise that the new sum still diverges even though we have made all the individual $a_i$'s finite. *No good way of fixing this is known.* It appears that the resulting series is in some sense an asymptotic expansion.

9. Ignore step 8, take only the first few terms and compare with experiment.

10. Depending on the results to step 9: Collect a Nobel prize or return to step 1.

There are many problems that arise in the above steps