Multi-nucleon correlations in Deep Inelastic Scattering at large Bjorken x_B

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Based on: A.A., Vary, Qiu, nucl-th/0701024

Outline

- **Introduction and overview**
  - CLAS “plateaus”
  - Factorization: nuclear and parton dynamics
- **Factorization of nuclear dynamics**
- **Collinear factorization (parton dynamics)**
- **Applications**
  - large x_B correlations
- **Conclusions and outlook**

  (more plots and details in the backup slides)

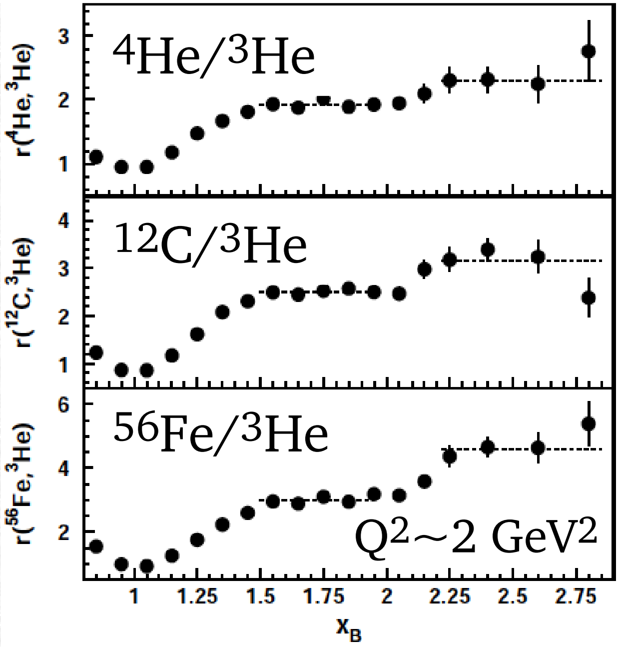
Based on: A.A., Vary, Qiu, nucl-th/0701024

Introduction

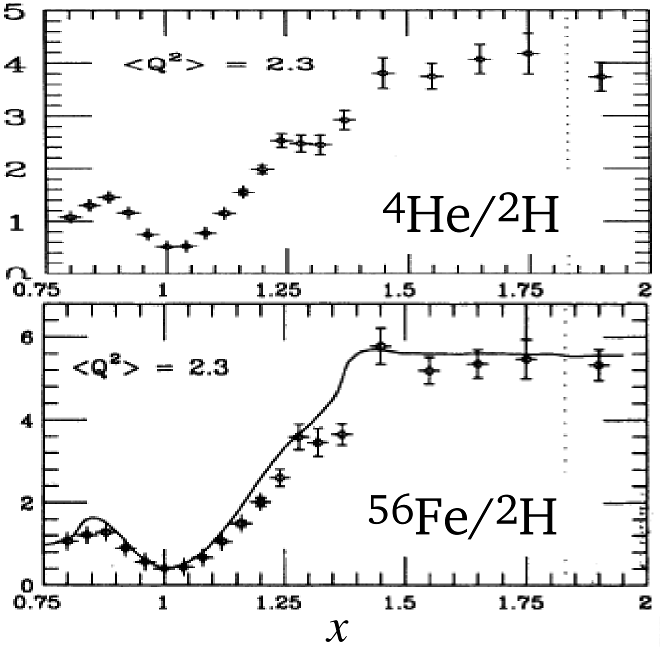
Plateaus at large x_B

- Per-nucleon cross-section ratios

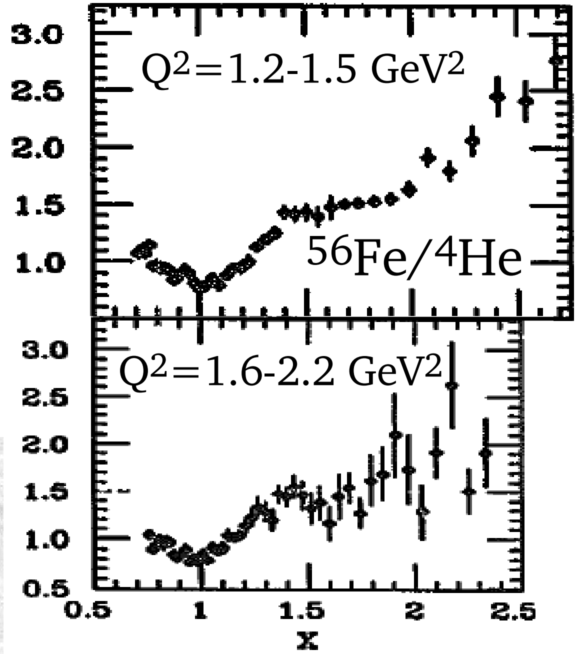
[CLAS, PRL96(06)082501]



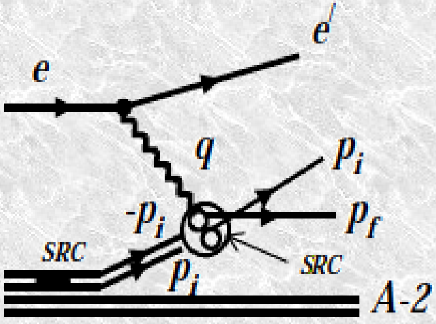
[SLAC: Frankfurt et al PRC '93]

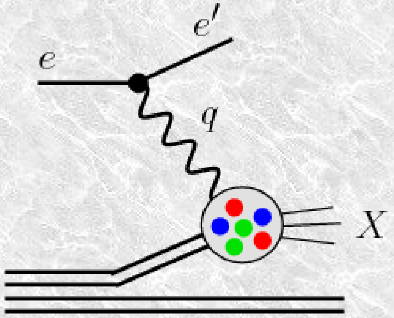


[SLAC: Day, NPA '88]



- “plateaus” generally ascribed to non-nucleonic degrees of freedom





SRC model [Frankfurt,Strikman '83]

quark-cluster model [Pirner, Vary, '81]

This work – central idea

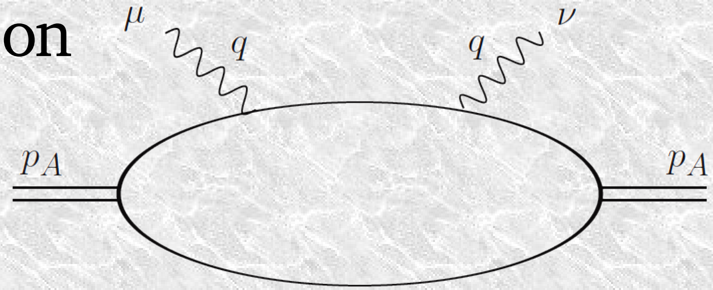
- Compute e+A cross-sections combining
  - realistic many-body nuclear wave function
    $\Rightarrow$ single-nucleon distributions (Fermi motion)
  - collinear factorization in QCD (parton dynamics)
  - exact treatment of kinematics (nucleus, nucleon, parton level)

- Minimalist approach to answer
  - How far can “conventional” physics explain CLAS data?
  - When do new degrees of freedom emerge?

- Wider applicability, e.g.,
  - nucleon PDF at large x in lepton-nucleus scatterings

This work – main result

- nDIS cross-section in 1-photon approximation determined by the **hadronic tensor**



$$W_A^{\mu\nu}(p_A,q) = \frac{1}{4\pi}\int d^4z\, e^{-iq\cdot z}\langle p_A|j^{\dagger\mu}(z)j^\nu(0)|p_A\rangle$$

- Factorization, 2 steps: nuclear and partonic dynamics

$$W_A^{\mu\nu} = \rho_A \otimes W_N^{\mu\nu} = \rho_A \otimes \mathcal{H}_f^{\mu\nu} \otimes \phi_{f/N}$$

nucleon distribution

bound nucleon tensor

partonic tensor

parton distribution function (PDF)

- Correspondingly, for structure functions

$$F_{iA}(x_B,Q^2) = \rho_A \otimes \Phi_i F_i^{(0)}(\xi_N,Q^2)$$

Factorization of nuclear dynamics

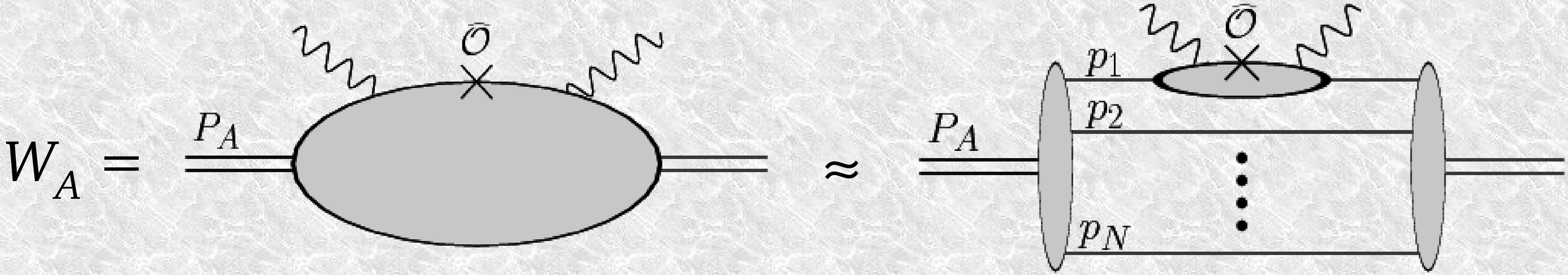
Model of the nucleus

- DIS determined by

$$W_A^{\mu\nu} = \langle P_A | \widehat{O}^{\mu\nu} | P_A \rangle$$

- **1^{st} assumption**
  nucleus = N nucleons with momenta p_1, p_2, … ,p_N and $\Sigma_i\, p_i = P_A$
  [nuclear Hilbert space $\mathcal{H}_A = \Pi_i\, \mathcal{H}_{Ni}$]

- **2^{nd} assumption**
  interaction involves only 1 nucleon [$\widehat{O}$ acts on single-nucleon $\mathcal{H}_{Ni}$]



Factorization of nuclear distribution

- Use of 2 completeness relations $1 = \int \prod_{i=1}^{A} \frac{d^4 p_i}{(2\pi)^4} |p_i\rangle\langle p_i|$ yields

$$W_A^{\mu\nu} = \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_1'}{(2\pi)^4} \rho_A^{\mathit{off}}(p_1, p_1') \langle p_1 | \widehat{\mathcal{O}}^{\mu\nu} | p_1' \rangle$$

with off-diagonal density matrix

$$\rho_A^{\mathit{off}}(p_1, p_1') = \int \prod_{i=2}^{A} \frac{d^4 p_i}{(2\pi)^4} \langle P_A | p_1, p_2, \ldots, p_A \rangle \langle p_1', p_2, \ldots, p_A | P_A \rangle$$

- Momentum conservation implies

$$\rho_A^{\mathit{off}}(p_1, p_1') = (2\pi)^4 \rho_A(p_1) \delta^{(4)}(p_1 - p_1')$$

so that, as promised,

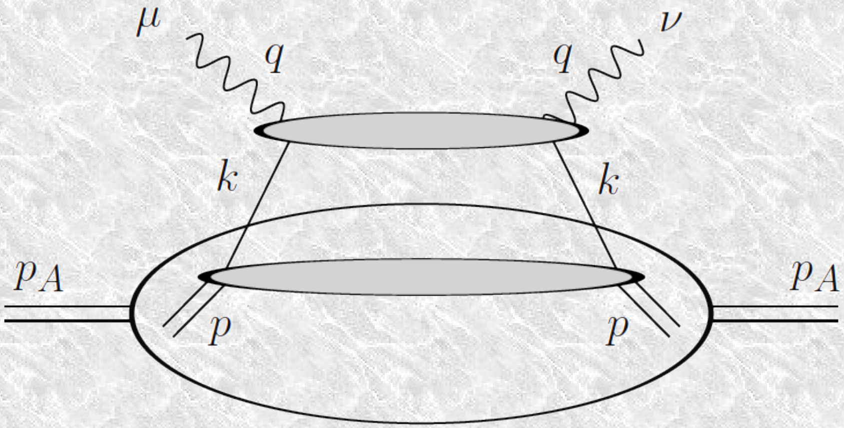
$$W_A^{\mu\nu} = \int \overbrace{\frac{d^4 p_1}{(2\pi)^4} \rho_A(p_1)}^{= d\mu_A \quad \textit{Fermi smearing measure}} \langle p_1 | \widehat{\mathcal{O}}^{\mu\nu} | p_1 \rangle = \rho_A \otimes W_N$$

Collinear factorization

(parton dynamics)

Impulse approximation

- Large Q^2, impulse approximation, $p_A{}^2 = (M_A/A)^2$



Note:
$p_A = P_A / A$

- Invariants:

$$x_B = \frac{-q^2}{2p_A \cdot q} \qquad \overline{m}^2 = p_A^2 = M_A^2/A^2 \qquad Q^2 = -q^2$$

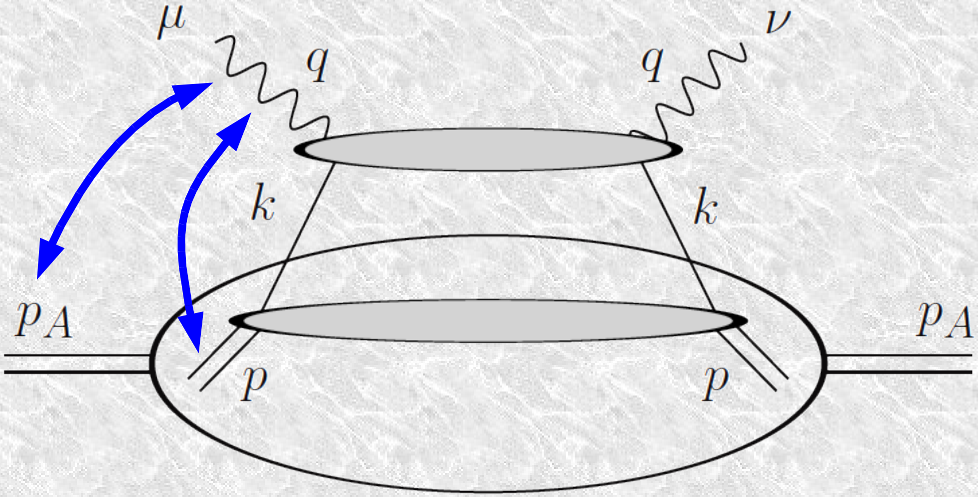
$$x_N = \frac{-q^2}{2p \cdot q} \qquad m^2 = p^2$$

$$\overline{x} = \frac{-q^2}{2k \cdot q}$$

- **remarks:**
  - non-zero mean nucleon mass
  - off-shell nucleon

Choice of frame - 1

- Need to define light-cone "+" and "–" directions.
  2 possibilities:
  - **A-frame**: "+" and "–" in the $\{q, p_A\}$ plane
  - **N-frame**: "+" and "–" in the $\{q, p\}$ plane



- N-frame useful to compute nuclear F_{iA} in terms of nucleon F_i
  - $x = k^+/p^+$ = fraction of nucleon momentum

Choice of frame - 2

- We choose the N-frame – momenta read:

$$p^\mu = p^+ \overline{n}^\mu + \frac{m^2}{2p^+} n^\mu$$

$$q^\mu = -\xi_A \omega p^+ \overline{n}^\mu + \frac{Q^2}{2\xi_A \omega p^+} n^\mu$$

$$p_A^\mu = \omega p^+ \overline{n}^\mu + \frac{\overline{m}_\perp^2}{2\omega p^+} n^\mu + \vec{p}_{A\perp}^{\,\mu}$$

light-cone definitions:

$$\overline{n} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

$$n = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$$

$$a^\pm = (a_0 \pm a_3)/\sqrt{2}$$

where we assume theoretically known:

$$\omega = p_A^+/p^+$$

$$m_T^2 = \vec{p}_T^{\,2} + \overline{m}^2$$

p^+ = boost parameter

p_A^+ , p_T = Fermi motion

- Nuclear Nachtmann variable $$\xi_A = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 \overline{m}_T^2/Q^2}}$$
- Nucleon Nachtmann variable $$\xi_N = \frac{2x_N}{1 + \sqrt{1 + 4x_N^2 m^2/Q^2}} = \xi_A \omega$$
- Free-nucleon limit A → 1 ⇒ TMC *à la* Ellis-Furmanski-Petronzio

[NPB 212(83)29]

Collinear factorization

- Expand parton *k* around its light-cone component:

$$k^\mu = x p^+ \overline{n}^\mu + O(k - x p^+ \overline{n}) \qquad \text{with} \qquad x = k^+/p^+$$

higher-twist corrections

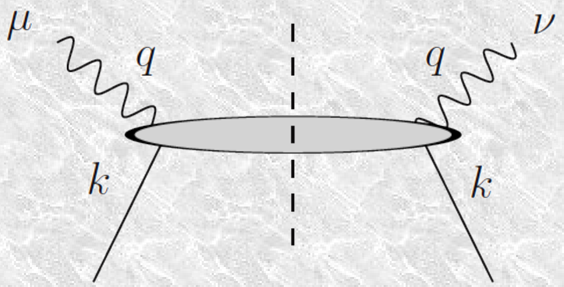
- Then [Collins, Soper, Sterman, '80s]:

$$W_N^{\mu\nu}(x_N, Q^2) = \sum_f \int \frac{dx}{x} \mathcal{H}_f^{\mu\nu}(\overline{x}, Q^2)\, \varphi_{f/N}(x, Q^2) + O(1/Q^2)$$

partonic tensor   bound nucleon PDF

where $\mathcal{H}^{\mu\nu}$ is the hadronic tensor for a parton target :

$$\mathcal{H}_f^{\mu\nu}(\overline{x}, Q^2) =$$



computable in pQCD order by order in α_s

on shell!   ($k^2 \sim 0$ for u,d,s)

and

$$\overline{x} = Q^2/2k \cdot q = (\xi_A \omega)/x$$

Nuclear structure function

- Nuclear hadronic tensor at Leading Twist, any order in α_s:

$$W_A^{\mu\nu}(x_B, Q^2) = \rho_A \otimes \mathcal{H}_f^{\mu\nu} \otimes \phi_{f/N}$$

$$= \sum_f \int d\mu_A \int \frac{dx}{x} \mathcal{H}_f^{\mu\nu}(\frac{\xi_A \omega}{x}, Q^2) \varphi_{f/N}(x, Q^2)$$

- Define "free massless nucleon" $F_i^{(0)}$ (set $m^2=0$ in our kinematics)
- Put everything together:
  nuclear F_{iA} in terms of nucleon $F_i^{(0)}$ and single nucleon Fermi motion

$$F_{1A}(x_B, Q^2) = \int d\mu_A \left\{ F_1^{(0)}(\xi_A\omega, Q^2) + \left[\frac{(1+\delta_\omega)^2}{(1+\delta_A)(1+\delta_n)} - 1 \right] \frac{F_2^{(0)}(\xi_A\omega, Q^2)}{4\xi_A\omega} \right\} \theta(1-\xi_A\omega)$$

$$F_{2A}(x_B, Q^2) = \frac{x_B}{1+\delta_A} \int d\mu_A \left[\frac{3(1+\delta_\omega)^2}{(1+\delta_A)(1+\delta_n)} - 1 \right] \frac{F_2^{(0)}(\xi_A\omega, Q^2)}{2\xi_A\omega} \theta(1-\xi_A\omega)$$

Note: $F_i^{(0)}$ evaluated at $\xi_N = \xi_A \omega$

Applications

Approximations

- Assume **on-shell nucleons** with $m^2 = \overline{m}^2$
  - $F_i^{(0)}$ ≡ massless free nucleon structure functions
  - from QCD global fits not already including TMC, e.g., CTEQ5

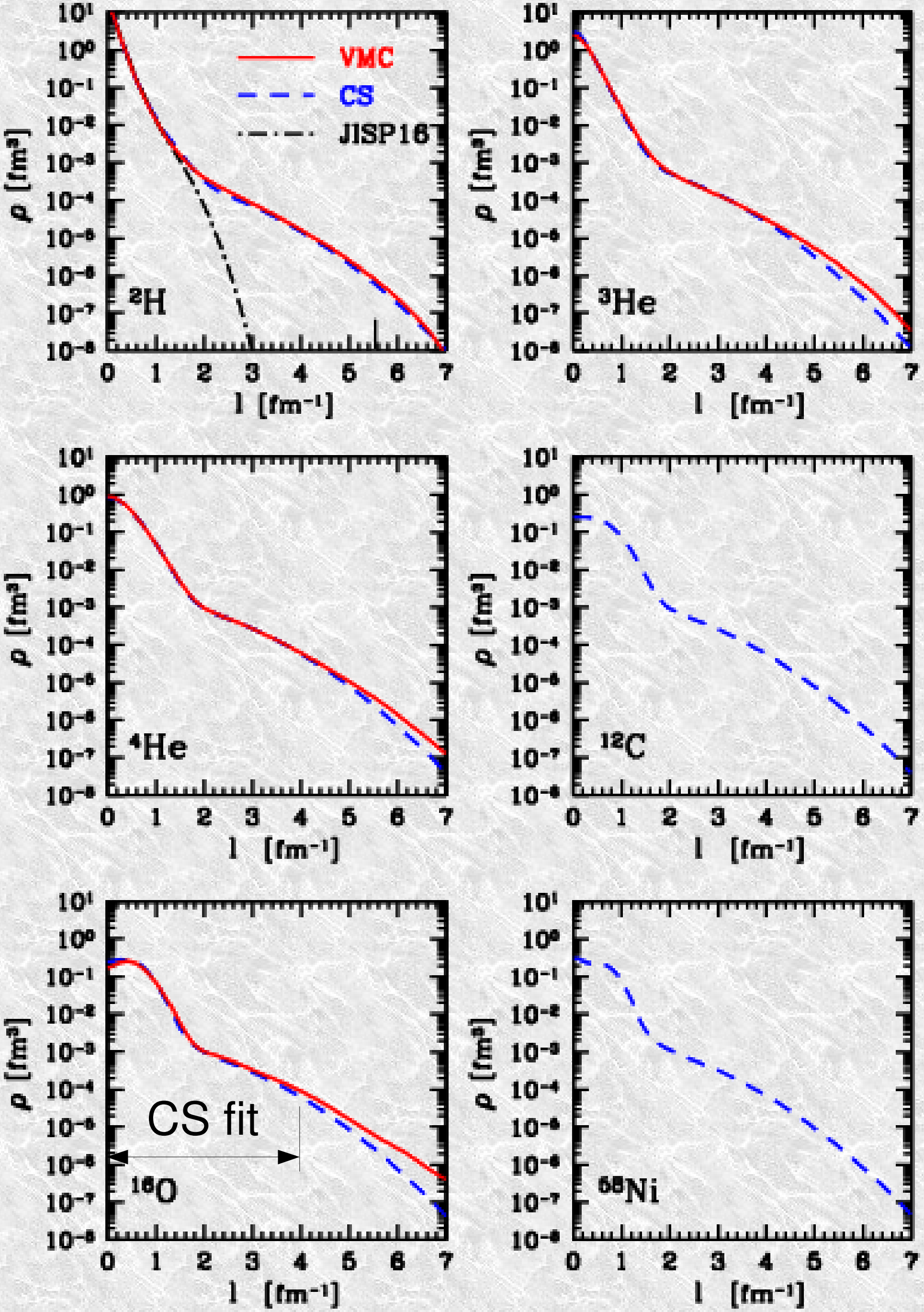
  [for off-shell corrections, Melnitchouk et al. PRD '94]

- Use **non-relativistic nucleon distributions**
  - realistic many body computations with NN and NNN potentials
  - fitted to low-E nuclear properties

$$\rho_A(p) \approx (2\pi)^4\, 2\sqrt{m^2 + \vec{p}^{\,2}}\, \delta(m^2 - \overline{m}^2)\, \rho_A^{nr}(\vec{p})$$

- **Note:**
  - "parameter free" computation (no tunable parameters)
  - Only freedom is the choice of nuclear distribution ρ_A
  - baseline computation for comparison to data

Nucleon distributions

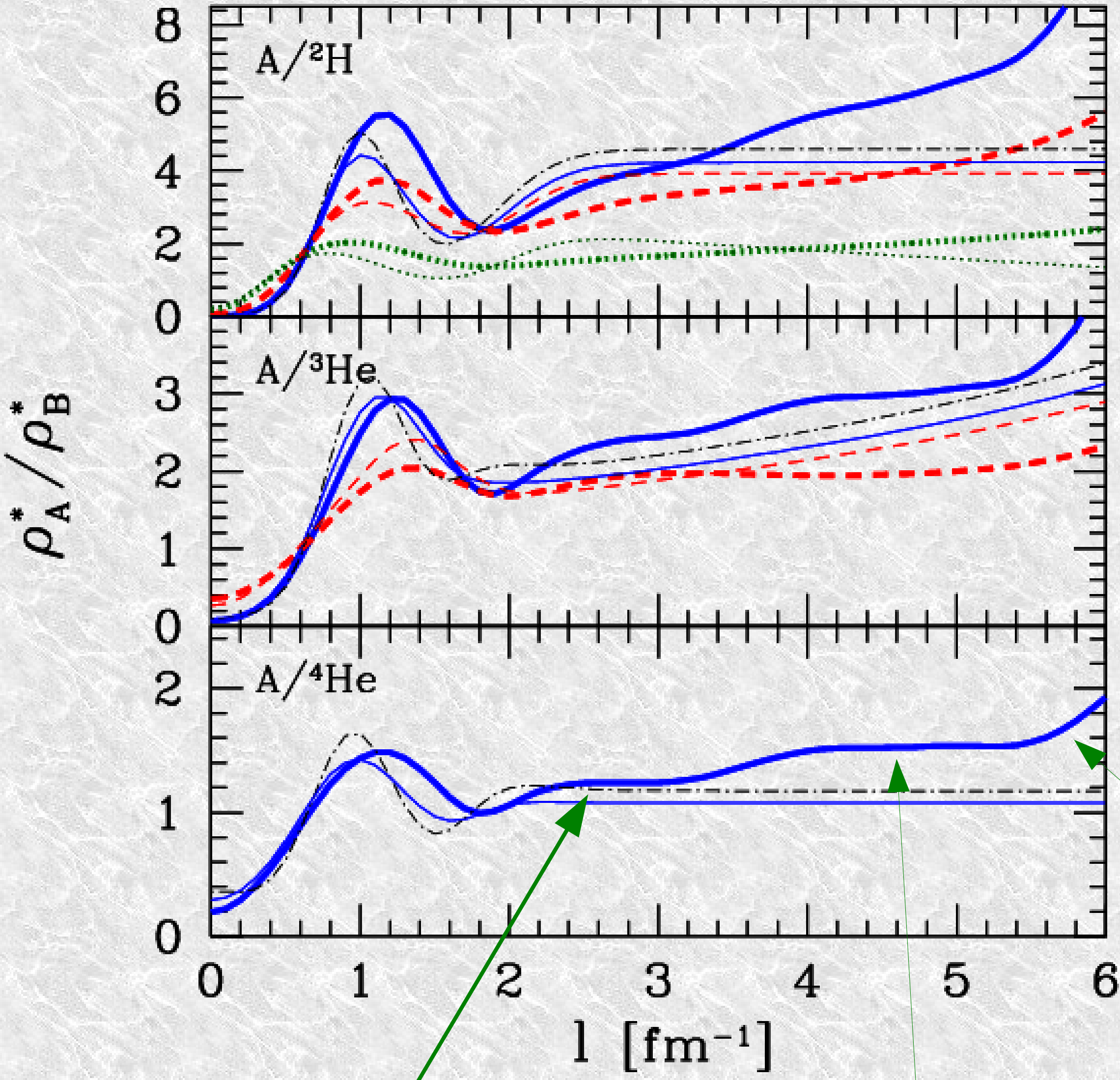


- **VMC**: Variational Monte Carlo [Pieper et al. PRC 46(92)17412]
  - NN + NNN potentials (AV18 + UIX)
  - NN + NNN correlations
- **CS**: Ciofi degli Atti and Simula [PRC 53(96)1689]
  - parametrization of several comps. at $l<4$ fm^{-1}
  - assumes universal NN correlation
  - no NNN correlations
- **JISP16**: [Shirokov et al. PLB 644(07)33]
  - non local NN potential
  - NN + NNN correlations

Note:

Potentials fitted to low-E nuke properties
$\Rightarrow$ uncertainties in large l tails

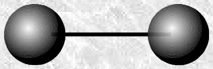
Nucleon distributions – A/B ratios



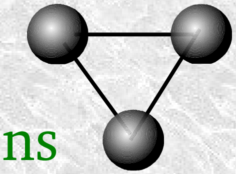
thick: VMC
thin: CS

- **VMC** (thick lines)
  - 2-plateau structure
- **CS** fit (thin lines)
  - no NNN correlation (by construction)
  - weaker NN correlation
  - weaker A-dependence

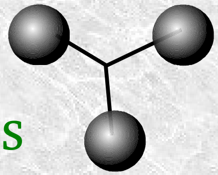
2N correlations



“indirect”
3N correlations



“direct”
3N correlations



I – Large-x_B correlations

Experimental settings

- I will analyze the following experimental kinematics:

CLAS	$Q^2=2$ GeV^2	$E_{lab}=4.5$ GeV
Hall-C	$\theta=18°$	$E_{lab}=5.8$ GeV
(E02-019)	$[Q^2\|_{xB=1}=2.5$ GeV$^2]$	
	$\theta=50°$	
	$[Q^2\|_{xB=1}=7.4$ GeV$^2]$	
CLAS12	$Q^2=5$ GeV^2	$E_{lab}=9$ GeV
EIC	$Q^2=100$ GeV^2	$E_{lab}=1000$ GeV

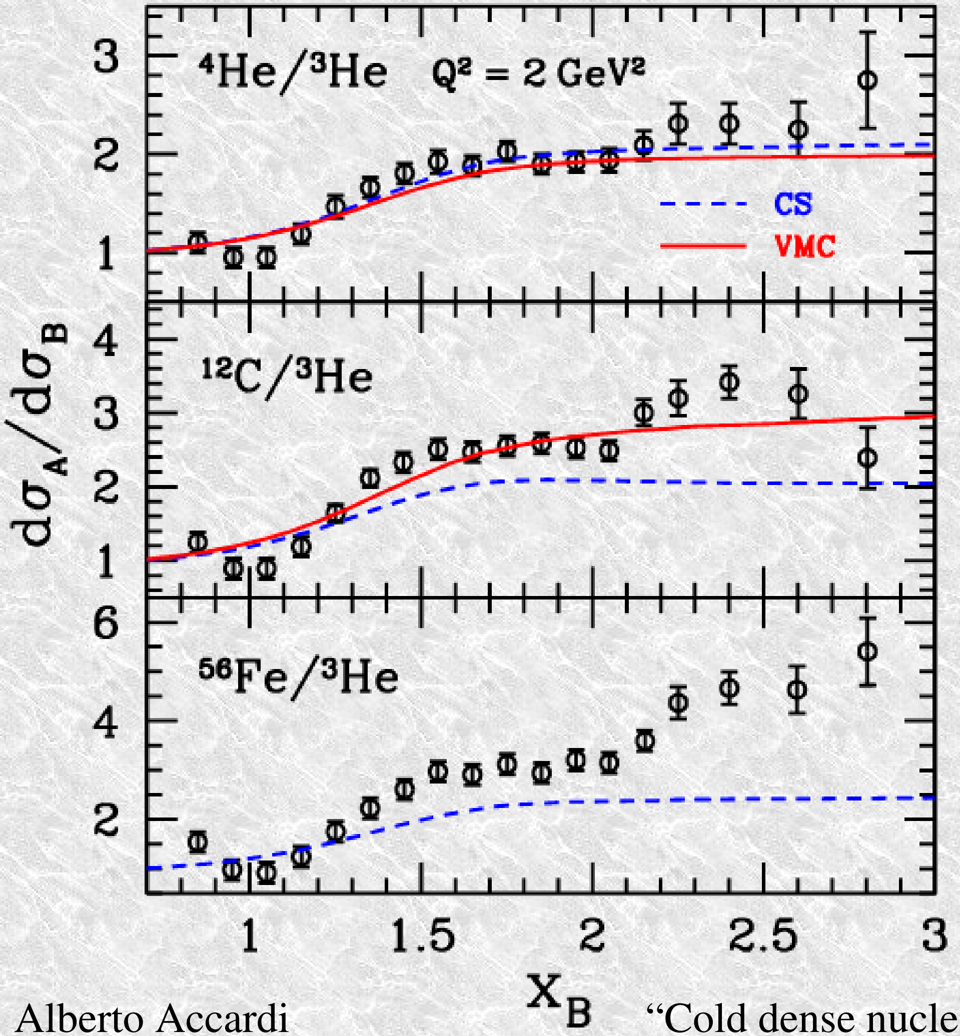
see backup slides

Cross section ratios – 1

- per-nucleon cross section:

$$\frac{d\sigma}{dQ^2 dx_B} = \frac{4\pi\alpha^2}{Q^4}\left\{\frac{1}{A}y^2 F_{1A}(x_B) + \left(1 - y - \frac{\overline{m}^2}{Q^2}x_B^2 y^2\right)\frac{F_{2A}(x_B)}{x_B}\right\}$$

CLAS [PRL 96(06)082501]



VMC

- 1st "plateau" explained by NN + indirect NNN correlations in the nuclear wave function
- does not describe 2nd "plateau" ⇒ **new degrees of freedom**

(no real plateau because of *l*-smearing with $\sigma_l \sim 1\ \mathrm{fm}^{-1}$)

CS

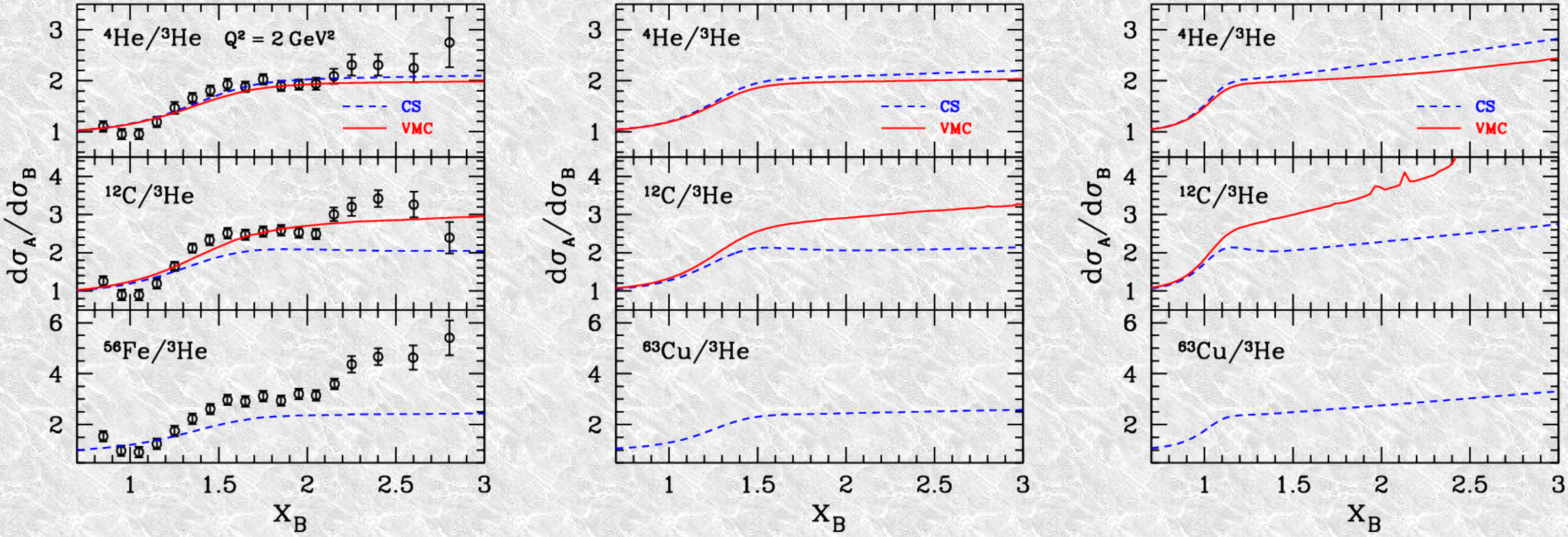
- does not describe the plateaus because of absence of NNN correlations in wave function

Cross section ratios – 2

CLAS

Hall C θ=18°

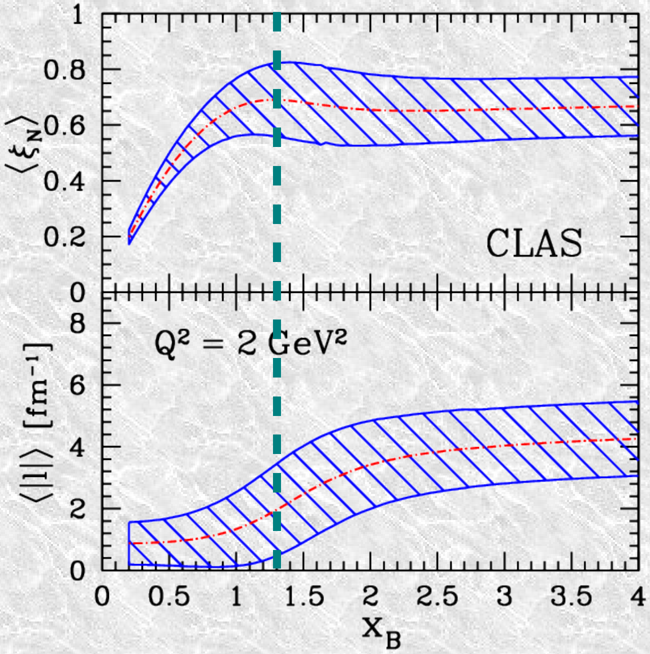
Hall C θ=50°



- As Q^2 increases:
  - onset of NN correlations narrows, moves to lower x_B,
  - slope of “plateaus” increases
  - difference VMC / CS becomes larger

Onset of NN correlations

- from CLAS experimental data:
  - onset of NN correlations at $x_B = 1.2 - 1.4$
- Computed average $\langle |l| \rangle$ and $\langle \xi_N \rangle$ with 1σ band



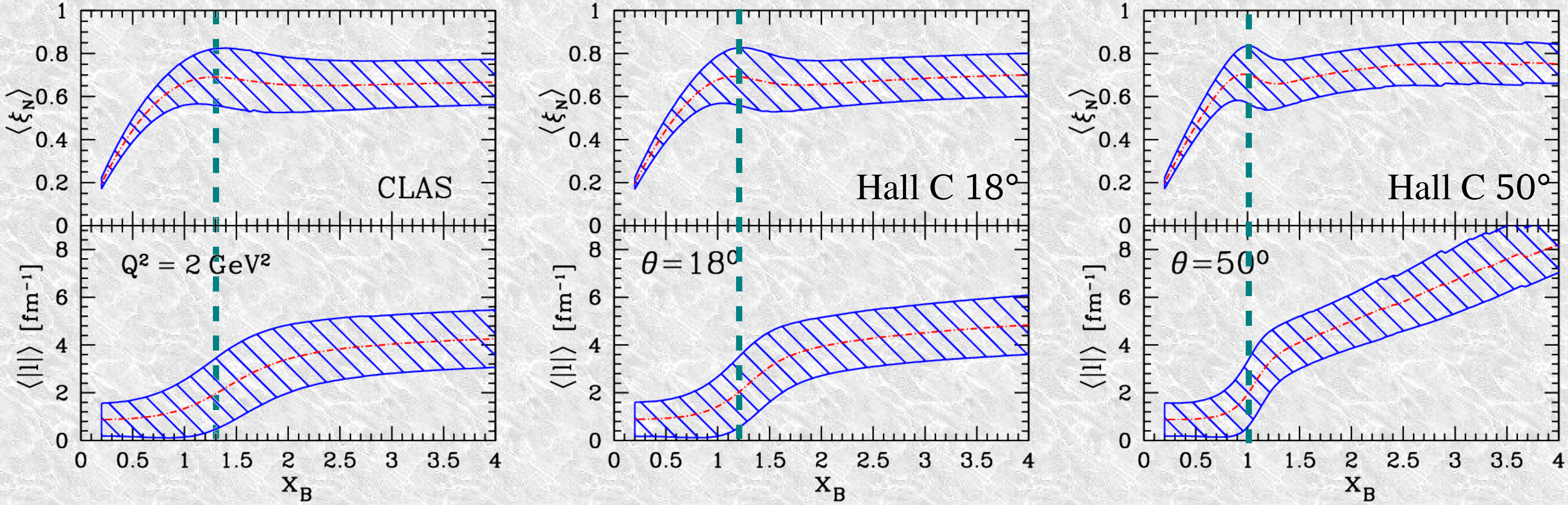
$x_B^{NN} \approx 1.3$

$l^{NN} \approx 2.0$ fm^{-1}

- Onset of NN correlations at CLAS
  $\leftrightarrow$ local max. of $\langle \xi_N \rangle$ $\leftrightarrow$ jump in $\langle |l| \rangle$
- Define onset x_B^{NN} (at any Q^2)
  as position of local max. $\Rightarrow x_B^{NN} = 1.3$
- Extract $l^{NN} = \langle |l| \rangle (x_B^{NN}) = 2.0 \pm 1$ fm^{-1}
  (Note large error bars, due to 1σ band)

Onset of NN correlations

- from CLAS experimental data:
  - onset of NN correlations at $x_B = 1.2 - 1.4$
- Computed average $\langle |l| \rangle$ and $\langle \xi_N \rangle$ with 1σ band



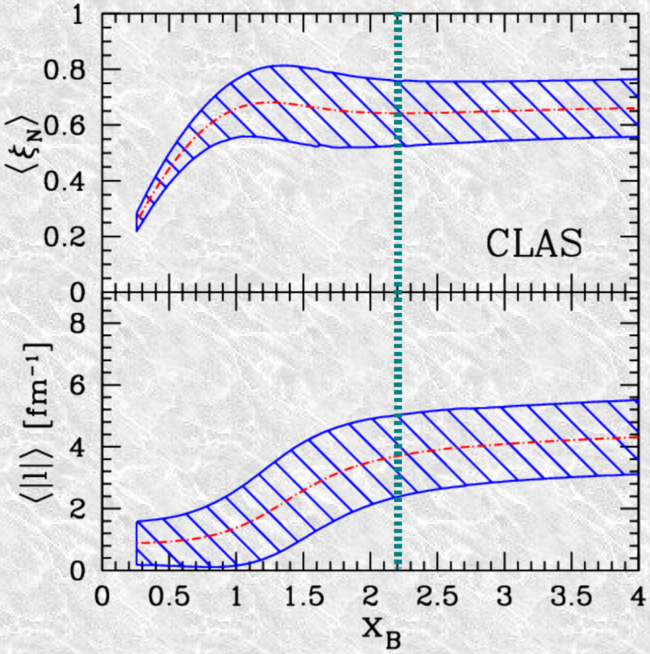
$x_B^{NN} \approx 1.3$ $x_B^{NN} \approx 1.2$ $x_B^{NN} \approx 1$

$l^{NN} \approx 2.0$ fm^{-1} $l^{NN} \approx 1.9$ fm^{-1} $l^{NN} \approx 2.0$ fm^{-1}

- **Note:** onset at constant *l*, corresponding to onset of hard tails in w.f.

Onset of new degrees of freedom

- from CLAS experimental data:
  - onset of new (non-nucleonic) degrees of freedom at $x_B = 2.1 - 2.3$
- Computed average $\langle|l|\rangle$ and $\langle\xi_N\rangle$ with 1σ band



$x_B^{new} \approx 2.2$

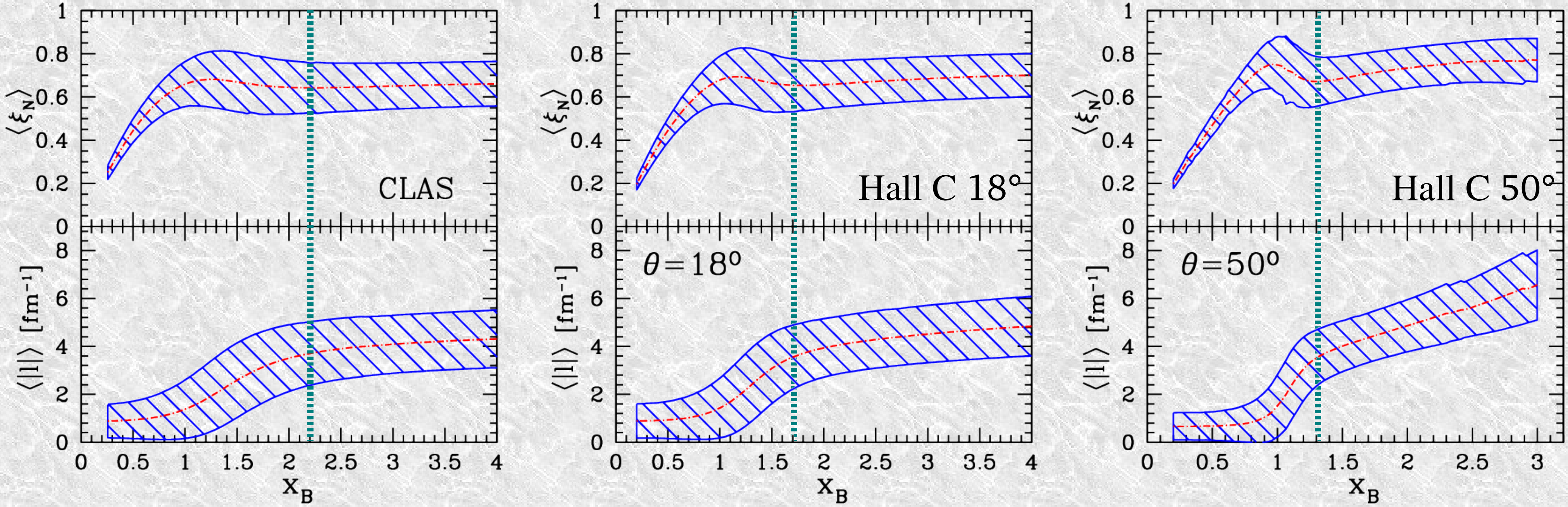
$l^{new} \approx 3.8$ fm^{-1}

- Assume 2nd plateau due to DIS on new d.o.f.
- at CLAS energy, take $x_B^{new}=2.2$ and determine $l^{new} = \langle|l|\rangle\,(x_B^{new})$
- Assume l^{new} independent of Q^2
- extract x_B^{new} at CLAS12, EIC

Onset of new degrees of freedom

- from CLAS experimental data:
  - onset of new (non-nucleonic) degrees of freedom at $x_B = 2.1 – 2.3$

- Computed average $\langle |l| \rangle$ and $\langle \xi_N \rangle$ with 1σ band

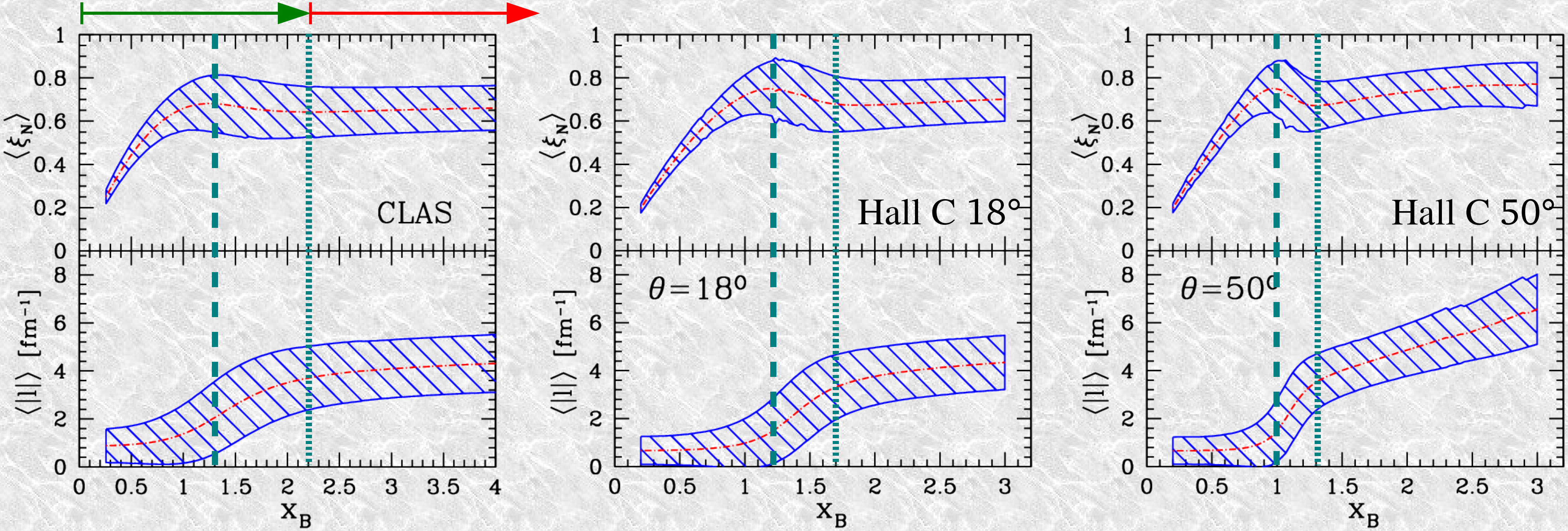


$x_B^{new} \approx 2.2$ $x_B^{new} \approx 1.7$ $x_B^{new} \approx 1.3$

$l^{new} \approx 3.8$ fm^{-1} $l^{new} \approx 3.8$ fm^{-1} $l^{new} \approx 3.8$ fm^{-1}

Onsets – summary

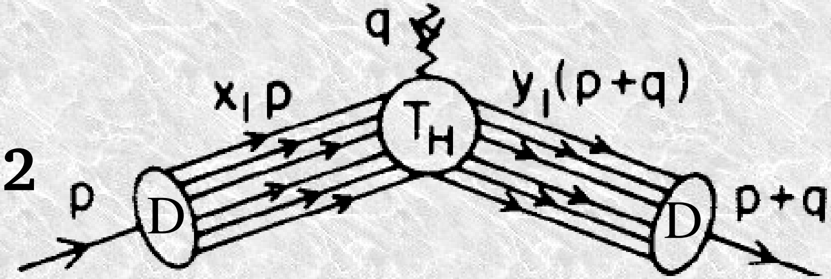
single nucleon new d.o.f.



	x_B^{NN}	l^{NN}	x_B^{new}	l^{new}
CLAS	1.3	1.9 fm^{-1}	2.2	3.8 fm^{-1}
Hall C 18°	1.2	2.0 fm^{-1}	1.7	3.8 fm^{-1}
Hall C 50°	1.0	1.9 fm^{-1}	1.3	3.8 fm^{-1}

2^{nd} plateau – other explanations (?)

- If 2^{nd} plateau not due to DIS on new d.o.f., onset may not run with Q^2
- Example:
  - quasi-elastic scattering on a "hidden color" deuteron state [Brodsky, Ji, Lepage, PRL51(83)83]
    - prob.(D) $\propto$ A(A-1) $\Rightarrow$ $\sigma_A^{QE}/A \propto A$
    - $x_D \approx 2$, $\langle l_D \rangle > 0 \Rightarrow$ $x_B \gtrsim 2$
  - "plateau" in σ ratios = **bump at $x_B \gtrsim 2$**
  - **height increases with A**
  - disappears as Q^2 increases



- Compare to Q.E. scattering on a nucleon:
    - prob.(D) $\propto$ A
    - $x_N \approx 1$, $\langle l_N \rangle \approx 0 \Rightarrow$ $x_B \approx 1$
    - Fermi motion at $l_N \approx 0$ increases with A
  - dip at $x_B \approx 1$, depth decreases with A

Conclusions

- **nDIS formalism** combines
  - QCD collinear factorization
  - nuclear many-body wave-functions
    ⇒ single-nucleon Fermi motion
  - exact kinematics at nuclear, nucleon, parton level
- no free parameters
  - only freedom is the choice of ρ_A
- **large-xB correlations:** if VMC is correct,
  - 1st plateau = single nucleon d.o.f. – large momentum due to NN+NNN correlations in nuclear wave function
  - 2nd plateau = non-nucleonic d.o.f. ("SRC") at $l^{new} \approx 3.8 \text{ fm}^{-1}$
- **Predictions for SRC – testable with Hall C data:**
  - Q^2-dependent onset ($x_B^{new} \approx 1.8, 1.3$ at $\theta=18°, 50°$)
    ⇒ DIS
  - fixed onset, disappears with Q^2
    ⇒ quasi-elastic scattering on "deuteron" – hidden color (??)

Outlook

- **Improvements of the formalism**
  - off-shell nucleons
  - EMC effect, impact at large x_B
  - quasi-elastic scattering, pionic cloud, ...

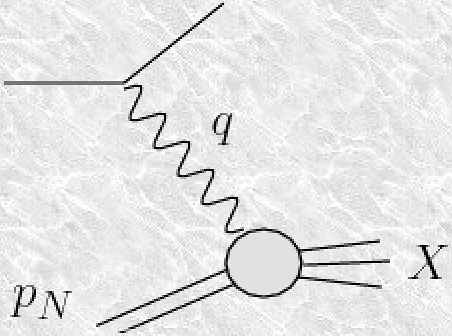
- **General purpose formalism:**
  - multi-nucleon interactions $\leftrightarrow$ non-diagonal matrix el.
  - higher-twist corrections $\big\}$ A-frame
  - definition of "nuclear PDF"
  - applications to p+A and A+A collisions
  - measurement of large-x PDF

- **How to validate a given ρ_A large-momentum tail?**
  - study of $d\sigma/dx_B$ in absolute value
  - exclusive measurements

The end

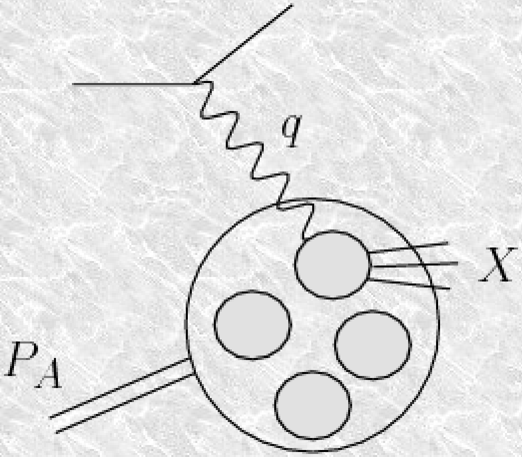
Backup slides

Why large x_B?



Bjorken invariant:

$$x_B = \frac{-q^2}{2p \cdot q} < 1$$



per-nucleon Bjorken invariant:

$$x_B = \frac{-q^2}{2\frac{P_A}{A} \cdot q} < A$$

- In a nucleus, x_B>1 if
  - nucleon has momentum larger than average
  - lepton scatters on non-nucleonic degrees of freedom

- Large x_B events select large-momenta in nuclear wave function
  - short distance NN repulsion
  - Short Range Correlations (SRC)
  - high-density fluctuations:
    1) color deconfinement
    2) chiral symmetry restoration

Factorization of nuclear distribution – 2

$$W_A^{\mu\nu} = \int \frac{d^4 p_1}{(2\pi)^4} \rho_A(p_1) \langle p_1 | \widehat{\mathcal{O}}^{\mu\nu} | p_1 \rangle = \rho_A \otimes W_N$$

- Gauge invariance at nuclear level then implies

$$q_\mu W_A^{\mu\nu} = 0 \quad \Rightarrow \quad q_\mu W_N^{\mu\nu} = 0$$

so we can define nuclear and nucleon structure functions as follows

$$W_A^{\mu\nu}(x_B, Q^2) = -\tilde{g}^{\mu\nu} F_{1A}(x_B, Q^2) + \frac{\tilde{p}_A^\mu \tilde{p}_A^\nu}{p_A \cdot q} F_{2A}(x_B, Q^2)$$

$$W_N^{\mu\nu}(x_N, Q^2) = -\tilde{g}^{\mu\nu} F_1(x_N, Q^2) + \frac{\tilde{p}^\mu \tilde{p}^\nu}{p \cdot q} F_2(x_N, Q^2)$$

- Finally, nucleon off-shellness is made explict by writing

$$d\mu_A = \frac{dm^2}{2\pi} \frac{d^3 p_1}{(2\pi)^3 2p_0} \rho_A(p) \Big|_{p^0 = \sqrt{m^2 + \vec{p}^2}}$$

Collinear factorization - 2

- Gauge invariance for on-shell partons:

$$q_\mu \mathcal{H}_f^{\mu\nu} = 0 \quad \Rightarrow \quad q_\mu W_N^{\mu\nu} = 0 \quad \Rightarrow \quad q_\mu W^{\mu\nu} = 0$$

  - justifies earlier decomposition of W_N on a microscopic level

- Tensor decomposition of $\mathcal{H}^{\mu\nu}$

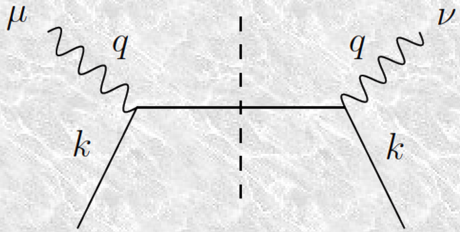
$$\mathcal{H}_f^{\mu\nu}(\overline{x}, Q^2) = -\tilde{g}^{\mu\nu}\, h_f^1(\overline{x}, Q^2) + \frac{\tilde{k}^\mu \tilde{k}^\nu}{k \cdot q}\, h_f^2(\overline{x}, Q^2)$$

$$\overline{x} = Q^2/2k \cdot q = (\xi_A \omega)/x$$

h_1 and h_2 computable in pQCD order by order in α_s

Remarks – 1

- Interpretation of Nachtmann variables at Leading Order:



$$\mathcal{H}_f^{\mu\nu}(\overline{x}, Q^2) = \qquad \propto \delta\big[(q + xp^+\overline{n})^2\big]$$

$$\Rightarrow \qquad x = \xi_A \omega = \xi_N$$

$$k^+ = \xi_N p^+ = \xi_A \, p_A^+$$

- Free-nucleon limit: $A \to 1\,, \quad x_N \to x_B\,, \quad \overline{m} \to m$

$$\xi_A \omega \longrightarrow \xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m^2/Q^2}}$$

  - target mass corrections *à la* Ellis-Furmanski-Petronzio [NPB 212(83)29]
  - A>1 ⇒ generalization to Fermi motion

Remarks – 2

- Why choosing N-frame?
  - Other than state $|p\rangle$ the definition of quark PDF at LO is the same as for a free nucleon:

$$\varphi_q(x) = \int \frac{dz^-}{2\pi} e^{-ixp^+z^-} \langle p|\overline{\psi}(z^- n)\,\frac{\gamma^+}{2}\,\psi(0)|p\rangle$$

  - Generalizes PDF to bound, off-shell nucleons
  - Would not be true in A-frame

- Formalism is quite general,
  - valid at leading twist, any order in α_s
  - only 1 assumption:
    single nucleon dynamics ⇔ diagonal nuclear matrix elements

Fermi measure

- Defining $p_A = p + l$ and using translation invariance,

$$d\mu_A = d^3 l \rho^*_A(\vec{l})$$

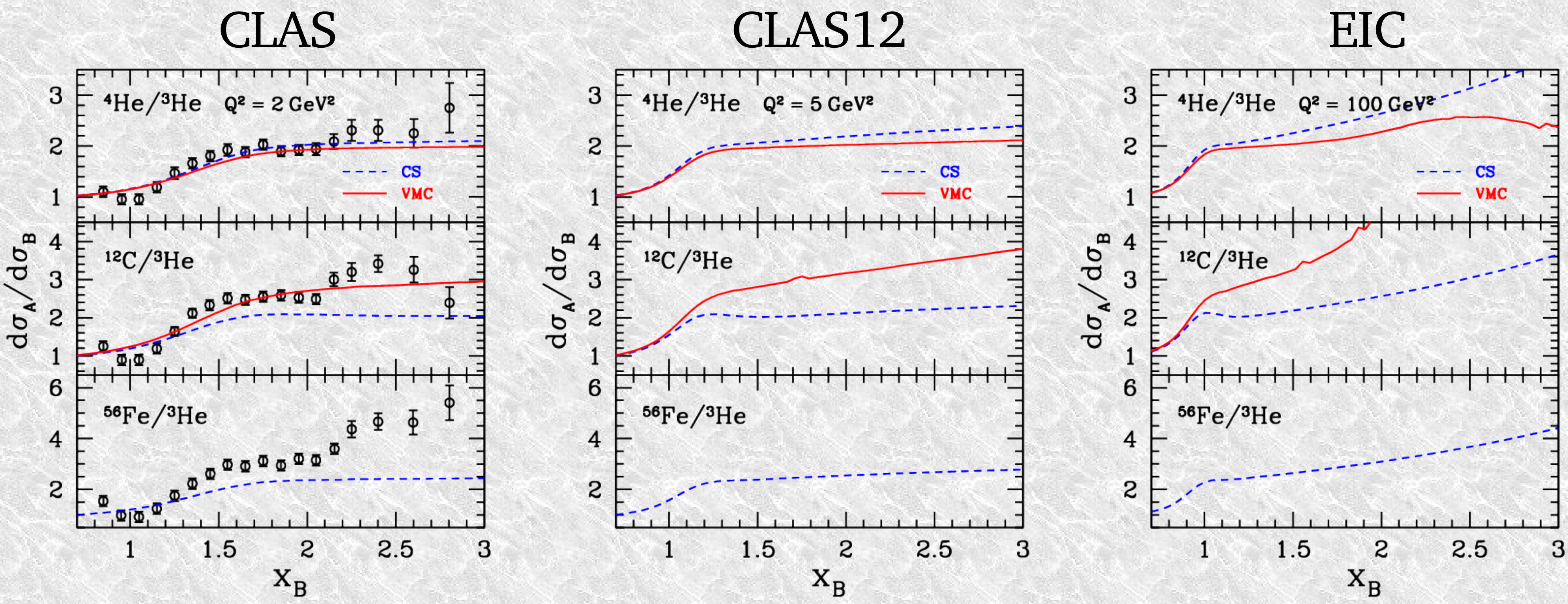
where $\rho^*_A(\vec{l})$ = nucleon distribution in nucleus rest frame

- The relative momentum l further appears in

$$\omega = (l_3 + \sqrt{l_3^2 + \overline{m}_\perp^2})/m$$

$$\overline{m}_\perp^2 = \overline{m}^2 + l_\perp^2$$

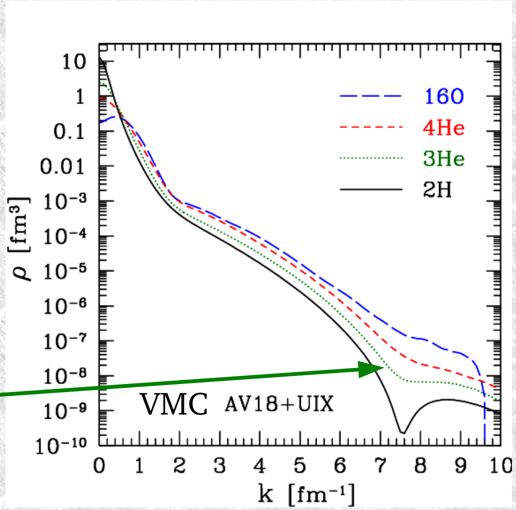
Cross section ratios – CLAS12 & EIC


CLAS

CLAS12

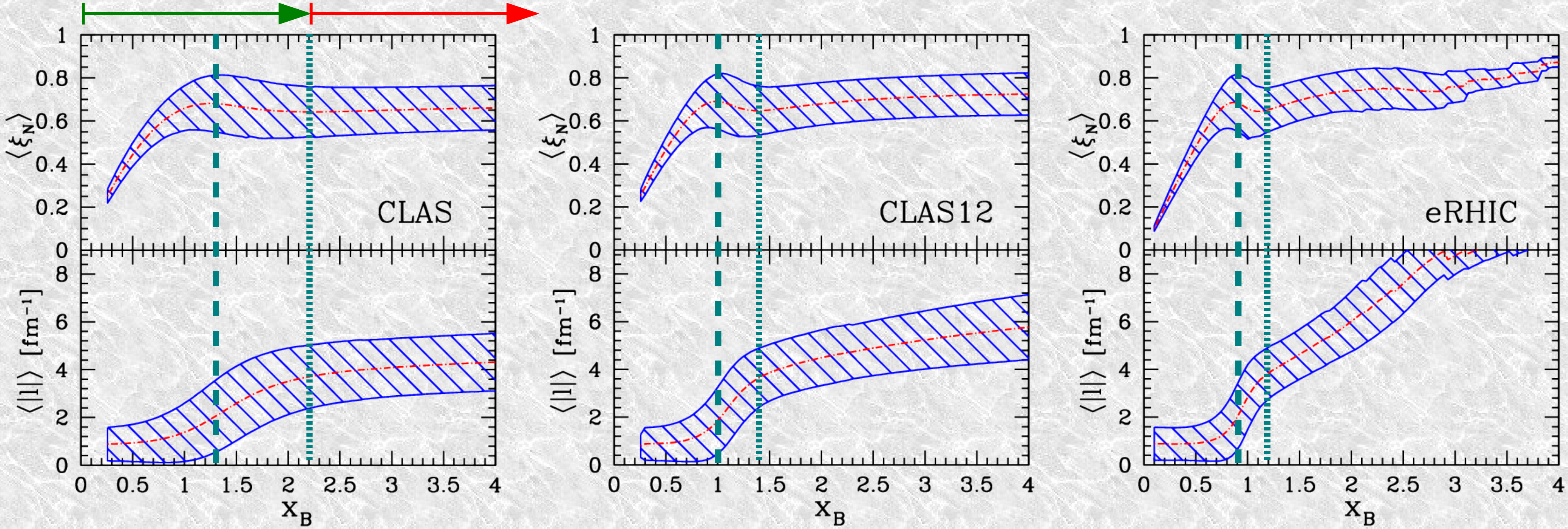
EIC

- As Q^2 increases:
  - onset of NN correlations narrows and moves to lower x_B,
  - slope of "plateau" increases
- at eRHIC, flattening of VMC ratio at $x_B > 2.3$ is a feature of $\rho_A(l)$ at l=7-8 fm^{-1}



Onsets at CLAS12 & EIC

single nucleon new d.o.f.

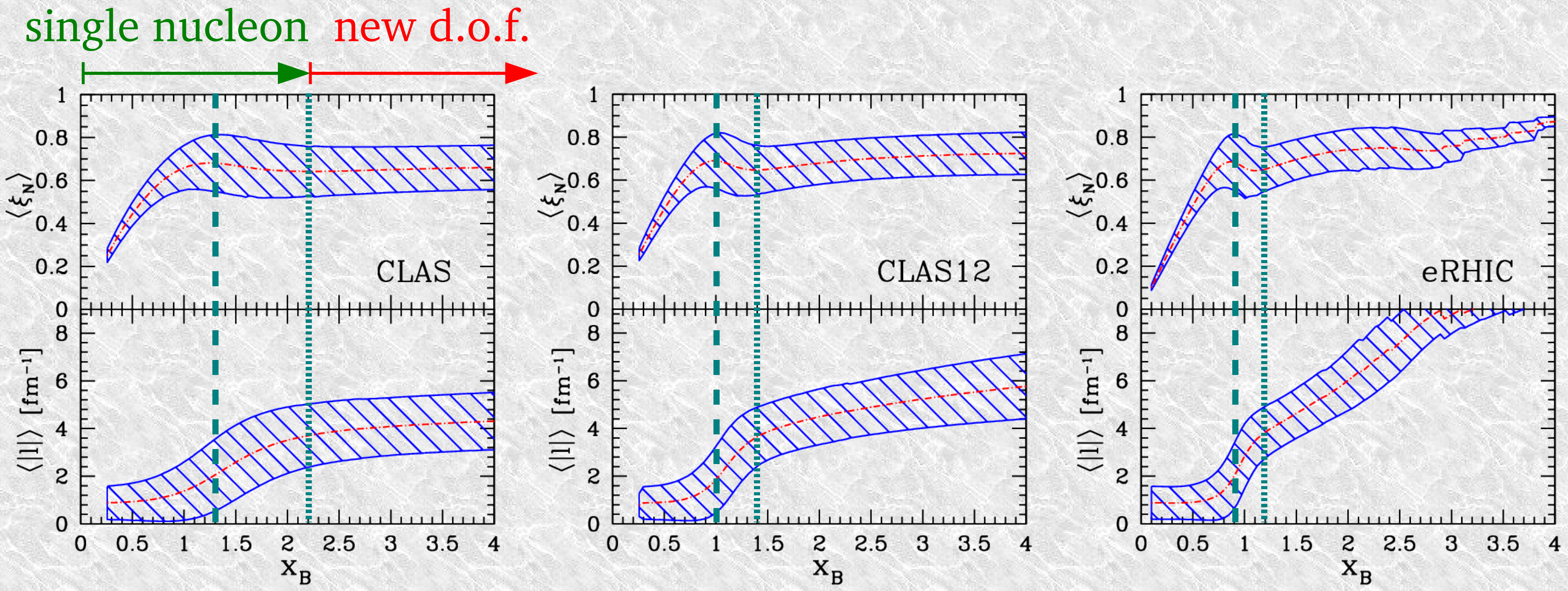


	x_B^{NN}	l^{NN}	x_B^{new}	l^{new}
CLAS	1.3	2.1 fm^{-1}	2.2	3.8 fm^{-1}
CLAS12	1.0	1.9 fm^{-1}	1.4	3.8 fm^{-1}
EIC	0.9	1.8 fm^{-1}	1.2	3.8 fm^{-1}

II – nucleon PDF at large x

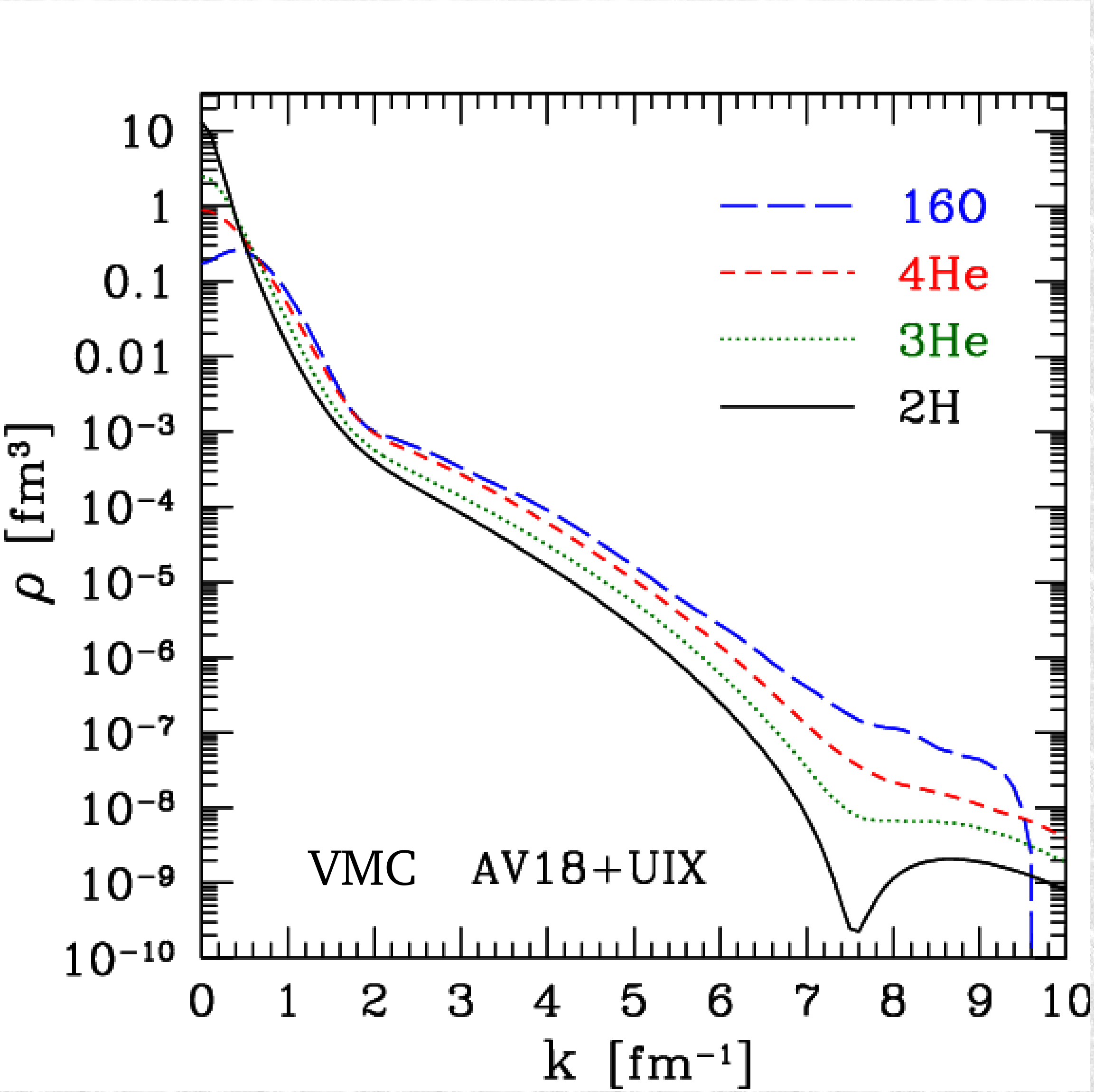
Nucleon PDF at large x

- Formalism useful to deconvolve Fermi motion and extract nucleon PDF from $F_i^{(0)}(\xi_N)$
- Useful range is $x_B < x_B^{new}$ only (otherwise non-nucleonic d.o.f)



- In this range, $\langle\xi_N\rangle < 0.8$ at all Q^2 (within 1σ).
  - for higher values, need to consider tail of $d\sigma/d\xi_N$ distribution

VMC distributions



Bound nucleon x_N