Physics Overview of Inclusive Electron-Nucleus Scattering at $x > 1$

Donal Day
University of Virginia

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Introduction

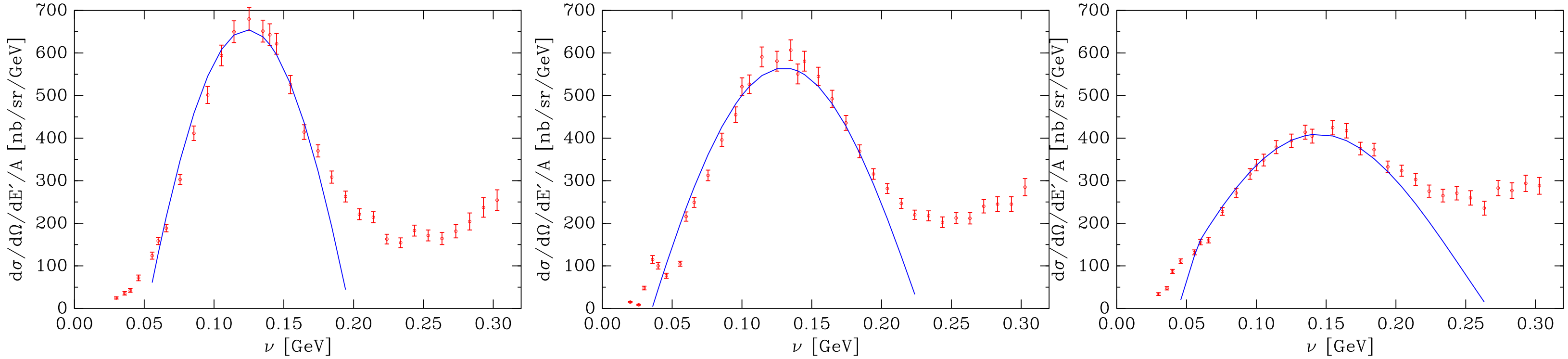
Inclusive electron scattering can be labeled as old–fashioned but it is clear that inclusive electron scattering from nuclei provides a rich, yet complicated mixture of physics that has yet to be fully exploited.

Very interesting physics including:

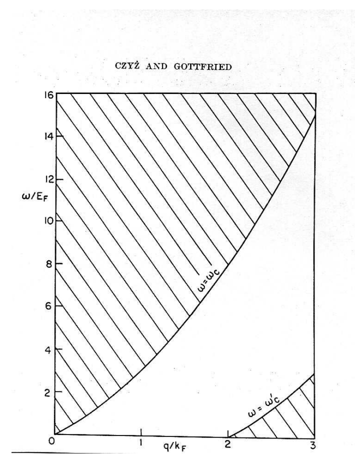
* Momentum distributions and the spectral function $P(\vec{k}, E)$.
* Short Range Correlations
* Scaling $(x, y, \psi', \xi\ )$
* Medium Modifications – tests of EMC
* Parton Recombination
* Color Transparency
* Bloom–Gilman Duality
* Structure Function Q^2 dependence and Higher Twists

The richness of inclusive scattering at large momentum transfers at once makes it alluring yet also makes it difficult but experiments over a range of Q^2 and with different A will help.

Early Quasielastic data

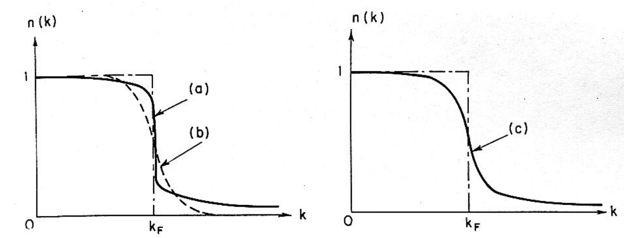


Nucleus	k_F	$\bar{\epsilon}$
^{6}Li	169	17
^{12}C	221	25
^{24}Mg	235	32
^{40}Ca	251	28
^{nat}Ni	260	36
^{89}Y	254	39
^{nat}Sn	260	42
^{181}Ta	265	42
^{208}Pb	265	44



Shaded domain where scattering is restricted solely to correlations. Czyz and Gottfried (1963)

$$w_c(q) = \frac{(k+q)^2}{2m} + \frac{q^2}{2m}$$
$$w'_c(q) = \frac{q^2}{2m} - \frac{qk_F}{m}$$



Czyz and Gottfried proposed to replace the Fermi $n(k)$ with that of an actual nucleus. (a) hard core gas; (b) finite system of noninteracting fermions; (c) actual large nucleus.

DIS at $x > 1$ or studying Superfast Quarks

* In the nucleus we can have $0 < x < A$
* In the Bjorken limit, $x > 1$ DIS tells us the virtual photon scatters incoherently from quarks
* Quarks can obtain momenta $x > 1$ by abandoning confines of the nucleon
  - deconfinement, color conductivity, parton recombination
  - multiquark configurations
  - correlations with a nucleon of high momentum (short range interaction)
* DIS at $x > 1$ is a filter that selects out those nuclear configurations in which the nucleon wave functions overlap. We are studying the dynamics of partons that have abandoned the confines of the nucleon.

  $$\langle r_{NN} \rangle \simeq 1.7 \text{ fm} \simeq 2 \times r_n = 1.6 \text{ fm}$$

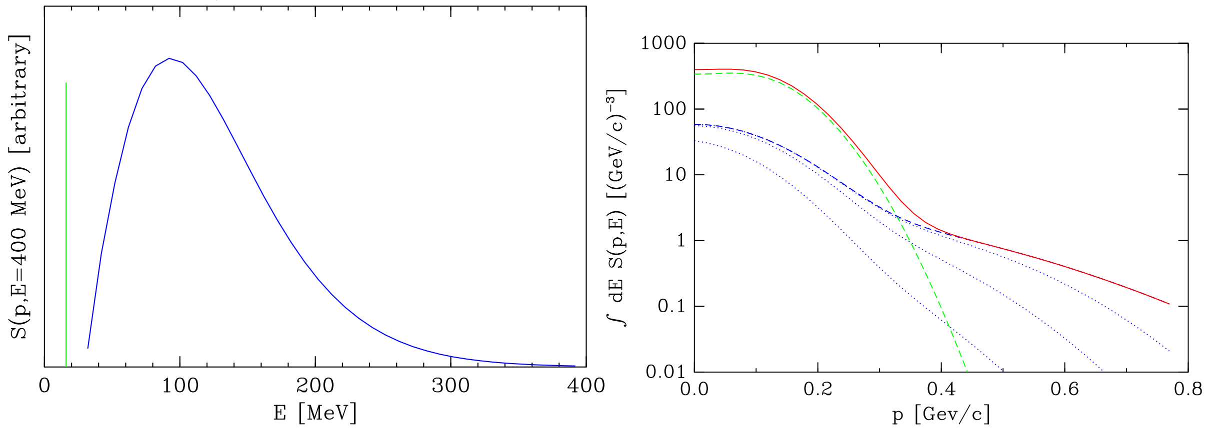
  The probability that nucleons overlap is large and at $x > 1$ we are kinematically selecting those configurations.

Spectral Functions

- The spectral function $P(k, E)$ represents the joint probability to find a nucleon in the nucleus with momentum $k \equiv |\mathbf{k}|$ and removal energy E.
- $P(k, E)$ is written (loosely) as follows:

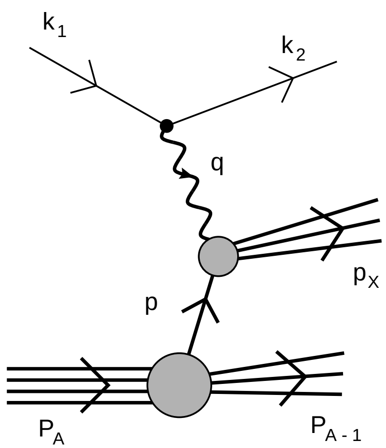
$$P(k, E) = P_0(k,) + P_1(k, E).$$

P_0 includes ground and one-hole states of the $(A - 1)$ system and P_1 includes the more complex configurations (mainly 1p-2h states) which arise from 2p-2h excitations produced by NN correlations.



$$\begin{aligned} P(k, E) &= \delta(E + \frac{k^2}{2M}), k < k_F \\ &= 0, k > k_F \end{aligned}$$

Inclusive Scattering



$$\frac{d^2\sigma}{dE'd\Omega} = \frac{4\alpha^2}{q_\mu^4} E'^2 [W_2^{(A)} \cos^2\frac{\theta}{2} + 2W_1^{(A)} \sin^2\frac{\theta}{2}]$$

(a) $P_x^2 = M_n^2$

$$\frac{d^2\sigma}{d\omega d\Omega_{e'}} = \sum_{i=1}^{A} \int d\vec{k} \int dE\, \sigma_{ei}\, P_i(k, E)$$
$$\times\, \delta(\omega - E + M_A - (M^2 + \vec{k}'^{\,2})^{1/2} - (M_{A-1}^2 + \vec{k}^2)^{1/2}).$$

(b) $P_x^2 > M_n^2$

$$W_1^A = Z \int d\vec{k} \int dE\, P_i(k, E) (W_1^p + \frac{W_2^p}{2M_p^2} (\vec{k}^2 - k_3^2))$$

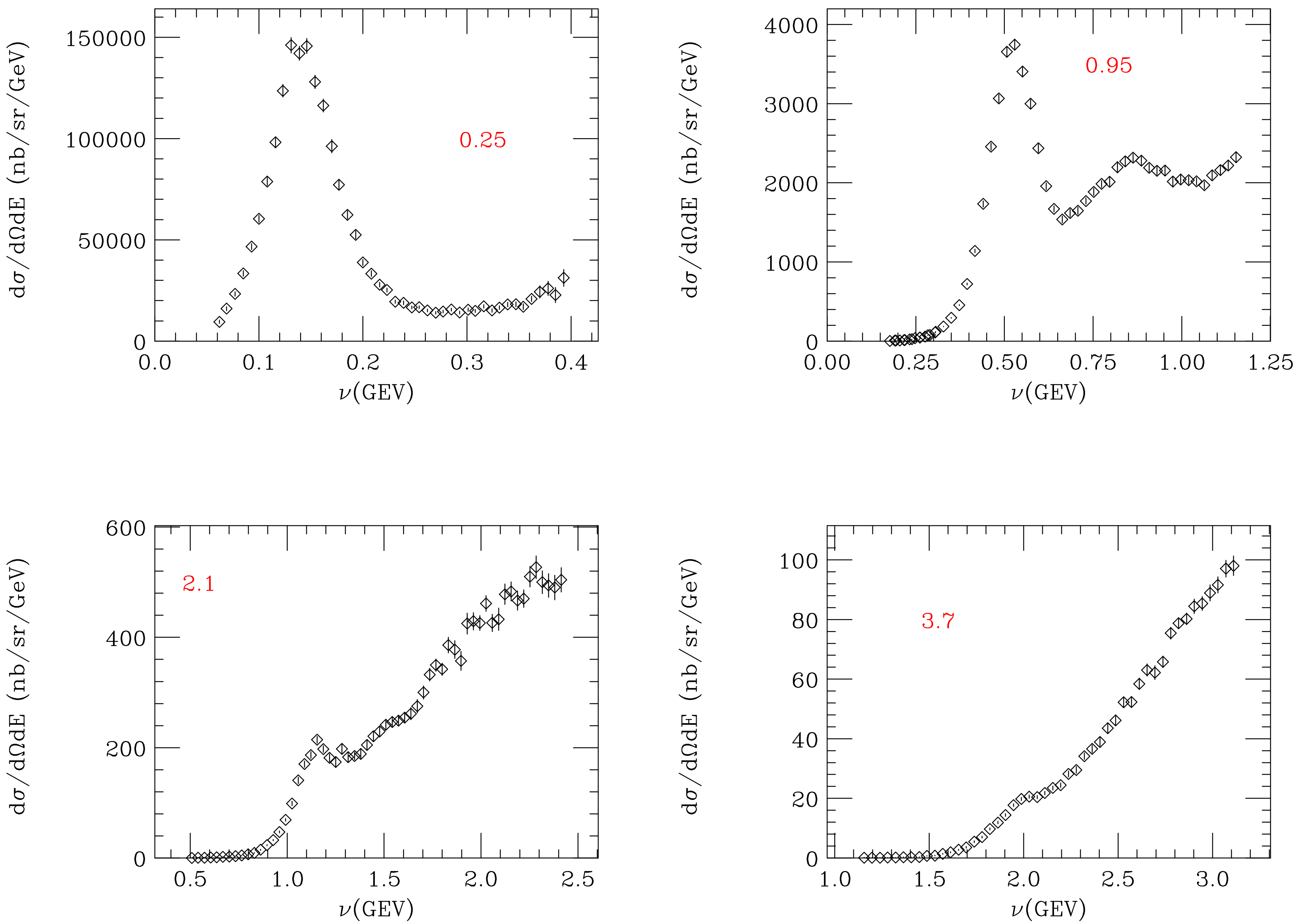
and

$$W_2^A = Z\int d\vec{k}\int dE\, P_i(k,E)\left[(1+\frac{k_3Q^2}{M_p\nu' q_3})^2(\frac{\nu'}{\nu})^2+\frac{\vec{k}^2-k_3^2}{2M_p^2}(\frac{Q^2}{q_3^2})\right]W_2^p$$

$W_{1,2}^{p,n}$ are the free proton and neutron inelastic structure functions. To $W_{1,2}^A$ we must add similar terms for the neutron.

- ✳ σ_{ei} and $W_{1,2}^{p,n}$ have different and distinct Q^2 behavior. The former falls with the elastic form factors while the latter has only $\ln Q^2$ behavior (in the DIS region).
- ✳ The limits on the integrals are determined by the kinematics. Specific (x, Q^2) select specific pieces of the spectral function.

Transition from QES to DIS



y scaling

At large q

$$\frac{d\sigma^2}{d\Omega dE'} = \sum_{ei} \sigma_{ei} \cdot K \cdot F(y)$$

$$F(y) = 2\pi \int_{-y}^{\infty} kdk \int_{E_{\text{min}}}^{E_{\text{max}}} dEP(k,E)$$

If $E_{\text{max}} = \infty$ then

$$F(y) = 2\pi \int_{-y}^{\infty} kdkn(k)$$

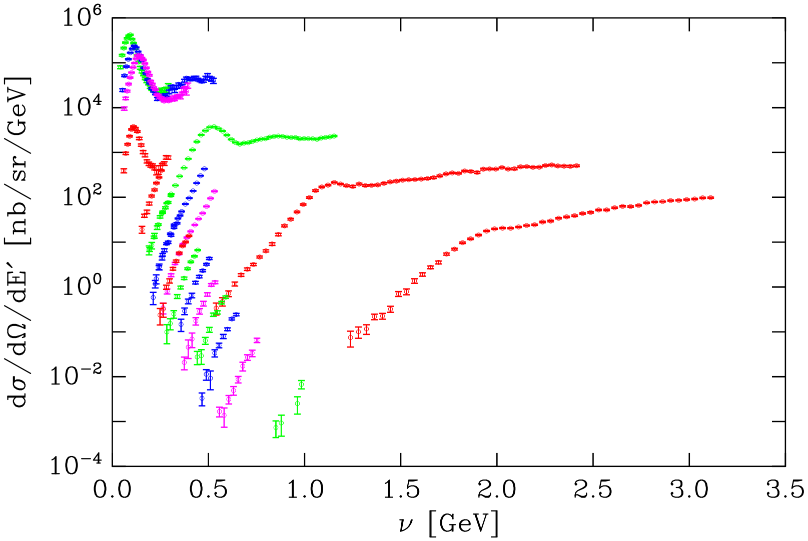
$F(y)$ is the longitudinal momentum distribution and QES would provide a direct measure of $n(k)$.

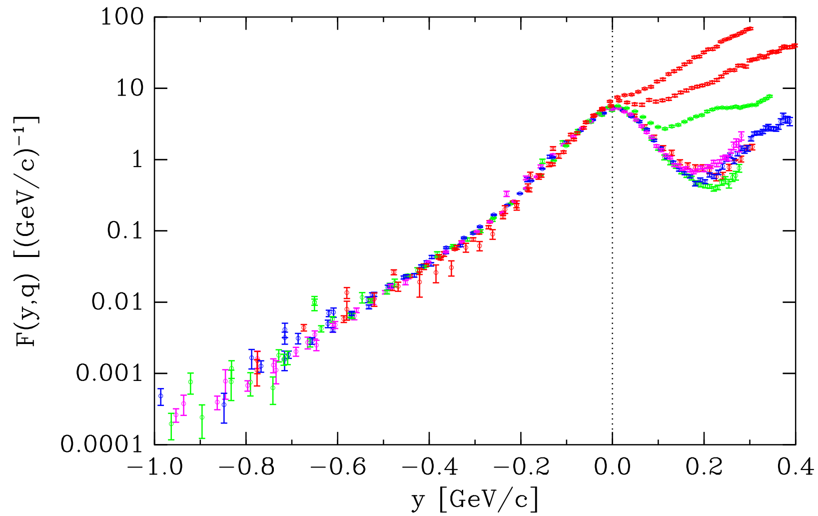
$$n(k) = -\frac{1}{2\pi y}\frac{df(y)}{dy}$$

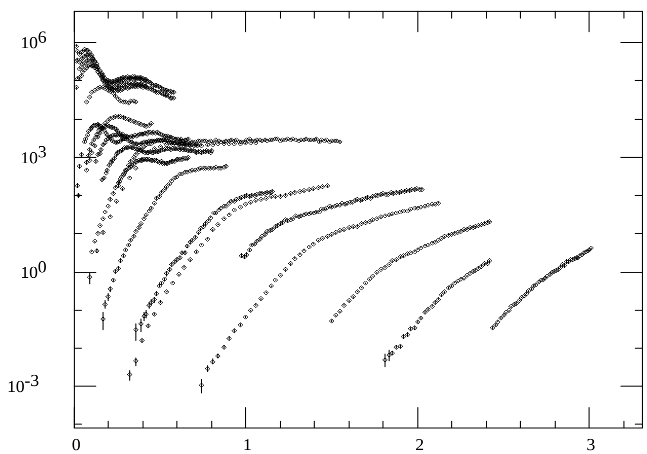
- ✳ y is determined from energy conservation
  $\nu + M_A = [(M_{A-1} + E^*_{A-1})^2 + k^2]^{1/2} + M^2 + (k^2 + q^2)^2]^{1/2}$
  when$E^*_{A-1} = E_{min}$.
- ✳ Scaling indicates the absence of interactions and a known Q^2 dependence of reaction mechanism.
- ✳ Scale breaking indicates either FSI or Q^2 dependence not expected

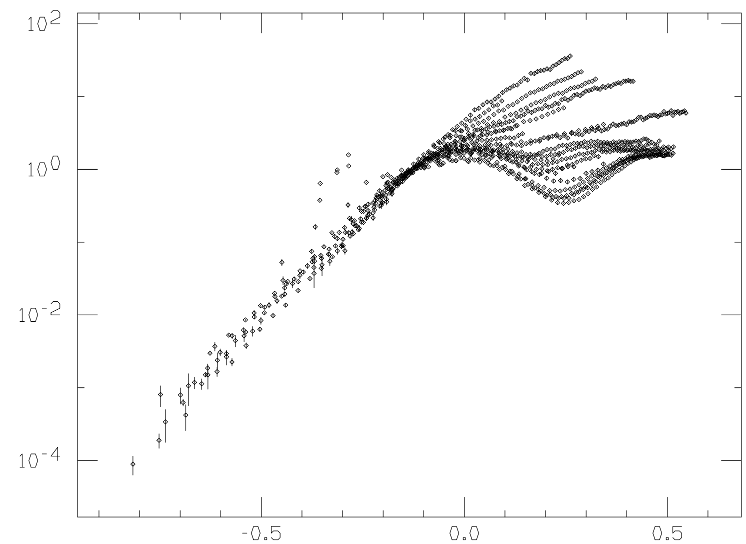
^{3}He and ^{12}C $\qquad F(y,|\mathbf{q}|) = \frac{\sigma^{\exp}}{(Z\tilde{\sigma}_p + N\tilde{\sigma}_n)} \cdot \frac{|\mathbf{q}|}{\sqrt{M^2+(y+|\mathbf{q}|)^2}}$

$$n(k) = -\frac{1}{2\pi y}\frac{df(y)}{dy}$$

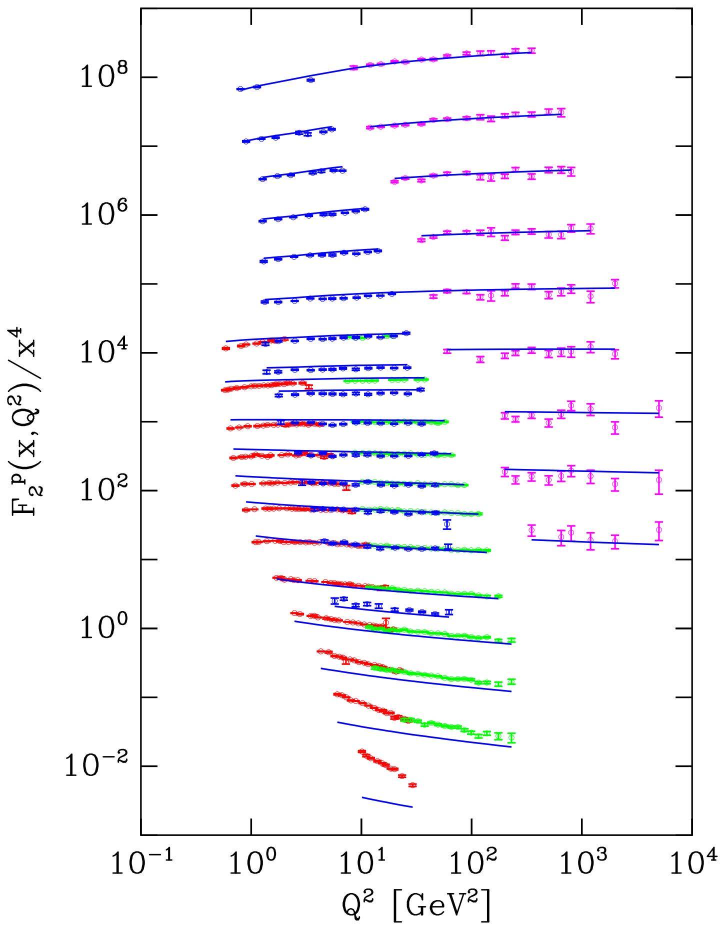




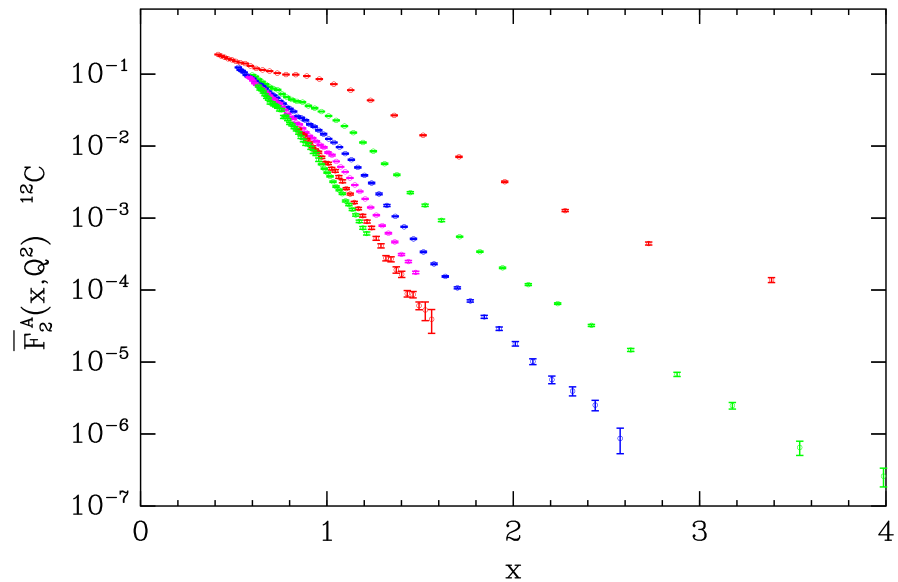




x scaling



Logarithmic scaling violations of proton structure function F_2^p. Data and NLO predictions using CTEQ distribution functions. $x = 0.85$ at bottom and $x = 0.008$ at top.



$$F_2^A = \nu \cdot \frac{\sigma^{\exp}}{\sigma_M}\left[1 + 2\tan^2(\theta/2)\cdot\left(\frac{1+\nu^2/Q^2}{1+R}\right)\right]^{-1}$$

ξ scaling

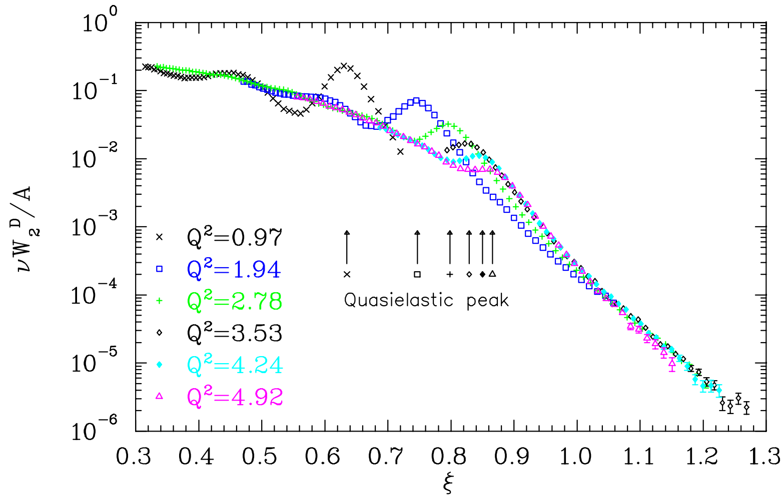
- ✳ F_2^N in resonance region falls with Q^2 as structure functions. Local duality (averaging over finite range in x.)
- ✳ This duality should also be valid for elastic peak at $x = 1$ if analyzed in ξ, the light cone momentum fraction.
- ✳ The Nachtmann variable (fraction ξ of nucleon *light cone* momentum p^+) is the variable in which logarithmic violations of scaling in DIS should be studied.

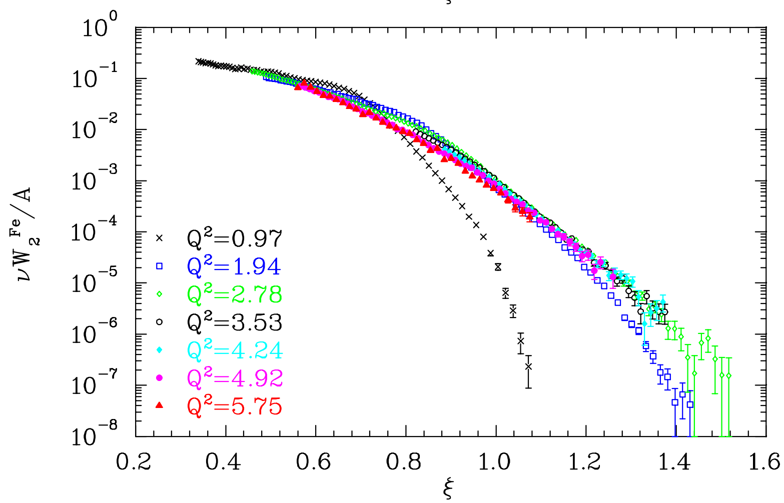
$$\xi \equiv -\frac{q^+}{p+} = \frac{|\vec{q}| - \nu}{M} = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \to x$$

- ✳ For nucleon local duality allows logarithmic Q^2 scaling violations and m^2/Q^2 higher twists to be studied.
- ✳ For a nucleus

$$F_2^A(\xi) = \underbrace{\int_\xi^A dz F(z) F_2^n(\xi/z)}_{\text{averaging}}$$

where $F(z)$ is the nucleon lightcone distribution and $F_2(\xi/z)$ is the free nucleon response.





$$F_2^A(\xi) = \underbrace{\int_\xi^A dz F(z) F_2^n(\xi/z)}_{\text{averaging}}$$

Evidently the inelastic and quasielastic contributions cooperate to produce ξ scaling.

Drell-Yan-West

Brings to mind the Drell-Yan-West relationship that connects the Q^2 dependence of the free Dirac elastic form factor with the shape of F_2^p near $x = 1$. For the dipole, with $1/Q^4$ dependence, Drell-Yan-West predicts $F_2^p \sim (1 - x)^3$.

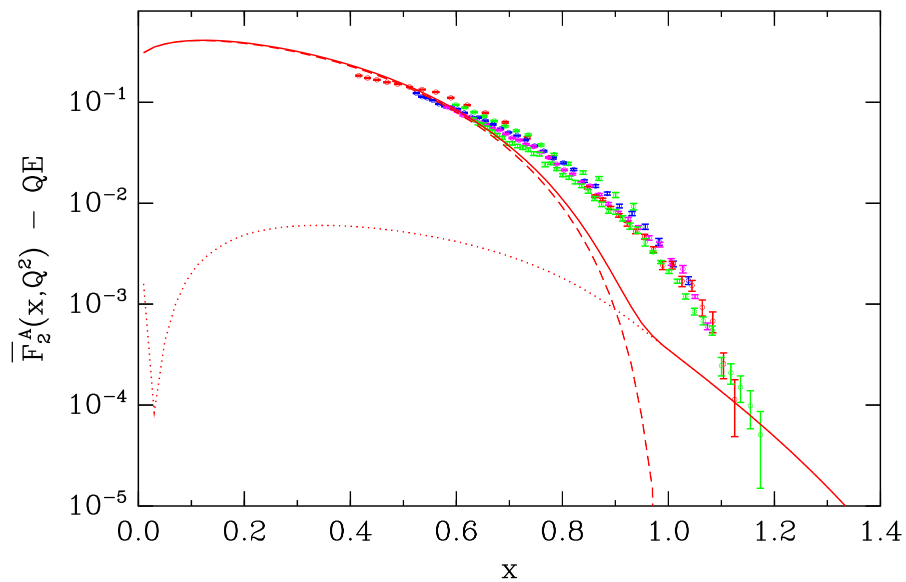
Fits of the form $(1 - \xi)^p$ for the data from the resonance region returned $p = 3$ within errors for C, Fe, and Au. This behavior in ξ rather than x indicates that ξ is the appropriate variable at finite kinematics.

Parton Recombination

- ✷ Explicit calculable mechanism to account for short range modification of nucleons structure in nucleus.
- ✷ Partons can recombine or fuse with partons of a neighboring nucleon to form a single parton with the sum of the initial momenta.
- ✷ Initial state recombination populates the $x > 1$ region with quarks that have absorbed momentum from neighboring gluons.

Modification to the valence parton distribution, and the nonsinglet part of F_2^N is determined by

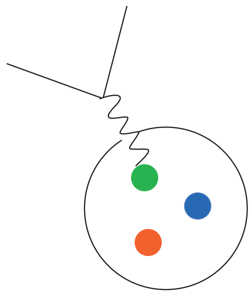
$$\begin{aligned}\Delta V(x, Q^2) &= \frac{3}{2} R_A \tilde{n}_A \frac{4\pi\alpha_s(Q^2)}{Q^2} \\ &\times \left[\int_x^1 dy V(y) G(x-y) \Gamma_{qq}(y, x-y, x) \right. \\ &\left. -V(x) \int_0^1 dy G(y) \Gamma_{qq}(x, y, x+y) \right]\end{aligned}$$



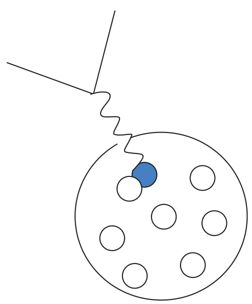
Inelastic contribution to $\bar{F}_2(x, Q^2)$ and a calculation of the parton recombination modification (dotted) for $Q_0 = 1$ GeV^2. The dashed line indicates the free nucleon structure function.

Cross section ratios and correlations

For a nucleon at rest, $x < 1$ as $x = 1$ is the elastic limit



For $e - A$ scattering x is not so restricted; $x > j - 1$ where j is the number of nucleons coming together. Recall for $k = k_F$, $x \leq 1.2$.



$x > 1 \Rightarrow$ 2 nucleons close together
$x > 2 \Rightarrow$ 3 nucleons close together
Further, when j nucleons are close together the $A - j$ nucleons have little influence.

The Spectral Functions with a high k nucleon can be represented as a sum over 2,3 $\cdots$ nucleon correlations
In the region where correlations should dominate, large x,

$$\begin{aligned}\sigma(x, Q^2) &= \sum_{j=1}^{A} A\frac{1}{j} a_j(A)\sigma_j(x, Q^2) \\ &= \frac{A}{2} a_2(A)\sigma_2(x, Q^2) + \\ &\quad \frac{A}{3} a_3(A)\sigma_3(x, Q^2) + \\ &\quad \vdots\end{aligned}$$

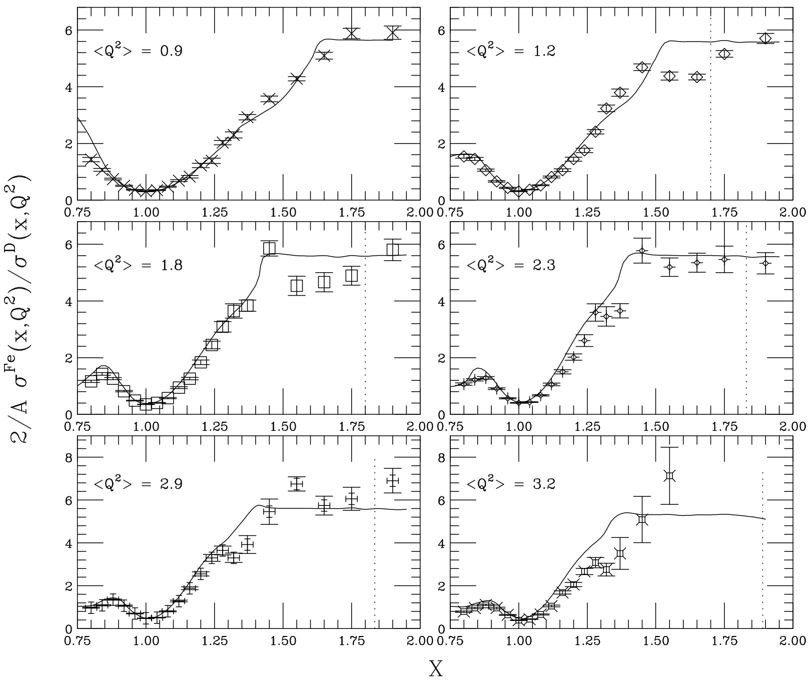
$a_j(A)$ are proportional to finding a nucleon in a j-nucleon correlation. It should fall rapidly with j as nuclei are dilute.
$\sigma_2(x, Q^2) = \sigma_{eD}(a, Q^2)$ and $\sigma_j(x, Q^2) = 0$ for $x > j$.

$$\Rightarrow \quad \frac{2}{A}\frac{\sigma_A(x, Q^2)}{\sigma_D(x, Q^2)} = a_2(A)\Big|_{1<x\leq 2}$$

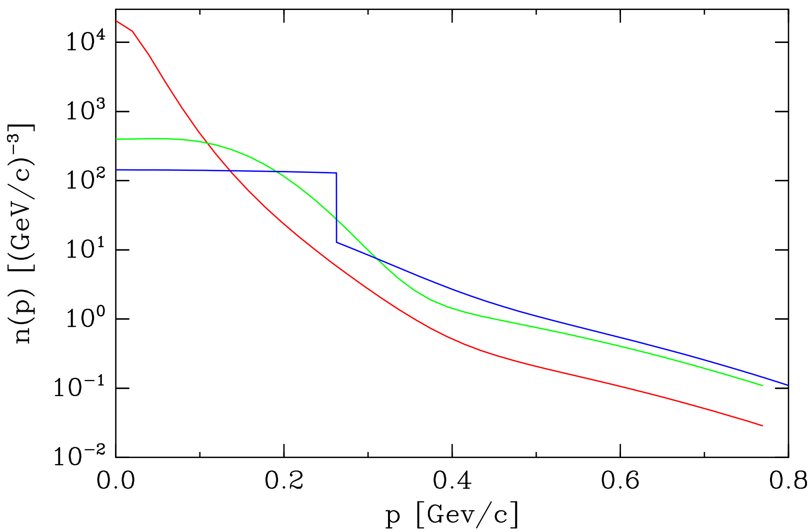
$$\frac{3}{A}\frac{\sigma_A(x, Q^2)}{\sigma_{A=3}(x, Q^2)} = a_3(A)\Big|_{2<x\leq 3}$$

In the ratios, offshell effects and FSI largely cancel.

Cross section ratios and correlations



$\frac{2\sigma^{\rm Fe}(x,Q^2)}{A\sigma^{\rm D}(x,Q^2)}$ versus x



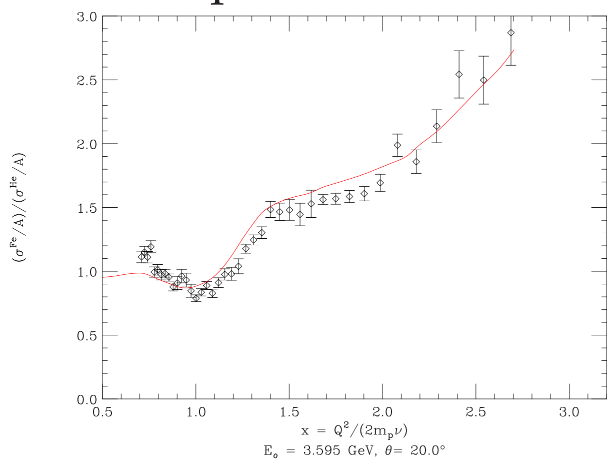
Ratios of momentum distributions at large momentum approach a constant

Alternative intepretations exist: Quark Cluster of Pirner and Vary

* Photon at high Q^2 absorbed in quasifree process in IMF
* Quark is constituent of a quasifree color singlet cluster in nucleus
* $3q$ cluster has critical radius R_c. $6, 9, \cdots q$ clusters are formed when $3q$ clusters are $< 2R_c$ apart.
* R_c is free parameter

$$\nu W_2(\nu, Q^2) = \sum_{\mathrm{quarks_j}} e_j^2 \frac{x}{A} \mathsf{P}_j(x) \text{ and } \mathsf{P}_\mathrm{j}(\mathrm{x}) = \sum_{\mathrm{iclusters}} \tilde{\mathrm{p}}_\mathrm{i} \overline{\mathrm{P}}_\mathrm{j}(\mathrm{x})$$

$\overline{P}_i(x)$ is x distribution of q in a $i - q$ cluster. $\tilde{p}_i$ is probablity to find $3, 6, 9$ q-clusters. Then $\mathsf{P}_j(x)$ is probablity that quark j carries $\frac{x}{A}$ of nucleon momentum. In this model the SR part of WF is replaced with multiquark clusters.



Summary

- ✳ High Q^2 scattering at $x > 1$ holds great promise. The data collected in this experiment will make a major contribution to realizing that promise.
- ✳ Window on wide variety of interesting physics.
- ✳ Provides access to SRC and high momentum components through y–scaling, ratios of heavy to light nuclei, ψ'–scaling
- ✳ Testing ground for EMC models of medium modifation, Color transparency
- ✳ Moment analysis of structure functions
- ✳ DIS is not completely dominant over QES at 6 GeV but should be at 12 GeV and at $Q^2 > 10 - 15(\mathrm{GeV/c})^2$.
- ✳ Experiments are relatively straightforward. JLAB at 12 GeV will significantly expand the coverage in $x - Q^2$.