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**Smearing Procedure to get Neutron S. S. F.'s**

Oscar A. Rondon, INPP - UVA

# 1 Introduction

We need to get the neutron spin structure functions from our RSS proton and deuteron data. This is an outline of the procedure.

# 2 Definitions

We start with the kinematics of off-shell scattering on the deuteron. Following [1] we consider a struck nucleon and a spectator one. The spectator nucleon is on-shell, and after the scattering it has the same momentum $\vec{p}_s$ as before the scattering. The off-shell nucleon has initial momentum $\vec{p}_i = -\vec{p}_s$. Its initial energy is $E_i = M_d - \sqrt{p_i^2 + M^2} = M_d - \sqrt{p_s^2 + M^2}$, where $M_d$ is the mass of the deuteron, and $M$ is the nucleon mass. Since we want to obtain neutron SSF's, $M = M_p$, because we need to subtract an off-shell proton from the deuteron.

From the above relation for $E_i$ and $\vec{p}_i$ we see that the off-shell structure functions of the bound proton in the deuteron will depend on the initial momentum distribution $|f(\vec{p}_i)|^2$ of the off-shell nucleon in the deuteron [2] , because it is no longer a target at rest, like the free proton. The mass of the final state produced on the off-shell neutron is $W'^2 = (P_i + q)^2$. The off-shell four-momentum $P_i = (\vec{p}_i, E_i)$ is different from the on-shell $P_i = (0, M)$. The virtual photon four-momentum $q = (\vec{q}, q_0) = (\vec{q}, \nu)$ is unchanged by the Fermi motion. The energy transfer to the moving off-shell nucleon is $\nu' = P_i q/M$. We then have

$$\vec{p}_i = -\vec{p}_s = \vec{p}$$
$$E_i = M_d - \sqrt{p_i^2 + M^2} = M_d - \sqrt{p^2 + M_p^2}$$

$$W'^2 = (P_i + q)^2 = (E_i^2 - p^2) + 2(\nu E_i - \vec{p} \cdot \vec{q}) - Q^2$$
$$\nu' = \frac{P_i q}{M} = \frac{1}{M}(\nu E_i - \vec{p} \cdot \vec{q}) \quad (1)$$

where $\nu$ and $Q^2$ are the ordinary virtual photon quantities. It can be seen that

$$W'^2 = (E_i^2 - p^2) + 2M\nu' - Q^2. \quad (2)$$

Since $\nu$ and $Q^2$ are independent kinematic variables, the off-shell kinematics involves three independent variables $p$, $\nu$ and $Q^2$, or $W'$, $\nu'$ and $Q^2$.

# 3 Atwood-West Smearing

The Atwood-West [3] smearing procedure states that off-shell nucleon quantities can be obtained from on-shell ones by forming the convolution of the momentum distribution and the on-shell quantities

$$F(Q^2, \nu) = \int_0^\infty |f(\vec{p})|^2 g(Q^2, W', \nu') d\vec{p} \quad (3)$$

For the specific case of the SSF's the on-shell quantities are the SSF's $g_1(Q^2, W', \nu')$ and $g_2(Q^2, W', \nu')$, which are combined to form the parallel and perpendicular cross section differences [4]

$$\Delta\sigma_{\|} = \frac{4\alpha^2}{Q^2} \frac{E'}{E}((E + E' \cos\theta) M G_1(Q^2, W) - Q^2 G_2(Q^2, W))$$
$$\Delta\sigma_{\perp} = \frac{4\alpha^2}{Q^2} \frac{E'}{E} E' \sin\theta \cos\phi (M G_1(Q^2, W) + 2E G_2(Q^2, W)) \quad (4)$$

The convolutions that give the smeared proton $\Delta\sigma_{\|}^s$ and $\Delta\sigma_{\perp}^s$ look like

$$\Delta\sigma_{\|}^s = \frac{4\alpha^2}{Q^2} \int_0^\infty |f(\vec{p})|^2 \frac{E - \nu'}{E} \Big((E + (E - \nu') \cos\theta) M G_1(Q^2, W', \nu') - Q^2 G_2(Q^2, W', \nu')\Big) d\vec{p}$$
$$\Delta\sigma_{\perp}^s = \frac{4\alpha^2}{Q^2} \int_0^\infty |f(\vec{p})|^2 \frac{E - \nu'}{E} (E - \nu') \sin\theta \cos\phi \Big(M G_1(Q^2, W', \nu') + 2E G_2(Q^2, W', \nu')\Big) d\vec{p}$$
$$(5)$$

Since we are solving these equations numerically, we only need to calculate $G_1(Q^2, W')$ and $G_2(Q^2, W')$ at values of $Q^2, W'(M_d, p)$ that correspond to our actual $Q^2, W$ [1]. For example, for $p$ =100 MeV/c, at our $W = 1.235$ GeV bin with $Q^2 = 1.406$ GeV/c$^2$, the corresponding $\nu = 1.093$ GeV, $E_i = 0.932$ GeV, $\nu' = 0.914$ GeV and $W' = 1.080$ GeV, which is near the free proton pion threshold. With $\tilde{\nu} = 0.902$, the corresponding off-shell correction would be $(\nu'/\tilde{\nu})^2 = 1.03$.

Another possible value for $\nu' = 1.2577$ comes from $\vec{p} \cdot \vec{q} = -pq$ so $W' = 1.3462$. I am not quite sure about how to handle the angular dependence of $\vec{p}$ relative to $\vec{q}$. Bodek and Ritchie just talk about a $pq$ product of magnitudes, eq.(3) of ref. [1].

# 4 Deuteron Fermi Momentum Distribution

The factor $|f(\vec{p})|^2$ was identified by West [3] as the squared deuteron wave function in momentum space $|\phi(\vec{p})|^2 = |u(\vec{p})|^2 + |w(\vec{p})|^2$, where $u(\vec{p})$ and $w(\vec{p})$ are the $S$ and $D$ state components

$$
\begin{aligned}
u(\vec{p}) &= \frac{1}{\sqrt{2}\pi} \int_0^\infty \frac{u(r)}{r} j_0(|\vec{p}|r) r^2 dr \\
w(\vec{p}) &= \frac{1}{\sqrt{2}\pi} \int_0^\infty \frac{w(r)}{r} j_2(|\vec{p}|r) r^2 dr
\end{aligned}
\tag{6}
$$

We can use the Paris potential to substitute for $u(r)$, $w(r)$ [5]. This reference also gives functions of $p$ directly:

$$
\begin{aligned}
U_a(p)/p &= \sqrt{\frac{2}{\pi}} \sum_{J=1}^{n} \frac{C_J}{p^2 + m_J^2} \\
W_a(p)/p &= \sqrt{\frac{2}{\pi}} \sum_{J=1}^{n} \frac{D_J}{p^2 + m_J^2}.
\end{aligned}
\tag{7}
$$

The wave function is normalized to 1, as a function of $p$, but there is a factor of $4\pi$ from the angular integrals. Since we are integrating the product of the squared wave function and the cross section differences over both $p$ and the angles, we need to be aware of this normalization.

[1] In Appendix C.1 of ref. [2] the off-shell $G_1, G_2(Q^2, W')$ are identified as the corresponding on-shell $G_1, G_2(Q^2, W)$, but off-shell corrections may be needed. One possible form of this correction given is Off-shell= $(\nu'/\tilde{\nu})^2\times$ On-shell, where $\tilde{\nu} = (W'^2 + Q^2 - M^2)/2/M$.

# 5 Cross Section Differences and Integration

To get $\Delta\sigma_{||}$ and $\Delta\sigma_{\perp}$ we first need to compute $A_1$ and $A_2$ from the fit to our data, and then convert to $G_1$ and $G_2$ using E. Christy's $F_1$. So we need to calculate $A_1(Q^2, W', \nu')$, $A_2(Q^2, W', \nu')$ and $F_1(Q^2, W', \nu')$:

- At a given central value of one of our $W$ bins we calculate $\nu(W, Q^2)$.
- For given values of $p$ and $\vec{p} \cdot \vec{q}$ we calculate $W'$, $\nu'$.
- We get the values of $A_1$, $A_2$ and $F_1$ at the corresponding $W'$. For $A_1$ and $A_2$ we call our $RSS$ fit at the corresponding $W = W'$ and $Q^2$. Same for Eric's $F_1$.
- Calculate $G_1$ and $G_2$ from the asymmetries, $F_1$ and the kinematic factor $\gamma^2 = \nu'^2/Q^2$.
- Form the cross section differences with $G_1$, $G_2$ and the kinematic factors.
- Loop over $\cos(\theta_{pq}) = (\vec{p} \cdot \vec{q})/(pq)$ from 0 to 1, computing new values of $\Delta\sigma_{||,\perp}(Q^2, W', \nu')$ at fixed $p$, summing them for the corresponding angular integral.
- Multiply the cross sections integrated over $\cos(\theta_{pq})$ times $|f(\vec{p})|^2$ and loop to sum over the $p^2dp$ integral. With the Paris momentum wave functions of eq. (7) the $p^2$ cancels with the $1/p^2$ in the wave function squared.

To get the smeared proton cross section differences we need to calculate the off-shell $G_1$ and $G_2$ for a reasonable range of $\vec{p}$, say from 0 to 300 MeV/c, in steps of 5 MeV/c or so. This gives the cross section difference at the "smeared" kinematics of the corresponding momentum probability $|f(\vec{p})|^2 \equiv |f(|\vec{p}|)|^2$. Then we multiply this integrand by the integration volume element $d\phi d(cos(\theta_{pq}))p^2dp$. A possible way of taking care of the $\theta_{pq}$ part is to calculate the off-shell $G_1$ and $G_2$ at fixed $|\vec{p}|$ for $0 \leq \theta_{pq} \leq \pi$ in steps of $\pi/18$ or $\pi/12$, or in $\cos(\theta_{pq})$ bins directly.

## 6 Subtraction

The results of the preceding procedure need to be subtracted from our measured deuteron $\Delta\sigma_{||}$ and $\Delta\sigma_{\perp}$ which we get from our $A_{||}$ and $A_{\perp}$ multiplied by the Hall C unpolarized cross section fit of I. Niculescu that we use in the MC for the deuteron dilution factor. The result is smeared neutron cross section differences.

## 7 Unsmeared neutron quantities

For this we'll use a similar fit to my proton fit: 3 or 4 Breit-Wigner resonances plus some DIS background with a reasonable $x_{Bjorken}$ dependence for $A_1^n$ and $A_2^n$. Working with $A_1$ and $A_2$ postpones the issue of the unpolarized neutron SF's to the last step: getting $g_1$ and $g_2$.

**I will refine this note with more details. This is to get us started. Inputs are welcome.**

## References


[1] A. Bodek and J. L. Ritchie, Phys. Rev. D **23**, 1070 (1981).

[2] A. Bodek *et al.*, Phys. Rev. D **20**, 1471 (1979).

[3] W. B. Atwood and G. B. West, Phys. Rev. D **7**, 773 (1973).

[4] http://spin.phys.virginia.edu/~or/e01006/analysis/aparaper.pdf

[5] M. Lacombe, B. Loiseau, R. Vinh Mau, J. Cote, P. Pires and R. de Tourreil, Phys. Lett. B **101**, 139 (1981).