Taken from Hall C proposal by X. Zheng and O. Rondon.
The elastic asymmetry is

$$A = \frac{-a\cos\theta^\star - bR\sin\theta^\star\cos\phi^\star}{R^2+c} \quad (1)$$

$$a = 2\tau\tan\frac{\theta}{2}\sqrt{1+\tau+f^2} \qquad \tau = \frac{Q^2}{4M^2} \quad (2)$$

$$b = 2\sqrt{\tau(1+\tau)}\tan\frac{\theta}{2} \quad (3)$$

$$c = \frac{\tau}{\epsilon} \qquad \epsilon = [1+2(1+\tau)\tan^2\frac{\theta}{2}]^{-1} \quad (4)$$

$$f = (1+\tau)\tan\frac{\theta}{2} \quad (5)$$

with the ratio of the proton form factors $R = \frac{G_E}{G_M}$ and $\theta^\star, \phi^\star$ the polar and azimuthal angles of the target proton spin vector relative to the $q$-vector in the $\vec{k}x\vec{k'}$ plane . $\vec{k}$ is the momentum vector of the incident electron and $\vec{k}$ is the momentum vector of the scattered electron.

For the perpendicular target field data, the target field was aligned 90° to the beam direction (pointing beam left). The beam energy was 5.7526 GeV. The central HMS spectrometer angle was 13.12° and define $\phi = 180°$. The elastic data was taken for HMS central momentum of 4.7 . Averaged over the data in the range 0.9 < W < 1.0 the electron polar angle $\theta = 13.22°$ and $\phi = 168.8°$. For the $q$-vector , $\theta_q = 50.43°$ and $\phi_q = 11.2°$. Relative to the $q$-vector , $\theta^\star = 40.87°$ and $\phi^\star = 162.73°$.

The derivative of A on R is

$$\frac{dA}{dR} = \frac{2XR + Y(R^2-c)}{(R^2+c)^2} \quad (6)$$

$$\frac{dA}{A} = \frac{dR}{R}\frac{R*(2XR - Yc + YR^2)}{(R^2+c)(-X-YR)} = \frac{dR}{R}f_R \quad (7)$$

with $X = a\cos\theta^\star$ and $Y = b\sin\theta^\star\cos\phi^\star$. For perpendicular target field, $f_R$ = -0.89.

To determine R from the asymmetry:

$$R^2 + BR + C = 0 \quad (8)$$

with

$$B = \frac{b}{A}\sin\theta^\star\cos\phi^\star \quad (9)$$

$$C = c + \frac{a\cos\theta^\star}{A} \quad (10)$$

So

$$R = \frac{1}{2}(-B + \sqrt{B^2-4C}) \quad (11)$$

chose '+' sign in front of square root since R is known not to be negative.

$$\Delta R = \frac{\Delta A}{2}\left(\frac{\partial B}{\partial A}\left(-1+\frac{B}{\sqrt{B^2-4C}}\right)-\frac{\partial C}{\partial A}\frac{2}{\sqrt{B^2-4C}}\right) \quad (12)$$

$$\frac{\partial B}{\partial A} = \frac{-B}{A} \qquad \frac{\partial C}{\partial A} = \frac{c-C}{A} \quad (13)$$

For R dependence on $\theta^\star$:

$$\frac{\partial R}{\partial \theta^\star} = \frac{1}{2}\left(\frac{\partial B}{\partial \theta^\star}\left(-1+\frac{B}{\sqrt{B^2-4C}}\right)-\frac{\partial C}{\partial \theta^\star}\frac{2}{\sqrt{B^2-4C}}\right) \quad (14)$$

$$\frac{\partial B}{\partial \theta^\star} = \frac{B}{\tan\theta^\star} \qquad \frac{\partial C}{\partial \theta^\star} = \frac{-a}{A}\sin\theta^\star \quad (15)$$

For perpendicular field, $\frac{\partial R}{\partial \theta^\star} = -0.01/\mathrm{deg}$, and with $\Delta\theta^\star = 0.1^\circ$, then $\Delta R = 0.002$ .

For R dependence on $\phi^\star$:

$$\frac{\partial R}{\partial \phi^\star} = \frac{1}{2}\frac{\partial B}{\partial \phi^\star}\left(-1+\frac{B}{\sqrt{B^2-4C}}\right) \quad (16)$$

$$\frac{\partial B}{\partial \phi^\star} = B\tan\phi^\star \quad (17)$$

For perpendicular field, $\frac{\partial R}{\partial \theta^\star} = -0.004/\mathrm{deg}$, and with $\Delta\theta^\star = 1^\circ$, then $\Delta R = 0.004$ .

For the $E$ and $E'$, one can ignore the $\theta$ dependence of $\tau$, $B$, and $C$. The derivative of B on $\tau$ is :

$$\frac{\partial B}{\partial \tau} = \frac{1}{2}\left(\frac{B}{\tau}+\frac{B}{1+\tau}\right) \quad (18)$$

The derivative of C on $\tau$ is :

$$\frac{\partial C}{\partial \tau} = \frac{\partial c}{\partial \tau}+\frac{\cos\theta^\star}{A}\frac{\partial a}{\partial \tau} \quad (19)$$

For $E$ and $E'$ dependence, $\tau$ and $\theta$ are independent so:

$$\frac{\partial a}{\partial \tau} = a\left\{\frac{1}{\tau}+\frac{1+2f\frac{\partial f}{\partial \tau}}{2(1+\tau+f^2)}\right\} \qquad \frac{\partial f}{\partial \tau} = \tan\left(\frac{\theta}{2}\right) \quad (20)$$

$$\frac{\partial c}{\partial \tau} = \frac{\epsilon-\tau\frac{\partial \epsilon}{\partial \tau}}{\epsilon^2} \qquad \frac{\partial \epsilon}{\partial \tau} = -2\epsilon^2\tan^2\left(\frac{\theta}{2}\right) \quad (21)$$

$$(22)$$

For derivative of R on $E$ ( for $E'$) :

$$\frac{\partial R}{\partial E} = \frac{1}{2}\left(\frac{\partial B}{\partial E}\left(-1+\frac{B}{\sqrt{B^2-4C}}\right)-\frac{\partial C}{\partial E}\frac{2}{\sqrt{B^2-4C}}\right) \quad (23)$$

$$\frac{\partial B}{\partial E} = \frac{\partial B}{\partial \tau}\frac{\partial \tau}{\partial E} \qquad \frac{\partial C}{\partial E} = \frac{\partial C}{\partial \tau}\frac{\partial \tau}{\partial E} \quad (24)$$

$$\frac{\partial \tau}{\partial E} = \frac{\tau}{E} \quad \text{or} \quad \frac{\partial \tau}{\partial E'} = \frac{\tau}{E'} \quad (25)$$

For perpendicular field, $\frac{\partial R}{\partial E} = 0.017/\mathrm{GeV}$ and with error of 5x10-4 on the beam energy this gives $\Delta R =$ 4.75e-5 . $\frac{\partial R}{\partial E^-} = 0.020/\mathrm{GeV}$ and with error of 1x10-3 on the scattered electron energy, this gives $\Delta R =$ 9.75e-5. Both are negligible.

The derivative of R on $\theta$ is

$$\frac{\partial R}{\partial \theta} = \frac{1}{2}\left(\frac{\partial B}{\partial \theta}\left(-1 + \frac{B}{\sqrt{B^2 - 4C}}\right) - \frac{\partial C}{\partial \theta}\frac{2}{\sqrt{B^2 - 4C}}\right) \tag{26}$$

$$\frac{\partial B}{\partial \theta} = \frac{B}{b}\frac{\partial b}{\partial \theta} \qquad \frac{\partial C}{\partial \theta} = \frac{\partial c}{\partial \theta} + \frac{\cos\theta^\star}{A}\frac{\partial a}{\partial \theta} \tag{27}$$

$$\frac{\partial a}{\partial \theta} = a\left[\frac{\partial \tau}{\partial \theta}\frac{1}{\tau} + \frac{1}{\sin\theta} + \frac{\frac{\partial \tau}{\partial \theta} + 2f\frac{\partial f}{\partial \theta}}{2(1 + \tau + f^2)}\right] \tag{28}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial \tau}{\partial \theta}\tan\left(\frac{\theta}{2}\right) + \frac{1 + \tau}{2\cos^2\left(\frac{\theta}{2}\right)} \tag{29}$$

$$\frac{\partial b}{\partial \theta} = b\left[\frac{1}{\sin\theta} + \frac{(1 + 2\tau)\frac{\partial \tau}{\partial \theta}}{2\tau(1 + \tau)}\right] \tag{30}$$

$$\frac{\partial c}{\partial \theta} = c\left[\frac{\partial \tau}{\partial \theta}\frac{1}{\tau} - \frac{\partial \epsilon}{\partial \theta}\frac{1}{\epsilon}\right] \qquad \frac{\partial \epsilon}{\partial \theta} = \frac{-2\tan^2\left(\frac{\theta}{2}\right)}{\epsilon^2}\left[\frac{\partial \tau}{\partial \theta} + \frac{2(1 + \tau)}{\sin\theta}\right] \tag{31}$$

$$\frac{\partial \tau}{\partial \theta} = \frac{EE'}{2M^2}\sin\theta \tag{32}$$

For the perpendicular field, $\frac{\partial R}{\partial \theta} = 0.0036$ /mr . The systematic error on the HMS angle is about 0.5mr so giving $\Delta R = .0018$ .