Radiation Thickness, Collisional Thickness, and Most Probable Collisional Energy Loss for E97110: The GDH Sum Rule, the Spin Structure of ^{3}He and the Neutron using Nearly Real Photons

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Abstract

Catalog of radiation lengths and materials for E97110. Also a catalog of collisional thickness and most probable energy loss. Still needs a list of uncertainties and a more complete list of references. Landau40 and Tsai71 for example. Better equation references. More discussion about the "1/18" term. Maybe explicitly write out bremmstrahlung spectrum? better table placement. reference/verify for GE180 composition. make comment about $\beta \approx 1$ and about the energy independence of B for our beam energies and momenta. references for densities of oxides. δ subscripts on stuff.

1 General Formulas for Calculating Radiation Lengths

Bremsstrahlung is radiation emitted when an electron is accelerated in the Coulombic fields of both the nucleus [1] and electrons [2, 3] of an atom. The amount of energy an electron loses to bremsstrahlung at a given frequency is given by:

$$d^2E = -h\nu \ [\mathrm{N}] \ \frac{d\sigma(\nu, E)}{d\nu} \ dx \ d\nu \tag{1}$$

where $h\nu$ is the energy of the radiated photons, [N] is the number density of the atoms, $d\sigma(\nu, E)/d\nu$ is differential cross section for the production of a photon, dx is the thickness of the material, and $d\nu$ is width of frequency range. To calculate the amount of energy lost per unit depth over all frequencies, one must integrate over the entire bremsstahlung frequency spectrum:

$$\int \frac{d^2E}{dx} = \frac{dE}{dx} = -[\mathrm{N}] \int_0^E h\nu \ \frac{d\sigma(\nu, E)}{d\nu} \ d\nu \tag{2}$$

It is convenient to change the integration variable to $u \equiv h\nu/E$, which is the fraction of the electron energy carried away by the radiated photons:

$$\frac{dE}{dx} = -[\mathrm{N}]E \int_0^1 u \ \frac{d\sigma(u, E)}{du} \ du \tag{3}$$

For electron energies over 50 MeV, the integrand is nearly independent of the electron energy E [4]:

$$\int_0^1 u \ \frac{d\sigma(u, E)}{du} \ du \approx \int_0^1 u \ \frac{d\sigma(u)}{du} \ du \equiv \sigma_{\mathrm{rad}} \tag{4}$$

Consequently σ_{rad}, to a very good approximation, depends only on the charge of the nucleus Z and is given by [5, 6, 7]:

$$\sigma_{\text{rad}}(Z) = 4\alpha r_e^2 \left[Z^2 \left(L_{\text{rad}}(Z) - f(Z\alpha)\right) + Z L'_{\text{rad}}(Z)\right] \quad (5)$$

$$4\alpha r_e^2 = 2.31787 \text{ millibarns} \quad (6)$$

where α is the fine structure constant and r_e is the classical electron radius. The term proportional to Z^2 is due to the field of the nucleus and the term proportional to Z is due to the field of the atomic electrons. When evaluated in the complete screening limit, the radiation integrals L_{rad} & L'_{rad} are given by:

$$L_{\text{rad}}(Z) = \left\{ \begin{array}{cc} 5.31 & Z=1 \\ 4.79 & Z=2 \\ 4.74 & Z=3 \\ 4.71 & Z=4 \\ \log\left(184.15 Z^{-1/3}\right) & Z \geq 5 \end{array} \right\} \quad (7)$$

$$L'_{\text{rad}}(Z) = \left\{ \begin{array}{cc} 6.144 & Z=1 \\ 5.621 & Z=2 \\ 5.805 & Z=3 \\ 5.924 & Z=4 \\ \log\left(1194 Z^{-2/3}\right) & Z \geq 5 \end{array} \right\} \quad (8)$$

where $f(z)$ is known as the "Coulomb correction" [8]:

$$f(z) = z^2 \sum_{\nu=1}^{\infty} \frac{1}{\nu\left(\nu^2 + z^2\right)} \approx z^2 \left[\frac{1}{1+z^2} + 0.20206 - 0.0369 z^2 + 0.0083 z^4 - 0.002 z^6\right] \quad (9)$$

and we have dropped the Z^2 "1/18" term altogether [4]. When the Coulomb correction is neglected, L_{rad} is approximated by $\log\left(183 Z^{-1/3}\right)$. If only the lowest terms in the Coulomb correction are kept, then L_{rad} is approximated by $\log\left(191 Z^{-1/3}\right)$. Note that the original calculation of L'_{rad} [2] contains an error [3] that results in this commonly used but incorrect form of $L'_{\text{rad}} = \log\left(1440 Z^{-2/3}\right)$.

Returning to the approximation that σ_{rad} is independent of the electron energy, the energy of the electron decays exponentially as it penetrates the material:

$$\frac{dE}{E} = -[\mathrm{N}]\sigma_{\text{rad}} dx \rightarrow E(x) = E(0) \exp\left(-[\mathrm{N}]\sigma_{\text{rad}}(Z) x\right) \quad (10)$$

If the material is a uniform mixture of different atoms and one makes the reasonable assumption that the electron interacts with only one atom at a time, then the cumulative energy loss is given by:

$$E(x) = E(0) \Pi_k \exp\left(-[\mathrm{N}]_k \sigma_{\text{rad}}(Z_k) x\right) = E(0) \exp\left(-\sum_k [\mathrm{N}]_k \sigma_{\text{rad}}(Z_k) x\right) \quad (11)$$

where k labels the atomic isotope. It is convenient to characterize a material by the thickness required for an electron to lose $1 - 1/e$ of its energy. This parameter is called the *radiation length* and is given by:

$$\bar{X}_0 \equiv \left[\sum_k [\mathrm{N}]_k \sigma_{\text{rad}}(Z_k)\right]^{-1} \quad (12)$$

In practice, it's more convenient to work with mass densities ρ then with number densities [N]:

$$\bar{X}_0 = \frac{1}{[\mathrm{N}]\sigma_{\text{rad}}(Z)} = \frac{A}{\rho N_A \sigma_{\text{rad}}(Z)} = \frac{X_0}{\rho} \quad (13)$$

where A is the molecular weight of the isotope and N_A is the Avogadro constant. Note that even though $\bar{X}_0$ and X_0 have different units, they are interchangeably called the radiation length in the literature [7, 9].

We will distinguish the two by a bar and their units. For a single isotope, the radiation length in mass per unit "area" is given by:

$$X_0 = \frac{A}{N_A \sigma_{\mathrm{rad}}(Z)} \tag{14}$$

For a composite material, the radiation length in mass per unit "area" is given by the sum:

$$X_0^{-1} = \sum_k \frac{w_k N_A \sigma_{\mathrm{rad}}(Z_k)}{A_k} = \sum_k \frac{w_k}{X_0^k} \tag{15}$$

where w_k is the fraction by mass for the k-th isotope. Finally the unitless *radiation thickness* is given by:

$$t = \sum_j \frac{\rho_j \ell_j}{X_0^j} \tag{16}$$

where the sum over j runs over each composite material with physical thickness ℓ_j and mass density ρ_j.

2 General Formulas for Calculating Collisional Energy Loss

Electrons can undergo elastic collisions with atomic electrons within the materials along the beam path. Very often these collisions result in the ionization of the struck atom. Consequently the process is interchangeably called "ionization loss," "collisional loss," and "loss to ionizing collisions." We will also use these term interchangealby. The mean energy loss is given by the celebrated Bethe-Bloch equation, while the most probable energy loss was first calculated by Landau. Both equations have the form of energy lost to collisions per unit mass density per unit length [9]:

$$\begin{aligned}
\left[\frac{\Delta}{\rho x}\right] &= \left[\frac{\xi}{\rho x}\right]\left[2\log\left(\frac{pc}{I}\right) - \delta(X) + g\right] && (17)\\
\left[\frac{\xi}{\rho x}\right] &= \frac{Za}{A\beta^2} && (18)\\
a &= 2\pi N_A r_e^2 m_e c^2 = 0.15353747\ \mathrm{MeV} \cdot \frac{\mathrm{cm}^2}{\mathrm{mol}} && (19)
\end{aligned}$$

where Z & A are the effective atomic number and weight (in g/mol) of the material, p & E are the electron's momentum and energy, ρ & x are the material's mass density and thickness, I is the mean excitation potential of the material, $\delta(X)$ is the density correction [10], and we'll call ξ the "collisional" thickness as used in TSAI71. The specific form of g depends on which energy loss is desired:

$$\begin{aligned}
\bar{g} &= \log(\gamma - 1) - F(\gamma) && \text{mean energy loss (Bethe} - \text{Bloch)} && (20)\\
g_{\mathrm{mp}} &= \log\left[\frac{2\xi}{m_e c^2}\right] - \beta^2 + 0.198 && \text{most probable (Landau)} && (21)\\
F(\gamma) &= \left[1 + \frac{2}{\gamma} - \frac{1}{\gamma^2}\right]\log(2) - \frac{1}{8}\left[1 - \frac{1}{\gamma}\right]^2 - \frac{1}{\gamma^2} \approx \log(2) - \frac{1}{8} = 0.568 \text{ for } \gamma \gg 1 && && (22)\\
\beta &= \frac{v}{c} = \frac{pc}{E} && && (23)\\
\gamma &= \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{m_e c^2} && && (24)
\end{aligned}$$

Note that constant term in g_{mp} was originally calculated by Landau to be 0.373 and subsequently recalculated more accurately [11] to be 0.198. The energy lost per unit length can be written alternatively as:

$$\begin{aligned}
\frac{dE}{dx} &= -(m_e c^2)[\mathrm{N}]\sigma_{\mathrm{coll}} && (25)\\
\sigma_{\mathrm{coll}} &= 2\pi r_e^2 \left(\frac{Z}{\beta^2}\right) B = \frac{A}{N_A m_e c^2}\left[\frac{\xi}{\rho x}\right] B && (26)\\
B &= \left[2\log\left(\frac{pc}{I}\right) - \delta(X) + g\right] && (27)
\end{aligned}$$

where $\sigma_{\rm coll}$ is the collisional cross section and B is called the stopping number when $g = \bar{g}$.

The density correction δ is given by [12]:

$$\delta(X) = \left\{ \begin{array}{ll} \delta\left(X_0^\delta\right) \times 10^{2\left(X - X_0^\delta\right)} & X \le X_0^\delta \\ 2\log(10)\left(X - X_a^\delta\right) + a_\delta \left(X_1^\delta - X\right)^{m_\delta} & X_0^\delta < X < X_1^\delta \\ 2\log(10)\left(X - X_a^\delta\right) & X \ge X_1^\delta \end{array} \right\} \tag{28}$$

$$X = \log_{10}\left(\frac{p}{m_e c}\right) \tag{29}$$

$$X_a^\delta = \frac{-C_\delta}{2\log(10)} \tag{30}$$

$$C_\delta = 2\log\left(\frac{\hbar\omega_p}{I}\right) - 1 \tag{31}$$

$$a_\delta = \frac{\delta\left(X_0^\delta\right) + 2\log(10)\left(X_a^\delta - X_0^\delta\right)}{\left(X_1^\delta - X_0^\delta\right)^{m_\delta}} \tag{32}$$

$$\delta(X_0^\delta) = \left\{ \begin{array}{ll} 0.0 & \text{insulators} \\ 0.06, 0.08, 0.10, 0.12, \text{ or } 0.14 & \text{conductors} \end{array} \right\} \tag{33}$$

where ω_p,X_0^δ, X_1^δ, m_δ , and $\delta(X_0^\delta)$ depend on the material. The density correction below X_0^δ depends on whether the material is an insulator or conductor [13]. The plasma frequency [14] is a function of the electron number density in the material, $[\mathrm{N}_e]$:

$$\omega_p = \sqrt{4\pi[\mathrm{N}_e]r_e c^2} \tag{34}$$

$$[\mathrm{N}_e] = \langle Z\rangle [\mathrm{N}] = \frac{\langle Z\rangle N_A \rho}{\langle A\rangle} = N_A\rho \left\langle \frac{Z}{A} \right\rangle \tag{35}$$

Note that above $X \ge X_1^\delta$, the mean energy loss becomes depends only logrithamically on energy and the most probable loss becomes independant of energy:

$$\frac{p}{m_e c} \ge \frac{p_1}{m_e c} = 10^{X_1^\delta} \tag{36}$$

$$\delta(p > p_1) = 2\log(10)\left(X - X_a^\delta\right) = 2\log\left(\frac{p}{p_a}\right) \tag{37}$$

$$\frac{p_a}{m_e c} = 10^{X_a^\delta} = \exp\left(-\frac{C_\delta}{2}\right) = \left(\frac{I}{\hbar\omega_p}\right)^2 \sqrt{e} \tag{38}$$

$$\left[\frac{\Delta}{\rho x}\right] = \left[\frac{\xi}{\rho x}\right] \left[2\log\left(\frac{p_a c}{I}\right) + g\right] \tag{39}$$

We use the density correction parameters from [12]. If the density of a material is different than that listed in [12], then the following substitutions are made [14]:

$$X_a^{\delta\prime} = X_a^\delta - \frac{1}{2}\log_{10}\left(\frac{[\mathrm{N}]}{[\mathrm{N}]_0}\right) \tag{40}$$

$$X_0^{\delta\prime} = X_0^\delta - \frac{1}{2}\log_{10}\left(\frac{[\mathrm{N}]}{[\mathrm{N}]_0}\right) \tag{41}$$

$$X_1^{\delta\prime} = X_1^\delta - \frac{1}{2}\log_{10}\left(\frac{[\mathrm{N}]}{[\mathrm{N}]_0}\right) \tag{42}$$

where $[\mathrm{N}]_0$ is the number density listed in table and [N] is the desired number density. This is particularly useful for gases because table blah uses the density for 20°C and 1 atm.

If the material is not listed, then one must calculate effective values for I & Z/A using Bragg's additivity law. If the constituents are elements, the mean ionization potential is underestimated because electrons are

Phase of Material and Conditions			X_0^δ	X_1^δ
Solids & Liquids	$I < 100$ eV	$C_\delta > -3.681$	0.2	2.0
		$C_\delta \leq -3.681$	$-0.326C_\delta - 1.0$	2.0
	$I \geq 100$ eV	$C_\delta > -5.215$	0.2	3.0
		$C_\delta \leq -5.215$	$-0.326C_\delta - 1.5$	3.0
Gases, 20°C at 1 atm		$-10.000 < C_\delta$	1.6	4.0
		$-10.500 < C_\delta \leq -10.000$	1.7	4.0
		$-11.000 < C_\delta \leq -10.500$	1.8	4.0
		$-11.500 < C_\delta \leq -11.000$	1.9	4.0
		$-12.250 < C_\delta \leq -11.500$	2.0	4.0
		$-13.804 < C_\delta \leq -12.250$	2.0	5.0
		$-13.804 \leq C_\delta$	$-0.326C_\delta - 2.5$	5.0

Table 1: General Parameterization of the Density Correction [14, 15]. C_δ is calculated from the plasma frequency. For all cases $m_\delta = 3.0$ and X_a^δ & a_δ are calculated using equations blah.

more tightly bound in molecules than in the free atoms of the constituent elements. Therefore we'll use the "13 percent rule" to compenstate [15]. If the constituents are molecules such as various types of glass, we do not incease the ionziation potentials. This is because the binding effect was accounted for when the molecular ionization potential were calculated from their constitutent elements. Consequently the effective Z/A and I values are given by [15]:

$$\left\langle \frac{Z}{A} \right\rangle = \frac{\sum_i n_i Z_i}{\sum_i n_i A_i} = \sum_i w_i \left\langle \frac{Z}{A} \right\rangle_i \tag{43}$$

$$\log \langle I \rangle = \sum_i \frac{n_i Z_i}{Z} \log (I')_i = \left\langle \frac{A}{Z} \right\rangle \sum_i w_i \left\langle \frac{Z}{A} \right\rangle_i \log (I)_i \tag{44}$$

where n_i are the number of atoms of the i-th type in the molecule, I' is the modified elemental ionization potential, w_i is the fraction of the i-th molecule by weight in the mixture, and I is the molecular ionization potential. Once the effective plasma frequency, ionization potential, and Z/A for the material have been calculated, then one uses Tab. (1) to select appropriate value of $X^\delta_{a,0,1}$. Finally the density correction δ, collisional thickness ξ, and energy loss are calculated only after first deriving effective density correction parameters for a given material.

3 Materials in the Path of the Beam

The electron beam exits the beam pipe and enters the target region through a 5 mil beryllium window. The target region is enclosed in a helium "bag." For now, we will assume that the helium gas displaces all of the air and that its temperature & pressure are 20°C & and 1 atm. The electron beam then enters the target chamber portion of the cell which is made of C1720 glass. The end "windows" are much thinner than the side "walls." About 10 amagats of polarized ^{3}He and 0.1 amg of unpolarized N_2 fill the target chamber. For now, we'll also make the traditional approximation that the scattering occurs at the center of the target. This effects the thickness of polarized ^{3}He, unpolarized N_2, and glass wall traversed by the scattered electron.

After scattering, the electron passes through more polarized ^{3}He, unpolarized N_2, and the side wall of the cell. The distance travelled through these two materials after scattering is inversely proportional to the sine of the central scattering angle. After exiting the cell, the electrons travel out of the helium “bag” and through the sieve slit box. At this point the electrons enter the bore of the septum magnet. Both the entrance and exit apertures of the septum bore were covered by 2 inch thick pieces of polystyrene “foam.” The density of this foam is discussed in detail in the next section. The sieve slit box is filled with helium gas. Howver there is some ambiguity regarding whether the septum bore and the small amount of space between the septum and the HRS were filled with air or helium gas.

The spacing of the sieve slit box and septum magnet were almost certainly changed during the six to nine degree switch. In addition, the path of the central ray within the septum bore is different bewteen six and nine degrees. The only consequence of these two differences is that the amount of helium gas traversed by the scattered electron is different. Since this makes up only a tiny fraction of the radiation thickness. we will assume the geometry for nine degrees is identical to the geometry for six degrees. Finally, according to E. Folts, the electrons enter the HRS through a 10 mil Kapton window.

4 Density of the Polystyrene Foam

Solid polystyrene, used in scintillators, has a density of 1.032 g/cm^3 [7]. The density of the polystyrene foam that we used was measured independently by V. Sulkosky and A. Deur:

$$\rho = \left(\frac{0.03151 + 0.03220}{2}\right) \text{g/cm}^3 = 0.03186 \text{ g/cm}^3 \tag{45}$$

Clearly a large fraction of the polystyrene foam is occupied by some unknown gas. In principle, this gas affects the radiation length of the foam:

$$\frac{1}{X_0} = \frac{w_p}{X_p} + \frac{w_g}{X_g} \tag{46}$$

where w is the fraction by mass and the subscipts p & g refer to the polystyrene & the gas. Taking advantage of $w_p + w_g = 1$ and $w_g = \rho_g/\rho$, we find:

$$\frac{X_p}{X_0} - 1 = \frac{\rho_g}{\rho}\left[\frac{X_p}{X_g} - 1\right] \tag{47}$$

If the gas is at room temperature and 1 atm, then the gas density is on order of 10^{-3} g/cm^3 and consequently the true radiation length of the foam differs from the radiation legnth of polystyrene at the level of a fraction of a percent. Since the difference in the two measurements for the density of the foam is larger than the estimated correction due to the trapped gas, we neglect it’s effect on the radiation length:

$$\left[\frac{\rho}{X_0}\right]_{\text{foam}} \approx \frac{\rho_{\text{foam}}}{X_{\text{polystyrene}}} \tag{48}$$

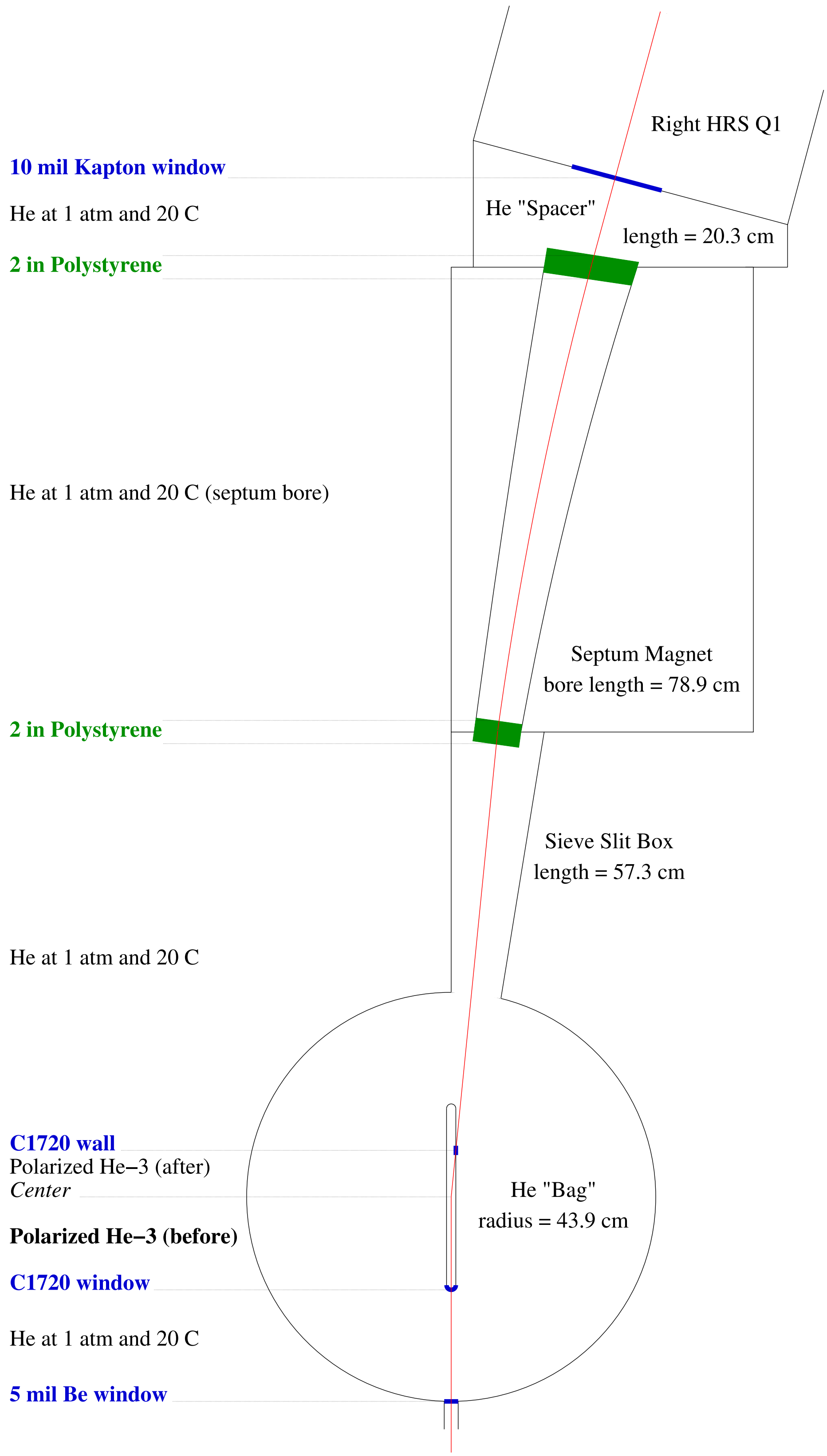

Figure 1: Scaled Geometry of Target Region.

parameter	value	units
α^{-1}	137.0359991	-
c	29979245800	cm/s
r_e	$2.817940325 \times 10^{-13}$	cm
$m_e c^2$	0.510998918	MeV
$\hbar$	$6.58211915 \times 10^{-16}$	$\mathrm{eV} \cdot \mathrm{s}$
R	8.314472	J/mol/K
N_A	6.0221415×10^{23}	#/mol
1 amg	2.6867773×10^{19}	#/cm^3
1 amg	4.4614981×10^{-5}	mol/cm^3
1 mil	0.001	inch

Table 2: Physical Constants and Conversions. [16]

Z -	**atom**	A amu	σ_{rad} barns	X_0 g/cm^2
1	H	1.00794	0.02655	63.0435
2	^{3}He	3.01603	0.07047	71.0736
2	He	4.00260	0.07047	94.3224
4	Be	9.01218	0.22956	65.1900
5	B	10.81100	0.34073	52.6868
6	C	12.01070	0.46711	42.6969
7	N	14.00670	0.61226	37.9879
8	O	15.99940	0.77597	34.2382
11	Na	22.98977	1.37637	27.7362
12	Mg	24.30500	1.61235	25.0315
13	Al	26.98154	1.86596	24.0112
14	Si	28.08550	2.13706	21.8231
19	K	39.09830	3.74958	17.3151
20	Ca	40.07800	4.12249	16.1434
33	As	74.92160	10.41949	11.9401
38	Sr	87.62000	13.51897	10.7624
56	Ba	137.32700	27.45233	8.3066

Table 3: Radiation Length by Atomic Species.

	Z/A	I eV	I' (g) eV	I' (l/s) eV	ρ g/cm^3	$-C$	X_0	X_1	a	m	$\delta(X_0)$
H	0.9922	19.2	19.2	19.2	8.3748E−05	9.5835	1.8639	3.2718	0.14095	5.7273	0.00
He	0.4997	41.8	41.8	47.2	1.6632E−04	11.1393	2.2017	3.6122	0.13443	5.8347	0.00
^{3}He	0.6631	41.8	41.8	47.2	1.2533E−04	11.1393	2.2017	3.6122	0.13443	5.8347	0.00
Be	0.4438	63.7	63.7	72.0	1.8480E+00	2.7847	0.0592	1.6922	0.76146	2.4339	0.14
B	0.4625	76.0	76.0	85.9	2.3700E+00	2.8477	0.0305	1.9688	0.56221	2.4512	0.14
C	0.4995	78.0	70.0	81.0	2.0000E+00	2.9925	-0.0351	2.4860	0.20489	3.0036	0.10
N	0.4998	82.0	82.0	82.0	1.1653E−03	10.5400	1.7378	4.1323	0.15955	3.2125	0.00
O	0.5000	95.0	97.0	106.0	1.3315E−03	10.7004	1.7541	4.3213	0.11778	3.2913	0.00
Na	0.4785	149.0	149.0	168.4	9.7100E−01	5.0526	0.2880	3.1962	0.07608	3.6452	0.08
Mg	0.4937	156.0	156.0	176.3	1.7400E+00	4.5297	0.1499	3.0668	0.08162	3.6166	0.08
Al	0.4818	166.0	166.0	187.6	2.6989E+00	4.2395	0.1708	3.0127	0.07934	3.6345	0.12
Si	0.4985	173.0	173.0	195.5	2.3300E+00	4.4351	0.2014	2.8715	0.14840	3.2546	0.14
K	0.4860	190.0	190.0	214.7	8.6200E−01	5.6423	0.3851	3.1724	0.20027	2.9233	0.10
Ca	0.4990	191.0	191.0	215.8	1.5500E+00	5.0396	0.3228	3.1191	0.15475	3.0745	0.14
As	0.4405	347.0	347.0	392.1	5.7300E+00	5.0510	0.1767	3.5702	0.06725	3.4176	0.08
Sr	0.4337	366.0	366.0	413.6	2.5400E+00	5.9867	0.4585	3.6778	0.07058	3.4435	0.14
Ba	0.4078	491.0	491.0	554.8	3.5000E+00	6.3153	0.4190	3.4547	0.18267	2.8906	0.14

Table 4: Density Corrections Parameters for Elemental Materials. I' are the ionization potentials to be used when calculating effective molecular ionization potentials. The densities are given for the natural form of the element (gas, diatomic gas, liquid, solid) at 1 atm and 20°C.

material	polystyrene	polyimide film (Kapton)
formula	$C_6H_5CH{=}CH_2$	$(C_{22}H_{10}N_2O_5)_n$
C by wt.	0.922582	0.691133
O by wt.	-	0.209235
N by wt.	-	0.073270
H by wt.	0.077418	0.026362
X_0 $(\mathrm{g/cm^2})$	43.7911	40.5761

Table 5: Radiation Length of Polymers. [7, 17]

material	**by wt.**	X_0 $(\mathrm{g/cm^2})$	**by** X_0
SiO_2	0.599	27.0460	0.594
Al_2O_3	0.182	27.9399	0.175
CaO	0.074	19.0098	0.104
MgO	0.088	28.0227	0.084
B_2O_3	0.047	38.4158	0.033
Na_2O	0.010	29.1660	0.009
K_2O	0.000	18.9019	negligible
As_2O_3	0.000	14.1807	negligible

$$X_0 = 26.8379\ \mathrm{g/cm^2}$$
$$\rho = 2.53\ \mathrm{g/cm^3}$$
$$\bar{X}_0 = 10.608\ \mathrm{cm}$$

Table 6: Radiation Length of Corning 1729 (C1720). [18]

material	**by wt.**	X_0 $(\mathrm{g/cm^2})$	**by** X_0
SiO_2	0.605	27.0460	0.435
BaO	0.183	9.0195	0.394
Al_2O_3	0.144	27.9399	0.100
CaO	0.065	19.0098	0.066
SrO	0.003	12.0367	0.005

$$X_0 = 19.4246\ \mathrm{g/cm^2}$$
$$\rho = 2.76\ \mathrm{g/cm^3}$$
$$\bar{X}_0 = 7.038\ \mathrm{cm}$$

Table 7: Radiation Length of GE180.

	Z/A	I eV	ρ g/cm^3	$-C$	X_0	X_1	a	m
Al_2O_3	0.4904	145.2	3.9700E+00	3.5682	0.0402	2.8665	0.08499	3.5458
As_2O_3	0.4549	276.6	3.8600E+00	4.9606	0.2000	3.0000	0.18402	3.0000
BaO	0.4174	451.1	5.7000E+00	5.6349	0.3370	3.0000	0.21620	3.0000
B_2O_3	0.4884	99.6	1.8120E+00	3.6027	0.1843	2.7379	0.11547	3.3832
CaO	0.4993	176.1	3.3000E+00	4.1209	-0.0172	3.0171	0.12127	3.1936
MgO	0.4962	143.8	3.5800E+00	3.6404	0.0575	2.8580	0.08313	3.5968
K_2O	0.4883	189.9	2.3200E+00	4.6463	0.0480	3.0110	0.16789	3.0121
SiO_2	0.4993	139.2	2.3200E+00	4.0029	0.1385	3.0025	0.08407	3.5064
Na_2O	0.4840	148.8	2.2700E+00	4.1892	0.1652	2.9793	0.07501	3.6943
SrO	0.4439	326.4	5.7000E+00	4.9259	0.2000	3.0000	0.18244	3.0000
C1720	0.4967	141.0	2.5300E+00	3.9477	0.2000	3.0000	0.13788	3.0000
GE180	0.4829	171.8	2.7600E+00	4.2834	0.2000	3.0000	0.15317	3.0000
kapton	0.5126	79.6	1.4200E+00	3.3497	0.1509	2.5631	0.15971	3.1921
polystyrene	0.5377	68.7	1.0600E+00	3.2999	0.1647	2.5031	0.16454	3.2224

Table 8: Density Correction Parameters for Composite Materials. For all these materials $\delta(X_0) = 0$.

	Z/A	I eV	ρ g/cm^3	$-C$	X_0	X_1	a	m
^{3}He	0.6631	41.8	1.4330E−03 1.4080E−03 1.4120E−03	11.1393	1.6726 1.6764 1.6758	3.0831 3.0869 3.0863	0.13443	5.8347
N_2	0.4998	82.0	1.3370E−04 1.3870E−04 1.4000E−04 1.1850E−02 1.1870E−02 1.1720E−02	10.5400	2.2080 2.2000 2.1980 1.2342 1.2338 1.2366	4.6025 4.5945 4.5925 3.6287 3.6283 3.6311	0.15350	3.2125
polystyrene	0.5377	68.7	3.1860E−02	3.2999	0.9257	3.2641	0.16454	3.2224

Table 9: Density Correction Parameters for Materials at Different Mass Densities.

cell	Penelope	Ref. Cell 1	Priapus	Ref. Cell 2
window, upstream (μm)	132 ± 2	127 ± 2	128 ± 2	131 ± 2
window, downstream (μm)	138 ± 2	120 ± 2	142 ± 2	150 ± 2
wall, beam right (μm)	622 ± 10	638 ± 10	600 ± 10	610 ± 10
wall, beam left (μm)	694 ± 10	693 ± 10	760 ± 10	711 ± 10
outer diameter (cm)	1.9249	1.920	1.9160	1.920
external length (cm)	39.6875	39.5288	39.3700	39.5288
$\left[{}^3He\right]$ (amg)	10.65	-	10.46/10.49	-
$[N_2]$ (amg)	0.107	9.48	0.111/0.112	9.50/9.38

Table 10: Cell Target Chamber Parameters. Densities for Priapus refer to 6/9 degrees.

material	X_0 g/cm^2	ρ g/cm^3	$\bar{X}_0$ cm	ℓ cm	t -	**fraction of tot** -
C1720 Window	26.8379	2.530E+00	1.0608E+01	1.320E−02	1.244E−03	0.5879
Polarized ^{3}He	71.0736	1.433E−03	4.9596E+04	1.983E+01	3.998E−04	0.1889
Beryllium Window	65.1900	1.848E+00	3.5276E+01	1.270E−02	3.600E−04	0.1701
Unpolarized N_2	37.9879	1.337E−04	2.8406E+05	1.983E+01	6.981E−05	0.0330
Helium Gas	94.3224	1.664E−04	5.6687E+05	2.409E+01	4.250E−05	0.0201
Total Before	2.116E-03					
C1720 Wall	26.8379	2.530E+00	1.0608E+01	5.951E−01	5.610E−02	0.8641
Polystyrene Foam	43.7911	3.186E−02	1.3745E+03	1.016E+01	7.392E−03	0.1139
Kapton Window	40.5761	1.420E+00	2.8575E+01	2.540E−02	8.889E−04	0.0137
Helium Gas	94.3224	1.664E−04	5.6687E+05	1.912E+02	3.373E−04	0.0052
Polarized ^{3}He	71.0736	1.433E−03	4.9596E+04	8.612E+00	1.737E−04	0.0027
Unpolarized N_2	37.9879	1.337E−04	2.8406E+05	8.612E+00	3.032E−05	0.0005
Total After	6.492E-02					

Table 11: Radiation Thicknesses Before & After Scattering from Penelope at 6 deg.

material	X_0 g/cm^2	ρ g/cm^3	$\bar{X}_0$ cm	ℓ cm	t -	**fraction of tot** -
Unpolarized N_2	37.9879	1.185E−02	3.2062E+03	1.975E+01	6.160E−03	0.7938
C1720 Window	26.8379	2.530E+00	1.0608E+01	1.270E−02	1.197E−03	0.1543
Beryllium Window	65.1900	1.848E+00	3.5276E+01	1.270E−02	3.600E−04	0.0464
Helium Gas	94.3224	1.664E−04	5.6687E+05	2.417E+01	4.264E−05	0.0055
Total Before	7.760E-03					
C1720 Wall	26.8379	2.530E+00	1.0608E+01	6.104E−01	5.754E−02	0.8359
Polystyrene Foam	43.7911	3.186E−02	1.3745E+03	1.016E+01	7.392E−03	0.1074
Unpolarized N_2	37.9879	1.185E−02	3.2062E+03	8.574E+00	2.674E−03	0.0389
Kapton Window	40.5761	1.420E+00	2.8575E+01	2.540E−02	8.889E−04	0.0129
Helium Gas	94.3224	1.664E−04	5.6687E+05	1.912E+02	3.374E−04	0.0049
Total After	6.883E-02					

Table 12: Radiation Thicknesses Before & After Scattering from Ref. Cell 1 at 6 deg.

material	X_0	ρ	$\bar{X}_0$	ℓ	t	**fraction of tot**
	g/cm^2	g/cm^3	cm	cm	-	-
C1720 Window	26.8379	2.530E+00	1.0608E+01	1.280E−02	1.207E−03	0.5827
Polarized ^{3}He	71.0736	1.408E−03	5.0496E+04	1.967E+01	3.896E−04	0.1881
Beryllium Window	65.1900	1.848E+00	3.5276E+01	1.270E−02	3.600E−04	0.1738
Unpolarized N_2	37.9879	1.387E−04	2.7383E+05	1.967E+01	7.184E−05	0.0347
Helium Gas	94.3224	1.664E−04	5.6687E+05	2.425E+01	4.278E−05	0.0207
Total Before	2.071E-03					
C1720 Wall	26.8379	2.530E+00	1.0608E+01	5.740E−01	5.411E−02	0.8599
Polystyrene Foam	43.7911	3.186E−02	1.3745E+03	1.016E+01	7.392E−03	0.1175
Kapton Window	40.5761	1.420E+00	2.8575E+01	2.540E−02	8.889E−04	0.0141
Helium Gas	94.3224	1.664E−04	5.6687E+05	1.913E+02	3.374E−04	0.0054
Polarized ^{3}He	71.0736	1.408E−03	5.0496E+04	8.591E+00	1.701E−04	0.0027
Unpolarized N_2	37.9879	1.387E−04	2.7383E+05	8.591E+00	3.137E−05	0.0005
Total After	6.293E-02					

Table 13: Radiation Thicknesses Before & After Scattering from Priapus at 6 deg.

material	X_0	ρ	$\bar{X}_0$	ℓ	t	**fraction of tot**
	g/cm^2	g/cm^3	cm	cm	-	-
Unpolarized N_2	37.9879	1.187E−02	3.1994E+03	1.975E+01	6.173E−03	0.7903
C1720 Window	26.8379	2.530E+00	1.0608E+01	1.310E−02	1.235E−03	0.1581
Beryllium Window	65.1900	1.848E+00	3.5276E+01	1.270E−02	3.600E−04	0.0461
Helium Gas	94.3224	1.664E−04	5.6687E+05	2.417E+01	4.264E−05	0.0055
Total Before	7.811E-03					
C1720 Wall	26.8379	2.530E+00	1.0608E+01	5.836E−01	5.501E−02	0.8295
Polystyrene Foam	43.7911	3.186E−02	1.3745E+03	1.016E+01	7.392E−03	0.1115
Unpolarized N_2	37.9879	1.187E−02	3.1994E+03	8.601E+00	2.688E−03	0.0405
Kapton Window	40.5761	1.420E+00	2.8575E+01	2.540E−02	8.889E−04	0.0134
Helium Gas	94.3224	1.664E−04	5.6687E+05	1.912E+02	3.374E−04	0.0051
Total After	6.632E-02					

Table 14: Radiation Thicknesses Before & After Scattering from Ref. Cell 2 at 6 deg.

material	X_0	ρ	$\bar{X}_0$	ℓ	t	**fraction of tot**
	g/cm^2	g/cm^3	cm	cm	-	-
C1720 Window	26.8379	2.530E+00	1.0608E+01	1.280E−02	1.207E−03	0.5822
Polarized ^{3}He	71.0736	1.412E−03	5.0352E+04	1.967E+01	3.907E−04	0.1885
Beryllium Window	65.1900	1.848E+00	3.5276E+01	1.270E−02	3.600E−04	0.1737
Unpolarized N_2	37.9879	1.400E−04	2.7138E+05	1.967E+01	7.249E−05	0.0350
Helium Gas	94.3224	1.664E−04	5.6687E+05	2.425E+01	4.278E−05	0.0206
Total Before	2.073E-03					
C1720 Wall	26.8379	2.530E+00	1.0608E+01	3.835E−01	3.616E−02	0.8050
Polystyrene Foam	43.7911	3.186E−02	1.3745E+03	1.016E+01	7.392E−03	0.1646
Kapton Window	40.5761	1.420E+00	2.8575E+01	2.540E−02	8.889E−04	0.0198
Helium Gas	94.3224	1.664E−04	5.6687E+05	1.943E+02	3.428E−04	0.0076
Polarized ^{3}He	71.0736	1.412E−03	5.0352E+04	5.740E+00	1.140E−04	0.0025
Unpolarized N_2	37.9879	1.400E−04	2.7138E+05	5.740E+00	2.115E−05	0.0005
Total After	4.492E-02					

Table 15: Radiation Thicknesses Before & After Scattering from Priapus at 9 deg.

material	X_0	ρ	$\bar{X}_0$	ℓ	t	**fraction of tot**
	g/cm^2	g/cm^3	cm	cm	-	-
Unpolarized N_2	37.9879	1.172E−02	3.2404E+03	1.975E+01	6.095E−03	0.7882
C1720 Window	26.8379	2.530E+00	1.0608E+01	1.310E−02	1.235E−03	0.1597
Beryllium Window	65.1900	1.848E+00	3.5276E+01	1.270E−02	3.600E−04	0.0466
Helium Gas	94.3224	1.664E−04	5.6687E+05	2.417E+01	4.264E−05	0.0055
Total Before	7.733E-03					
C1720 Wall	26.8379	2.530E+00	1.0608E+01	3.899E−01	3.676E−02	0.7795
Polystyrene Foam	43.7911	3.186E−02	1.3745E+03	1.016E+01	7.392E−03	0.1568
Unpolarized N_2	37.9879	1.172E−02	3.2404E+03	5.747E+00	1.774E−03	0.0376
Kapton Window	40.5761	1.420E+00	2.8575E+01	2.540E−02	8.889E−04	0.0188
Helium Gas	94.3224	1.664E−04	5.6687E+05	1.943E+02	3.428E−04	0.0073
Total After	4.716E-02					

Table 16: Radiation Thicknesses Before & After Scattering from Ref. Cell 2 at 9 deg.

material	ρ g/cm^3	ℓ cm	ξ MeV	mp MeV	dE MeV
Polarized ^{3}He	1.433E−03	1.983E+01	2.893E−03	0.064	0.102
C1720 Window	2.530E+00	1.320E−02	2.547E−03	0.038	0.072
Beryllium Window	1.848E+00	1.270E−02	1.599E−03	0.024	0.046
Helium Gas	1.664E−04	2.409E+01	3.076E−04	0.007	0.012
Unpolarized N_2	1.337E−04	1.983E+01	2.035E−04	0.004	0.008
Total Before			7.550E-03	0.137	0.240
C1720 Wall	2.530E+00	5.951E−01	1.148E−01	2.151	3.208
Polystyrene Foam	3.186E−02	1.016E+01	2.672E−02	0.577	0.861
Helium Gas	1.664E−04	1.912E+02	2.441E−03	0.060	0.092
Kapton Window	1.420E+00	2.540E−02	2.839E−03	0.044	0.081
Polarized ^{3}He	1.433E−03	8.612E+00	1.257E−03	0.027	0.044
Unpolarized N_2	1.337E−04	8.612E+00	8.838E−05	0.002	0.003
Total After			1.481E-01	2.861	4.289

Table 17: Before ($p = 2134.3$ MeV) & After ($p = 1806.4$ MeV) Scattering from Penelope at 6 deg.

material	ρ g/cm^3	ℓ cm	ξ MeV	mp MeV	dE MeV
Unpolarized N_2	1.185E−02	1.975E+01	1.796E−02	0.399	0.601
C1720 Window	2.530E+00	1.270E−02	2.450E−03	0.036	0.069
Beryllium Window	1.848E+00	1.270E−02	1.599E−03	0.024	0.046
Helium Gas	1.664E−04	2.417E+01	3.086E−04	0.007	0.012
Total Before			2.232E-02	0.466	0.728
C1720 Wall	2.530E+00	6.104E−01	1.178E−01	2.209	3.290
Polystyrene Foam	3.186E−02	1.016E+01	2.672E−02	0.577	0.861
Unpolarized N_2	1.185E−02	8.574E+00	7.795E−03	0.167	0.260
Helium Gas	1.664E−04	1.912E+02	2.442E−03	0.060	0.092
Kapton Window	1.420E+00	2.540E−02	2.839E−03	0.044	0.081
Total After			1.576E-01	3.057	4.584

Table 18: Before ($p = 2134.3$ MeV) & After ($p = 1806.4$ MeV) Scattering from Ref. Cell 1 at 6 deg.

material	ρ g/cm^3	ℓ cm	ξ MeV	mp MeV	dE MeV
Polarized ^{3}He	1.408E−03	1.967E+01	2.819E−03	0.063	0.101
C1720 Window	2.530E+00	1.280E−02	2.470E−03	0.037	0.070
Beryllium Window	1.848E+00	1.270E−02	1.599E−03	0.024	0.046
Helium Gas	1.664E−04	2.425E+01	3.096E−04	0.007	0.012
Unpolarized N_2	1.387E−04	1.967E+01	2.094E−04	0.005	0.008
Total Before			7.407E-03	0.136	0.237
C1720 Wall	2.530E+00	5.740E−01	1.108E−01	2.071	3.102
Polystyrene Foam	3.186E−02	1.016E+01	2.672E−02	0.577	0.863
Helium Gas	1.664E−04	1.913E+02	2.442E−03	0.060	0.092
Kapton Window	1.420E+00	2.540E−02	2.839E−03	0.044	0.081
Polarized ^{3}He	1.408E−03	8.591E+00	1.231E−03	0.026	0.043
Unpolarized N_2	1.387E−04	8.591E+00	9.146E−05	0.002	0.003
Total After			1.441E-01	2.780	4.184

Table 19: Before ($p = 3145.3$ MeV) & After ($p = 1941.5$ MeV) Scattering from Priapus at 6 deg.

material	ρ g/cm^3	ℓ cm	ξ MeV	mp MeV	dE MeV
Unpolarized N_2	1.187E−02	1.975E+01	1.800E−02	0.400	0.609
C1720 Window	2.530E+00	1.310E−02	2.528E−03	0.038	0.072
Beryllium Window	1.848E+00	1.270E−02	1.599E−03	0.024	0.046
Helium Gas	1.664E−04	2.417E+01	3.086E−04	0.007	0.012
Total Before			2.244E-02	0.469	0.739
C1720 Wall	2.530E+00	5.836E−01	1.126E−01	2.107	3.154
Polystyrene Foam	3.186E−02	1.016E+01	2.672E−02	0.577	0.863
Unpolarized N_2	1.187E−02	8.601E+00	7.836E−03	0.168	0.261
Helium Gas	1.664E−04	1.912E+02	2.442E−03	0.060	0.092
Kapton Window	1.420E+00	2.540E−02	2.839E−03	0.044	0.081
Total After			1.524E-01	2.956	4.451

Table 20: Before ($p = 3145.3$ MeV) & After ($p = 1941.5$ MeV) Scattering from Ref. Cell 2 at 6 deg.

material	ρ	ℓ	ξ	mp	dE
	g/cm^3	cm	MeV	MeV	MeV
Polarized ^{3}He	1.412E−03	1.967E+01	2.827E−03	0.063	0.101
C1720 Window	2.530E+00	1.280E−02	2.470E−03	0.037	0.070
Beryllium Window	1.848E+00	1.270E−02	1.599E−03	0.024	0.046
Helium Gas	1.664E−04	2.425E+01	3.096E−04	0.007	0.012
Unpolarized N_2	1.400E−04	1.967E+01	2.113E−04	0.005	0.008
Total Before			7.417E-03	0.136	0.237
C1720 Wall	2.530E+00	3.835E−01	7.400E−02	1.354	2.075
Polystyrene Foam	3.186E−02	1.016E+01	2.672E−02	0.577	0.864
Helium Gas	1.664E−04	1.943E+02	2.481E−03	0.061	0.093
Kapton Window	1.420E+00	2.540E−02	2.839E−03	0.044	0.081
Polarized ^{3}He	1.412E−03	5.740E+00	8.250E−04	0.017	0.029
Unpolarized N_2	1.400E−04	5.740E+00	6.166E−05	0.001	0.002
Total After			1.069E-01	2.054	3.144

Table 21: Before ($p = 3219.9$ MeV) & After ($p = 2007.0$ MeV) Scattering from Priapus at 9 deg.

material	ρ	ℓ	ξ	mp	dE
	g/cm^3	cm	MeV	MeV	MeV
Unpolarized N_2	1.172E−02	1.975E+01	1.777E−02	0.395	0.602
C1720 Window	2.530E+00	1.310E−02	2.528E−03	0.038	0.072
Beryllium Window	1.848E+00	1.270E−02	1.599E−03	0.024	0.046
Helium Gas	1.664E−04	2.417E+01	3.086E−04	0.007	0.012
Total Before			2.221E-02	0.464	0.732
C1720 Wall	2.530E+00	3.899E−01	7.524E−02	1.378	2.110
Polystyrene Foam	3.186E−02	1.016E+01	2.672E−02	0.577	0.864
Unpolarized N_2	1.172E−02	5.747E+00	5.170E−03	0.109	0.173
Helium Gas	1.664E−04	1.943E+02	2.480E−03	0.061	0.093
Kapton Window	1.420E+00	2.540E−02	2.839E−03	0.044	0.081
Total After			1.124E-01	2.169	3.321

Table 22: Before ($p = 3219.9$ MeV) & After ($p = 2007.0$ MeV) Scattering from Ref. Cell 2 at 9 deg.

cell	**angle** deg.	p MeV	**before** mp	dE	**after** mp	dE	**total** mp (MeV)	dE (MeV)
penelope	6	400.0	0.137	0.226	2.860	4.065	2.998	4.291
penelope	6	2134.3	0.138	0.239	2.861	4.314	2.998	4.553
priapus	6	400.0	0.135	0.222	2.780	3.957	2.914	4.178
priapus	6	2134.9	0.135	0.234	2.780	4.199	2.915	4.433
priapus	6	2844.8	0.135	0.236	2.780	4.241	2.915	4.477
priapus	6	4208.9	0.135	0.239	2.780	4.297	2.915	4.536
priapus	9	400.0	0.135	0.222	2.054	2.972	2.189	3.194
priapus	9	1147.3	0.135	0.230	2.054	3.086	2.189	3.316
priapus	9	2233.9	0.135	0.235	2.054	3.157	2.189	3.392
priapus	9	3318.8	0.135	0.238	2.054	3.199	2.189	3.437
priapus	9	3775.5	0.135	0.239	2.054	3.213	2.189	3.452
priapus	9	4404.2	0.135	0.240	2.054	3.230	2.189	3.470
refcell 1	6	400.0	0.466	0.689	3.056	4.345	3.522	5.033
refcell 1	6	2134.3	0.467	0.727	3.057	4.610	3.524	5.337
refcell 2	6	400.0	0.468	0.692	2.955	4.209	3.423	4.901
refcell 2	6	2134.9	0.469	0.731	2.956	4.466	3.424	5.197
refcell 2	6	2844.8	0.469	0.737	2.956	4.510	3.424	5.247
refcell 2	6	4208.9	0.469	0.746	2.956	4.570	3.424	5.316
refcell 2	9	400.0	0.463	0.685	2.167	3.139	2.630	3.824
refcell 2	9	1147.3	0.464	0.709	2.168	3.259	2.632	3.968
refcell 2	9	2233.9	0.464	0.724	2.168	3.334	2.632	4.058
refcell 2	9	3318.8	0.464	0.733	2.168	3.378	2.632	4.111
refcell 2	9	3775.5	0.464	0.736	2.168	3.393	2.632	4.128
refcell 2	9	4404.2	0.464	0.739	2.168	3.410	2.632	4.149

Table 23: Collisional Energy Loss for Penelope, Priapus, and the Ref. Cells for Different Electron Momenta. Note that the energy loss is insensensitive to the electon momentum. Because of this insensitivity in our momentum range, we use average momenta for each cell and angle in the calculation of the collisional thickness ξ, the most probable energy loss mp, and the mean energy loss dE.

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