# $SU(3)$ Chiral Effective Field Theories

# — A Status Report —

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## Basic idea of an effective field theory

- treat active, light particles as collective degrees of freedom
- heavy particles are frozen and reduced to static sources

**Dynamics are described by an effective Lagrangian which incorporates all relevant symmetries of the underlying fundamental theory**

# Chiral effective theory

- spontaneous chiral symmetry breaking leads to characteristic gap $\Delta \sim 1$ GeV in hadronic spectrum

- hadron physics at low energies $E \ll \Delta$ is governed by softest excitations of QCD vacuum, the 8 Goldstone bosons: pions ($\pi$), kaons ($K$), eta ($\eta$)

Construct effective Lagrangian containing Goldstone bosons and incorporating symmetries and symmetry breaking patterns of QCD:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{sb}}$$

- $\mathcal{L}_0$ invariant under chiral transformations $SU(3)_L \times SU(3)_R$

- $\mathcal{L}_{\text{sb}}$ incorporates chiral symmetry breaking patterns due to non-zero quark masses $(m_u, m_d, m_s)$;
  these are small $\Rightarrow$ treat perturbatively

- consequence of confinement:
  quarks and gluons do not show up as explicit degrees of freedom in $\mathcal{L}$

# Construction of the chiral effective Lagrangian

8 Goldstone bosons most conveniently summarized in matrix $U(x) \in SU(3)$

$$U(x) = u^2(x) = \exp\left(i\frac{\sqrt{2}}{f_\pi}\,\phi(x)\right).$$

$f_\pi \simeq 93$ MeV,
pion decay constant

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

The effective Lagrangian is a function of $U, \partial_\mu U$ and the quark mass matrix $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$.

$$\mathcal{L} = \mathcal{L}(U, \partial U, \partial^2 U, \ldots, \mathcal{M})$$

Yields expansion in powers of $\mathcal{M}$ and $\partial_\mu U$
$\sim$ powers of meson momenta.

$$\mathcal{L}_0 = \mathcal{L}_0^{(2)} + \mathcal{L}_0^{(4)} + \ldots$$

with $(i)$ being $i^{\text{th}}$ chiral order.

## Leading term

$$\mathcal{L}_0^{(2)} = \frac{f_\pi^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U \rangle$$
$$= \frac{1}{2}\langle \partial_\mu \phi \partial^\mu \phi \rangle + \frac{1}{12 f_\pi^2}\langle [\phi, \partial_\mu \phi][\phi, \partial^\mu \phi] \rangle + \ldots$$

At higher chiral orders new, additional coupling constants appear that need to be determined by experiment.
$\mathcal{L}_0$ has the form of a Taylor expansion.

Explicit chiral symmetry breaking due to quark masses

$$\mathcal{L}_{\text{sb}} = \mathcal{L}_{\text{sb}}^{(2)} + \mathcal{L}_{\text{sb}}^{(4)} + \ldots$$

with the leading term

$$\mathcal{L}_{\text{sb}}^{(2)} = B_0 \frac{f_\pi^2}{2}\langle \mathcal{M}(U + U^\dagger) \rangle$$

# Inclusion of Baryons

Ground state $SU(3)$ baryon octet

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Lagrangian is extended to include baryons

$$\mathcal{L} = \mathcal{L}(U, \partial U, \ldots, B, \partial B, \ldots, \mathcal{M})$$

At leading order

$$\boxed{\begin{aligned} \mathcal{L}^{(1)}_{\phi B} = {} & i\langle \bar{B}\gamma_\mu D^\mu B\rangle - M_0\langle \bar{B}B\rangle \\ & -\tfrac{i}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle - \tfrac{i}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle \end{aligned}}$$

$M_0$ octet baryon mass in chiral limit, $\mathcal{M} = 0$.
$D, F$: axial vector couplings.
Determined from hyperon beta decays:

$$D \approx 0.80, \qquad F \approx 0.46$$

Inclusion of external vector and axial vector fields:

Chiral Symmetry is treated as a *local* symmetry.

Replace partial derivative $\partial_\mu$ by gauge covariant derivatives which involve external vector ($v_\mu$) and axial vector ($a_\mu$) fields.

Baryons:

$$\partial_\mu B \to D_\mu B = \partial_\mu B + [\Gamma_\mu, B]$$

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}(u^\dagger r_\mu u + u l_\mu u^\dagger)$$

$$r_\mu = v_\mu + a_\mu \ , \qquad l_\mu = v_\mu - a_\mu$$

Mesons:

$$\partial_\mu U \to \nabla_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$$

$$u_\mu = i u^\dagger \nabla_\mu U \, u$$

Expand $u_\mu$ in meson fields

$$u_\mu = -\frac{\sqrt{2}}{f_\pi}\partial_\mu \phi - 2 i a_\mu + \dots$$

Yields (in the chiral limit) **generalized Goldberger-Treiman** relations, *e.g.*, $(g_A^{pn} = D + F = 1.26)$

$$g_{\pi NN} = \frac{g_A^{pn} M_N}{f_\pi}$$

agreement better than 2%

Inclusion of baryons introduces a new scale, $M_0$,

$$\frac{M_0}{\Lambda_\chi} \sim 1 \qquad \Lambda_\chi = 4\pi f_\pi \sim 1.2\text{GeV},$$

$\Lambda_\chi$: scale of spontaneous chiral symmetry breaking

$\Rightarrow$ chiral counting scheme, *i.e.* higher loops correspond to higher chiral powers, is spoilt

• Can be reestablished by considering the fermions to be very heavy (nonrelativistic framework of **heavy baryon ChPT**)

(Jenkins, Manohar '91)

• or: evaluate loops in relativistic framework with **infrared regularization**, isolating infrared singularities due to Goldstone boson masses

(Ellis, Tang '98, Becher, Leutwyler '99)

## Inclusion of strange quarks

Is the strange quark light?

Current quark masses at $\mu = 2$ GeV in $\overline{\text{MS}}$

| | |
|---|---|
| $m_u$=1.5–4.5 | MeV |
| $m_d$= 5–8.5 | MeV |
| $m_s$= 80–155 | MeV |

- $m_u, m_d$ light compared to any hadronic scale, *e.g.*, $m_u, m_d \ll \Lambda_{QCD} \sim 150$ MeV
- On the other hand: $m_s \sim \Lambda_{QCD}$
  ... Convergence of chiral series ???
- Alternatively, treat $m_s$ as heavy state and integrate out
  a) $\Rightarrow SU(2)$ ChPT
  b) $\Rightarrow$ heavy kaon ChPT, expansion parameter $\frac{m_\pi^2}{m_K^2}$

# Baryon masses

At next-to-leading order quark masses enter:

$$\begin{aligned}\mathcal{L}_{\phi B}^{(2)} =& 4B_0 b_0 \langle \bar{B}B\rangle\langle\mathcal{M}\rangle + 4B_0 b_D \langle \bar{B}\{\mathcal{M}, B\}\rangle \\ &+ 4B_0 b_F \langle \bar{B}[\mathcal{M}, B]\rangle\end{aligned}$$

$b_0, b_F, b_D$ unknown parameters.
To be determined from experiment.

## Mass splittings of the baryon octet at leading order in symmetry breaking

(Work in limit $m_u = m_d$: $\Rightarrow$ 4 different baryon masses $M_N, M_\Lambda, M_\Sigma, M_\Xi$)

$$\begin{aligned} M_N &= \tilde{M}_0 - 4m_K^2 b_D + 4(m_K^2 - m_\pi^2) b_F \\ M_\Lambda &= \tilde{M}_0 - \tfrac{4}{3}(m_K^2 - m_\pi^2) b_D \\ M_\Sigma &= \tilde{M}_0 - 4m_\pi^2 b_D \\ M_\Xi &= \tilde{M}_0 - 4m_K^2 b_D - 4(m_K^2 - m_\pi^2) b_F \end{aligned}$$

4 octet baryon masses are represented in terms of effectively 3 parameters

⇒ Sum Rule (Gell-Mann and Okubo)

$$M_\Sigma - M_N = \frac{1}{2}(M_\Xi - M_N) + \frac{3}{4}(M_\Sigma - M_\Lambda)$$

Experimentally: 254 MeV = 248 MeV

Satisfied at the 3% level.

Let's go to higher chiral order:

- Chiral expansion of masses

$$\boxed{M_B = M_0 + \sum_q b_q m_q + \sum_q c_q m_q^{3/2} + \sum_q d_q m_q^2 + \ldots}$$

(LNAC)

- Complete one-loop calculation in heavy baryon approach to $4^{th}$ chiral order ($m_u = m_d$) (BB, Meißner '97)

$$\begin{aligned} M_N &= M_0(1 + 0.34 - 0.35 + 0.24) \\ M_\Lambda &= M_0(1 + 0.69 - 0.77 + 0.54) \\ M_\Sigma &= M_0(1 + 0.81 - 0.70 + 0.44) \\ M_\Xi &= M_0(1 + 1.10 - 1.16 + 0.78) \end{aligned}$$

$p^2$ $p^3$ $p^4$

Large nonanalytic terms at $\mathcal{O}(p^3)$ arise from the integral

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{[k_0 + i\epsilon]\,[k^2 - m^2 + i\epsilon]} = i\delta_{ij}\frac{I(m)}{24\pi}$$

• In dimensional regularization: $I_{dim.reg.}(m) = m^3$

$\Rightarrow$ **large loop contributions**

<u>But:</u> baryons are treated as point like particles, although they have a finite size $\sim 1$ fm

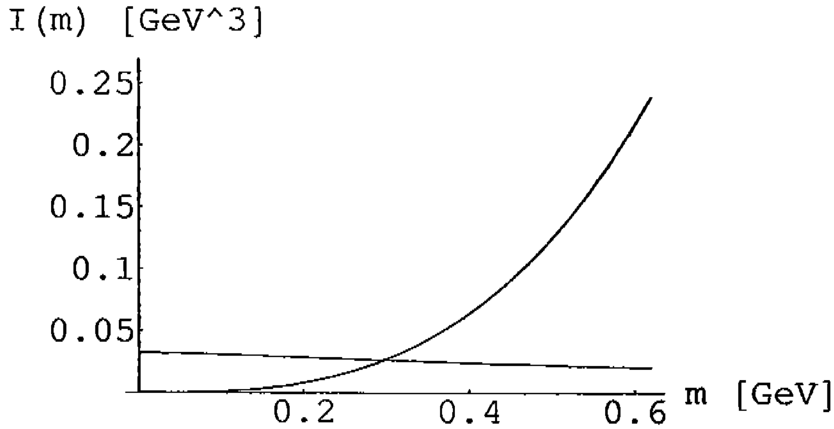
• Suppress short distance portion of loop integral which is not described appropriately by chiral physics

Utilize, *e.g.*, simple dipole regulator (Donoghue, Holstein, BB '98)

$$\left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2$$

Regulator suppresses short-distance physics. One obtains

$$I_\Lambda(m) = \Lambda^4 \frac{2m + \Lambda}{2(m + \Lambda)^2}$$




• Power divergences $\propto \Lambda^3$ and $\Lambda$ can be absorbed into $M_0$ and $b_0, b_D, b_F$
$\Rightarrow$ chiral symmetry is maintained

Nonanalytic contributions (in GeV) to baryon masses

| | dim. reg. | $\Lambda = 300$ MeV | $\Lambda = 400$ MeV |
|---|---|---|---|
| $N$ | $-0.31$ | 0.02 | 0.03 |
| $\Lambda$ | $-0.66$ | 0.03 | 0.06 |
| $\Sigma$ | $-0.62$ | 0.03 | 0.05 |
| $\Xi$ | $-1.03$ | 0.04 | 0.08 |

Phenomenologically relevant cutoffs

$$\frac{1}{\langle r_B \rangle} \leq \Lambda \sim 300 - 600 \text{ MeV}$$

# $\sigma$ terms

$$\begin{aligned}
\sigma_{\pi N}(t) &= \tfrac{1}{2}(m_u + m_d)\langle p'|\bar{u}u + \bar{d}d|p\rangle \\
\sigma^{(1)}_{KN}(t) &= \tfrac{1}{2}(\hat{m} + m_s)\langle p'|\bar{u}u + \bar{s}s|p\rangle \\
\sigma^{(2)}_{KN}(t) &= \tfrac{1}{2}(\hat{m} + m_s)\langle p'| - \bar{u}u + 2\bar{d}d + \bar{s}s|p\rangle
\end{aligned}$$

with

$$t \equiv (p' - p)^2, \qquad \hat{m} \equiv \frac{1}{2}(m_u + m_d)$$

Empirical value deduced from extrapolation of low-energy pion nucleon scattering data (Gasser, Leutwyler, Sainio '91 )

$$\sigma_{\pi N}(0) = (45 \pm 8)\text{MeV}$$

- Strangeness contribution to nucleon mass:

$$\begin{aligned}
m_s\langle p|\bar{s}s|p\rangle = &\left(\tfrac{1}{2} - \tfrac{m_\pi^2}{4m_K^2}\right)\left(3\sigma^{(1)}_{KN}(0) + \sigma^{(2)}_{KN}(0)\right) \\
&+ \left(\tfrac{1}{2} - \tfrac{m_K^2}{m_\pi^2}\right)\sigma_{\pi N}(0)
\end{aligned}$$

- Strangeness fraction

$$y = \frac{2\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

To leading order in quark masses and using $SU(3)$ baryon wave functions $(m_s/\hat{m} \sim 25)$

$$\sigma_{\pi N}(0) = \frac{\hat{m}}{m_s - \hat{m}} \frac{M_\Xi + M_\Sigma - 2M_N}{1-y}$$
$$= \frac{26\,\text{MeV}}{1-y} \qquad \Rightarrow \qquad \boxed{y = 0.42}$$

Two orders higher in the chiral expansion (BB, Meißner '97)

$$\sigma_{\pi N}(0) = \frac{(36 \pm 7)\text{MeV}}{1-y} \qquad \Rightarrow \qquad \boxed{y = 0.2 \pm 0.2}$$

Compatible with zero but tendency for non-zero admixture of strange quarks in the nucleon.

**But:** new $\pi N$ scattering data from TRIUMF and PSI became available

More recent extractions of $\sigma_{\pi N}(0)$ range from

$$\sigma_{\pi N}(0) = 45 \ldots 80 \text{ MeV}$$

corresponding to a strangeness fraction

$$y = 0.2 \ldots 0.5 \quad !!$$

- Results for $KN$ $\sigma$ terms (in cutoff scheme) (BB '99)

$$\boxed{\begin{aligned} \sigma^{(1)}_{KN}(0) &= 380 \pm 50 \text{ MeV} \\ \sigma^{(2)}_{KN}(0) &= 250 \pm 40 \text{ MeV} \\ y &= 0.25 \pm 0.05 \\ m_s\langle p|\bar{s}s|p\rangle &= 150 \pm 50 \text{ MeV} \end{aligned}}$$

# Axial vector couplings

Hadronic axial current for the semileptonic decay $B_i \to B_j l \bar{\nu}_l$ can be written in the form

$$\langle B_j | A_\mu | B_i \rangle = \bar{u}(p_j) \Big( g_1(q^2)\gamma_\mu\gamma_5 - \frac{i g_2(q^2)}{M_i + M_j}\sigma_{\mu\nu}q^\nu\gamma_5 + \frac{g_3(q^2)}{M_i + M_j} q_\mu \gamma_5 \Big) u(p_i)$$

- Axial vector couplings: $g_A \equiv g_1(0)$

Axial vector couplings $D, F$ in $\mathcal{L}^{(1)}_{\phi B}$ provide good fit to experimentally measured $g_A$: $D = 0.80,\ F = 0.46$

$SU(3)$ breaking effects in data $< 10\%$.

| | | |
|---|---|---|
| $g_A^{pn}$ | $= D + F = 1.26$ | (1.267) |
| $g_A^{p\Lambda}$ | $= -\frac{1}{\sqrt{6}}(D + 3F) = -0.89$ | (−0.89) |
| $g_A^{\Lambda\Sigma^-}$ | $= \frac{2}{\sqrt{6}}D = 0.65$ | (0.60) |
| $g_A^{\Xi^0\Xi^-}$ | $= D - F = 0.34$ | |
| $g_A^{\Lambda\Xi^-}$ | $= -\frac{1}{\sqrt{6}}(D - 3F) = 0.24$ | (0.30) |
| $g_A^{n\Sigma^-}$ | $= D - F = 0.34$ | (0.34) |
| $g_A^{\Sigma^0\Xi^-}$ | $= \frac{1}{\sqrt{2}}(D + F) = 0.89$ | (0.93) |

• LNACs from chiral loops lead to significant $SU(3)$ breaking ....in disagreement with experiment (Bijnens, Sonoda, Wise '85)

• Employ cutoff regularization (absorbing power divergences $\propto \Lambda$ into phenomenological parameters $D, F$)
$\Rightarrow$ chiral corrections are under control

(Donoghue, Holstein, BB '97)

- Nonanalytic corrections to $g_A$

| | dim. reg. | $\Lambda = 300$ MeV | $\Lambda = 400$ MeV |
|---|---|---|---|
| $g_A^{pn}$ | $0.92$ | $0.20$ | $0.28$ |
| $g_A^{p\Lambda}$ | $-0.95$ | $-0.18$ | $-0.27$ |
| $g_A^{\Lambda\Sigma^-}$ | $0.62$ | $0.12$ | $0.18$ |
| $g_A^{n\Sigma^-}$ | $0.19$ | $0.04$ | $0.05$ |
| $g_A^{\Lambda\Xi^-}$ | $0.44$ | $0.08$ | $0.12$ |
| $g_A^{\Sigma^0\Xi^-}$ | $1.44$ | $0.21$ | $0.31$ |

# Nonleptonic weak hyperon decays

Dominant hadronic decay mode of hyperons: $\underline{B \to B'\pi}$

There are seven such decays

$$\Lambda \to \pi^0 n, \quad \Lambda \to \pi^- p$$
$$\Sigma^+ \to \pi^+ n, \quad \Sigma^+ \to \pi^0 p, \quad \Sigma^- \to \pi^- n$$
$$\Xi^0 \to \pi^0 \Lambda, \quad \Xi^- \to \pi^- \Lambda$$

- Matrix element:

$$\boxed{\mathcal{A}(B \to B'\pi) = \bar{u}_{B'}(p')(A + B\gamma_5)u_B(p)}$$

$A$ : parity-violating $s$ wave
$B$ : parity-conserving $p$ wave

## $\Delta I = 1/2$ rule:

$\Delta I = 3/2$ amplitudes are suppressed with respect to $\Delta I = 1/2$ counterparts by a factor of twenty or so (also in kaon nonleptonic decays)
Still no simple explanation for its validity available

- Isopsin symmetry of strong interactions implies three relations for $s$ and $p$ waves

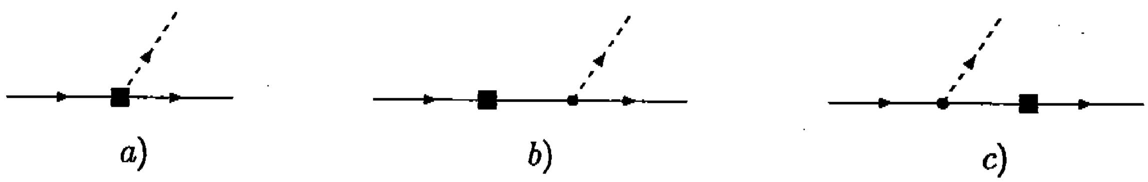
$\Rightarrow$ 8 independent decay amplitudes

- **$s$ and $p$ wave problem:**

Chiral effective field theory at tree level

Lowest order *weak* Lagrangian:

$$\mathcal{L}_W^{(0)} = d_W\langle\bar{B}\{u^\dagger\lambda_6 u, B\}\rangle + f_W\langle\bar{B}[u^\dagger\lambda_6 u, B]\rangle$$




$s$ wave amplitudes $\qquad$ $p$ wave amplitudes

**No simultaneous fit to both $s$ and $p$ waves possible**

- good $s$ wave fit possible, but poor $p$ wave description

→ Include chiral corrections

- (Bijnens, Sonoda, Wise '85): leading nonanalytic corrections to both $s$ and $p$ waves do *not* agree with data

- (Jenkins '92): inclusion of decuplet fields within heavy baryon formulation

| | |
|---|---|
| $s$ waves: | good agreement |
| $p$ waves: | no satisfactory description |

• (BB, Holstein '99): *complete* one-loop calculation

Introduces more coupling constants than there exist data

Reduce number of counterterms by resonance saturation principle

⇒ Exact fit possible, but not unique

• (Le Yaouanc et al. '79): possible solution to $s$ and $p$ wave problem

Appending pole contributions from $SU(6)$ $(70, 1^-)$ states to $s$ waves within a constituent quark model appears to provide a possible resolution of the $s$ and $p$ wave dilemma

• Consider validity of this approach within the framework of ChPT (BB, Holstein '99)

Tree level results and contributions from lowest

$1/2^-$ $(\Lambda(1405), N(1535), ...)$ and

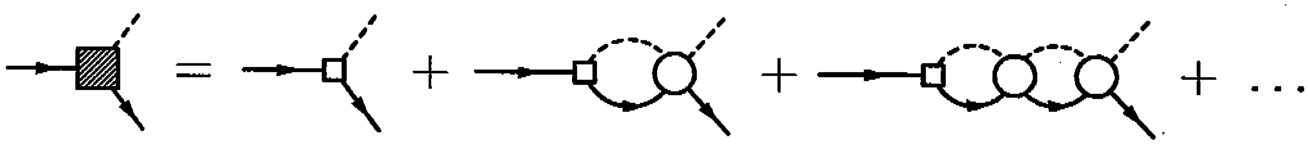
$1/2^+$ $(N(1440), ...)$ resonances

*a)* *b)*

⇒ satisfactory picture of nonleptonic hyperon decays can be provided

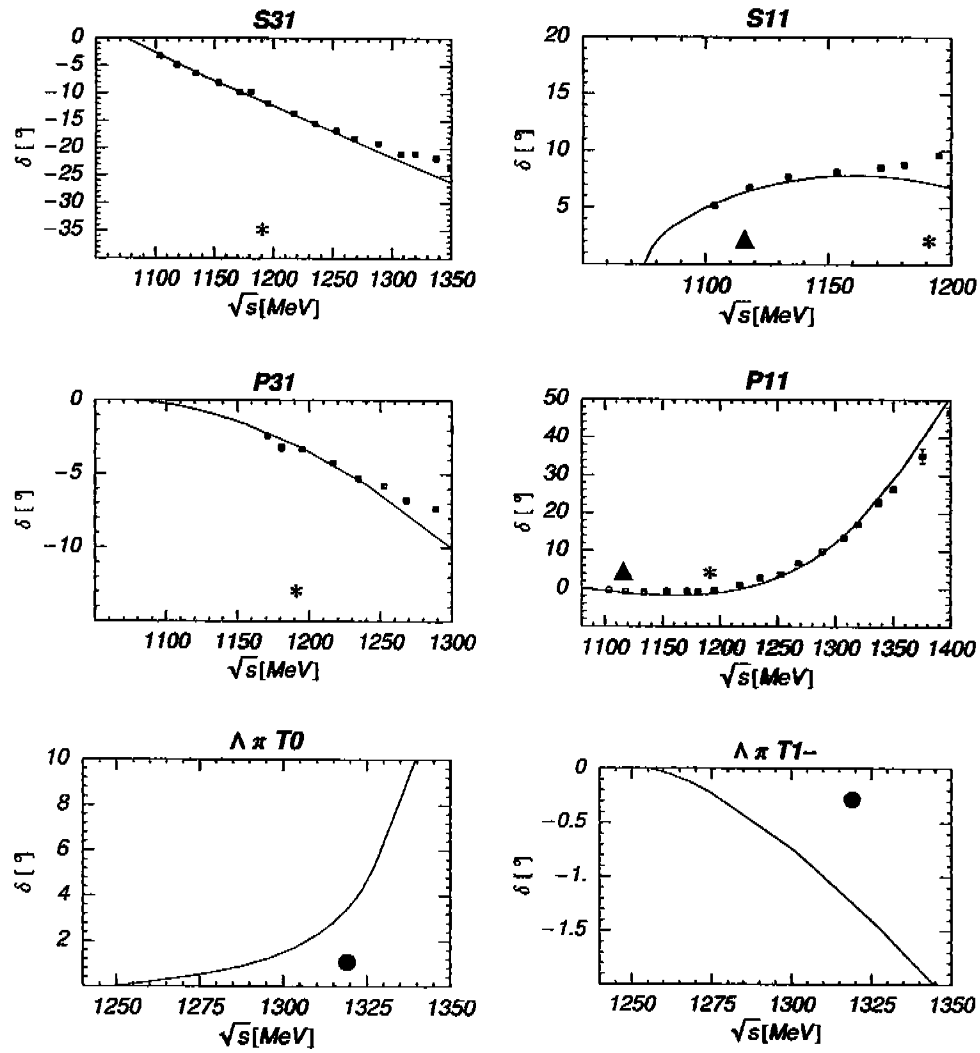
- Importance of final state interactions has been investigated within a chiral unitary approach based on coupled channels

(BB, Marco '03)



- Resonances are not included explicitly, they are generated dynamically

• Phase shifts of $\pi N$ scattering are reproduced at the relevant energies




⇒ accurate inclusion of final state interactions

## Observations:

FSI have sizeable effects $> 10\%$
and should not be omitted

Inclusion of FSI improves overall lowest order fit to data
But: does not enable a reasonable fit

$\Rightarrow$ importance of higher order weak counterterms
which include effects from higher energies

# Nonleptonic radiative hyperon decays

The weak radiative hyperon decays

$$\Sigma^+ \to p\gamma, \quad \Sigma^0 \to n\gamma, \quad \Lambda \to n\gamma$$
$$\Xi^0 \to \Sigma^0\gamma, \quad \Xi^0 \to \Lambda\gamma, \quad \Xi^- \to \Sigma^-\gamma$$

are described by

$$\mathcal{A}(B \to B'\gamma) = -\frac{i}{2(M_B + M_{B'})} \times \bar{u}_{B'}(p')\sigma_{\mu\nu}F^{\mu\nu}(A_{B'B} + B_{B'B}\gamma_5)u_B(p)$$

$A$ : parity-conserving $M1$ amplitude
$B$ : parity-violating $E1$ amplitude

**Hara's theorem:**

In $SU(3)$ limit $B_{B'B}$ must vanish for decays between states of common $U$-spin multiplet ($s \leftrightarrow d$): $\Sigma^+ \to p\gamma, \quad \Xi^- \to \Sigma^-\gamma$

• *Real World:* $\sim$ 20% $SU(3)$ breaking effects are expected

$\Rightarrow$ **small** photon asymmetry $\quad \alpha \equiv -\frac{2\mathrm{Re}A^* B}{|A|^2 + |B|^2}$

**But:** $\alpha_{\Sigma^+ p} = -0.76 \pm 0.08$

$(\alpha_{\Xi^- \Sigma^-} = 1.0 \pm 1.3)$

- At lowest order in ChPT: *Pole diagrams*

- Parity-violating amplitude $B$ vanishes

$\Rightarrow$ asymmetry parameter $\alpha = 0$ for all decays

- one-loop calculation (Neufeld '93)

$|\alpha_{\Sigma^+ p}| \leq 0.21$ !!

- *Quark model* (Gavela et al. '81)

Inclusion of $(70, 1^-)$ states provided a solution for the problem with Hara's theorem

- *Chiral framework* (BB, Holstein '99)

Lowest $1/2^-$ and $1/2^+$ resonant contributions yield relatively large asymmetries (no additional unknown parameters)

# Summary

- $SU(3)$ ChPT is an appropriate tool to investigate properties and decays of hyperons (also: hyperon polarizabilities, hypernuclear decay, kaon photoproduction)

- convergence problems of chiral series due to strange quark mass can be overcome by utilizing a cutoff regularization

- extraction of $\sigma_{\pi N}(0)$ from $\pi N$ scattering data will help to provide a more precise value of the strangeness content of the nucleon

- despite a few promising investigations, the problems related to the $s$ and $p$ wave description of nonleptonic hyperon decays and Hara's theorem for radiative hyperon decays still lacks a definite solution

  $\Rightarrow$ importance of resonances and physics at energies beyond the range of ChPT

- *not discussed:* importance of decuplet states $(\Delta(1232), \Sigma(1385), \Xi(1530), \Omega(1672))$

  $\Rightarrow$ combined $1/N_c$ and chiral expansions

  (Jenkins, Manohar, Lebed)