Strange nuclear structures with high density formed by single/double K^- meson

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1. Introduction
2. New framework of Antisymmetrized Molecular Dynamics
    - K^-p-$\bar{K}^0n$ mixing
    - J & T projections
3. Results

    $ppnK^-$, $pppK^-$, $pppnK^-$, $^6BeK^-$, $^9BK^-$ and $^{11}CK^-$
4. Results = extremely proton-rich nuclei =

    $ppppK^-$, $pppK^-K^-$ and $ppppK^-K^-$
5. Summary & Future plan

HYP2003,
'03.10.18 at Jlab, America

Introduction

Studies of deeply bound kaonic nuclei by Akaishi & Yamazaki

- phenomenological $\bar{K}N$ potential
  1. free KN scattering data
  2. X-ray data of kaonic hydrogen atom
  3. binding energy and width of $\Lambda(1405)$

Strongly attractive.

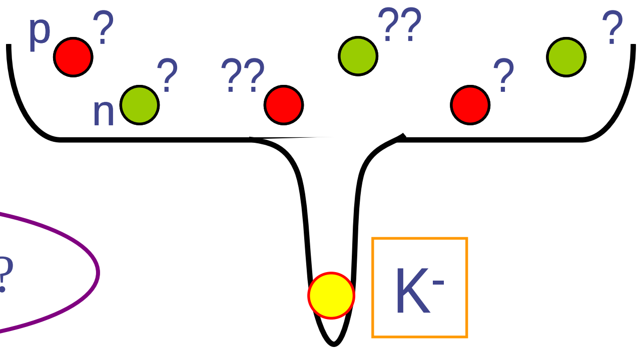
$$V^{\bar{K}N}_{I=0}$$

- Analysis of ^{8}BeK$^-$ with $\alpha\alpha$K$^-$ model ⇨

Binding energy of K^- = 113 MeV
Central density = $5\rho_0$

Kaonic nucleus

K$^-$ deeply bound?



Peculiar structure?

Highly dense?

What kind of structure does A-nucleons + K$^-$ system favor?

Formalism

Kaonic nuclei ...

- unknown structure
- peculiar structure   ex) isovector deformation

⇨ ☆ Method : Antisymmetrized Molecular Dynamics

no assumption on clusters, shape, etc.

$V^{\bar{K}N}_{I=0}$ plays an essential role in kaonic nuclei.

Improving

- K^-p-$\bar{K}^0n$ mixing

$$K^-p \rightleftarrows \boxed{I=0\ \bar{K}N\ \text{int.}} \rightleftarrows \bar{K}^0 n$$

- Angular momentum & isospin projections

Wave function

Essence of $K^-p/\overline{K}^0n$ mixing

$$|N\rangle = a|\text{proton}\rangle + b|\text{neutron}\rangle$$

$$|K\rangle = x|K^-\rangle + y|\overline{K^0}\rangle$$

Nucleon's wave function

$$|\varphi_\iota\rangle = \sum_{\alpha=1}^{Nn} C_\alpha^i \exp\left[-\upsilon\left(\mathbf{r} - \frac{\mathbf{Z}_\alpha^i}{\sqrt{\upsilon}}\right)^2\right]|\sigma_\alpha^i \tau_\alpha^i\rangle$$

$$|\sigma_\alpha^i\rangle = |\uparrow\rangle \text{ or } |\downarrow\rangle$$

$$|\tau_\alpha^i\rangle = \left(\frac{1}{2} + \gamma_\alpha^\iota\right)|p\rangle + \left(\frac{1}{2} - \gamma_\alpha^\iota\right)|n\rangle$$

p-n mixing

Anti-kaon's wave function

$$|\varphi_K\rangle = \sum_{\alpha=1}^{Nk} C_\alpha^K \exp\left[-\upsilon\left(\mathbf{r} - \frac{\mathbf{Z}_\alpha^K}{\sqrt{\upsilon}}\right)^2\right]|\tau_\alpha^K\rangle$$

$$|\tau_\alpha^K\rangle = \left(\frac{1}{2} + \gamma_\alpha^K\right)|\overline{K^o}\rangle + \left(\frac{1}{2} - \gamma_\alpha^K\right)|K^-\rangle$$

$\overline{K}^0$-K^- mixing

Total wave function

$$|\Phi\rangle = \det[|\varphi_\iota\rangle] \otimes |\varphi_K\rangle$$

$$|\Phi^\pm\rangle = |\Phi\rangle \pm |P\Phi\rangle$$

Charge projection

$$|P_M\Phi^\pm\rangle = \int d\alpha \exp\left[-i\alpha\left(\hat{T}_Z - M\right)\right]|\Phi^\pm\rangle$$

Use $|P_M\Phi^\pm\rangle$ as a trial function

J & T projections (VBP)

After obtaining $\Phi^{\pm}$ by Tz-projected AMD cooling, it is projected onto the eigen state of the angular momentum J and the isospin T.

$$\left| P^{J}_{MK} P^{T}_{TzTz'} \Phi^{\pm} \right\rangle = \underline{\int d\Omega_{Ang} \; D^{J*}_{MK}(\Omega_{Ang}) \, \hat{R}_{Ang}(\Omega_{Ang})}$$

$$\times \underline{\int d\Omega_{iso} \; D^{T*}_{TzTz'}(\Omega_{iso}) \, \hat{R}_{iso}(\Omega_{iso})} \left| \Phi^{\pm} \right\rangle$$

J projection

T projection

$$\hat{R}_{Ang}(\Omega) = \exp\left[-i\alpha \hat{J}_{z}\right] \exp\left[-i\beta \hat{J}_{y}\right] \exp\left[-i\gamma \hat{J}_{z}\right]$$

$$\hat{R}_{iso}(\Omega) = \exp\left[-i\alpha \hat{T}_{z}\right] \exp\left[-i\beta \hat{T}_{y}\right] \exp\left[-i\gamma \hat{T}_{z}\right]$$

We calculate various expectation values with $\left| P^{J}_{MK} P^{T}_{TzTz'} \Phi^{\pm} \right\rangle$.

Formalism

1. Hamiltonian $\hat{H} = \hat{T} + \hat{V}_{NN} + \hat{V}_{KN} + \hat{V}_{Coulomb} - \hat{T}_G$

2. Variational parameters $\left\{X_\alpha^i\right\} = \left\{C_\alpha^i,\ \mathbf{Z}_\alpha^i,\ \gamma_\alpha^i,\ C_\alpha^{\mathrm{K}},\ \mathbf{Z}_\alpha^{\mathrm{K}},\ \gamma_\alpha^{\mathrm{K}}\right\}$
   are determined by Frictional cooling eq. with constraint.

3. G-matrix method ⇒ Effective interaction $\hat{V}_{NN}$, $\hat{V}_{KN}$
   bare NN int = Tamagaki potential (OPEG)
   bare KN int = AY potential

given density and starting energy of $\bar{K}$
→ G-matrix

AMD calculation
→ density and starting energy of $\bar{K}$

Repeat until getting consistency

Results ─ ppnK$^-$ ─

Model space

2 Gaussian / nucleon
5 Gaussian / kaon

Assumption of G-matrix

E(K)=112.8MeV, central density=1.50fm^{-3}

Results

Projecting onto J=1/2 and T=0. Parity is positive.

	E(K)	Width	ρ (0)	Rrms
JT projection	110.3	21.2	1.50	0.72
simple AMD	105.2	23.7	1.39	0.72
BHF	108	20		0.97

G-matrix consistency : OK !

Quantum numbers

J(J+1)
$$= \frac{1}{2}\cdot\left(\frac{1}{2}+1\right)$$

	After	Before
J2 (total sys.)	0.75	1.36
J2 (N sys.)	[illegible]78	1.22
L2 (N	[illegible]	[illegible]
S2 (N	[illegible]	[illegible]
L2 (kaon)	[illegible]03	0.14
T2	0.00	0.02
Tz	0.00	0.00

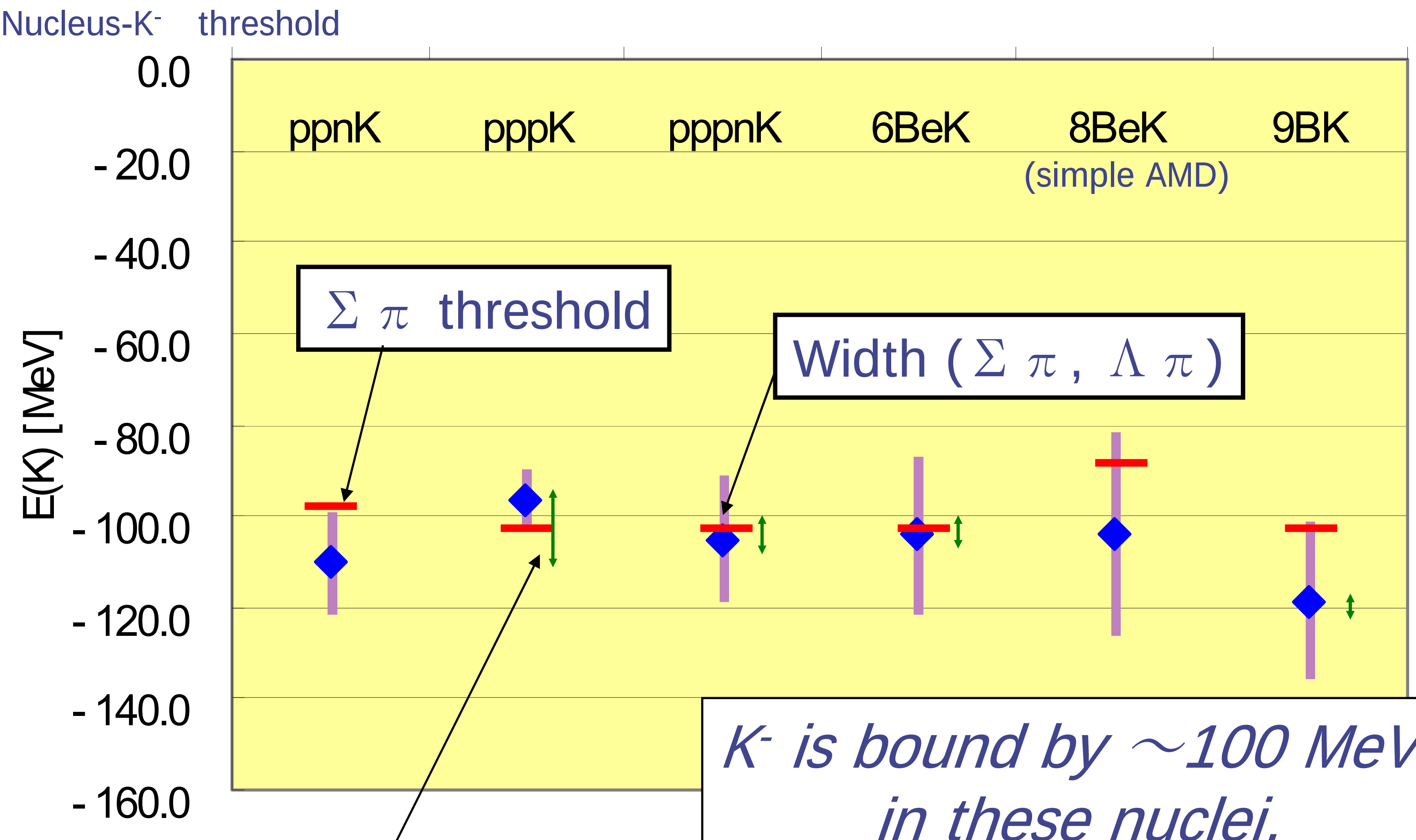
JT projection : OK !

Particle numbers

Proton	1.51
Neutron	1.49
K-	0.51
K0bar	0.49

$$\mathrm{ppnK}^- : \mathrm{pnn}\overline{\mathrm{K}^0} = 1:1$$

Binding energy of Kbar and width



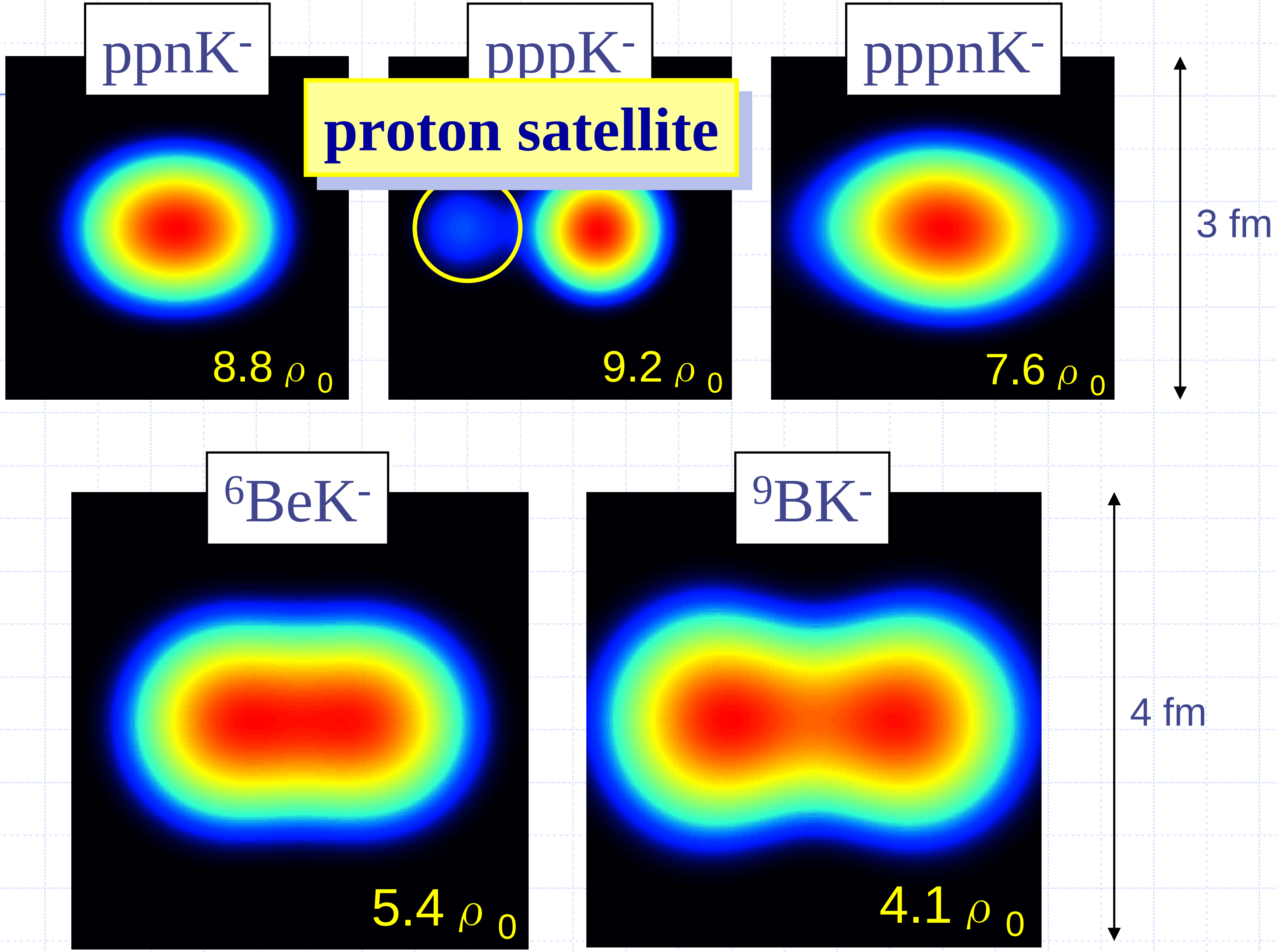
$\Sigma\ \pi$ threshold

Width ($\Sigma\ \pi$, $\Lambda\ \pi$)

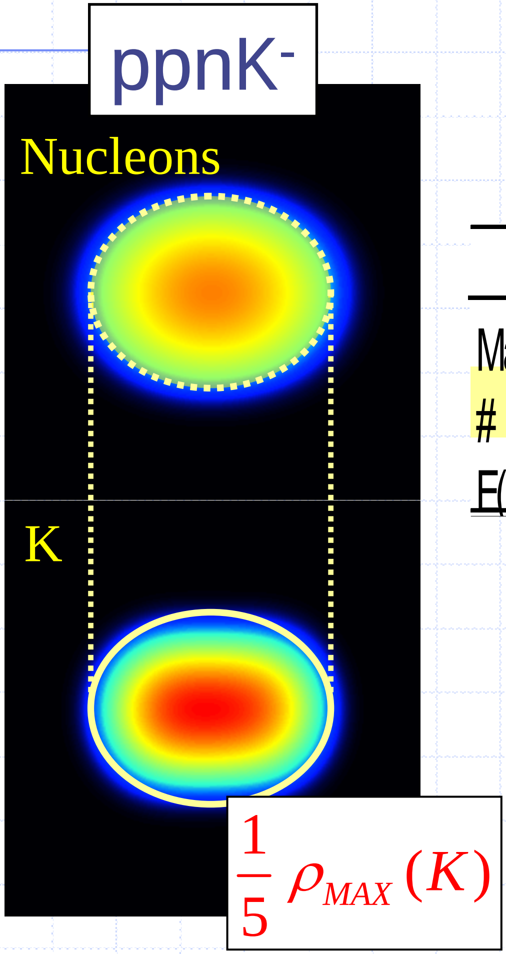
K$^-$ is bound by ～100 MeV in these nuclei.

Uncertainty due to the inconsistency between the obtained results and the G-matrix used in the calculation

Nucleon density distribution



Number of nucleons near K^- meson



$$\frac{1}{5}\rho_{MAX}(K)$$

one center like

two center like

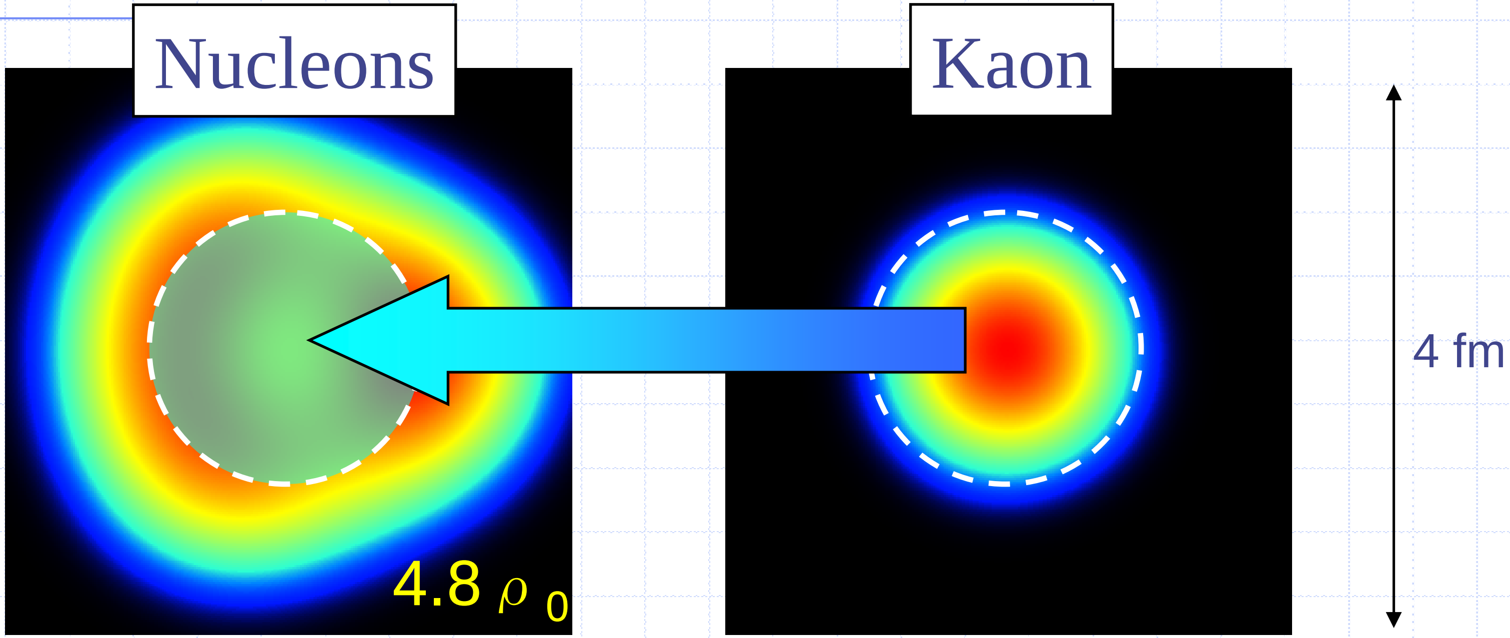
	ppnK	pppK	pppnK	6BeK	9BK
Max Kdens	0.67	1.48	0.62	0.34	0.35
# of nucleons	1.67	1.14	1.78	2.55	2.53
E(K)	110.3	96.7	105.0	104.2	117.0

strange structure

Single K^- meson can interact with limited numbers of nucleons?

→ *Saturation of E(K)*

Larger nucleus case = $^{11}\mathrm{CK}^-$ =



Results

E(K) = 117 MeV

Γ = 37 MeV

R_{rms} = 1.49 fm

LS force : off

A K^- meson stays at the center of the system.

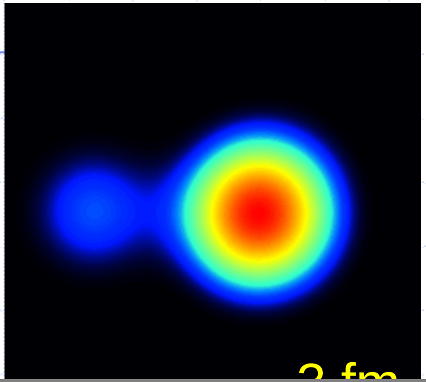
⇩

It attracts three clusters.

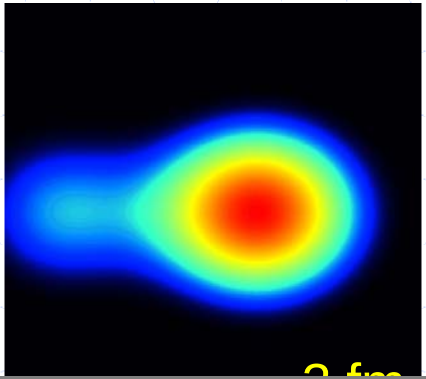
Results of extremely proton-rich kaonic nuclei

$pppK^-$

total B.E.	96.7 MeV
E(K)	96.7 MeV
Γ	12.5 MeV
Rrms	0.81 fm



Proton added



$ppppK^-$

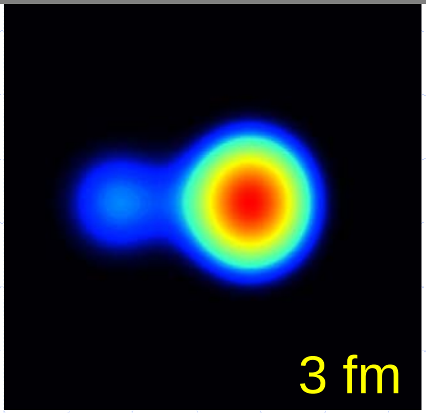
total B.E.	75.1 MeV
E(K)	75.1 MeV
Γ	161.8 MeV
Rrms	0.92 fm

These kaonic nuclei have proton-satellite structure.

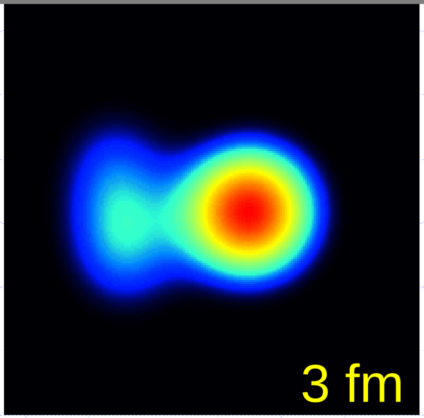
But not so deeply bound.

$pppK^-K^-$

total B.E.	205.7 MeV
E(K)	102.8 MeV
Γ	51.9 MeV
Rrms	0.65 fm
ρ_{max}	21.6 ρ_0



Proton added



$ppppK^-K^-$

total B.E.	217.3 MeV
E(K)	108.7 MeV
Γ	41.2 MeV
Rrms	0.73 fm
ρ_{max}	14.1 ρ_0

tentative

Summary & Future plan

- We have improved AMD so that we can treat $K^-p/\bar{K}^0n$ mixing and perform J & T projections.

- We have calculated various kaonic nuclei – ppnK$^-$, pppK$^-$, pppnK$^-$, ^{6}BeK$^-$, ^{9}BK$^-$ and ^{11}CK$^-$.
  Our results are ↓

	E(K) [MeV]	width [MeV]	ρ (0) [fm^-3]	Rrms [fm]	β	γ [deg.]
ppnK-	110.3	21.2	1.50	0.72	0.22	9.2
pppK-	96.7	12.5	1.56	0.81	0.70	11.8
pppnK-	105.0	25.9	1.29	0.97	0.54	3.8
6BeK-	104.2	33.3	0.91	1.17	0.44	0.3
9BK-	118.5	33.0	0.71	1.45	0.46	20.8
11CK-	117.0	37.0	0.82	1.49	0.36	46.4

K$^-$ is very deeply bound
and forms very highly dense state.

- Saturation of E(K) is related to the number of nucleons with which a K$^-$ can interact?

- Even if the KN interaction is very attractive, extremely proton-rich nuclei are not always deeply bound.

- Excited states ?
- KK interaction ?

Introduction

Remarks

$$v^{I}_{\bar{K}N}(r) = v^{I}_{D}\exp\left[-\left(r/0.66\,\mathrm{fm}\right)^{2}\right],$$

$$v^{I}_{\bar{K}N,\pi\Sigma}(r) = v^{I}_{C_1}\exp\left[-\left(r/0.66\,\mathrm{fm}\right)^{2}\right],$$

$$v^{I}_{\bar{K}N,\pi\Lambda}(r) = v^{I}_{C_2}\exp\left[-\left(r/0.66\,\mathrm{fm}\right)^{2}\right]$$

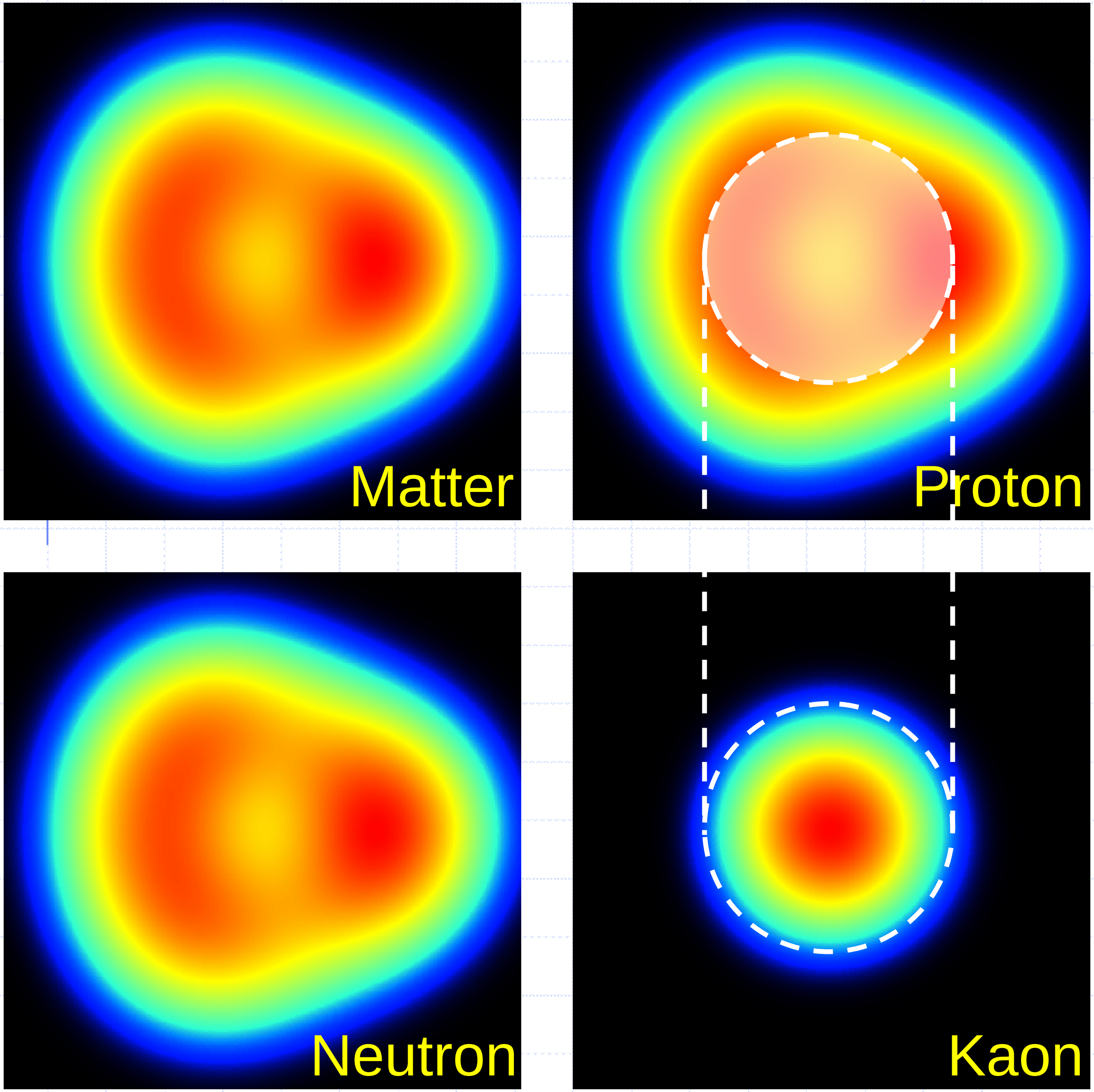
$$v^{I=0}_{D} = -436\ \mathrm{MeV},\quad v^{I=0}_{C_1} = -412\ \mathrm{MeV},\quad v^{I=0}_{C_2} = \mathrm{none},$$

$$v^{I=1}_{D} = -62\ \mathrm{MeV},\quad v^{I=1}_{C_1} = -285\ \mathrm{MeV},\quad v^{I=1}_{C_2} = -285\ \mathrm{MeV}.$$

Y. Akaishi and T. Yamazaki, PRC 65 (2002) 044005

- Not single channel, but coupled channel.
- Same property as KN interaction derived from Chiral theory.

$^{11}\mathrm{CK}^{-}$



$E(K) = 117$ MeV
$\Gamma = 37$ MeV
$\rho_{\mathrm{MAX}} = 0.82\ \mathrm{fm}^{-3}$
$R_{\mathrm{rms}} = 1.49$ fm

LS force : off

size : 4fm x 4fm

Introduction

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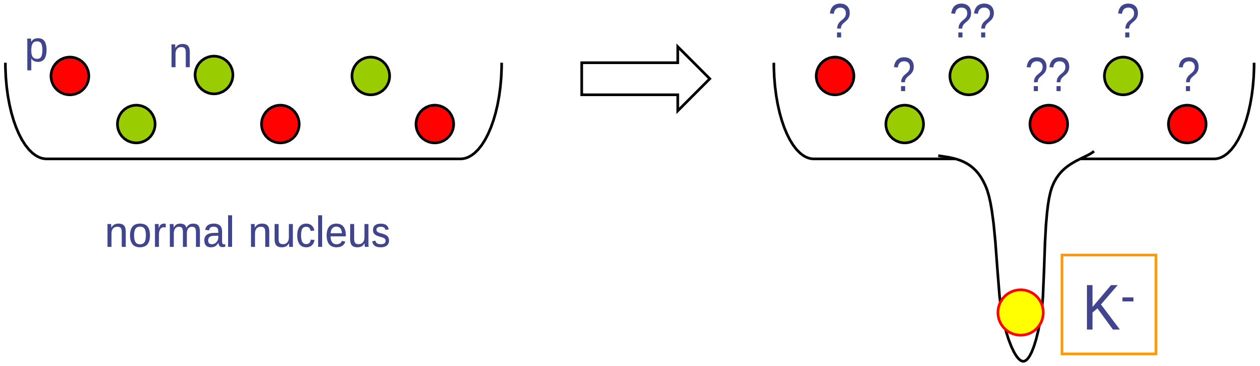
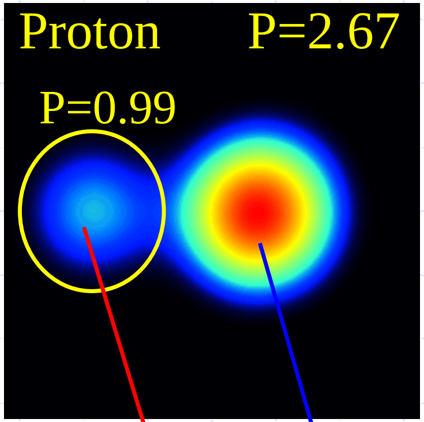


What kind of structure does A-nucleons + K$^-$ system favor?

K$^-$ deeply bound? highly dense? peculiar structure?

Proton satellite in pppK$^-$

pppK$^-$



$$|{}^{3}_{K}\mathrm{He}\rangle = |p\rangle \otimes |{}^{2}_{K}\mathrm{H}\rangle$$

Isospin

$$|1,1\rangle = |1/2,1/2\rangle \otimes |1/2,1/2\rangle$$

T,Tz

$$\text{proton} : 2.66 = 1 + 1.66 = 1 + \frac{2}{3} \times 2 + \frac{1}{3} \times 1$$

$$\text{neutron} : 0.33 = \frac{1}{3} \times 1$$

$$\Uparrow$$

$$\boxed{\sqrt{\frac{2}{3}}\,|pp \otimes K^-\rangle + \sqrt{\frac{1}{3}}\,|pn \otimes \overline{K}^0\rangle}$$

$$\Updownarrow$$

$$\Leftrightarrow \quad \sqrt{\frac{2}{3}}\,|1,1\rangle \otimes |1/2,-1/2\rangle + \sqrt{\frac{1}{3}}\,|1,0\rangle \otimes |1/2,+1/2\rangle$$

Clebsch-Gordan coefficient

Introduction

According to our study of various kaonic nuclei …

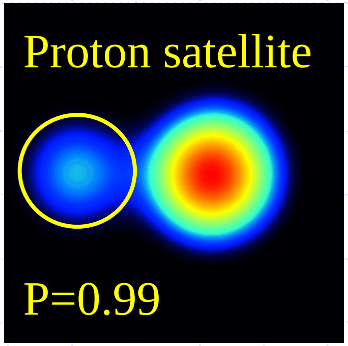
- K^- is deeply bound in light nuclei. E(K) ≒ 100 MeV.

- Since the main decay channel, $\Sigma\pi$, is closed, the deeply bound state is a discrete state.

- Due to the strong attraction from a K-, nuclei are drastically shrunk to form dense state. $\rho = 4 \sim 8\ \rho_0$

- Structures peculiar to kaonic nuclei are found.

  1. Isovector deformation in $^8BeK^-$
  2. Proton satellite in ppK^-



Question

The strong attraction from K^- can make extremely proton-rich nuclei bound?

How are their structures?

Kaonic nucleiに期待できること

１、ハドロン物理の観点

K^-が核子を引き寄せ、非常に高密度な状態が核内に。

高密度だが、割と冷たい。

- 高密度核物質、カイラル対称性の回復
- 高密度下でのＮＮ／ＫＮ相互作用
- Ｋ凝縮、strange matter

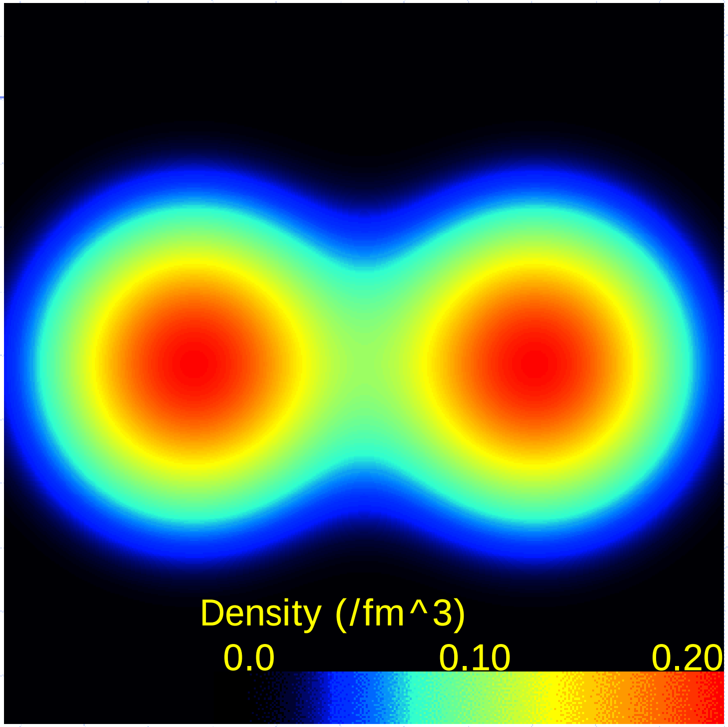
２、核構造の観点

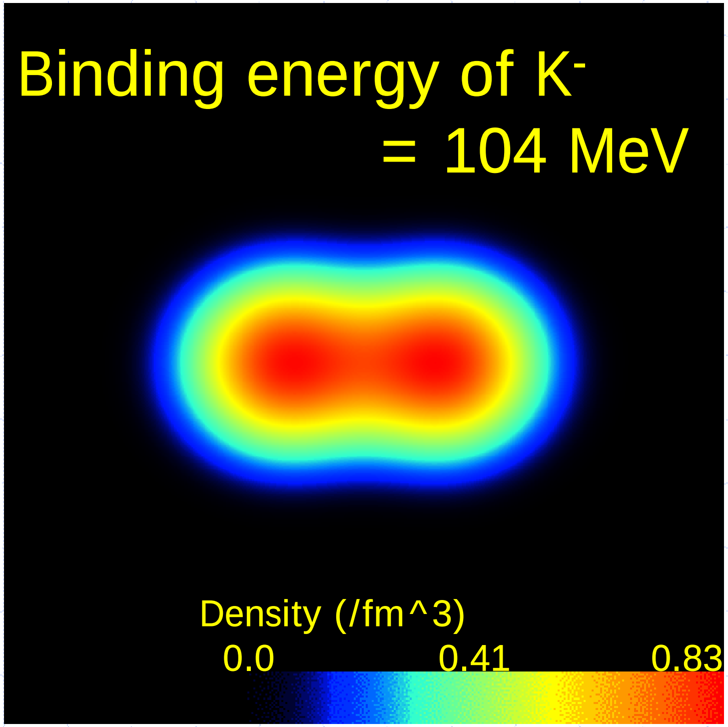
K^-からのアイソスピンに依存した強い引力

- 従来のクラスター構造が変化
  ・・・原子核がshrink
- isovector deformation
  ・・・K^-は陽子をより強くひきつける
- extreme proton-rich nuclei
  ・・・ｐｐＫ、ｐｐｐＫ、ｐｐｐｐＫ、ｐｐｐＫＫ、ｐｐｐｐＫＫなど

今までの原子核にない新しい構造様式？
多彩な構造？

Results // ^{8}Be and ^{8}BeK$^-$ //




Binding energy of K$^-$ = 104 MeV

7 fm

^{8}Be

rmsR = 2.46 fm

β = 0.63

Central density = 0.10 /fm^3

force: MV1 case 3

^{8}BeK$^-$

rmsR = 1.42 fm

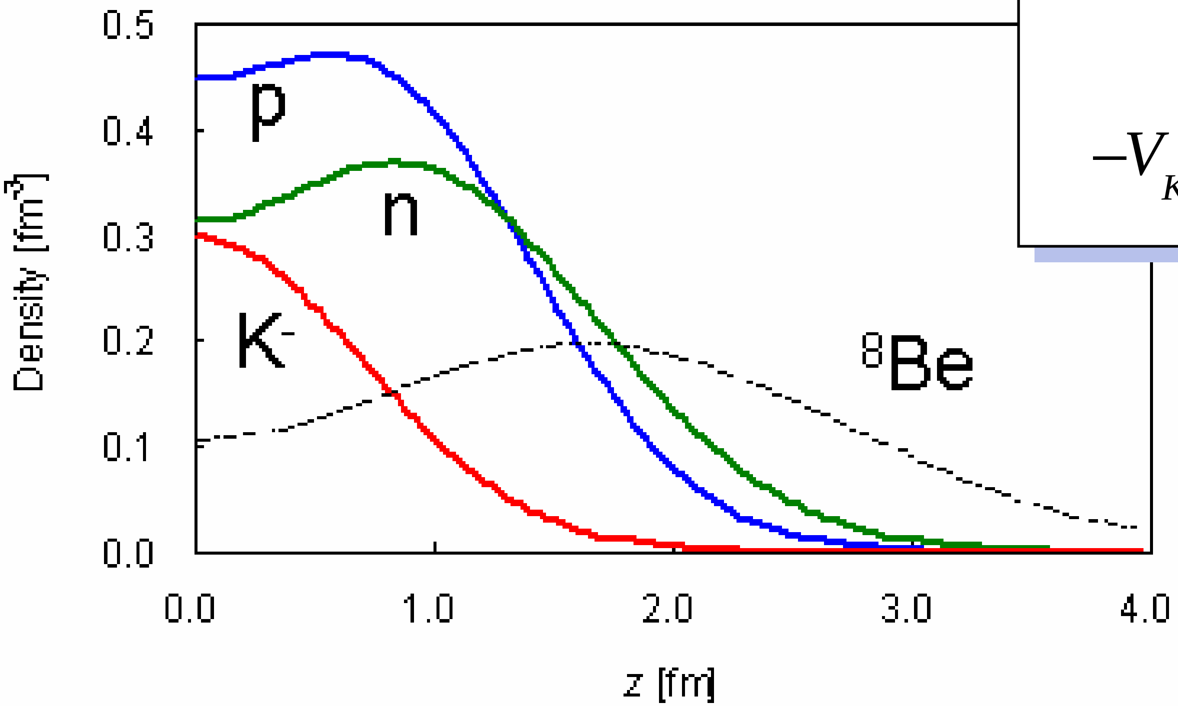
β = 0.55

Central density = 0.76 /fm^3

4.5 normal density

Isovector deformation in ^{8}BeK$^-$

density distributions along Z-axis



$$-V^{\bar{K}N}_{I=0} >> -V^{\bar{K}N}_{I=1}$$

$$\Downarrow$$

$$-V_{K^-p} >> -V_{K^-n}$$

New type of neutron skin

Neutrons are peripherally distributed, yet with quite high density.

Double kaonic nuclei

Motivation

- We have a simple question:
  How do kaonic nuclei behave, if extra-one K^- is added?

- Multi K^- system is related to K^- condensation, strange quark matter, and so on.
  Double kaonic nuclei, which contain two K-'s, are the simplest case of Multi K^- systems.

Formalism

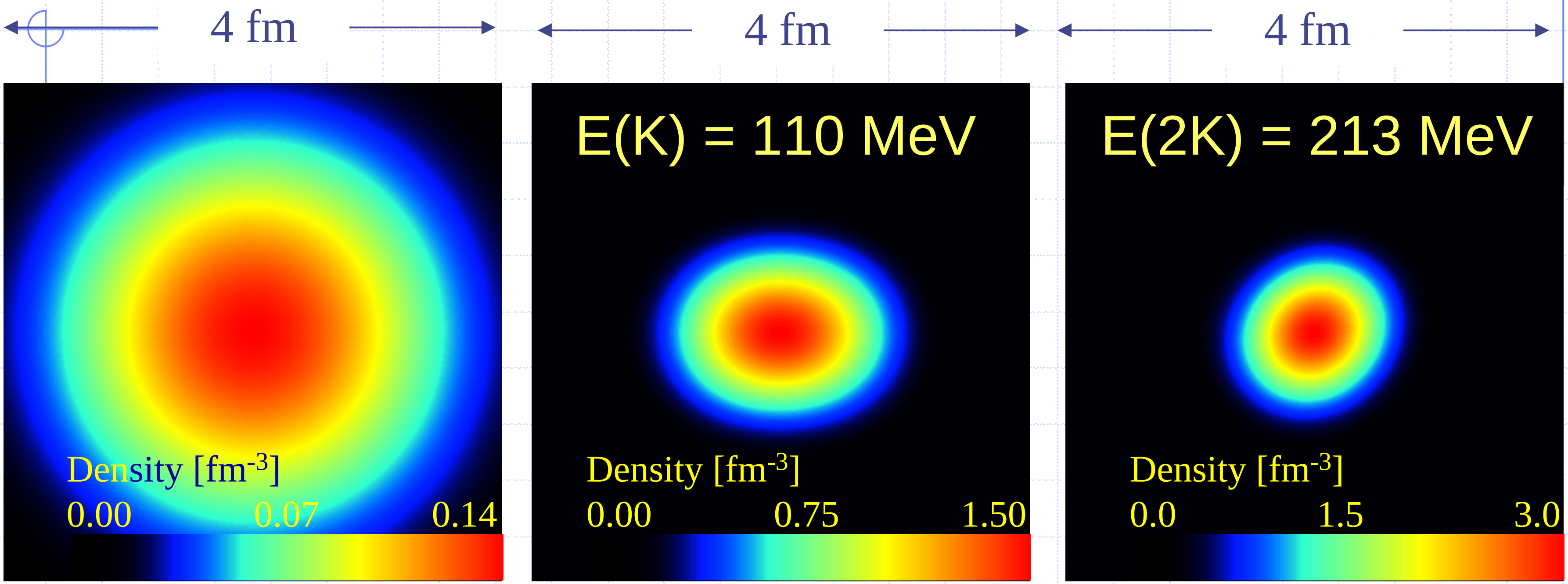
Wave func.: $\left|\Phi\right\rangle = \det\left[\mathrm{NN}\cdots\mathrm{N}\right]\otimes S\left[\mathrm{KK}\right]$

Symmetrized

calculate in quite the same way as single kaonic nuclei.

$V_{KK} = 0$ since we have no information in the present stage.

Double kaonic nucleus // ppnK^-K^- //



ppn

total B.E. = 6.0 MeV
central density = 0.14 fm^{-3}
R_{rms}= 1.59 fm

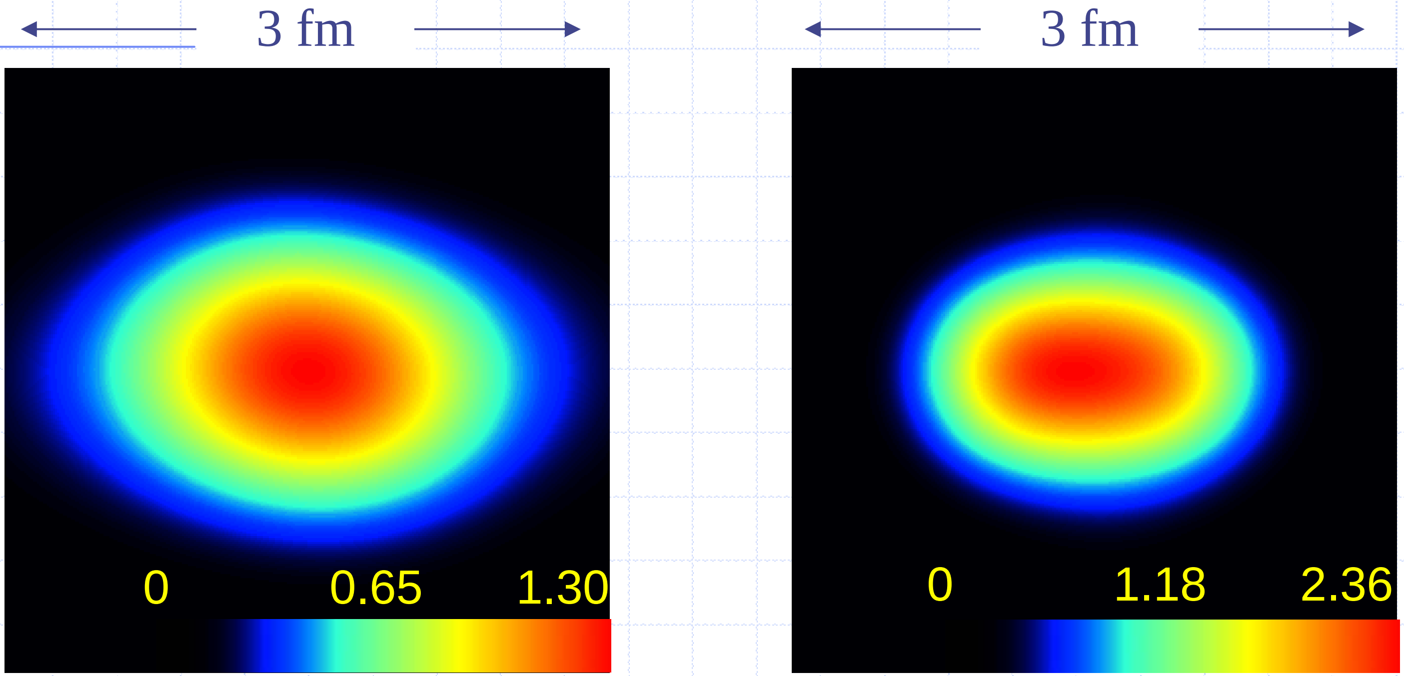
ppnK^-

total B.E. = 118 MeV
central density = 1.50 fm^{-3}
R_{rms}= 0.72 fm

ppnK^-K^-

total B.E. = 221 MeV
central density = 3.01 fm^{-3}
R_{rms}= 0.69 fm

Double kaonic nucleus // pppnK$^-$K$^-$ //



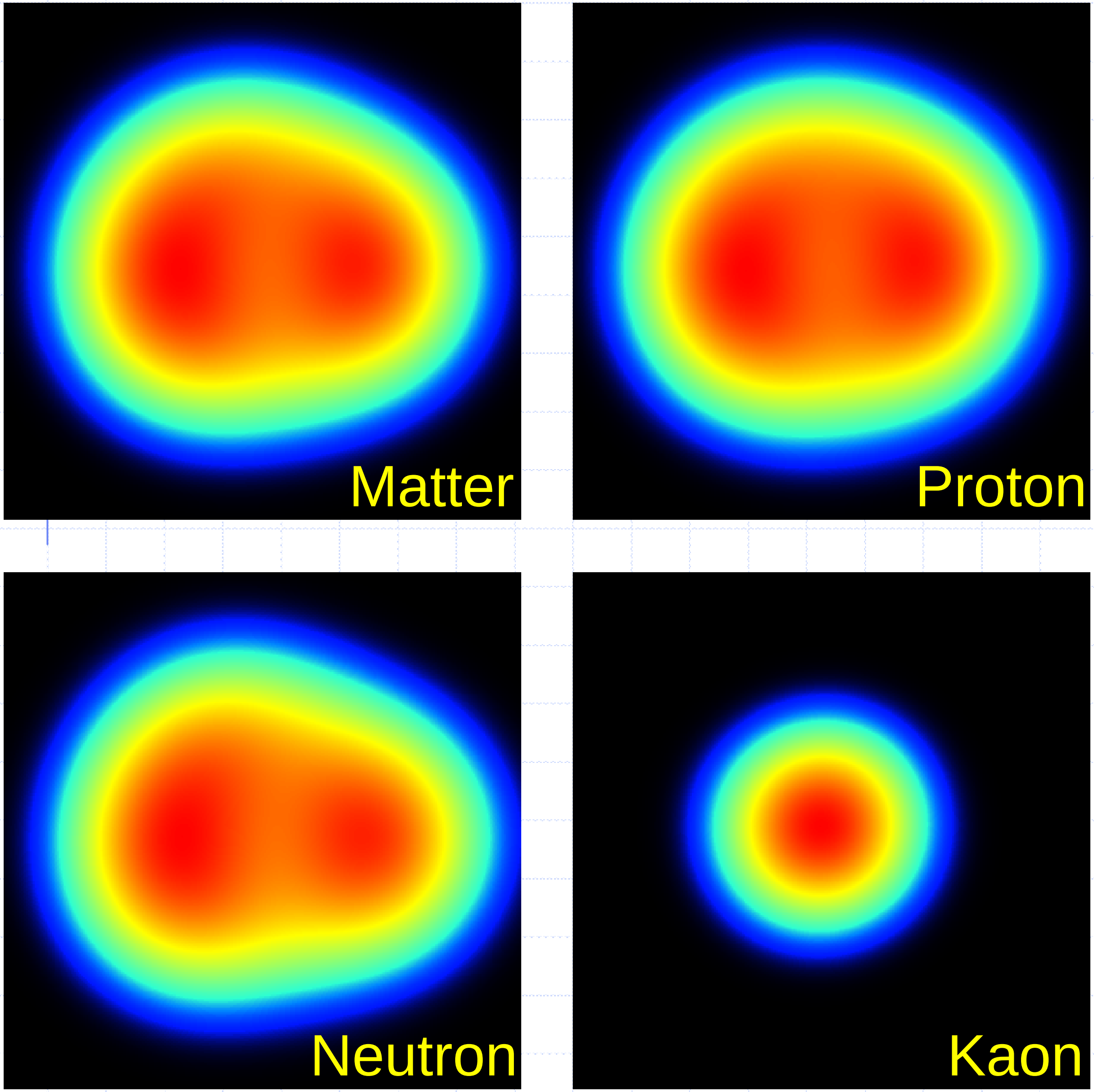
pppnK$^-$

E(K) = 105 MeV
central density = 1.29 fm^{-3}
R_{rms} = 0.97 fm

pppnK$^-$K$^-$

E(2K) = 222 MeV
central density = 2.33 fm^{-3}
R_{rms} = 0.73 fm

$^{11}\mathrm{C}K^{-}$



$E(K) = 137$ MeV
$\Gamma = 33$ MeV
$\rho_{\mathrm{MAX}} = 0.80\ \mathrm{fm}^{-3}$
$R_{\mathrm{rms}} = 1.43$ fm

LS force : on

size : 4fm x 4fm

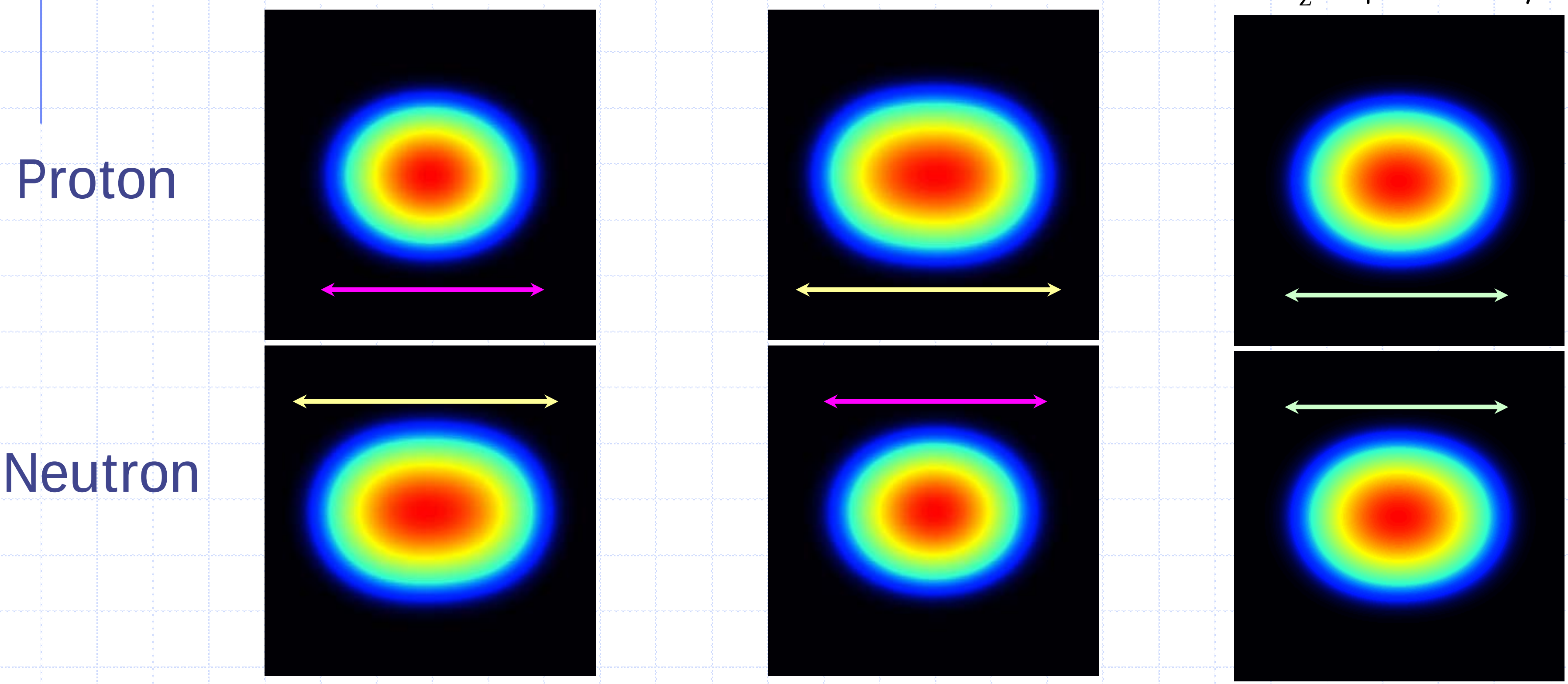
$$\left|{}^{3}_{\mathrm{K}}\mathrm{H}\right\rangle = P_{T_Z=0}\left|\Phi_N\Phi_K\right\rangle$$

$$= P_{T_Z=0}\left[\left(\sum_{m=-3/2}^{+3/2} P_{T_Z^N=m}\right)\left|\Phi_N\right\rangle \otimes \left(P_{T_Z^K=+1/2}+P_{T_Z^K=-1/2}\right)\left|\Phi_K\right\rangle\right]$$

$$= \underbrace{P_{T_Z^N=+1/2}\left|\Phi_N\right\rangle \otimes P_{T_Z^K=-1/2}\left|\Phi_K\right\rangle}_{\left|\mathrm{ppnK}^{-}\right\rangle} + \underbrace{P_{T_Z^N=-1/2}\left|\Phi_N\right\rangle \otimes P_{T_Z^K=+1/2}\left|\Phi_K\right\rangle}_{\left|\mathrm{pnn}\overline{\mathrm{K}}^{0}\right\rangle}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$P_{T_Z^N=+1/2}\left|\Phi_N\right\rangle$ $\quad$ $P_{T_Z^N=-1/2}\left|\Phi_N\right\rangle$ $\quad$ $P_{T_Z=0}\left|\Phi_N\Phi_K\right\rangle$



$$\left|{}^{3}_{\mathrm{K}}\mathrm{H}\right\rangle = P_{T_Z=0}\left|\Phi_N\Phi_K\right\rangle$$

; AMD wave function

$$= \left|\mathrm{ppnK^{-}}\right\rangle + \left|\mathrm{pnn}\overline{\mathrm{K}}^{0}\right\rangle$$

Here,

$${}_N\left\langle \mathrm{ppnK^{-}} \middle| {}^{3}_{\mathrm{K}}\mathrm{H}\right\rangle_N = 0.71 \simeq \frac{1}{\sqrt{2}}$$

$${}_N\left\langle \mathrm{pnn}\overline{\mathrm{K}}^{0} \middle| {}^{3}_{\mathrm{K}}\mathrm{H}\right\rangle_N = 0.70 \simeq \frac{1}{\sqrt{2}}$$

$$\Rightarrow \left|{}^{3}_{\mathrm{K}}\mathrm{H}\right\rangle_N = \frac{1}{\sqrt{2}}\left|\mathrm{ppnK^{-}}\right\rangle_N + \frac{1}{\sqrt{2}}\left|\mathrm{pnn}\overline{\mathrm{K}}^{0}\right\rangle_N$$

normalized wave function

In addition,

$${}_N\left\langle \mathrm{pnn}\overline{\mathrm{K}}^{0} \middle| e^{i\pi\widehat{T}_y} \middle| \mathrm{ppnK^{-}}\right\rangle_N = 0.999 \Rightarrow \left|\mathrm{pnn}\overline{\mathrm{K}}^{0}\right\rangle_N = e^{i\pi\widehat{T}_y}\left|\mathrm{ppnK^{-}}\right\rangle_N$$

Therefore,

Intrinsic state in isospin space

$$\left|{}^{3}_{\mathrm{K}}\mathrm{H}\right\rangle_N = \sum_{\theta=0,\pi} \frac{1}{\sqrt{2}} e^{i\theta\widehat{T}_y} \left|\mathrm{ppnK^{-}}\right\rangle_N$$

holds good in AMD calculation of ${}^{3}_{\mathrm{K}}\mathrm{H}(\mathrm{T}=0)$.

Treatment of $K^{-}p\,/\,\overline{K}^{0}n$ mixing

Usual practice = Coupled channel ex) ppnK- $\left|\Phi\right\rangle = \sum_a C_a \left| ppnK^- \right\rangle_a + \sum_b D_b \left| pnn\overline{K}^0 \right\rangle_b$

⇔ Multi-Slater determinants

But some problems ...

1. common $\{\mathbf{Z}_i\}$ for all slater det., ${}_A C_Z + {}_A C_{Z-1}$ slater determinants !
2. different $\{\mathbf{Z}_i^{(a)}\}$ for each slater det., how many slater determinants ?
3. How is the effective frictional cooling for multi-slater det. ?
4. Calculation of non-diagonal matrix elements is somewhat tedious. $\left\langle ppnK^- \middle| \hat{V} \middle| pnn\overline{K}^0 \right\rangle$

Single slater determinat with charge-mixed s.p. wave func.

In the single-particle state,
p and n are mixed, and
K- and K0bar are mixed.

$$\left|N\right\rangle = a\left|p\right\rangle + b\left|n\right\rangle$$

$$\left|K\right\rangle = x\left|K^-\right\rangle + y\left|\overline{K}^0\right\rangle$$

$\left|\Phi\right\rangle = \left|NNNK\right\rangle$ contains $ppnK^-$ and $pnn\overline{K}^0$

But total Tz is restored with the Tz-projection.

Treatment of $K^{-}p/\overline{K}^{0}n$ mixing

Usual practice = Coupled channel

ex) ppnK- $\left|\Phi\right\rangle = C\left|ppnK^{-}\right\rangle + D\left|pnn\overline{K}^{0}\right\rangle$

But in AMD treatment non-diagonal matrix element can't be calculated.

Calculation of $\left\langle ppnK^{-}\left|\hat{V}\right|pnn\overline{K}^{0}\right\rangle$ needs the inverse matrix of the overlap matrix $B_{ij} = \left\langle \varphi_{\iota} \middle| \varphi_{j} \right\rangle$

$$B = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} p & p & n & \\ \end{array}$$

$$B = \left(\begin{array}{ccc} \alpha & \beta & 0 \\ 0 & 0 & \gamma \\ 0 & 0 & \delta \end{array}\right)\begin{array}{c} p \\ n \\ n \end{array} \Rightarrow B^{-1}$$

does not exist !

In the Slater determinant $\left|\Phi\right\rangle = \left|NNNK\right\rangle$, $ppnK^{-}$ and $pnn\overline{K}^{0}$ are mixed.

In the single-particle state,
p and n are mixed, and
K- and K0bar are mixed.

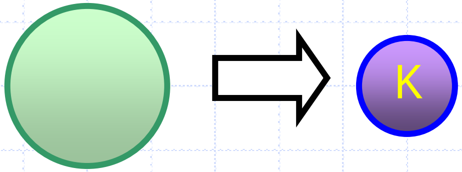
$$\left|N\right\rangle = a\left|p\right\rangle + b\left|n\right\rangle$$
$$\left|K\right\rangle = x\left|K^{-}\right\rangle + y\left|\overline{K}^{0}\right\rangle$$

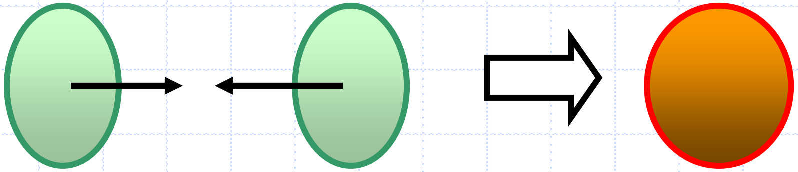
But total Tz is restored with the Tz-projection.

Introduction

If $\bar{K}N$ interaction is very strongly attractive...

- We can obtain a highly dense state by implanting K- into normal nuclei.

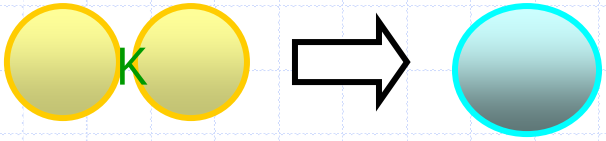




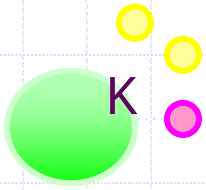
low temperature, high density

high temperature, high density

- Change of nuclear structure
  : well-developed clustering structure vanishes ??

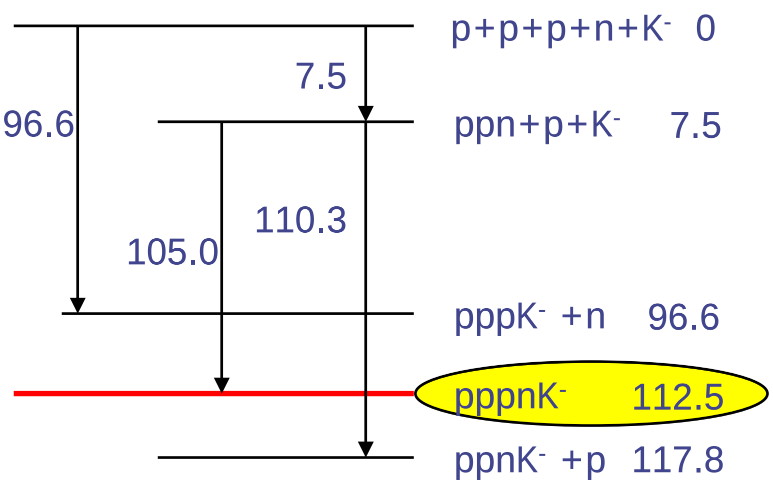


- Extending the drip line of unstable nuclei ???



Stability for strong interaction

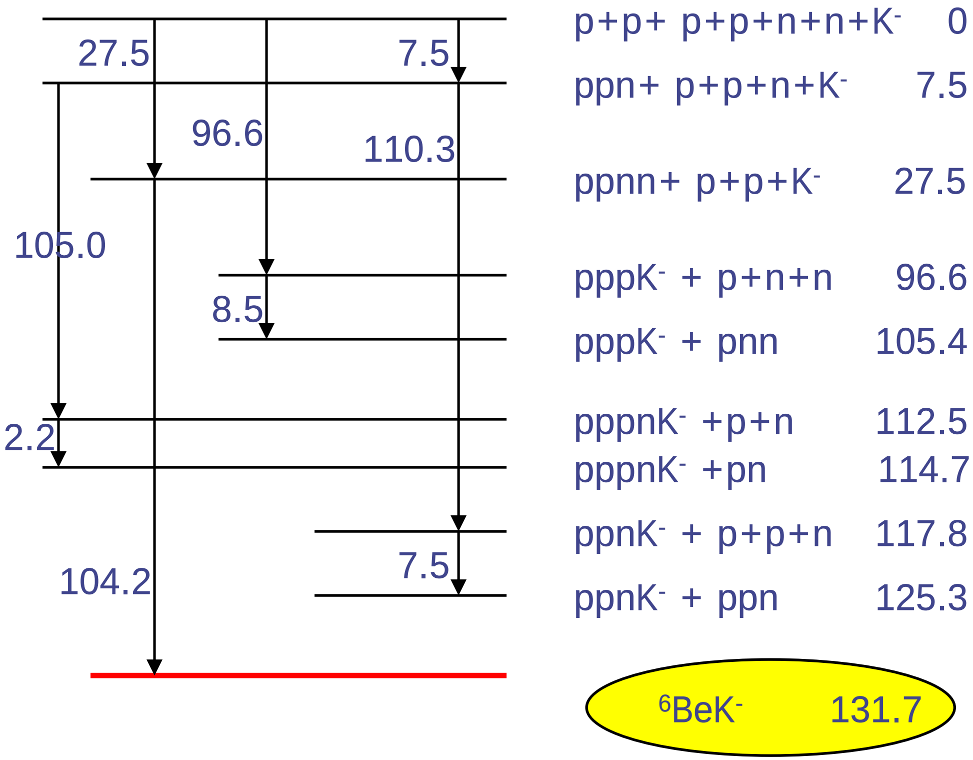
pppnK$^-$



p+p+p+n+K$^-$	0
ppn+p+K$^-$	7.5
pppK$^-$ +n	96.6
pppnK$^-$	112.5
ppnK$^-$ +p	117.8

pppnK$^-$ is unstable
for strong interaction.

^{6}BeK$^-$



p+p+ p+p+n+n+K$^-$	0
ppn+ p+p+n+K$^-$	7.5
ppnn+ p+p+K$^-$	27.5
pppK$^-$ + p+n+n	96.6
pppK$^-$ + pnn	105.4
pppnK$^-$ +p+n	112.5
pppnK$^-$ +pn	114.7
ppnK$^-$ + p+p+n	117.8
ppnK$^-$ + ppn	125.3
^{6}BeK$^-$	131.7

^{6}BeK$^-$ is stable
for strong interaction.

Formalism

- Hamiltonian

$$\hat{H} = \hat{T} + \hat{V}_C + \hat{V}_{Coul.} + \hat{V}_{KN} - \hat{T}_G$$

In kinetic energy part, taking the mass difference between a nucleon and K- into consideration.

- Frictional cooling eq. with constraint

$$\dot{X}^{i}_{\alpha} = (\lambda + i\mu)\frac{1}{i\hbar}\left[\frac{\partial H}{\partial X^{i*}_{\alpha}} + \eta\frac{\partial W_G}{\partial X^{i*}_{\alpha}}\right] \quad \text{and c.c.}$$

negative

$$W_G = \left|\langle \mathbf{R}_G \rangle\right|^2 + \left|\langle \mathbf{P}_G \rangle\right|^2$$

Constraint condition

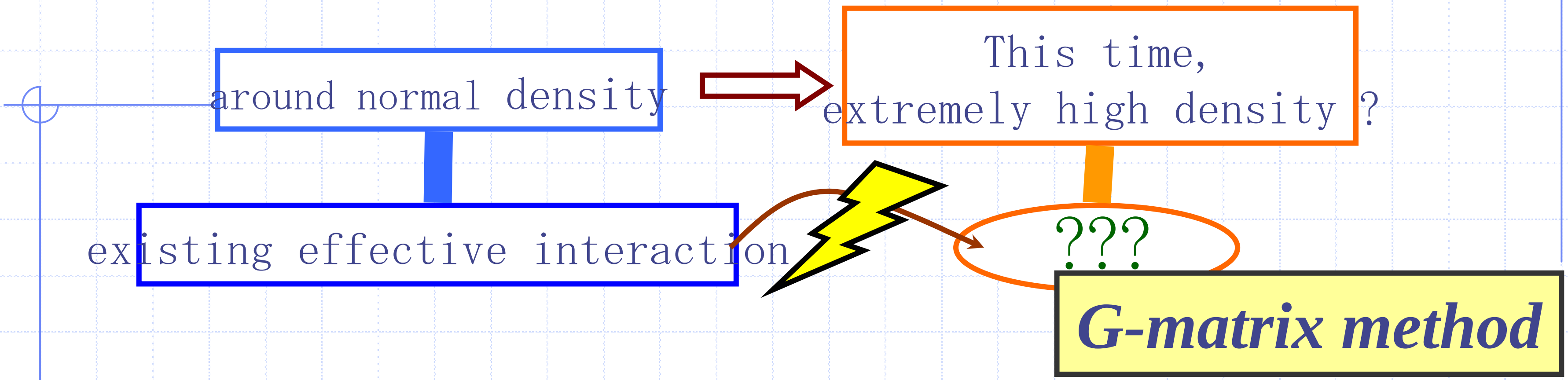
The center of mass of the system is fixed to the origin.

$$\left\{X^{i}_{\alpha}\right\} = \boxed{\left\{C^{i}_{\alpha}, \mathbf{Z}^{i}_{\alpha}, C^{K}_{\alpha}, \mathbf{Z}^{K}_{\alpha}\right\}}$$

complex numbers

$$\mathbf{R}_G = \frac{\sum_{i=1}^{A} m_N \mathbf{r}_i + m_K \mathbf{r}_K}{A m_N + m_K}, \quad \mathbf{P}_G = \sum_{i=1}^{A} \mathbf{p}_i + \mathbf{p}_K$$

G-matrix calculation


around normal density

This time,
extremely high density ?

existing effective interaction

???

G-matrix method

- bare interaction and model space

NN int. : Tamagaki potential (OPEG)
KN int. : Akaishi-Yamazaki potential
model space : 2 Gaussian wave packets for one nucleon,
5 Gaussian wave packets for one anti-kaon.

- procedure

given density and starting energy of $\bar{K}$
→ G-matrix

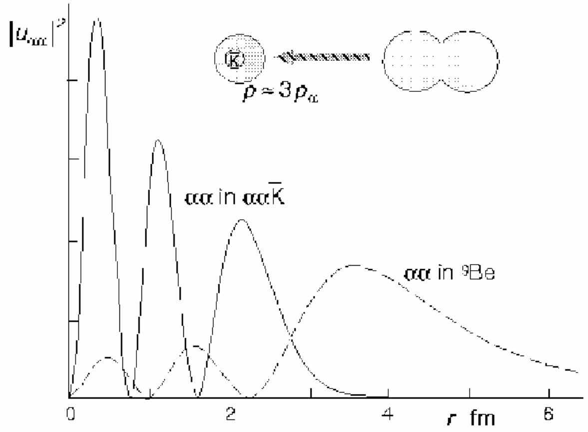
AMD calculation
→ density and starting energy of $\bar{K}$

Repeat
until getting
a consistent result.

Introduction

Changing the matter density of 8Be

Y.Akaishi solved $\alpha\ \alpha$ K^- system with ATMS method.
NN interaction = Hasegawa-Nagata No.1 which has 1.6 GeV repulsive core.
KN interaction = Y.A. & T.Y. force.



The distribution $|u_{\alpha\alpha}(r)|^2$ of the $\alpha\ \alpha$ relative motion in $\alpha\ \alpha$ K system.

Binding energy = 113 MeV
$\Gamma_{\Lambda+\pi}$ = 38 MeV
Central density = 5 ρ_0

K^- causes 8Be to be shrunk.
8Be implanted K^- loses two α structure.

Akaishi-Yamazaki $\bar{K}N$ interaction

1, free $\bar{K}N$ scattering data

A.D.Martin, Nucl.Phys.B179(1981)33

$a^{I=0}$= -1.76 + i 0.46 fm, $a^{I=1}$= 0.37 + i 0.60 fm

cf) A.D.Martin :
$a^{I=0}$= -1.70 + i 0.68 fm, $a^{I=1}$= 0.37 + i 0.60 fm

2, X-ray data of kaonic hydrogen atom

M.Iwasaki et al. Phys.Rev.Lett.78(1997)3067
T.M.Ito et al. Phys.Rev.C58(1998)2366

$a_{K\text{-}p}$= ($a^{I=0}$ +$a^{I=1}$) /2 = -0.70 + i 0.53 fm

cf) T.M.Ito et al. :
$a_{K\text{-}p}$= (-0.78 $\pm$ 0.15 $\pm$ 0.03) + i (0.49 $\pm$ 0.25 $\pm$ 0.12) fm

3, Binding energy and decay width of Λ(1405)

B.E.= -29.7 MeV, Γ = 40 MeV cf) Λ(1405) exists 27 MeV below K^-+p threshold

regarding Λ(1405) as I=0 bound state of $\bar{K}N$

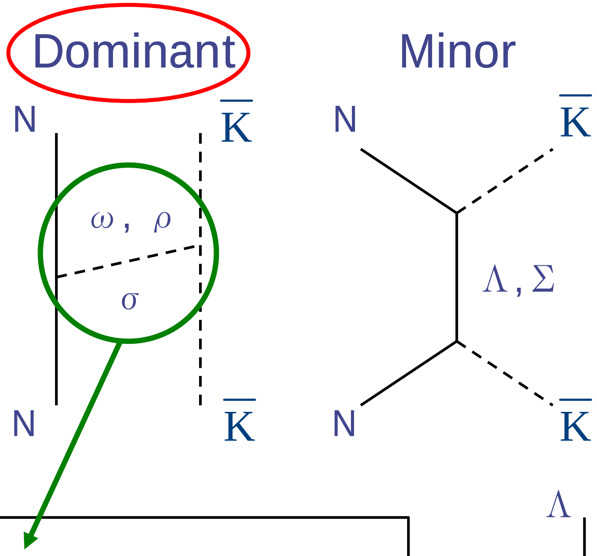
$V_{I=0}^{KN}$ is much more attractive than $V_{I=1}^{KN}$!!

いろいろな$\overline{\text{Ｋ}}$Ｎポテンシャルの作り方

	W.Weise	Julichグループ (K. Holinde)	赤石・山崎
基礎理論	Chiral SU(3) effective Lagrangian	meson exchange	------
目的	$\overline{\text{Ｋ}}$Ｎ系全体を合わす	NN,YN,KN,$\overline{\text{KN}}$全てを合わす	$\overline{\text{Ｋ}}$Ｎのbound stateにのみ着目
Channel	$pK^-, n\overline{K}^0, \Lambda\pi^0, \Sigma^0\pi^0, \Sigma^+\pi^-, \Sigma^-\pi^+$	(同左)　+　$N\overline{K}^*, \Delta\overline{K}, \Delta\overline{K}^*$	$pK^-, n\overline{K}^0, \Lambda\pi^0, \Sigma^0\pi^0, \Sigma^+\pi^-, \Sigma^-\pi^+$
ポテンシャルの導入	$\overline{\text{KN}}$ threshold下にΛ（１４０５）という resonance。カイラル摂動論で処理しようとすると、ある種のダイヤグラムを無限オーダーとらねばならない。 ⇩ ポテンシャルで処理 q2まで取ったカイラルラグランジアンからBorn近似で求めたｓ波散乱長と等しい散乱長を持つようなポテンシャル。 但しポテンシャルの形は仮定 local：　Yukawa型 separable：　$\frac{\alpha_i^2}{\alpha_i^2+k_i^2}\cdot\frac{\alpha_j^2}{\alpha_j^2+k_j^2}$	相互作用ラグランジアン$W=-\int d^3x\, L_{int}$ から W：２次　one boson exchange, resonance diagram W：４次　box diagram のポテンシャル ＫＮ相互作用からG-parity変換で決まるものはそれを用いる。その他はfree parameter。 vertexのform factorは仮定。	とにかくポテンシャルの形はGauss型としてしまえ！ 全チャネルで $v_x^I = v_{cx}^I \exp[-(r/0.66)^2]$
再現	• Λ（１４０５）の位置と幅 • $\overline{\text{Ｋ}}$Ｎ低エネルギー散乱の全断面積 (K: 60～300 MeV/c in LAB) • Branching ratio	• Λ（１４０５）の位置と幅 • $\overline{\text{Ｋ}}$Ｎ低エネルギー散乱の全断面積 (K: 60～300 MeV/c in LAB)	• Λ（１４０５）の位置と幅 • $\overline{\text{Ｋ}}$Ｎ低エネルギー散乱の散乱長 • kaonic hydrogen atomのスペクト 核物質中での散乱振幅の振る舞いはWeiseとconsistent
論文	Nucl.Phys.A594(1995)325	Nucl.Phys.A513(1990)557	Phys.Rev.C65(2002)044005

Julich $\bar{\mathrm{K}}$N Quasi-potential

$$p_{\bar{K}}^{\mathrm{LAB}} = 60 \sim 300\ \mathrm{MeV}/c$$



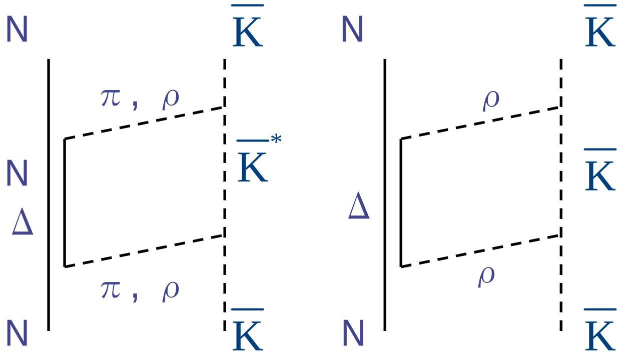
$\bar{\mathrm{K}}$Nの場合、
全てのメソンが引力的に働く。

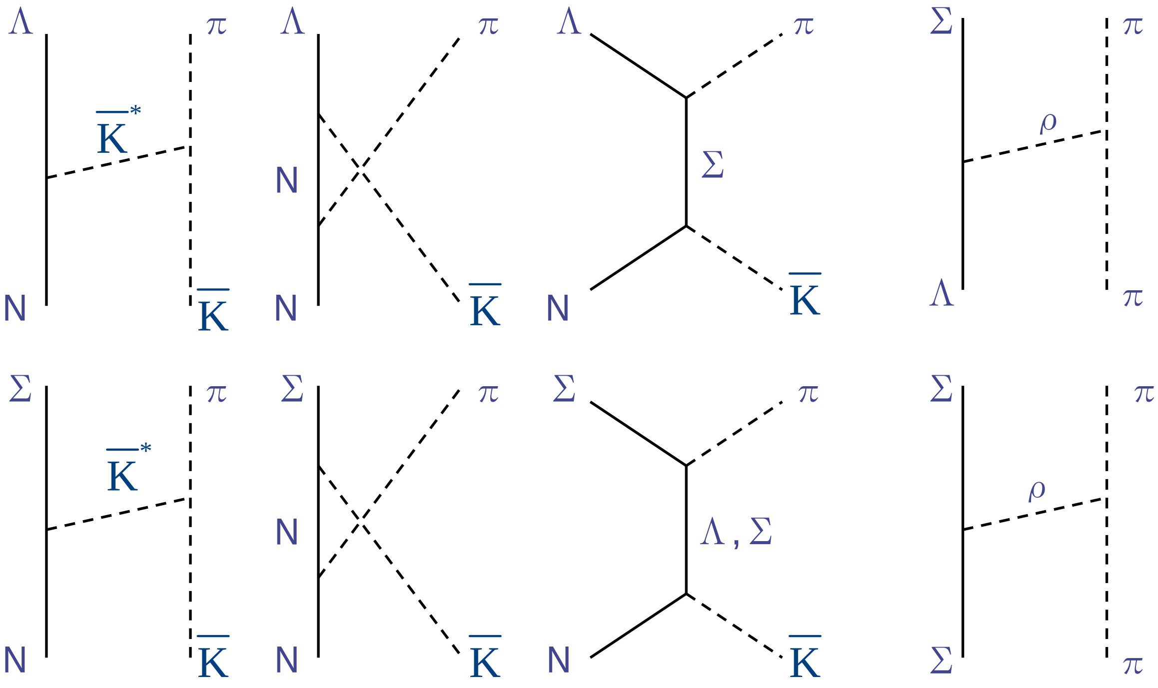
⇩

Λ（１４０５）の形成

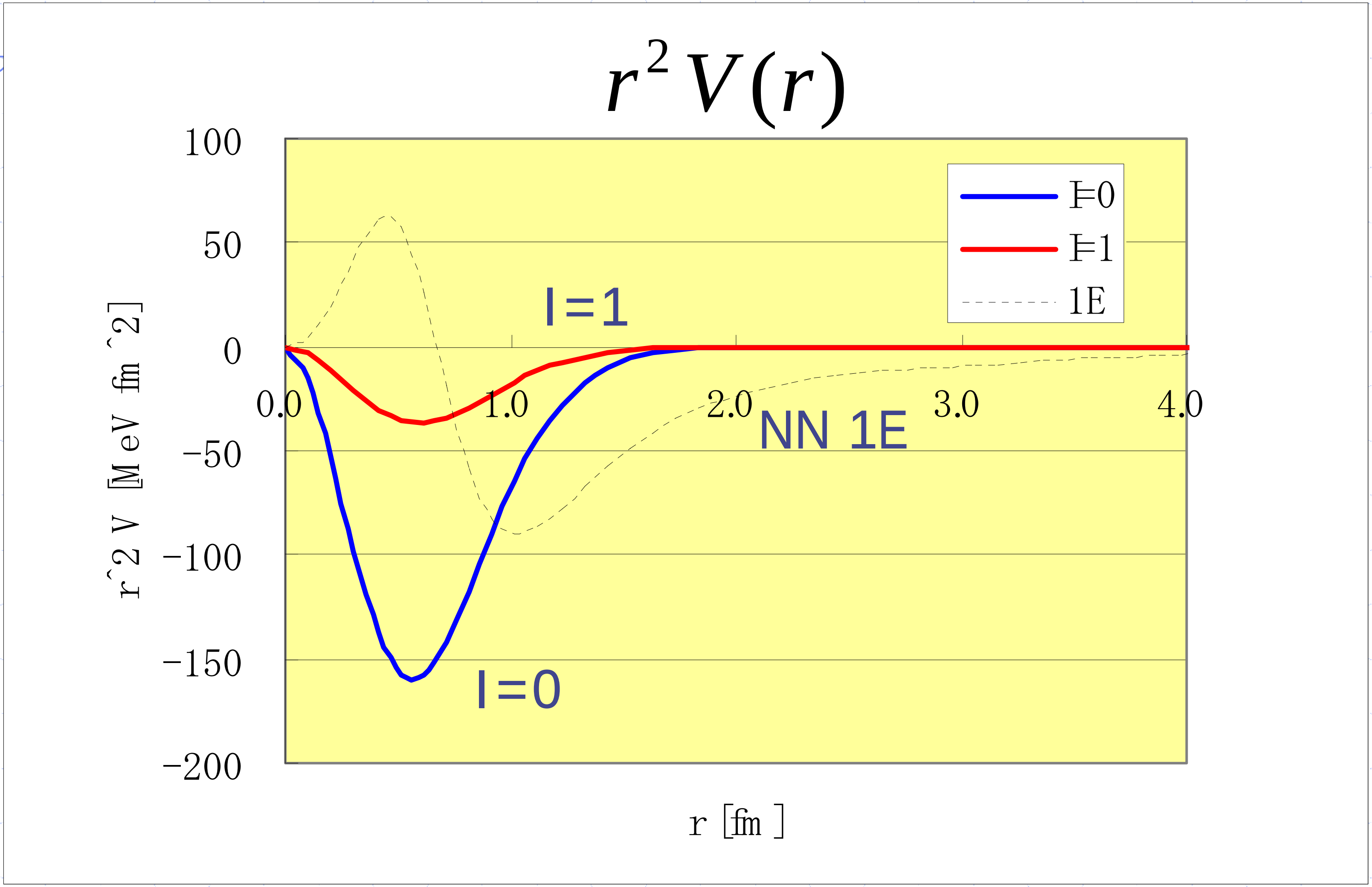
Weise とも consistent

local potentialを用いた場合、
レンジパラメータはこれらの
メソン質量に対応。
Λ（１４０５）を作る。



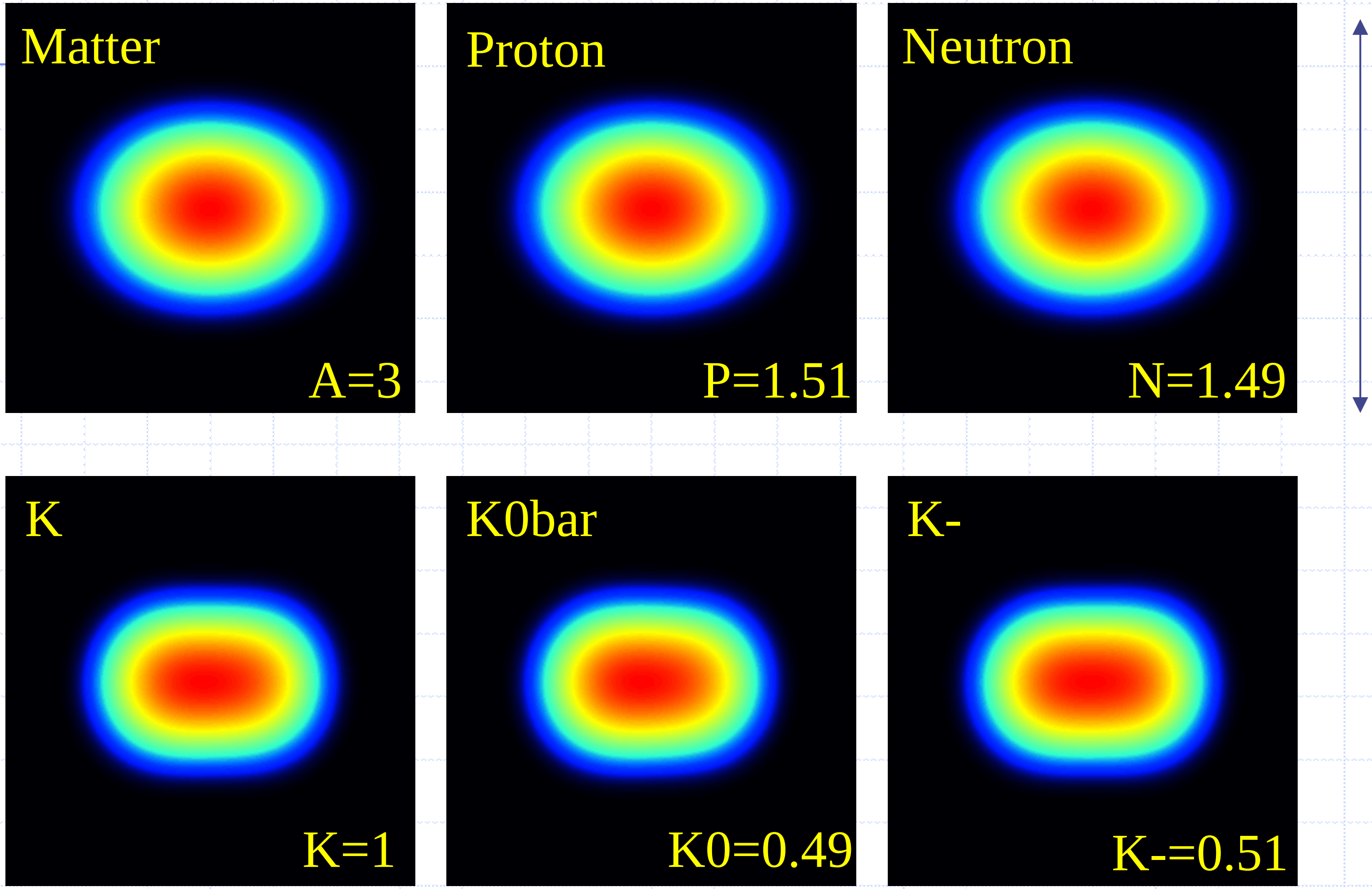


KN potential (G-matrix) for ^{6}BeK$^-$



ppnK$^-$

Density distributions of $|P_{Tz}\Phi\rangle$ in ZX plane.



$\rho(\mathbf{0}) = 1.50\mathrm{fm}^{-3}$ ··· central density

$$\boxed{\mathrm{ppnK}^- : \mathrm{pnn}\overline{\mathrm{K}^0} = 1:1}$$

model space

2 Gaussian / nucleon
5 Gaussian / kaon

force

E(K)=94.5MeV, central density=$1.49\mathrm{fm}^{-3}$

Results

Projecting onto J=3/2 and T=1. Parity is negative.

	B.F.	width	dens0	rmsR	beta	gamma
JT projection	96.68	12.45	1.56	0.81	0.70	11.78

G-matrix consistency is a little violated.

Quantum numbers

	After	Before
J2 (total sys.)	3.73	4.20
J2 (N sys.)	3.69	3.96
L2 (N sys.)	1.98	3.15
S2 (N sys.)	0.75	0.81
L2 (kaon)	0.14	0.24
T2	2.00	2.00
Tz	1.00	1.00

Particle numbers

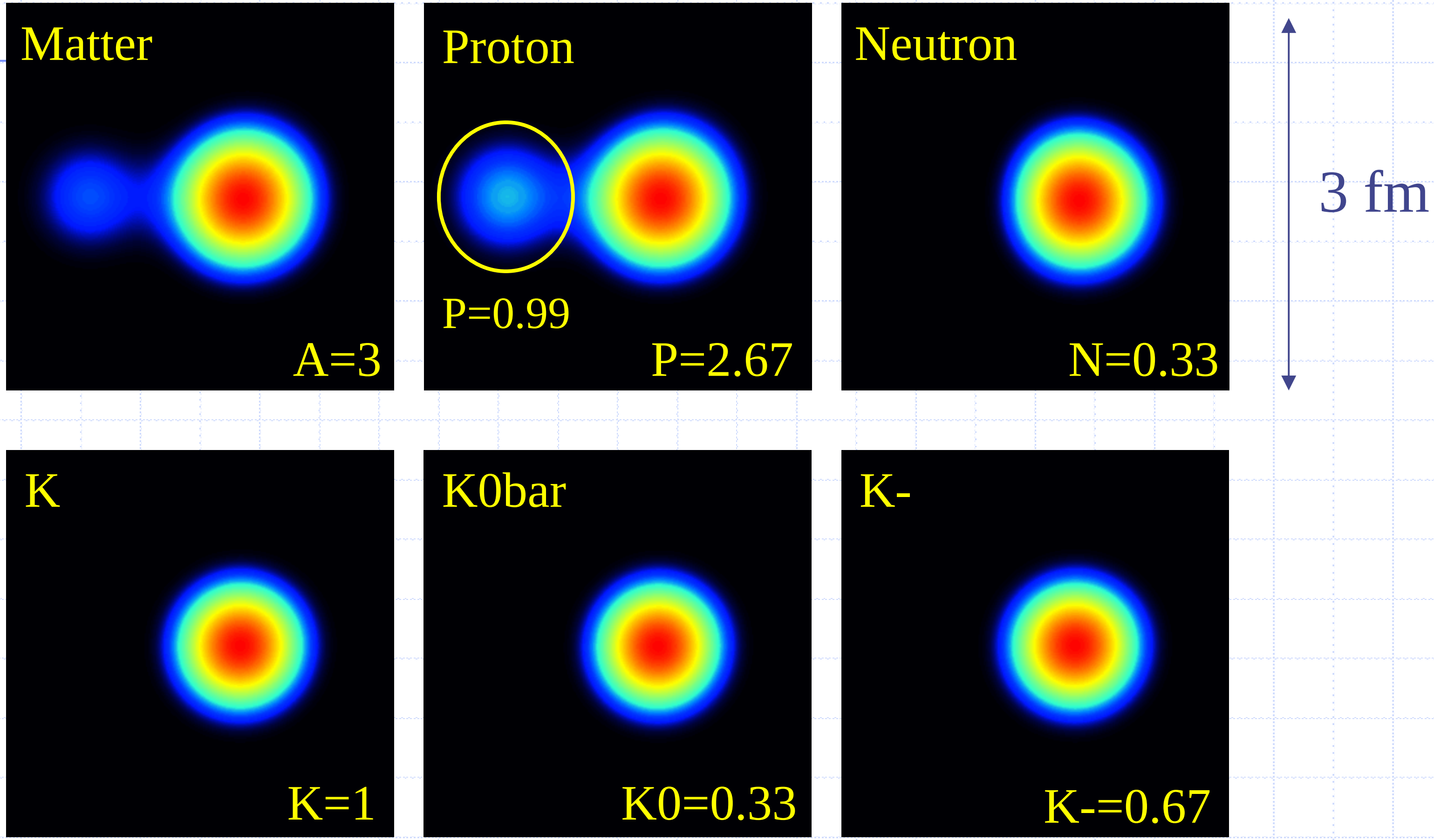
Proton	2.67
Neutron	0.33
K-	0.67
K0bar	0.33

Even if we start AMD cooling from various initial value γ which is concerned to the isospin, most of all solutions are converged to this result.

JT projection is correctly working.

pppK$^-$

Density distributions of $\left|P_{Tz}\Phi\right\rangle$ in ZX plane.



$$\left|{}^{3}_{K}\mathrm{He}\right\rangle = \left|\mathrm{p}\right\rangle \otimes \left|{}^{2}_{K}\mathrm{H}\right\rangle$$

$$\sqrt{\frac{2}{3}}\left|\mathrm{pp}\otimes \mathrm{K}^{-}\right\rangle - \sqrt{\frac{1}{3}}\left|\mathrm{pn}\otimes \overline{\mathrm{K}^{0}}\right\rangle$$

$\rho(\mathbf{0}) = 1.57\mathrm{fm}^{-3}$ $\cdots$ central density model space

2 Gaussian / nucleon
5 Gaussian / kaon

force

E(K)=99.6MeV, central density=1.31fm^{-3}

Results

Projecting onto J=1 and T=1/2. Parity is negative.

	B.E.	width	dens0	rmsR	beta	gamma
JT projection	105.01	25.85	1.29	0.97	0.54	3.80

G-matrix consistency is a little violated.

Quantum numbers

	After	Before
J2 (total sys.)	1.97	5.49
J2 (N sys.)	2.01	5.31
L2 (N sys.)	2.04	3.26
S2 (N sys.)	2.02	2.05
L2 (kaon)	0.05	0.18
T2	0.75	0.79
Tz	0.50	0.50

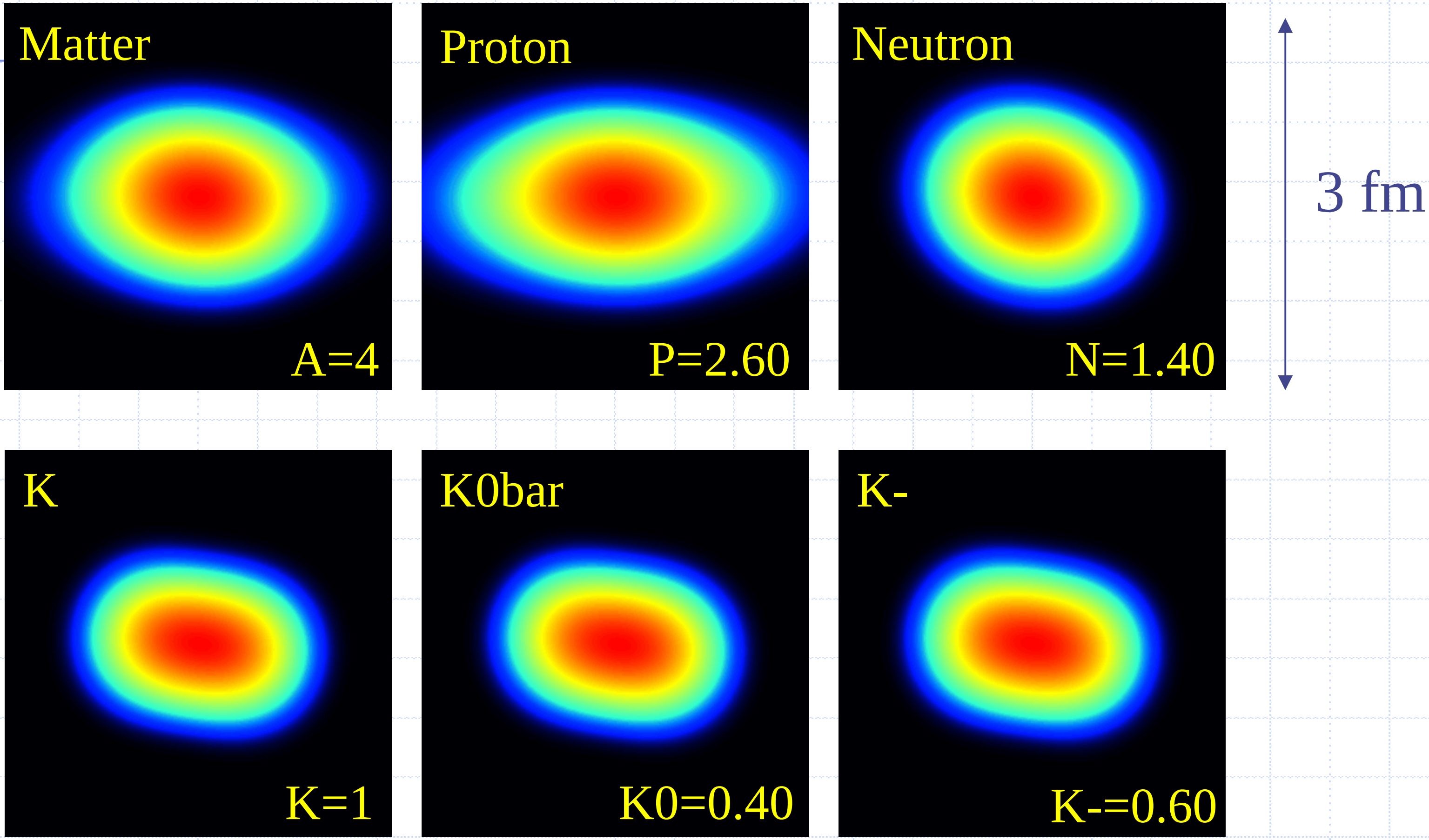
Particle numbers

Proton	2.60
Neutron	1.40
K-	0.60
K0bar	0.40

JT projection is correctly working.

pppnK$^-$

Density distributions of $\left|P_{Tz}\Phi\right\rangle$ in ZX plane.



$\rho(\mathbf{0}) = 1.29\text{fm}^{-3}$ ⋯ central density

Results — ^{6}BeK$^-$ —

model space

2 Gaussian / nucleon
5 Gaussian / kaon

force

E(K)=106MeV, central density=0.96fm^{-3}

Results

Projecting onto J=0 and T=1/2. Parity is negative.

	B.E.	width	dens0	rmsR	beta	gamma
JT projection	104.24	33.40	0.91	1.17	0.44	0.30

G-matrix consistency is accomplished.

Quantum numbers

	After	Before
J2 (total sys.)	- 0.06	4.86
J2 (N sys.)	0.03	4.60
L2 (N sys.)	0.03	4.58
S2 (N sys.)	0.00	0.02
L2 (kaon)	0.09	0.27
T2	0.75	0.97
Tz	0.50	0.50

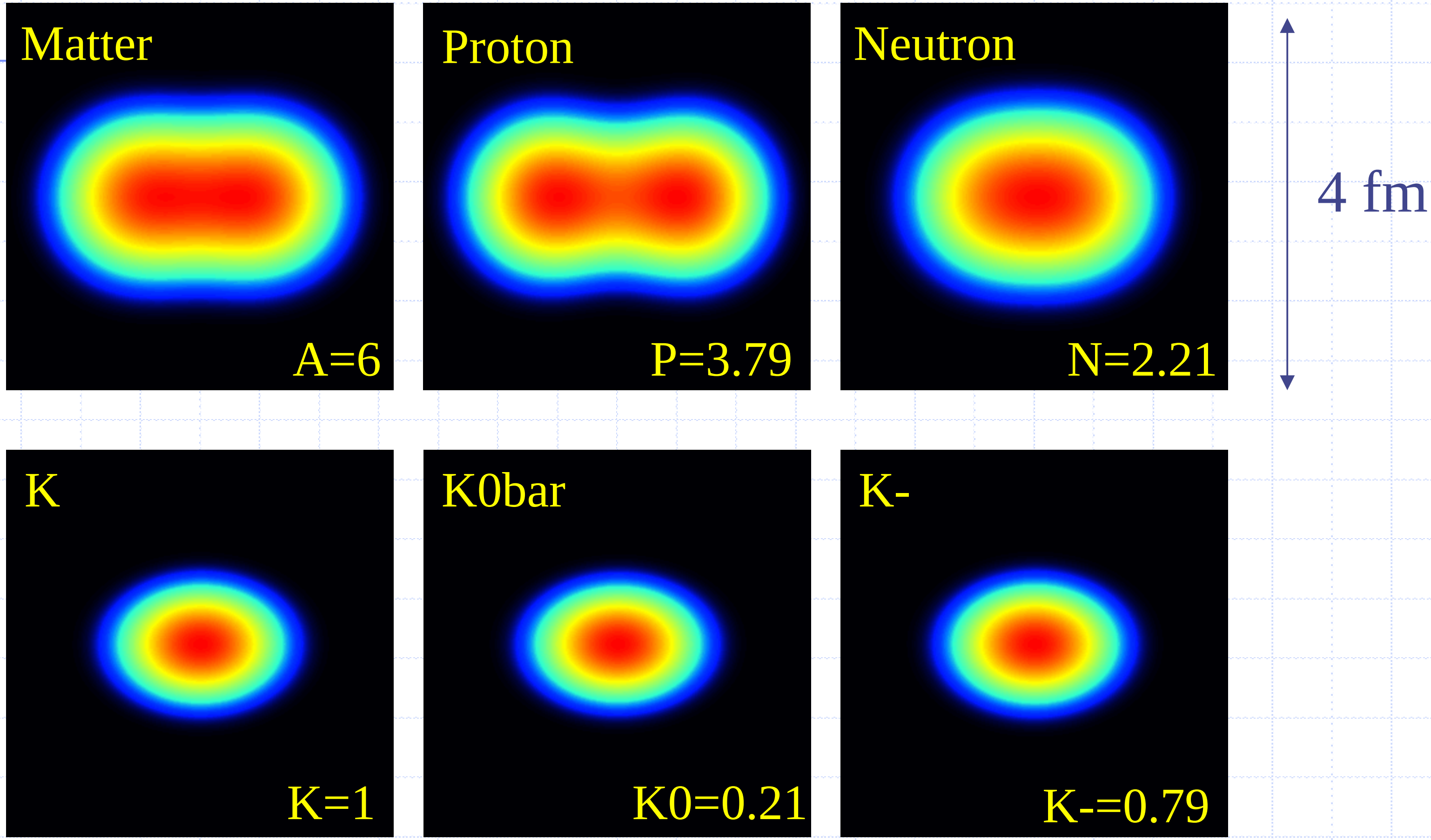
Particle numbers

Proton	3.79
Neutron	2.21
K-	0.79
K0bar	0.21

JT projection is correctly working.

^{6}BeK^{-}

Density distributions of $\left|P_{Tz}\Phi\right\rangle$ in ZX plane.



$\rho(\mathbf{0}) = 0.91\mathrm{fm}^{-3}$ $\cdots$ central density

^{9}BK$^-$

model space

2 Gaussian / nucleon
5 Gaussian / kaon

force

E(K)=106MeV, central density=0.96fm^{-3}
for ^{6}BeK$^-$

Results

Projecting onto J=3/2 and T=0. Parity is negative.

	E(K)	width	dens0	rmsR	beta	gamma
JT projection	99.6	35.5	0.67	1.46	0.46	16

G-matrix consistency is violated.

Quantum numbers

	After	Before
J2 (total sys.)	3.68	9.74
J2 (N sys.)	3.71	9.55
L2 (N sys.)	2.77	8.75
S2 (N sys.)	0.76	0.78
L2 (kaon)	0.08	0.18
T2	0.00	0.24
Tz	0.00	0.00

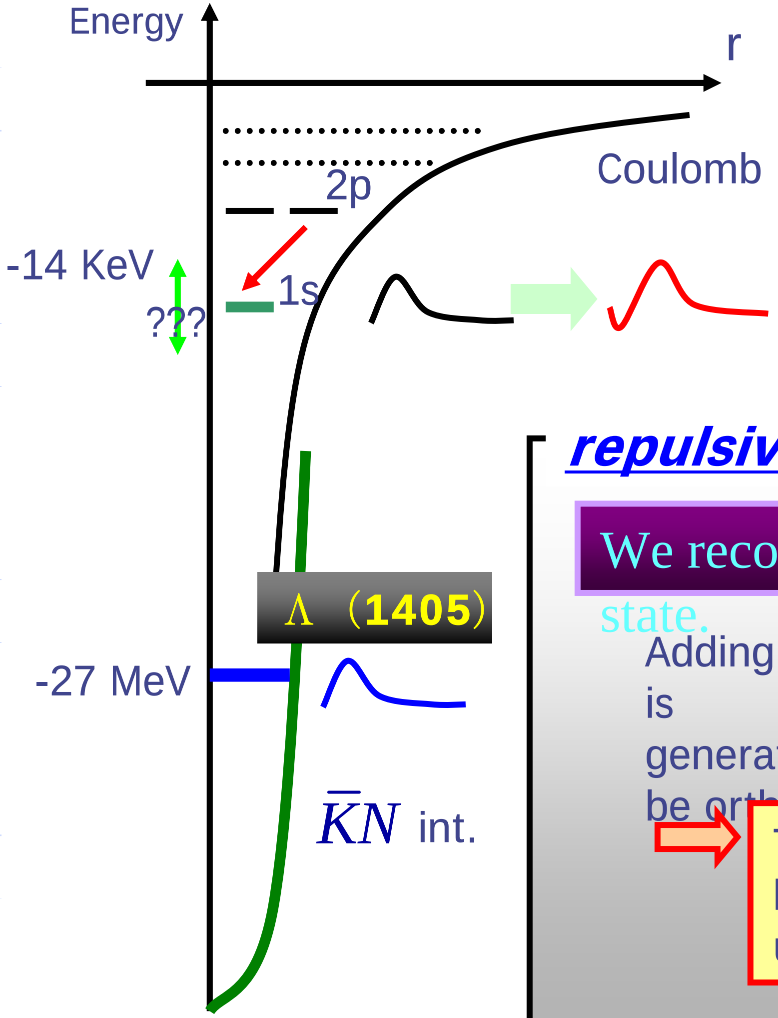
Particle numbers

Proton	4.69
Neutron	4.31
K-	0.69
K0bar	0.31

JT projection is almost correctly working.

KN interaction

Is $\bar{K}N$ interaction very strongly attractive ?



“kaonic hydrogen puzzle”

atomic 1s shift caused by KN interaction

- atomic 2p → 1s X-ray measurement : attractiv
- analysis of low energy data : repulsive

Prof. Iwasaki investigated very precise

→ “repulsive” !!

*: M.Iwasaki et.al. Phys.Rev.Lett.78(1997)3067

repulsive ≡ attractive ???

Λ (1405) = 3 quark state ??

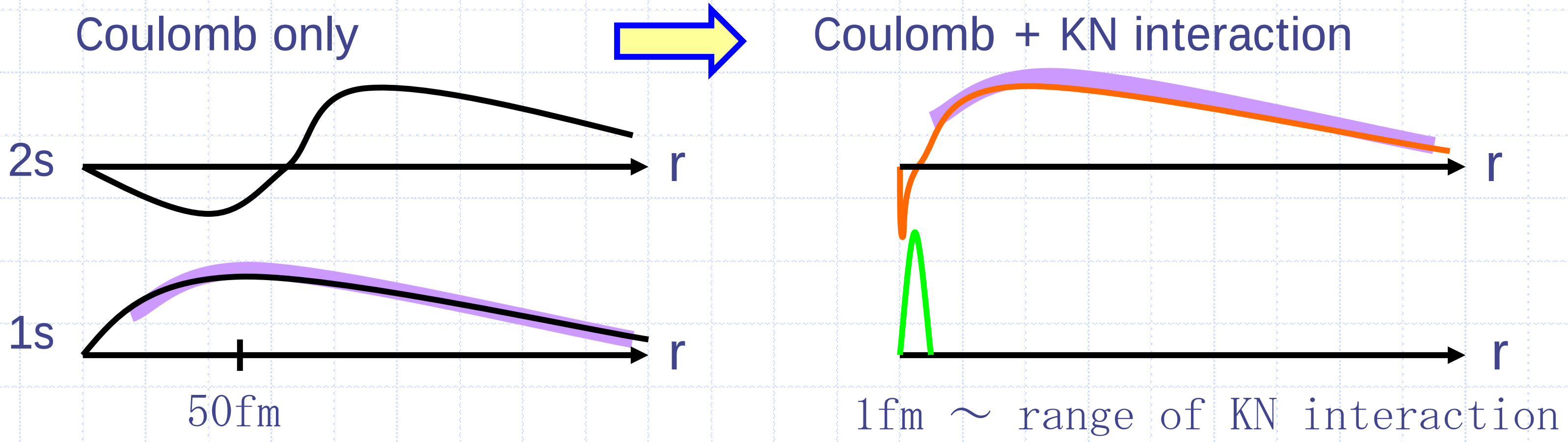
We recognize Λ (1405) is K$^-$p bound state.

Adding $\bar{K}N$ int., a new bound state (= Λ (1405)) is generated below the atomic 1s state. Then it must be orthogonal to the new bound state.

The original atomic 1s state has a node and its kinetic energy increases. As a result it shifts upward. = repulsive-like

kaonic hydrogen atom

about atomic 1s state shifted by KN interaction



2s state lowered by K^-N interaction is very similar to the original atomic 1s state.
If ignoring a node. this lowered 2s state seems to be the solution obtained by changing the boundary condition a little --- ϕ =0 at r=0 fm → ϕ =0 at r=1 fm ---
So lowered 2s state appears energetically near the original 1s state.

lowered 2s = 1s’ （１ｓもどき!!)

Seeing with nuclear scale, it has a node. So it is 2s state.
Seeing with atomic scale, the shape of its wave function is almost that of original 1s state.

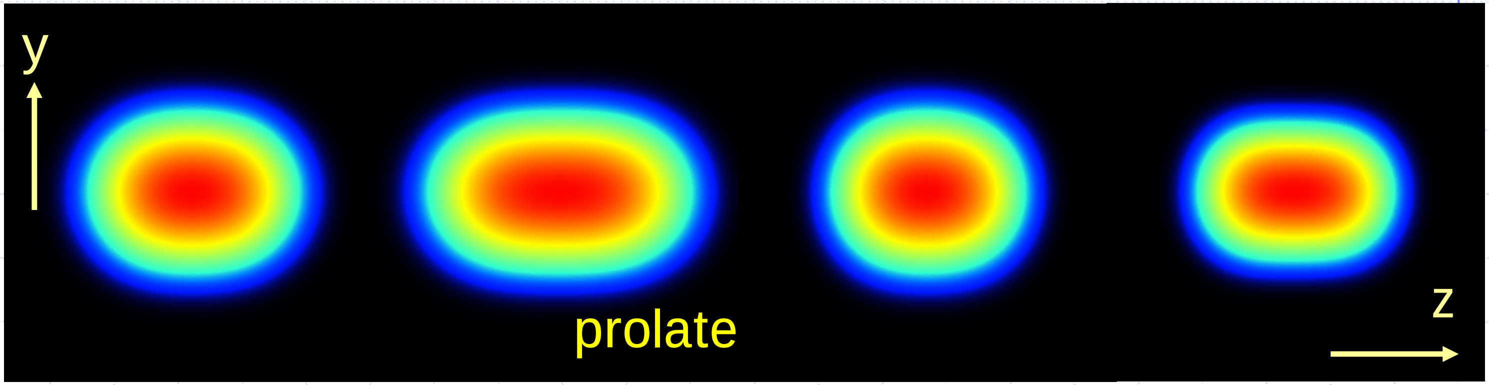
ppnK- // dependency on the number of wave packets //

Density distribution in YZ plane

Matter Neutron Proton K-

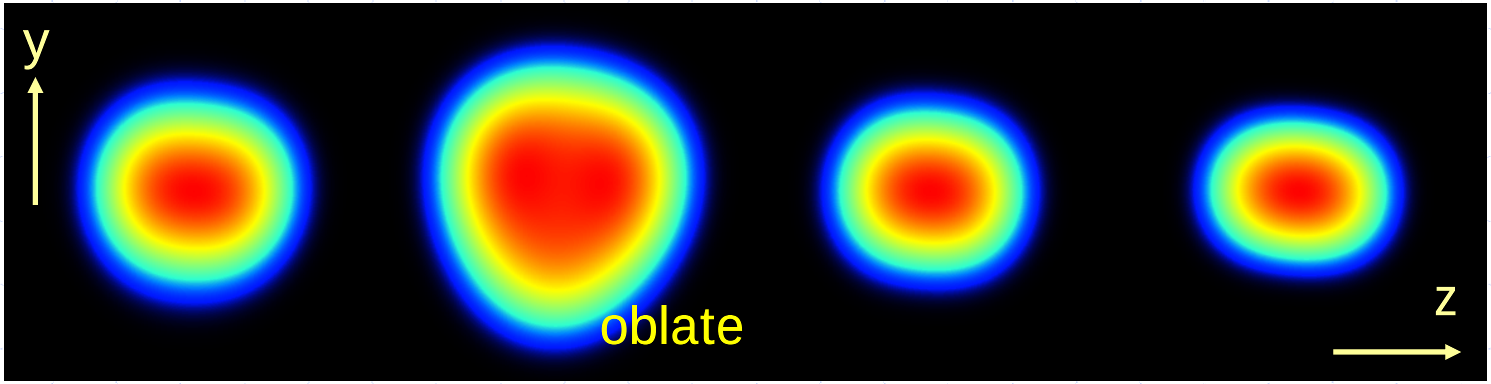
2 Gaussian

$\beta = 0.19$
$\gamma = 0^\circ$



3 Gaussian

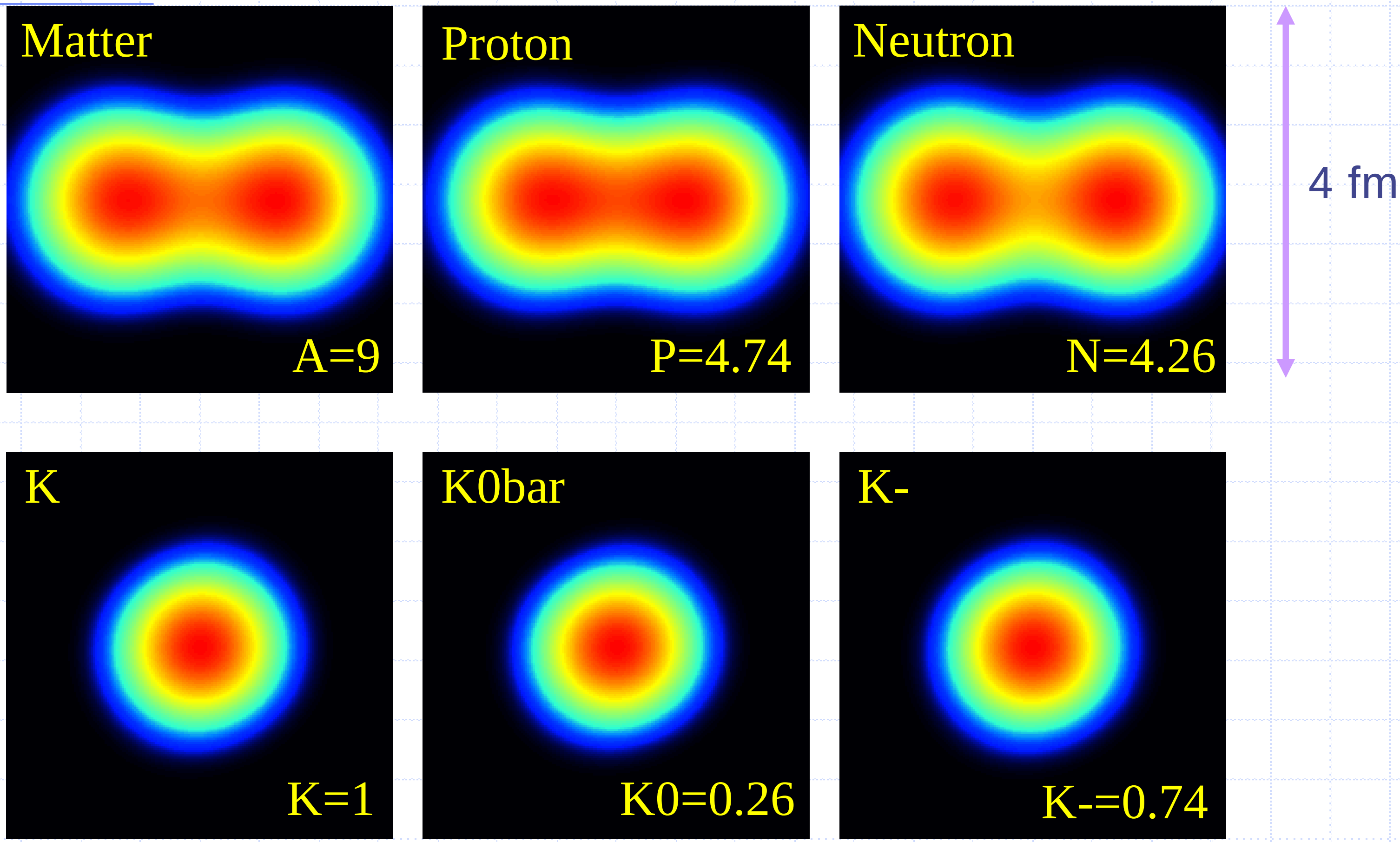
$\beta = 0.13$
$\gamma = 55^\circ$



As expected, the neutron distribution is strongly dependent on the number of wave packets.
As a result, the shape of the system is largely changed.

^{9}BK^{-} (tentative)

Density distributions of $|P_{Tz}\Phi\rangle$ in ZX plane.



Similar structure to ^{8}BeK^{-}