Interactions between Octet Baryons in the SU_6 Quark Model and their Applications to Light Hypernuclei *

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The recent quark-model baryon-baryon interaction by the Kyoto-Niigata group is applied to the triton, hypertriton, $2\alpha\Lambda$ and $2\Lambda\alpha$ systems, in which a new three-cluster Faddeev formalism, using the 2-cluster resonating-group method (RGM) kernel, is developed for the exact treatment of the Pauli forbidden states between clusters.

1. 3-CLUSTER FADDEEV FORMALISM USING THE 2-CLUSTER RGM KERNEL

The QCD-inspired spin-flavor SU_6 quark model for the baryon-baryon interaction, proposed by the Kyoto-Niigata group, is a unified model for the complete baryon octet ($B_8 = N$, Λ, Σ and Ξ), which has achieved very accurate descriptions of the NN and YN interactions. [1–3] In particular, the nucleon-nucleon (NN) interaction of the most recent model fss2 [2] is accurate enough to compare with the modern realistic meson-exchange models. These quark-model interactions can be used for realistic calculations of few-baryon and few-cluster systems, once an appropriate three-body equation is developed for the pairwise interactions described by the RGM kernel. The desired 3-cluster equation should be able to deal with the non-locality and the energy-dependence intrinsically involved in the quark-exchange RGM kernel. Furthermore, the quark-model description of the hyperon nucleon (YN) and the hyperon hyperon (YY) interactions in the full coupled-channel formalism sometimes involves a Pauli forbidden state, which excludes the most compact spatial configuration, resulting in the strongly repulsive nature of the interactions in some particular channels. We have recently formulated a new 3-cluster equation which uses two-cluster RGM kernels explicitly. [4] This equation exactly eliminates 3-cluster redundant components by the orthogonality of the total wave function

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to the pairwise two-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently determined. This equation is entirely equivalent to the Faddeev equation which uses a modified singularity-free T-matrix (which we call the RGM T-matrix) constructed from the two-cluster RGM kernel. We first applied this formalism to a 3-dineutron system and the 3α system, and obtained complete agreement between the Faddeev calculations and the variational calculations which use the translationally invariant harmonic-oscillator (h.o.) basis. [4,5] For the 3α system, the input is the 3-range Minnesota force (MN force) with the exchange mixture $u = 0.946869$, and the h.o. width parameter, $\nu = 0.257\ \mathrm{fm}^{-2}$, is used for the $(0s)^4$ α-clusters. The 2α phase shifts are nicely reproduced in the 2α RGM. We find that the 3α ground-state energies obtained by solving the present 3α Faddeev equations are only about $1.0 \sim 1.7$ MeV higher than those of the full microscopic 3α RGM calculations. [5]

2. TRITON AND HYPERTRITON FADDEEV CALCULATIONS

The present 3-cluster RGM formalism was applied to the Faddeev calculations of the $3N$ bound state [6] and $(2N\Lambda)$-$(2N\Sigma)$ system for the hypertriton, employing the off-shell T-matrices which are derived from the non-local and energy-dependent RGM kernel for our quark-model NN and YN interactions of fss2 and FSS. The model fss2 yields the triton energy $E(^3\mathrm{H}) = -8.519$ MeV in the 50 channel calculation, when the np interaction is employed for any NN pairs in the isospin basis. [7] The charge rms radii for ^{3}H and ^{3}He are also correctly reproduced. These results are the closest to the experiments among many Faddeev calculations employing modern realistic NN interactions. A characteristic description of the short range correlations in the quark model is essential to reproduce the large binding energy and the correct size of the three-nucleon bound state without reducing the D-state probability of the deuteron. For the hypertriton calculation, the exact treatment of the ΛN-ΣN coupling and the resulting Pauli forbidden state with the SU_3 $(11)_s$ symmetry is very important to obtain the precise result. In the final calculation with 150 ΛNN and ΣNN channels included, we find $B_\Lambda(^3_\Lambda\mathrm{H}) = 289$ keV and the ΣNN component $P_{\Sigma NN} = 0.805\,\%$ for the fss2 prediction. For our previous model FSS, we obtain $B_\Lambda(^3_\Lambda\mathrm{H}) = 878$ keV and $P_{\Sigma NN} = 1.36\,\%$. Since ${B_\Lambda}^{\mathrm{exp}} = 130 \pm 50$ keV, the fss2 result is slightly overbound, which implies that the $^1S_0$ attraction of the ΛN interaction is slightly too attractive in comparison to the $^3S_1$ attraction. From these results, we can extrapolate the desired difference of the $^1S_0$ and $^3S_1$ phase shifts at the maximum values. It turns out to be $3^\circ \sim 7^\circ$ more attractive in the $^1S_0$ state, which is consistent with the result in [8] which uses more simplified effective interactions.

3. $2\alpha\Lambda$ FADDEEV CALCULATION FOR $^9_\Lambda$Be

The formalism is now applied to the $2\alpha\Lambda$ Faddeev calculation for $^9_\Lambda$Be, by using the 2α RGM kernel and the $\Lambda\alpha$ folding potentials for various ΛN effective forces. The effective ΛN force, denoted by SB (Sparenberg-Baye potential) in Table 1, is constructed from the $^1S_0$ and $^3S_1$ phase shifts predicted by the YN sector of the model fss2 [3], by using an inversion method based on supersymmetric quantum mechanics. [9] These are simple 2-range Gaussian potentials which reproduce the low-energy behaviour of the ΛN phase shifts obtained by the full coupled-channel calculations. Since any central and single-channel effective ΛN force leads to the well-known overbinding problem of ${}^{5}_{\Lambda}$He [10] by about 2 MeV (in the present case, it is 1.63 MeV), the attractive part of the ${}^{3}S_{1}$ ΛN potential is adjusted to reproduce the correct binding energy $E^{\rm exp}({}^{5}_{\Lambda}{\rm He}) = -3.12 \pm 0.02$ MeV. The odd-state ΛN force is assumed to be zero. The partial waves up to $\lambda_{\rm Max} = \ell_{1{\rm Max}} = 6$ are included both in the 2α and $\Lambda\alpha$ channels. The direct and exchange Coulomb kernel between two α-clusters is introduced at the nucleon level with the cut-off radius, $R_C = 14$ fm (central case) or 10 fm (ℓs included). Table 1 shows the ground-state (0^+) and the excited-state (2^+) energies of ${}^{9}_{\Lambda}$Be, predicted by the SB and the other various ΛN potentials used by Hiyama *et al.* [11]. In the present calculations using only the central force, the SB potential with the pure Serber character can reproduce the ground-state and excited-state energies within the accuracy of 100 - 200 keV. Table 1 also shows simple estimates of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, due to the spin-orbit interaction predicted by fss2 and FSS.

4. $2\Lambda\alpha$ FADDEEV CALCULATION FOR ${}^{6}_{\Lambda\Lambda}$He

Next, we use the $\Lambda\alpha$ T-matrix, used in the $2\alpha\Lambda$ Faddeev calculation, to calculate the ground-state energy of ${}^{6}_{\Lambda\Lambda}$He. The full coupled-channel T-matrices of fss2 and FSS with the strangeness $S = -2$ and the isospin $I = 0$ [3] are employed for the $\Lambda\Lambda$ RGM T-matrix. We find that the Hiyama's 3-range Gaussian $\Lambda\Lambda$ potential and our Faddeev calculation using FSS yield very similar results with the large $\Delta B_{\Lambda\Lambda}$ values (defined by $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}{\rm He}) - 2B_{\Lambda}({}^{5}_{\Lambda}{\rm He})$) about 3.6 - 3.7 MeV, since the $\Lambda\Lambda$ phases shifts predicted by both interactions increase up to about 40°. The improved quark model fss2 yields $\Delta B_{\Lambda\Lambda} = 1.41$ MeV. (If we use the $\Lambda\Lambda$ single-channel T-matrix, this number is reduced to $\Delta B_{\Lambda\Lambda} = 1.14$ MeV.) If we use a simple 2-range Gaussian potential, $V_{\Lambda\Lambda}$(SB), derived from the fss2 ${}^{1}S_{0}$ $\Lambda\Lambda$ phase shift by the supersymmetric inversion method, we obtain $\Delta B_{\Lambda\Lambda} = 1.90$ MeV. We think that the 0.5 MeV difference between our fss2 result and

Table 1
The ground-state energy $E_{\rm gr}(0^+)$ and the 2^+ excitation energy $E_{\rm x}(2^+)$ in MeV, calculated by solving the Faddeev equation for the $2\alpha\Lambda$ system. In the last column, ΔE is a simple estimate of the ℓs splitting for the $5/2^+$ and $3/2^+$ excited states, using the $\boldsymbol{P}$=0 Wigner transform of the fss2 and FSS ΛN LS interactions. The model fss2 involves an extra σ-meson contribution, which is indicated by "$+\sigma$".

$V_{\Lambda N}$	$E_{\rm gr}(0^+)$ (MeV)			$E_{\rm x}(2^+)$	ΔE (keV)	
	ours	Hiyama [11]	diff.	(MeV)	fss2	FSS
SB	-6.837	$-$	$-$	2.915	$103+\sigma$	164
NS	-6.742	-6.81	0.07	2.916	$97+\sigma$	154
ND	-7.483	-7.57	0.09	2.935	$106+\sigma$	169
NF	-6.906	-7.00	0.09	2.930	$86+\sigma$	138
JA	-6.677	-6.76	0.08	2.919	$86+\sigma$	138
JB	-6.474	-6.55	0.08	2.911	$85+\sigma$	136
Exp't	-6.62 ± 0.04			3.029(3)/3.060(3)	43 ± 5	

the $V_{\Lambda\Lambda}$(SB) result is probably because we neglected the full coupled-channel effect of the $\Lambda\Lambda\alpha$ channel to the $\Xi N\alpha$ and $\Sigma\Sigma\alpha$ channels. We should also keep in mind that in all of these 3-cluster calculations the Brueckner rearrangement effect [10] of the α-cluster with the magnitude of about -1 MeV (repulsive) is very important. It is also reported in [12] that the quark Pauli effect among the α cluster and the Λ hyperons gives a non-negligible repulsive contribution of the order of 0.1 - 0.2 MeV for the Λ separation energy of ${}^{\ 6}_{\Lambda\Lambda}$He, even when we assume a rather compact ($3q$) size of $b = 0.6$ fm. Taking all of these effects into consideration, we can conclude that the present results by fss2 are in good agreement with the recent experimental value $\Delta B^{\rm exp}_{\Lambda\Lambda} = 1.01 \pm 0.20$ MeV [13] deduced from the Nagara event.

5. SUMMARY

The 3-cluster Faddeev formalism using the 2-cluster RGM kernel opens a way to solve few-baryon systems interacting by the quark-model baryon-baryon interaction without spoiling the essential features of the RGM kernel; i.e., the non-locality, the energy dependence and the existence of the pairwise Pauli-forbidden state. It can also be used for the 3-cluster systems involving α-clusters, like the ${}^{9}_{\Lambda}$Be system. A nice point of this formalism is that the underlying NN and YN interactions are more direcly related to the structure of the hypernuclei than the models assuming simple 2-cluster potentials. In particular, we have found that the most recent quark-model interaction, the model fss2, yields a realistic description of many systems including the triton, hypertriton, ${}^{9}_{\Lambda}$Be and ${}^{\ 6}_{\Lambda\Lambda}$He.

REFERENCES

1. Y. Fujiwara, C. Nakamoto and Y. Suzuki, Phys. Rev. C54 (1996) 2180.
2. Y. Fujiwara, T. Fujita, M. Kohno, C. Nakamoto and Y. Suzuki, Phys. Rev. C65 (2002) 014002.
3. Y. Fujiwara, M. Kohno, C. Nakamoto and Y. Suzuki, Phys. Rev. C64 (2001) 054001.
4. Y. Fujiwara, H. Nemura, Y. Suzuki, K. Miyagawa and M. Kohno, Prog. Theor. Phys. 107 (2002) 745.
5. Y. Fujiwara, Y. Suzuki, K. Miyagawa, M. Kohno and H. Nemura, Prog. Theor. Phys. 107 (2002) 993.
6. Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki and H. Nemura, Phys. Rev. C66 (2002) 021001 (R).
7. Y. Fujiwara, K. Miyagawa, M. Y. Suzuki, M. Kohno and H. Nemura, Nucl. Phys. A721 (2003) 983c.
8. H. Nemura, Y. Akaishi and Y. Suzuki, Phys. Rev. Lett. **89** (2002) 142504.
9. J.-M. Sparenberg and D. Baye, Phys. Rev. C55 (1997) 2175.
10. M. Kohno, Y. Fujiwara and Y. Akaishi, Phys. Rev. C68 (2003) 034302.
11. E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Prog. Theor. Phys. 97 (1997) 881.
12. Y. Suzuki and H. Nemura, Prog. Theor. Phys. 102 (1999) 203.
13. H. Takahashi et al., Phys. Rev. Lett. 87 (2001) 212502.