Three- and four-body structure of light double Λ hypernuclei

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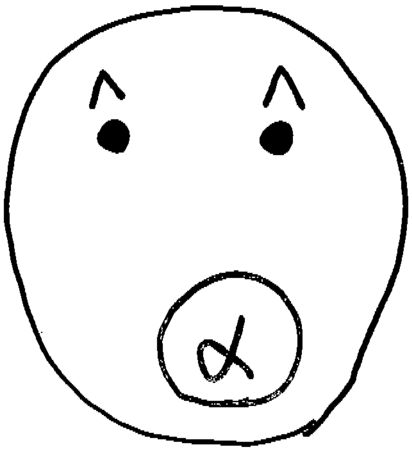
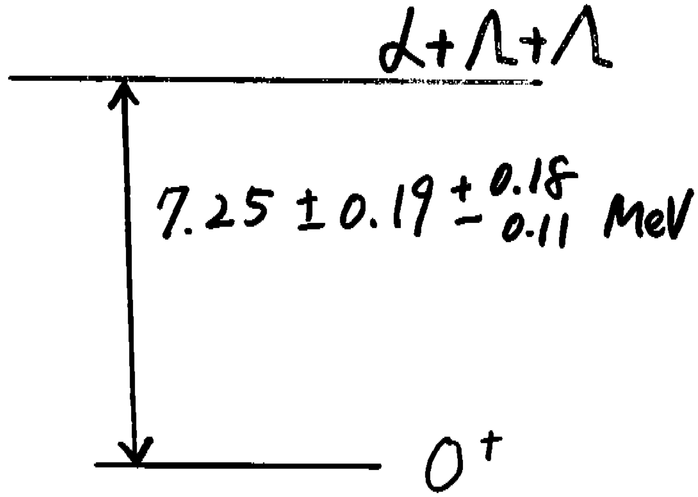
$^{6}_{\Lambda\Lambda}\mathrm{He}$, $^{7}_{\Lambda\Lambda}\mathrm{He}$, $^{7}_{\Lambda\Lambda}\mathrm{Li}$, $^{8}_{\Lambda\Lambda}\mathrm{Li}$, $^{9}_{\Lambda\Lambda}\mathrm{Li}$, $^{9}_{\Lambda\Lambda}\mathrm{Be}$, $^{10}_{\Lambda\Lambda}\mathrm{Be}$

Observation of $^{6}_{\Lambda\Lambda}\mathrm{He}$: NAGARA event

H. Takahashi et al., Phys. Rev. Lett. <u>87</u>, 212502 (2001).



$^{6}_{\Lambda\Lambda}\mathrm{He}$



$$\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20 ^{+0.18}_{-0.11} \ \mathrm{MeV}$$

This observation is giving a great contribution to the study of the structure of double Λ hypernuclei as the entrance to the multi-strangeness world.

The analysis of KEK-E373: in progress

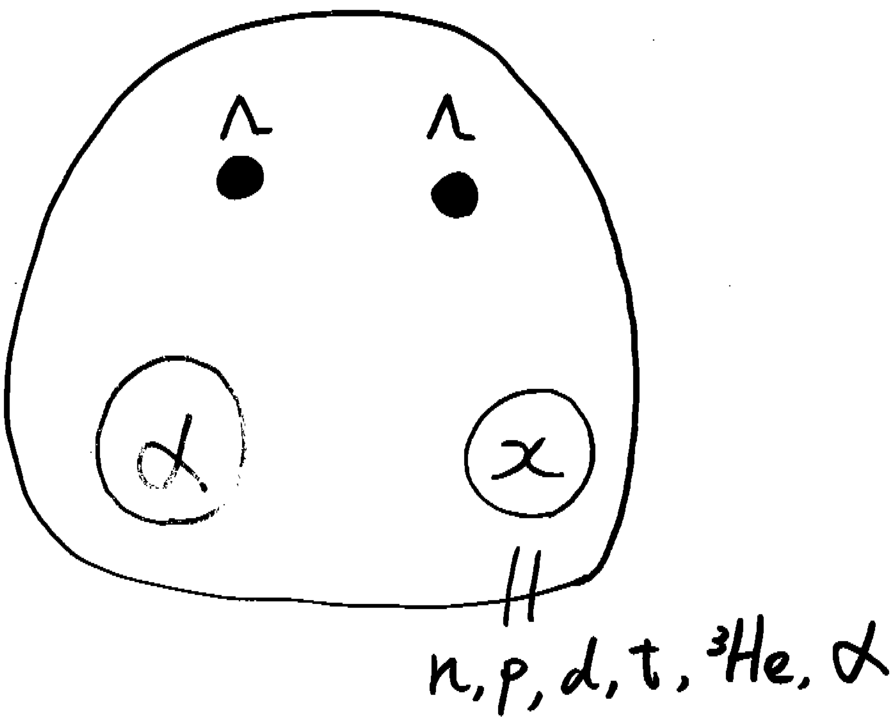
It is possible to observe the ground state as well as the excited states as bound states of more double Λ hypernuclei.

However, we cannot determine spin-parities of the resultant double Λ hypernuclei.

Furthermore, it will be planned to produce many double Λ hypernuclei in the future at J-PARC and GSI facilities.

Therefore, it is required to predict binding energies of the ground states as well as the excited states precisely before measurement.

Hoping, to measure much more double Λ hypernuclei at the analysis of E373, J-PARC and GSI in the future, I predicted the level structure of A=7 ~ 10 double Λ hypernuclei within the framework of α+x+Λ+Λ 4-body model.



E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Phys. Rev. C 66, 024007 (2002)

Gaussian Expansion Method

developed by kyushu Univ.
kamimura

(1) 3-cluster structure of light nuclei

(2) Coulomb 3-body muonic molecular ions appearing in the muon-catalyzed fusion cycles (1987～)

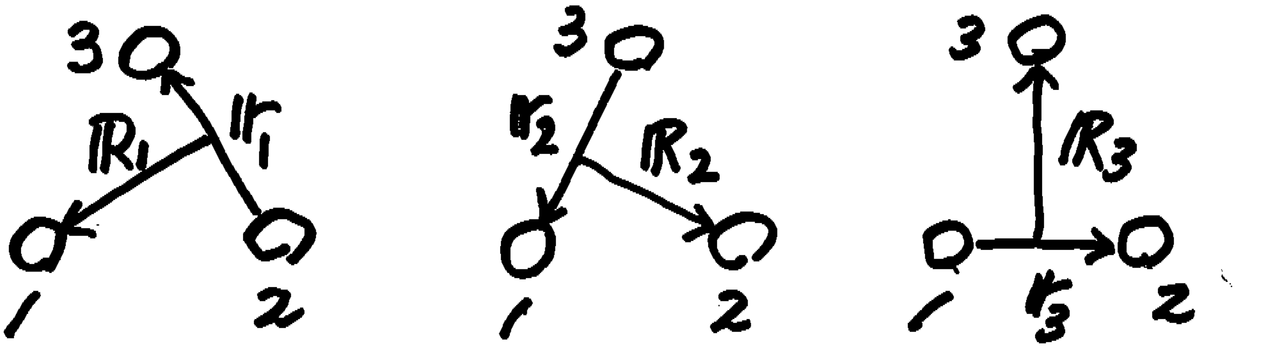
(3) 3-nucleon bound states with realistic NN and 3N force (1988)

(4) Meta-stable anti-protonic helium atom (He^{++}+p+e)(1995～)

Gaussian Expansion Method (GEM)

1988~ by Kyushu Group

3-Body case



c=1 c=2 c=3

$$\Psi_{JM} = \Phi^{(1)}_{JM}(\mathbf{r}_1, \mathbf{R}_1) + \Phi^{(2)}_{JM}(\mathbf{r}_2, \mathbf{R}_2) + \Phi^{(3)}_{JM}(\mathbf{r}_3, \mathbf{R}_3)$$

c=1, 2, 3

$$\Phi^{(c)}_{JM}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{n\ell NL} A^{(c)}_{n\ell NL} \left[\phi_{n\ell}(\mathbf{r}_c) \otimes \chi_{NL}(\mathbf{R}_c) \right]_{JM}$$

Gaussian basis functions

$$\phi_{n\ell m}(\mathbf{r}) = r^{\ell} e^{-\left(\frac{r}{r_n}\right)^2} Y_{\ell m}(\hat{\mathbf{r}})$$

$$\chi_{NLM}(\mathbf{R}) = R^{L} e^{-\left(\frac{R}{R_N}\right)^2} Y_{LM}(\hat{\mathbf{R}})$$

Range: geometric progression

$$r_n = r_1 a^{n-1} \quad (n = 1 \sim n_{max})$$

$$R_N = R_1 A^{N-1} \quad (N = 1 \sim N_{max})$$

Schröd. Eq. $$(H - E)\Psi_{JM} = 0$$

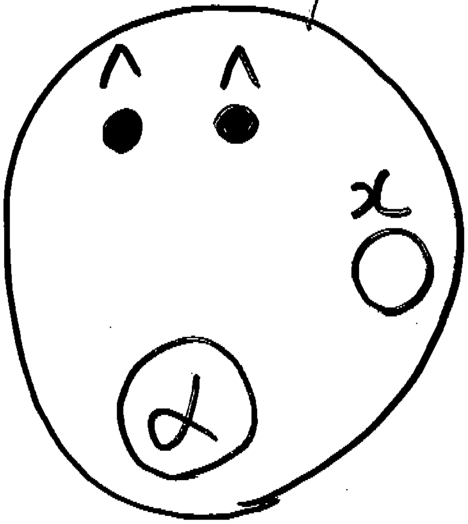
is solved with Rayleigh-Ritz variational princip.

I developed this calculational method to 4-body systems.

More details of the method and its applications are written in this invited review paper:

E. Hiyama, Y. Kino and M. Kamimura.
"Gaussian Expansion Method for Few-Body Systems"
Progress of Particle and Nuclear Physics
<u>52</u> (2003) 223-307.

4-body model



$$\underbrace{\alpha + x}_{\text{core nucleus}} + \Lambda + \Lambda$$

$$^{5}\mathrm{He} = \alpha + n$$

$$^{5}\mathrm{Li} = \alpha + p$$

$$^{6}\mathrm{Li} = \alpha + d$$

$$^{7}\mathrm{Li} = \alpha + t$$

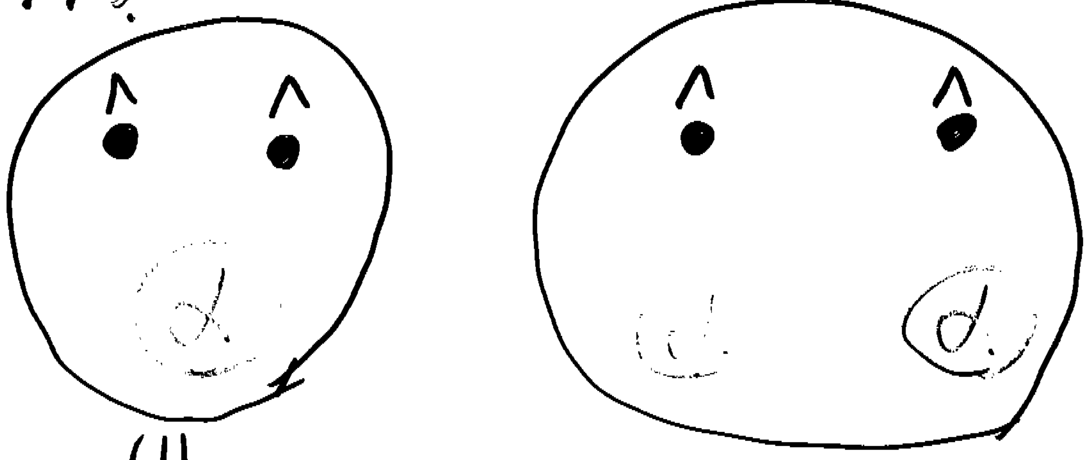
$$^{7}\mathrm{Be} = \alpha + {}^{3}\mathrm{He}$$

$$^{8}\mathrm{Be} = \alpha + \alpha$$

$$\Downarrow$$

Often employed in the cluster-model study of light nuclei

- H. Takaki, Wang Xi-cang, and H. Bandō, Prog. Theor. Phys. 83, 13 (1989)
- A. R. Bodmer, Q. N. Usmani and J. Carlson, Nucl. Phys. A422, 510 (1984)
- A. R. Bodmer and Q. N. Usmani, Nucl. Phys. A468, 653 (1987)
- E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Prog. Theor. Phys. 97, 881 (1997)

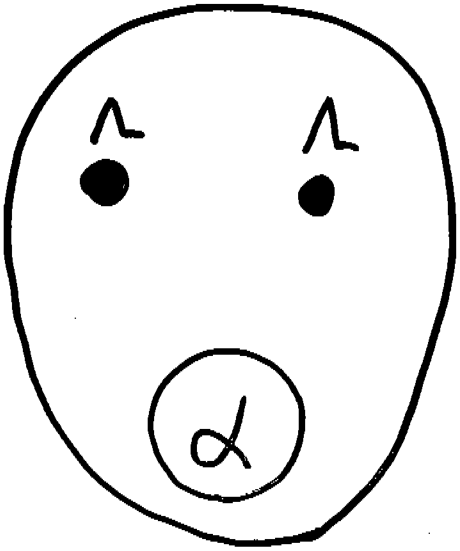


$^{6}_{\Lambda\Lambda}$He $\qquad$ $^{10}_{\Lambda\Lambda}$Be

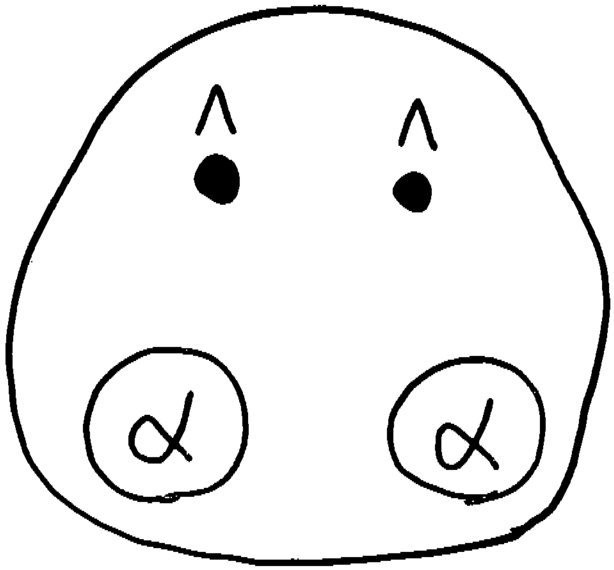
Eariler data $^{6}_{\Lambda\Lambda}$He : $\Delta B_{\Lambda\Lambda} = 4.6 \pm 0.5$ MeV

$^{10}_{\Lambda\Lambda}$Be : $\Delta B_{\Lambda\Lambda} = 4.28 \pm 0.4$ MeV

I. N. Filikhin and A. Gal., Phys. Rev. C <u>65</u> 041001(R) (2002)



$^{6}_{\Lambda\Lambda}$He



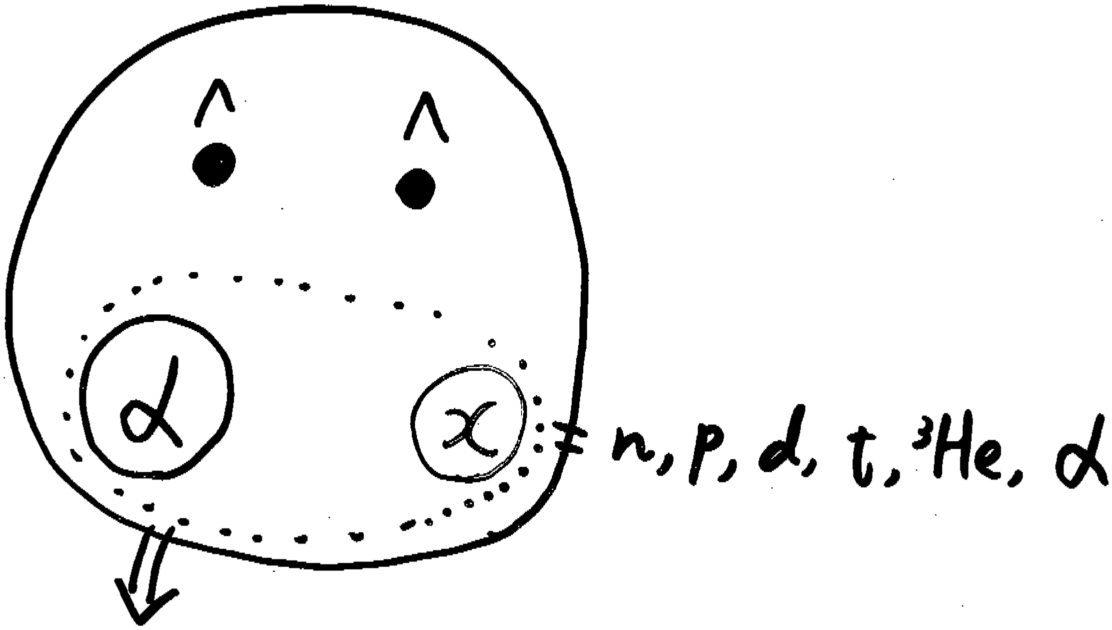
$^{10}_{\Lambda\Lambda}$Be

Faddeev Yakubovsky

calculation

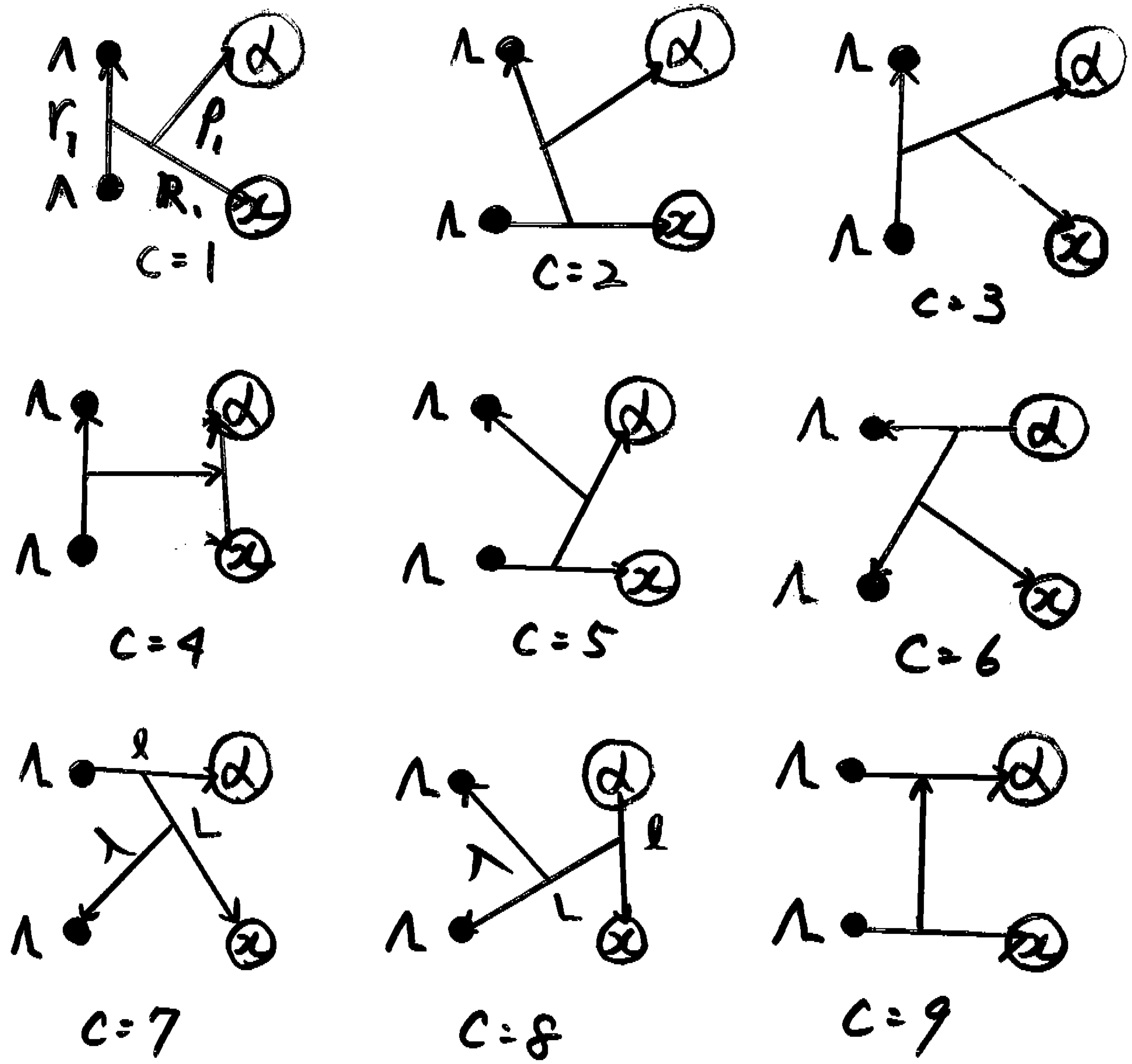
NAGARA event $\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.2$ MeV

E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, and Y. Yamamoto, Phys. Rev. C 66, 024007 (2002)



nuclear parts are well represented by $\alpha + x$ cluster model.

The extensive calculations are presented for the first time for $A = 7 \sim 9$ double Λ hypernuclei.



$$\Psi_{JM}({}^{A}_{\Lambda}Z) = \sum_{c=1}^{9} \Phi_{JM}(\mathbf{r}_c, \mathbf{R}_c, \boldsymbol{\rho}_c)$$

<u>Spatial part of each amplitude</u>

$$\Phi_{JM}(\mathbf{r}, \mathbf{R}, \boldsymbol{\rho}) = \sum_{n\ell NL\nu\lambda} \underline{C_{n\ell, NL, \nu\lambda}} \left[\left[\phi_\ell(\mathbf{r})\, \psi_L(\mathbf{R})\right]_I \xi_\lambda(\boldsymbol{\rho})\right]_{JM}$$

$$\phi_{\ell m}(\mathbf{r}) = r^\ell e^{-(r/r_n)^2} Y_{\ell m}(\hat{\mathbf{r}})$$

$$\psi_{LM}(\mathbf{R}) = R^L e^{-(R/R_N)^2} Y_{LM}(\hat{\mathbf{R}})$$

$$\xi_{\lambda\mu}(\rho) = \rho^{\lambda} e^{-(\rho/\rho_\nu)^2} Y_{\lambda\mu}(\hat{\rho})$$

$$r_n = r_1 a^{n-1}, \quad n = 1 \sim n_{max}$$

$$R_N = R_1 A^{N-1}, \quad N = 1 \sim N_{max}$$

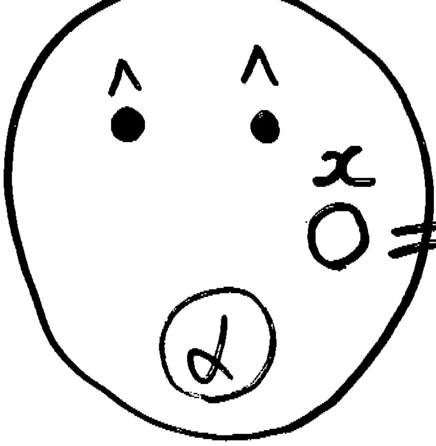
$$\rho_\nu = \rho_1 \alpha^{\nu-1}, \quad \nu = 1 \sim \nu_{max}$$

$$(H - E)\Psi = 0 \quad \text{Rayleigh-Ritz}$$

For the angular-momentum component of the wavefunction, the approximation with $l, L, \lambda \leq 2$ was found to be sufficient to obtain satisfactory convergence of the binding energies

But, no truncation of the interaction is made in the angular-momentum space.

4-body model



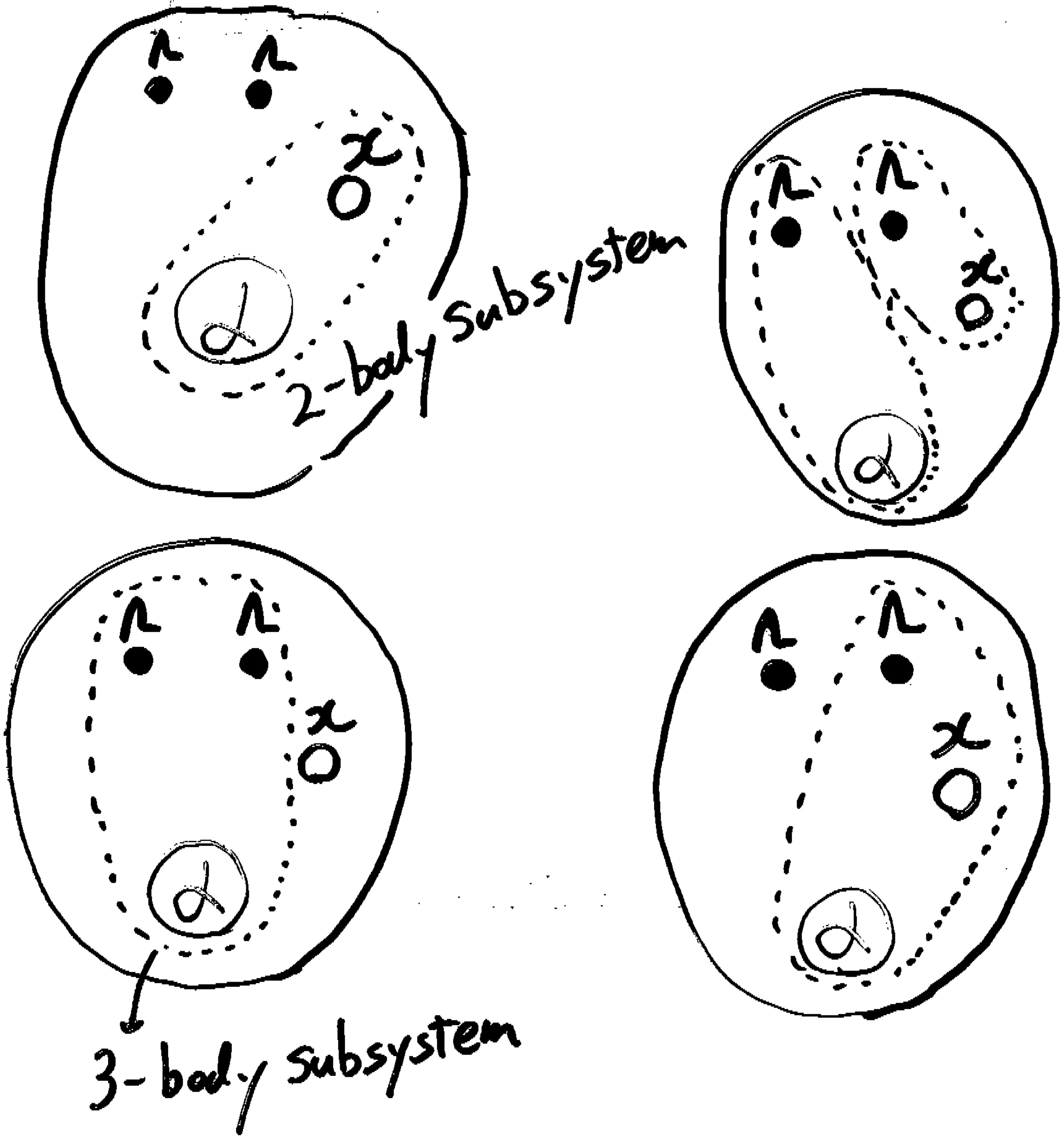
$\Rightarrow n, p, t, {}^3\mathrm{He}, \alpha$

$V_{\alpha\text{-}x}$: potentials popularly used in the cluster model

$V_{\alpha\text{-}\Lambda}$, $V_{x\text{-}\Lambda}$: derived by folding a familiar YNG ΛN interaction into the α or x-cluster densities

$V_{\Lambda\Lambda}$: adjusted the strength of ΛΛ interaction so as to reproduce the newly observed binding energy, $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

reproduce the observed energies of low lying levels of any 2- and 3-body subsystems



$x = n, p, d$
$t, {}^3He, \alpha$

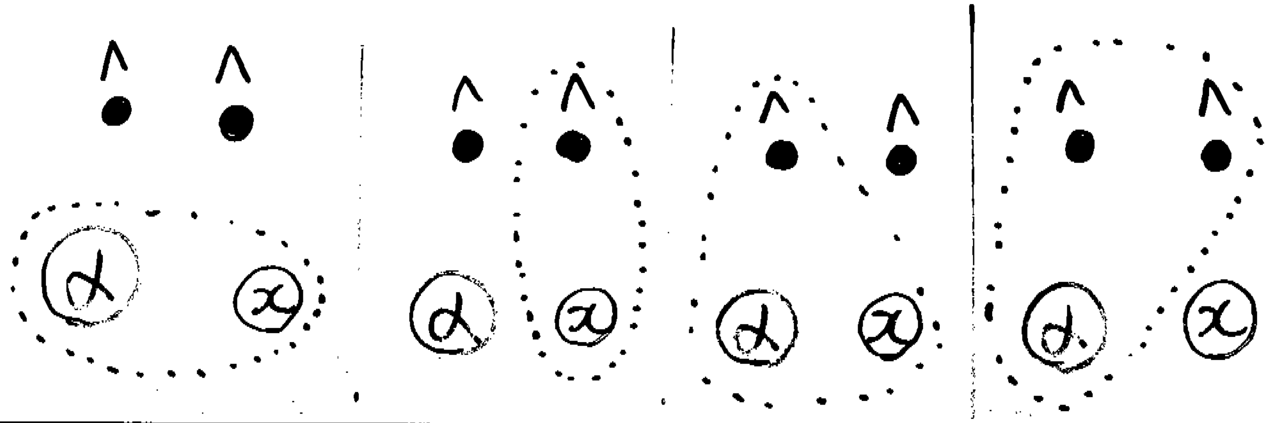
In the study of double Λ hypernuclei based on the α+x+Λ+Λ 4-body model, it is absolutely necessary to examine before 4-body calculation, whether the model with the interactions adopted is able to reproduce reasonably well the following observed quantities of the subsystems:

(i) Energies of the low-lying states and scattering phase shifts of the α+x nuclear systems

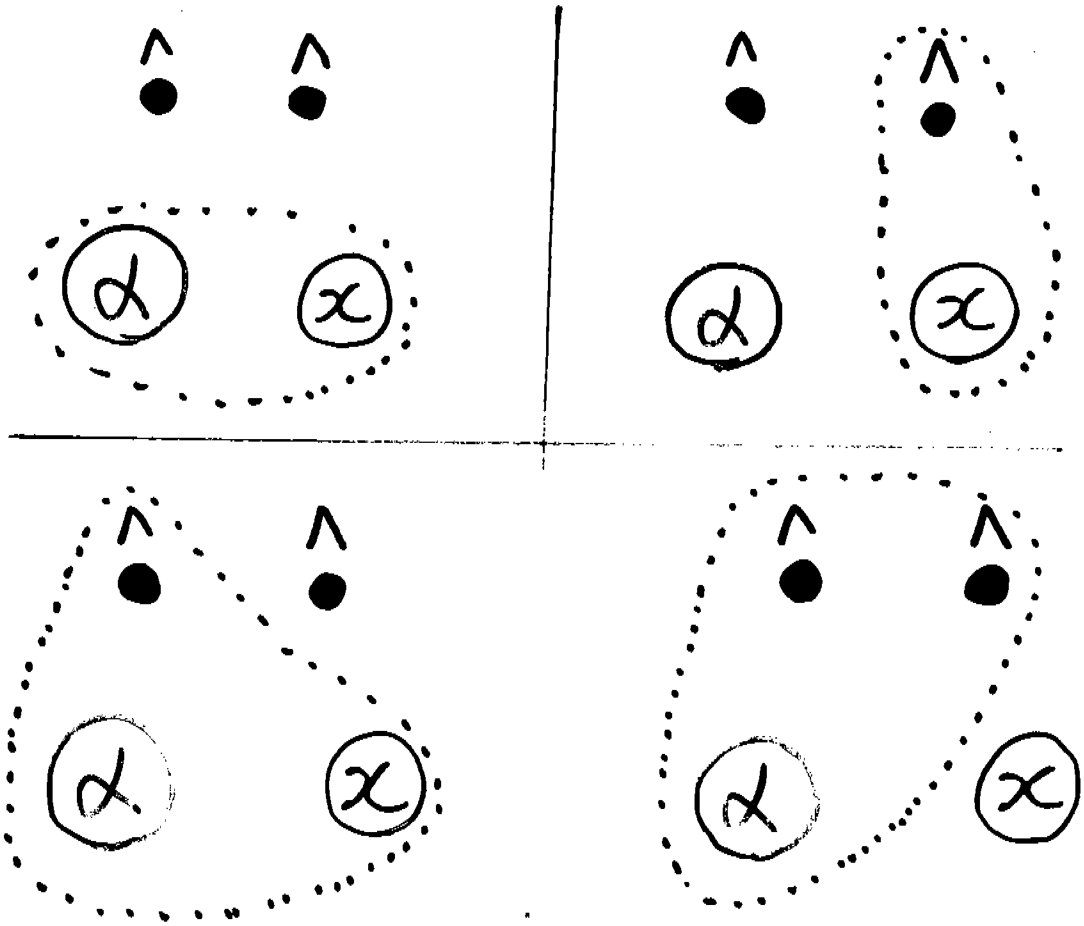
(ii) B_Λ of hypernuclei composed of x+Λ, x being n, p, d, t, ^{3}He and α

(iii) B_Λ of hypernuclei composed of α+x+Λ, x being n, p, d, t, ^{3}He and α

(iv) $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He$=\alpha+\Lambda+\Lambda$



In our model, the observed low-energy properties of the α+x nuclei and the existing Λ-binding energies of the x+Λ and α+x+Λ hypernuclei have been reproduced accurately.



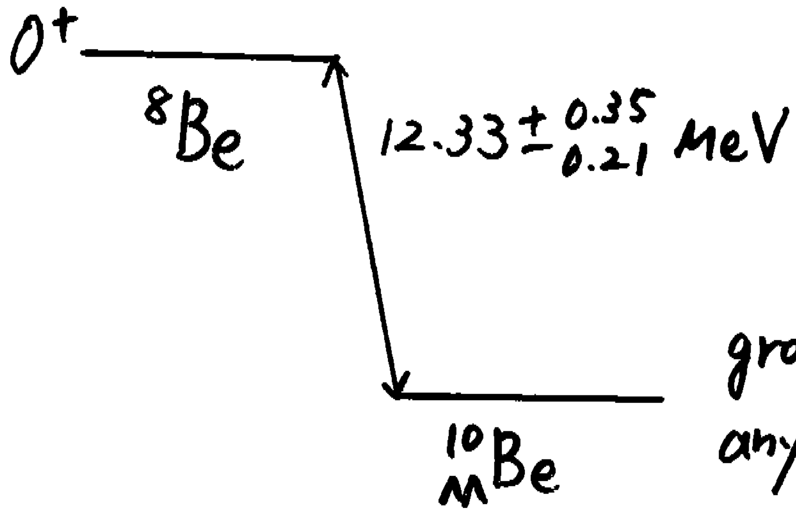
This encourages us to perform the 4-body calculation, with NO adjustable parameters at this stage, expecting high reliability of the results.

KEK-E373

$^{10}_{\Lambda}$Be

(i) Demachi-Yanagi event

discussed by Prof. Nakazawa in the Wednesday's plenary session



ground state ?
any excited state ?

(ii) Observation as the ground state some 40 years ago

M. Danysz et al., Nucl. Phys. **49**, 121 (1963).

$$B_{\Lambda} = 17.7 \pm 0.4 \text{ MeV} : {}^{10}_{\Lambda}\text{Be}(0^+) \to {}^{9}_{\Lambda}\text{Be}(1/2^+) + p + \pi^-$$

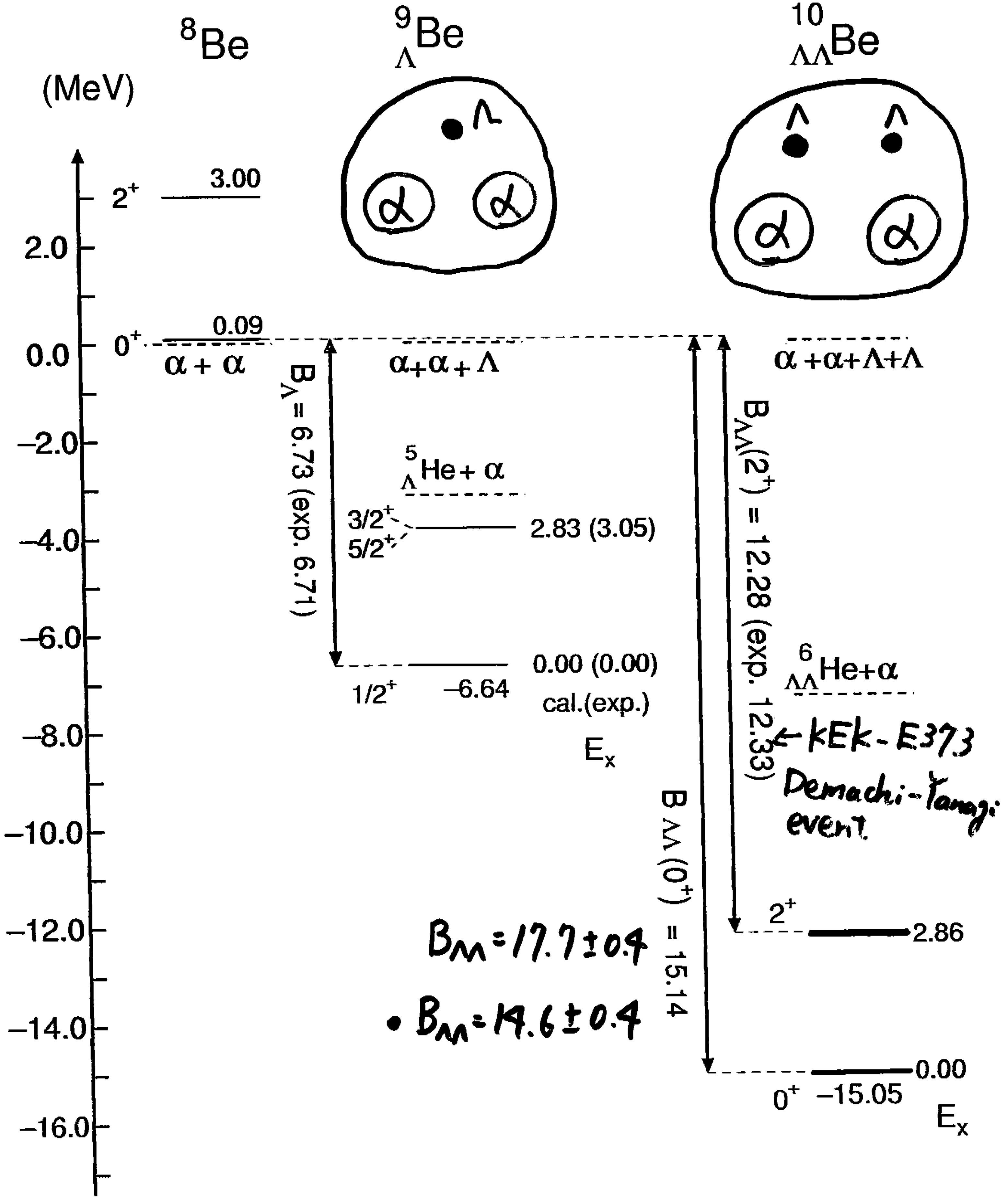
$$B_{\Lambda} = 14.6 \pm 0.4 \text{ MeV} : {}^{10}_{\Lambda}\text{Be}(0^+) \to {}^{9}_{\Lambda}\text{Be}(3/2^+, 5/2^+) + p + \pi^-$$

I. N. Filikhin and A. Gal., nucl-th/0203036
Nucl. Phys. A707 (2002) 491

(1) Does the $\Lambda\Lambda$ interaction which is designed so as to reproduce the binding energy of ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ reproduce the Demachi-Yamanagi event consistently?

(2) Which ground-state-candidate does this $\Lambda\Lambda$ interaction reproduce?

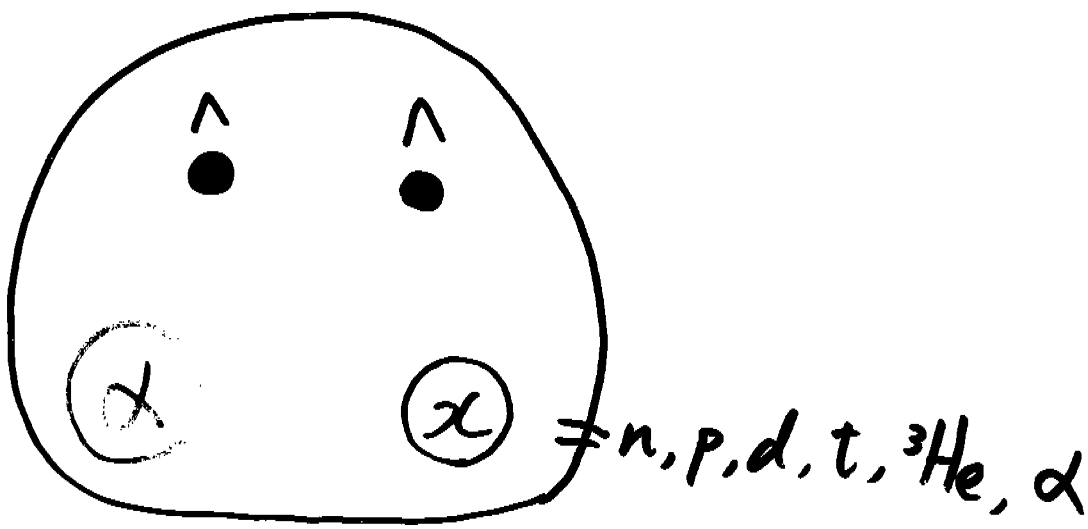
- $B_{\Lambda\Lambda} = 17.7 \pm 0.4$ MeV
- $B_{\Lambda\Lambda} = 14.6 \pm 0.4$ MeV

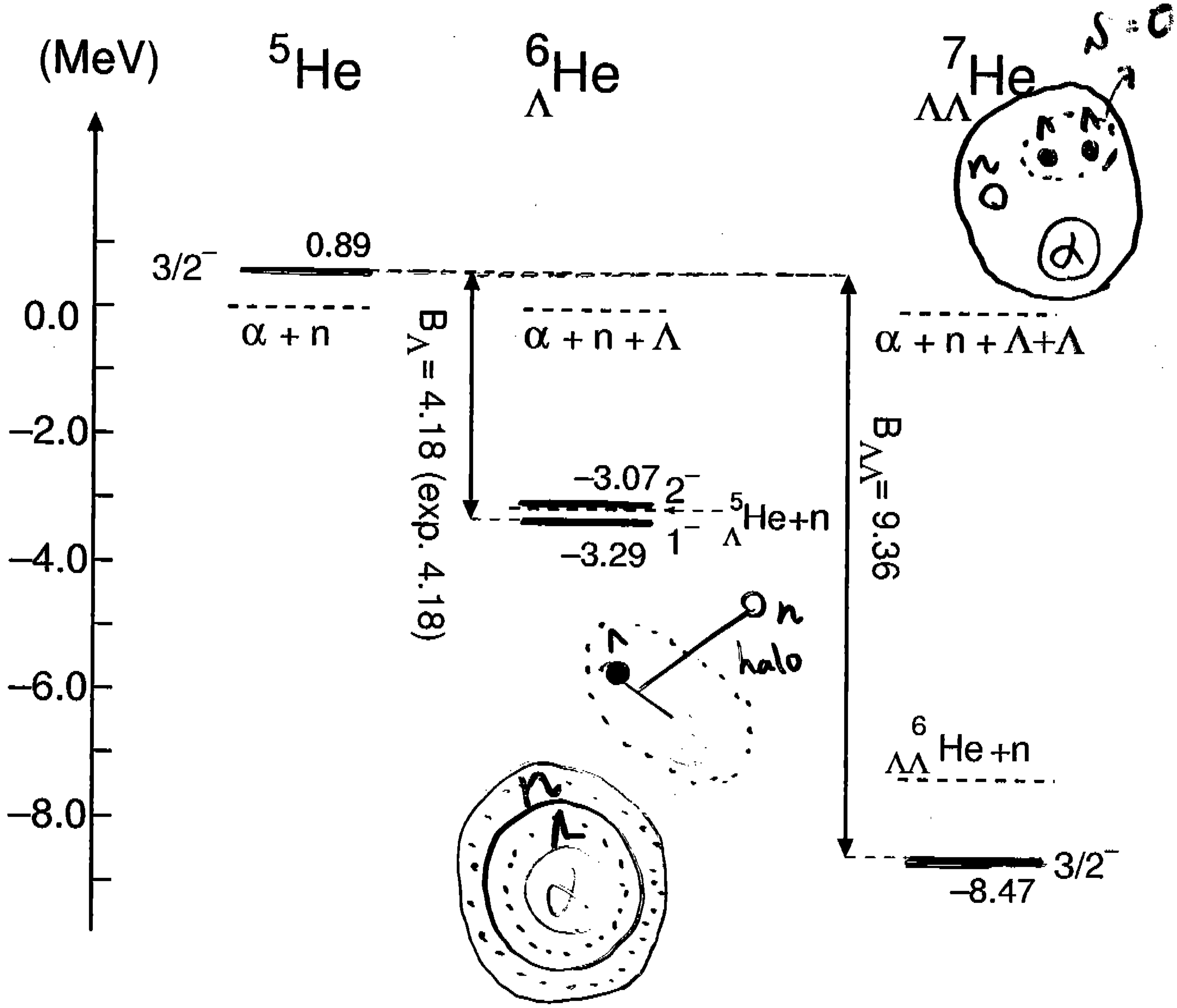


^{8}Be
$^{9}_{\Lambda}Be$
$^{10}_{\Lambda\Lambda}Be$
(MeV)
Λ
Λ Λ
α α
α α
2⁺ 3.00
0⁺ 0.09
α + α
α+α+Λ
α+α+Λ+Λ
B_Λ = 6.73 (exp. 6.71)
$^{5}_{\Lambda}He + \alpha$
3/2⁺
5/2⁺
2.83 (3.05)
0.00 (0.00)
1/2⁺ −6.64
cal.(exp.)
E_x
$B_{\Lambda\Lambda}(2^+)$ = 12.28 (exp. 12.33)
$^{6}_{\Lambda\Lambda}He+\alpha$
← KEK-E373
Demachi-Yanagi event
$B_{\Lambda\Lambda}(0^+)$ = 15.14
$B_{\Lambda\Lambda}$ = 17.7 ± 0.4
• $B_{\Lambda\Lambda}$ = 14.6 ± 0.4
2⁺ 2.86
0⁺ −15.05 0.00
E_x
2.0
0.0
−2.0
−4.0
−6.0
−8.0
−10.0
−12.0
−14.0
−16.0

Our 4-body calculation is predictive.

Therefore, hoping to observe many double Λ hypernuclei in the future experiment, we have predicted level structure for the double Λ hypernuclei with A=7～9 within the framework of an α+x+Λ+Λ 4-body model.

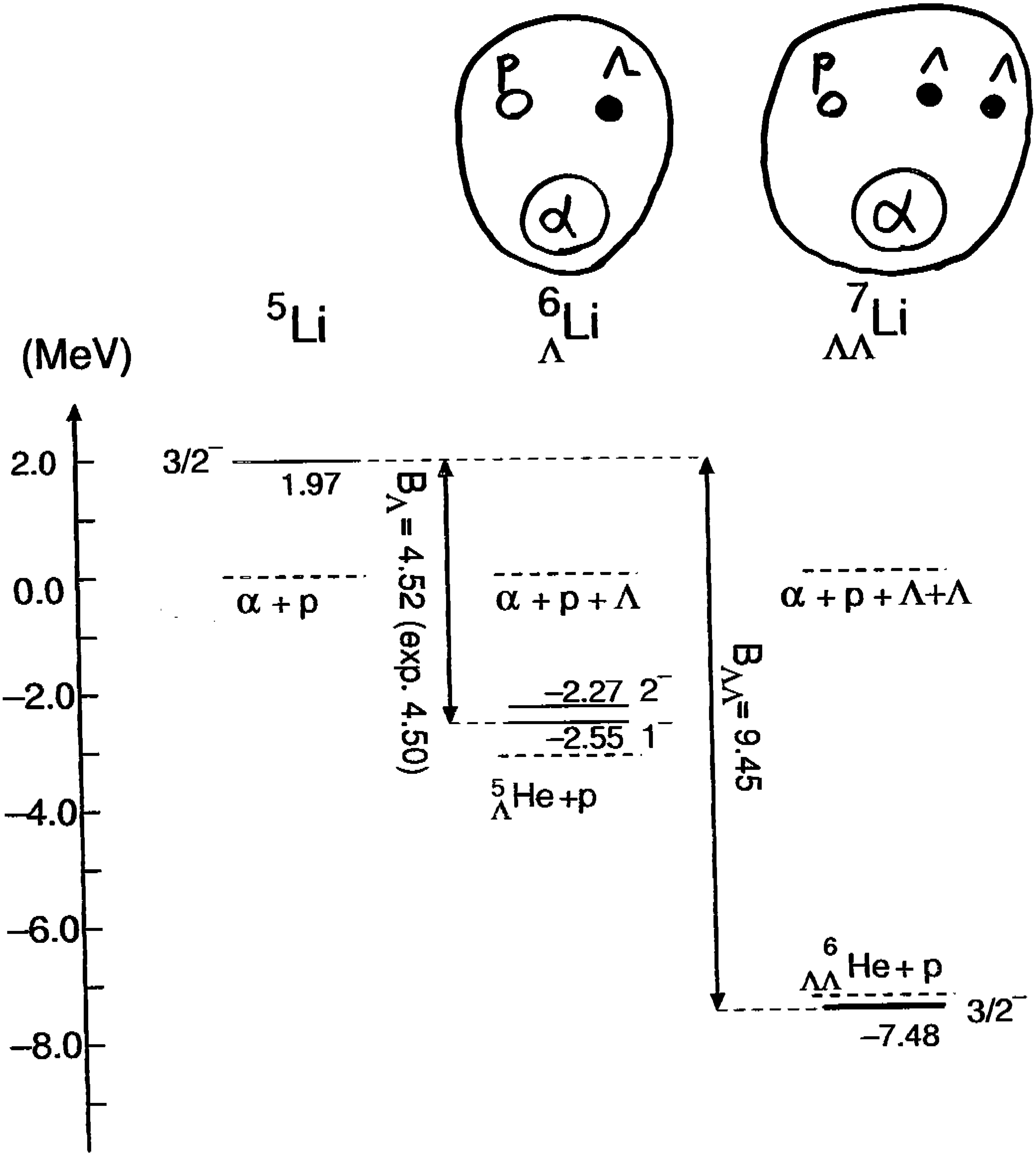




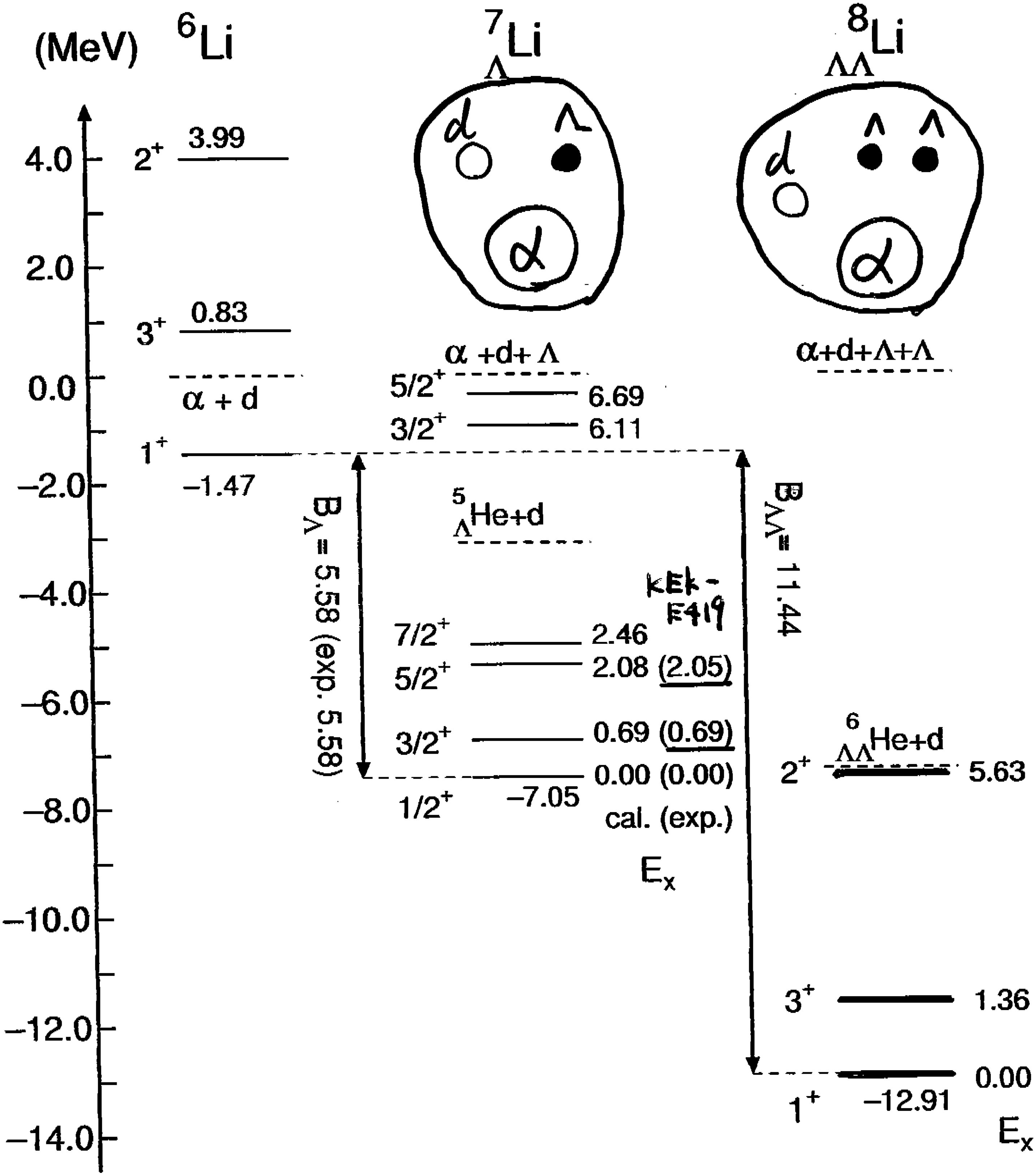
3層構造

Three-layer-Structure

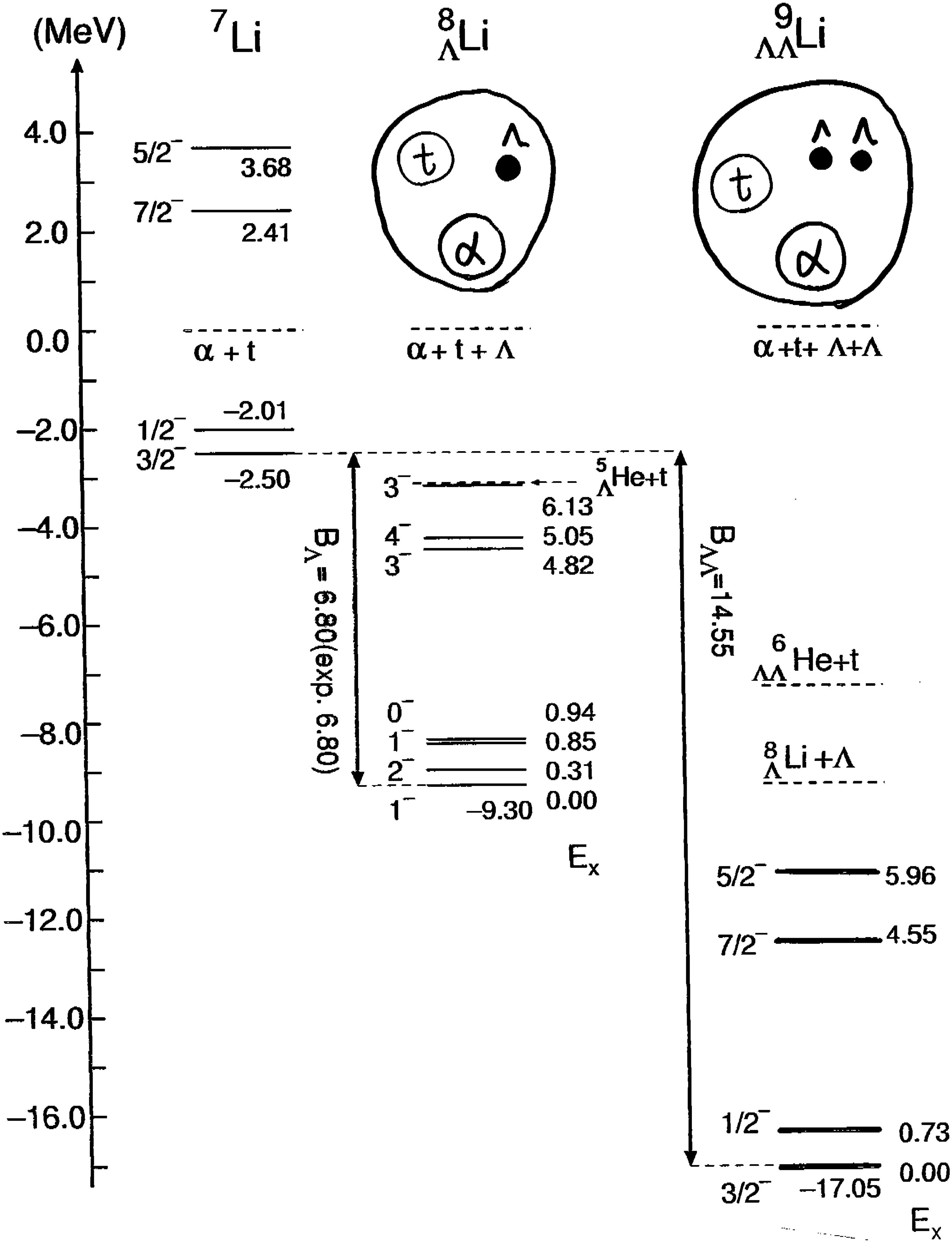
Phys. Rev. C53, 2075 (1996)



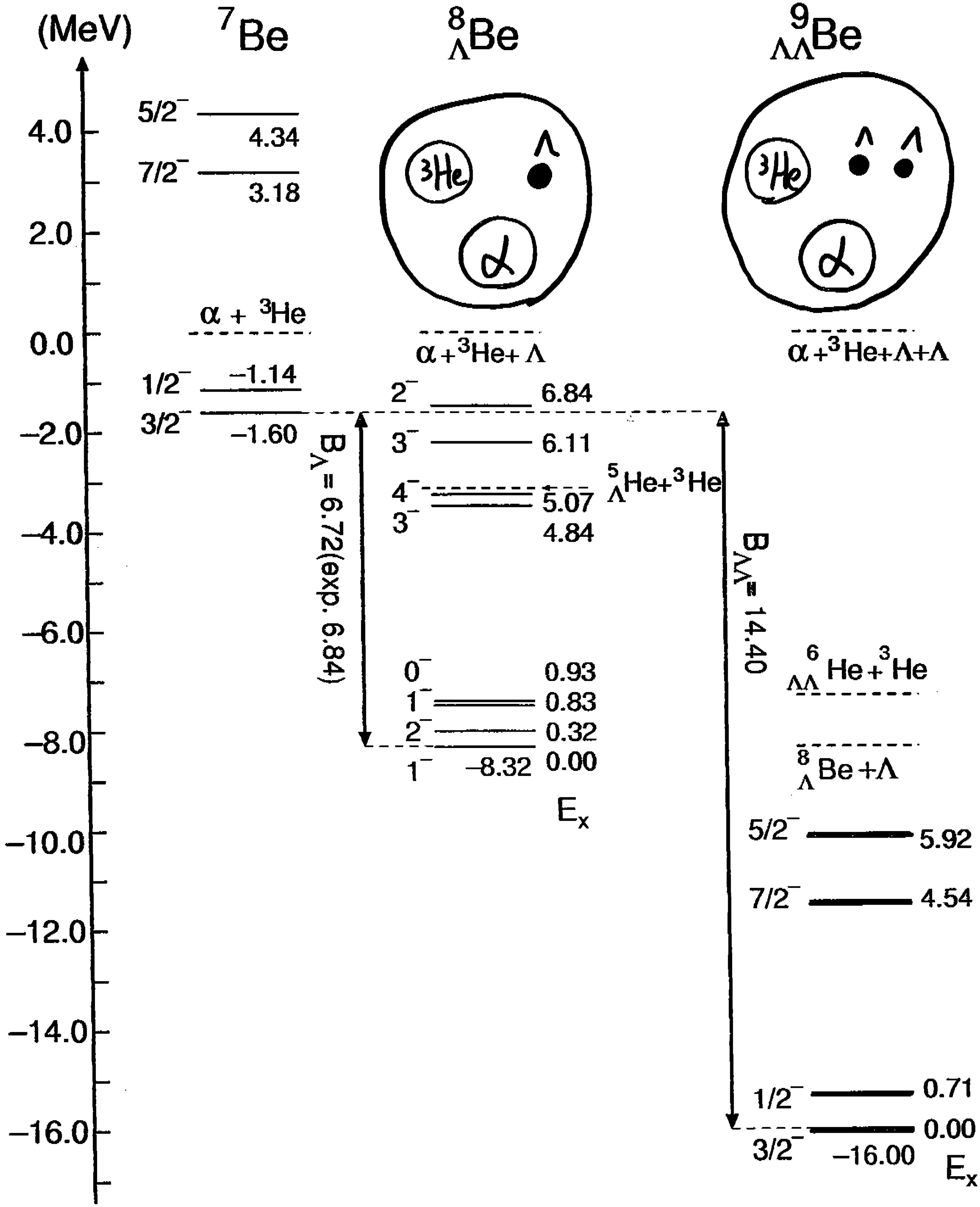
p
Λ
α
p
Λ
Λ
α
^{5}Li
$^{6}_{\Lambda}$Li
$^{7}_{\Lambda\Lambda}$Li
(MeV)
2.0
0.0
−2.0
−4.0
−6.0
−8.0
3/2^{-}
1.97
α + p
B_{Λ} = 4.52 (exp. 4.50)
α + p + Λ
−2.27 2^{-}
−2.55 1^{-}
$^{5}_{\Lambda}$He + p
$B_{\Lambda\Lambda}$ = 9.45
α + p + Λ + Λ
$^{6}_{\Lambda\Lambda}$He + p
3/2^{-}
−7.48



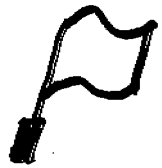
(MeV)
^{6}Li
$^{7}_{\Lambda}$Li
$^{8}_{\Lambda\Lambda}$Li
d
Λ
α
4.0
2.0
0.0
−2.0
−4.0
−6.0
−8.0
−10.0
−12.0
−14.0
2+ 3.99
3+ 0.83
α + d
1+ −1.47
α +d+ Λ
5/2+ 6.69
3/2+ 6.11
B_Λ= 5.58 (exp. 5.58)
$^{5}_{\Lambda}$He+d
KEK-E419
7/2+ 2.46
5/2+ 2.08 (2.05)
3/2+ 0.69 (0.69)
0.00 (0.00)
1/2+ −7.05
cal. (exp.)
E_x
α+d+Λ+Λ
$B_{\Lambda\Lambda}$= 11.44
$^{6}_{\Lambda\Lambda}$He+d
2+ 5.63
3+ 1.36
0.00
1+ −12.91
E_x



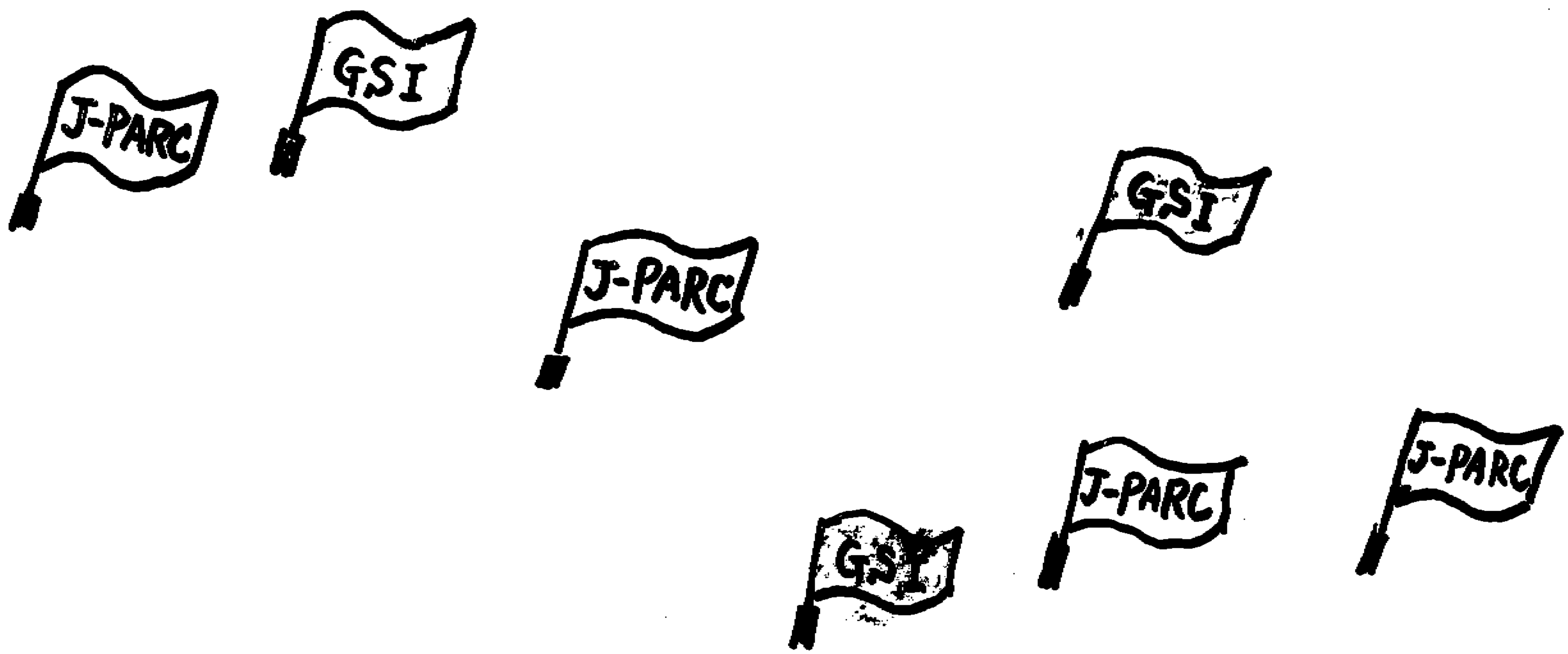
(MeV)
^{7}Li
$^{8}_{\Lambda}$Li
$^{9}_{\Lambda\Lambda}$Li
4.0
2.0
0.0
−2.0
−4.0
−6.0
−8.0
−10.0
−12.0
−14.0
−16.0
5/2⁻ 3.68
7/2⁻ 2.41
α + t
1/2⁻ −2.01
3/2⁻ −2.50
α + t + Λ
$^{5}_{\Lambda}$He+t
3⁻ 6.13
4⁻ 5.05
3⁻ 4.82
0⁻ 0.94
1⁻ 0.85
2⁻ 0.31
1⁻ −9.30 0.00
E_x
B_Λ = 6.80(exp. 6.80)
α +t+ Λ+Λ
$B_{\Lambda\Lambda}$=14.55
$^{6}_{\Lambda\Lambda}$He+t
$^{8}_{\Lambda}$Li +Λ
5/2⁻ 5.96
7/2⁻ 4.55
1/2⁻ 0.73
3/2⁻ −17.05 0.00
E_x
t
α
Λ

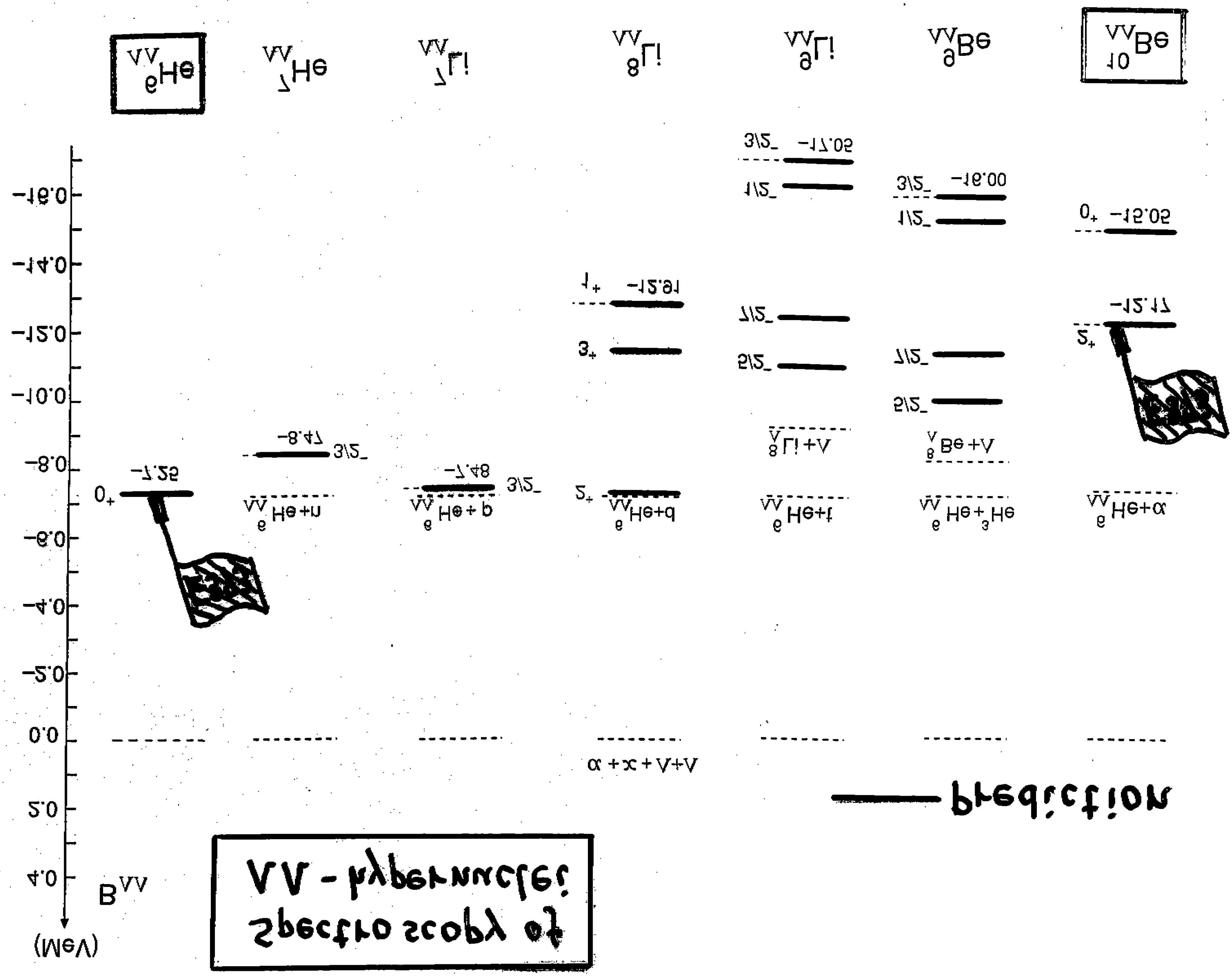


(MeV)
^{7}Be
$^{8}_{\Lambda}$Be
$^{9}_{\Lambda\Lambda}$Be
4.0
2.0
0.0
−2.0
−4.0
−6.0
−8.0
−10.0
−12.0
−14.0
−16.0
5/2⁻ 4.34
7/2⁻ 3.18
α + ^{3}He
1/2⁻ −1.14
3/2⁻ −1.60
B_{Λ} = 6.72(exp. 6.84)
α + ^{3}He+ Λ
2⁻ 6.84
3⁻ 6.11
4⁻ 5.07
3⁻ 4.84
$^{5}_{\Lambda}$He+^{3}He
0⁻ 0.93
1⁻ 0.83
2⁻ 0.32
1⁻ −8.32 0.00
E_x
3He
α
Λ
α + ^{3}He+Λ+Λ
$B_{\Lambda\Lambda}$ = 14.40
$^{6}_{\Lambda\Lambda}$He + ^{3}He
$^{8}_{\Lambda}$Be + Λ
5/2⁻ 5.92
7/2⁻ 4.54
1/2⁻ 0.71
3/2⁻ −16.00 0.00
E_x



Future experiments
at J-PARC and GSI





Spectroscopy of ΛΛ - hypernuclei
$B_{\Lambda\Lambda}$ (MeV)
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
-10.0
-12.0
-14.0
-16.0
$^{\Lambda\Lambda}_{6}He$
$^{\Lambda\Lambda}_{7}He$
$^{\Lambda\Lambda}_{7}Li$
$^{\Lambda\Lambda}_{8}Li$
$^{\Lambda\Lambda}_{9}Li$
$^{\Lambda\Lambda}_{9}Be$
$^{\Lambda\Lambda}_{10}Be$
0^+ -7.25
-8.47 $3/2^-$
$^{\Lambda\Lambda}_{6}He+n$
-7.48 $3/2^-$
$^{\Lambda\Lambda}_{6}He+p$
1^+ -12.91
3^+
2^+
$^{\Lambda\Lambda}_{6}He+d$
$\Lambda+\Lambda+\alpha+x$
$3/2^-$ -17.05
$1/2^-$
$7/2^-$
$5/2^-$
$^{\Lambda}_{8}Li+\Lambda$
$^{\Lambda\Lambda}_{6}He+t$
$3/2^-$ -16.00
$1/2^-$
$7/2^-$
$5/2^-$
$^{\Lambda}_{8}Be+\Lambda$
$^{\Lambda\Lambda}_{6}He+{}^{3}He$
0^+ -15.05
-12.17
2^+
$^{\Lambda\Lambda}_{6}He+\alpha$
Prediction

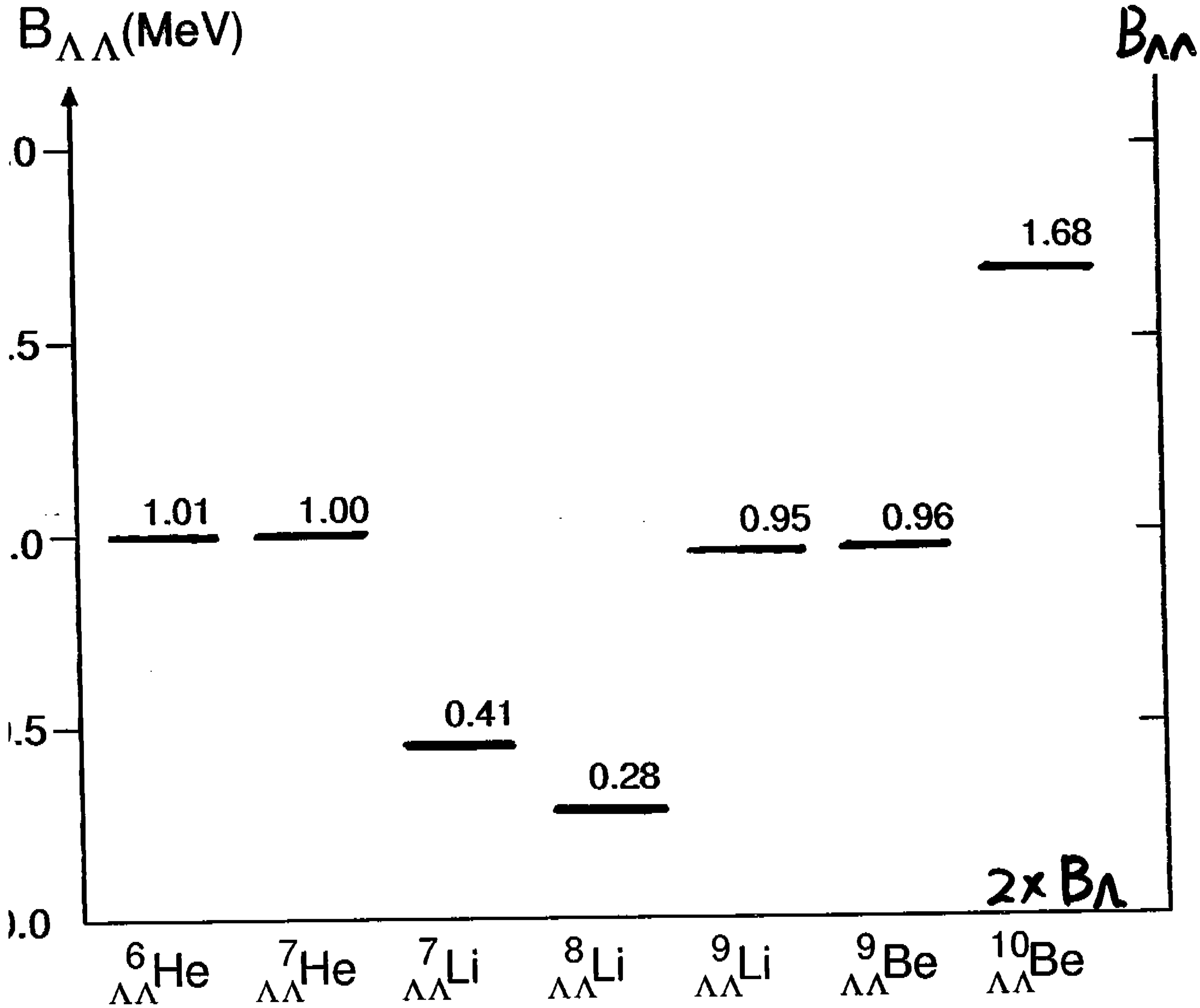
Spectroscopic experiments of double Λ hypernuclei are useful to obtain information on $\Lambda\Lambda$ interaction.

So far, in order to estimate the $\Lambda\Lambda$ interaction strength, the $\Delta B_{\Lambda\Lambda}$ has been usually used.

$$\underbrace{\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2 \times B_{\Lambda}}_{\| \atop \Lambda\Lambda \text{ bond energy}}$$

How is the calculated $\Delta B_{\Lambda\Lambda}$ of $A=6 \sim 10$ double Λ hypernuclei?

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$$

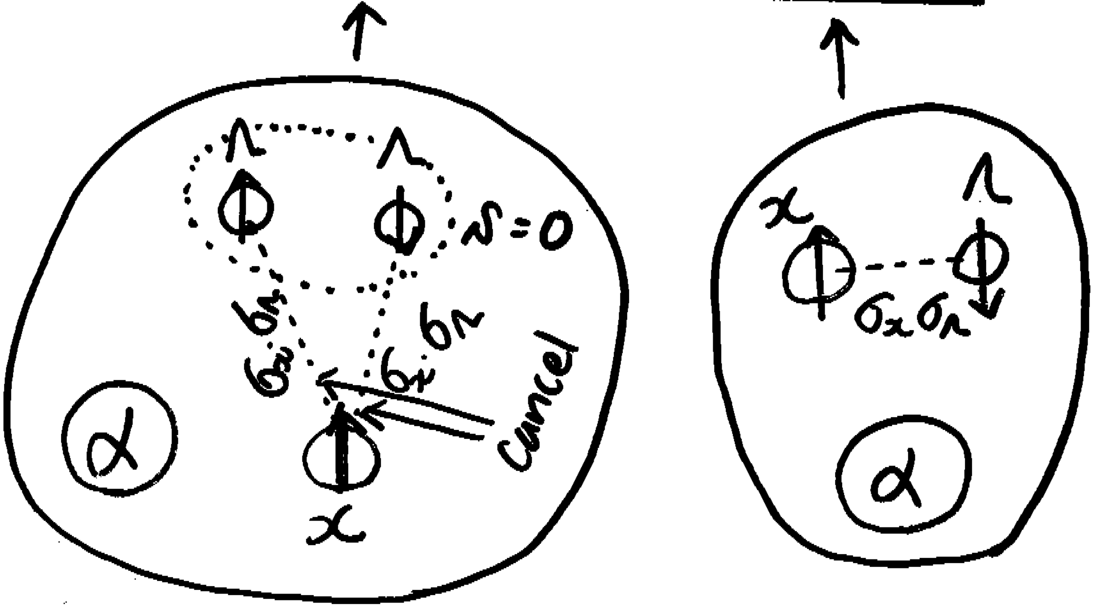


$\Delta B_{\Lambda\Lambda}$ is strongly dependent on individual hypernuclei.

How should we consider about this interesting figure?

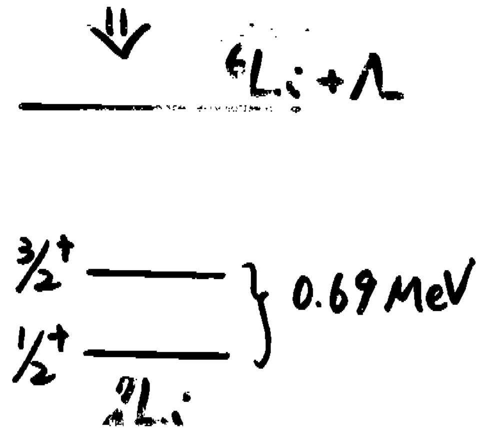
Answer: mass-number-dependence comes from contribution of spin-spin part of Λ N interaction in A=7 ～ 10 double Λ hypernuclei

$$\Delta B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = \underline{B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)} - 2\,\underline{B_{\Lambda}({}^{A-1}_{\Lambda}Z)}$$



There is <u>NO</u> contribution of $\sigma_x \cdot \sigma_\Lambda$.

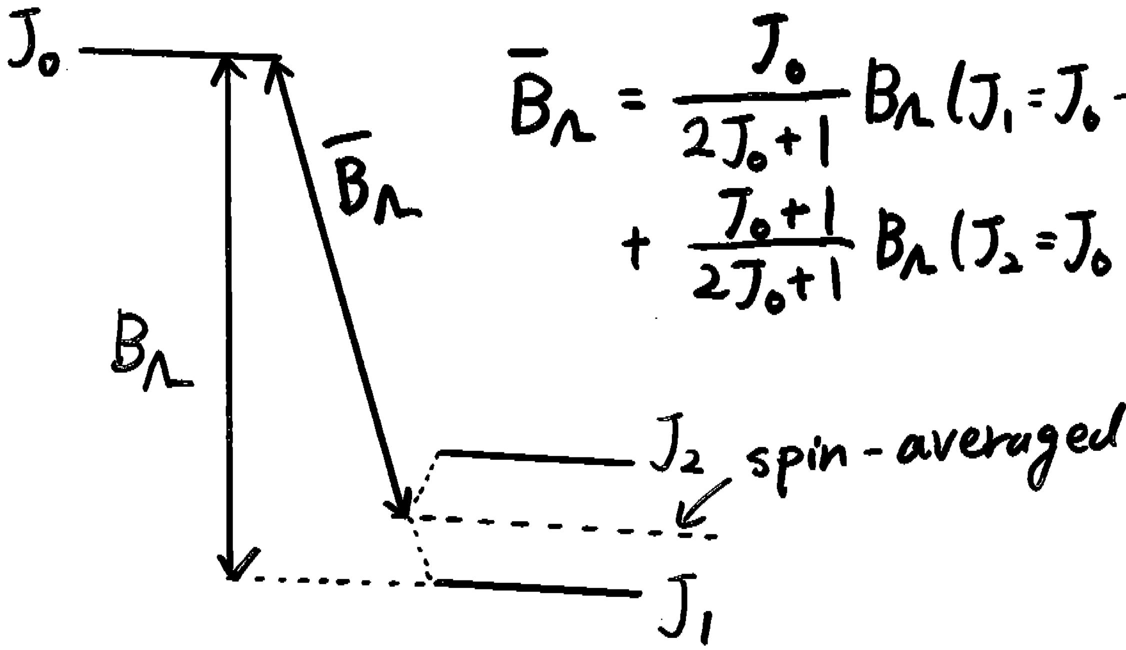
There is contribution of $\sigma_x \cdot \sigma_\Lambda$.



M. Danysz et al., Nucl. Phys. <u>49</u>, 121 (1963)

"This definition is of simple meaning only when the nuclear core is spinless."

Core nucleus



$$\bar{B}_\Lambda = \frac{J_0}{2J_0+1} B_\Lambda(J_1 = J_0 - 1/2) + \frac{J_0+1}{2J_0+1} B_\Lambda(J_2 = J_0 + 1/2)$$

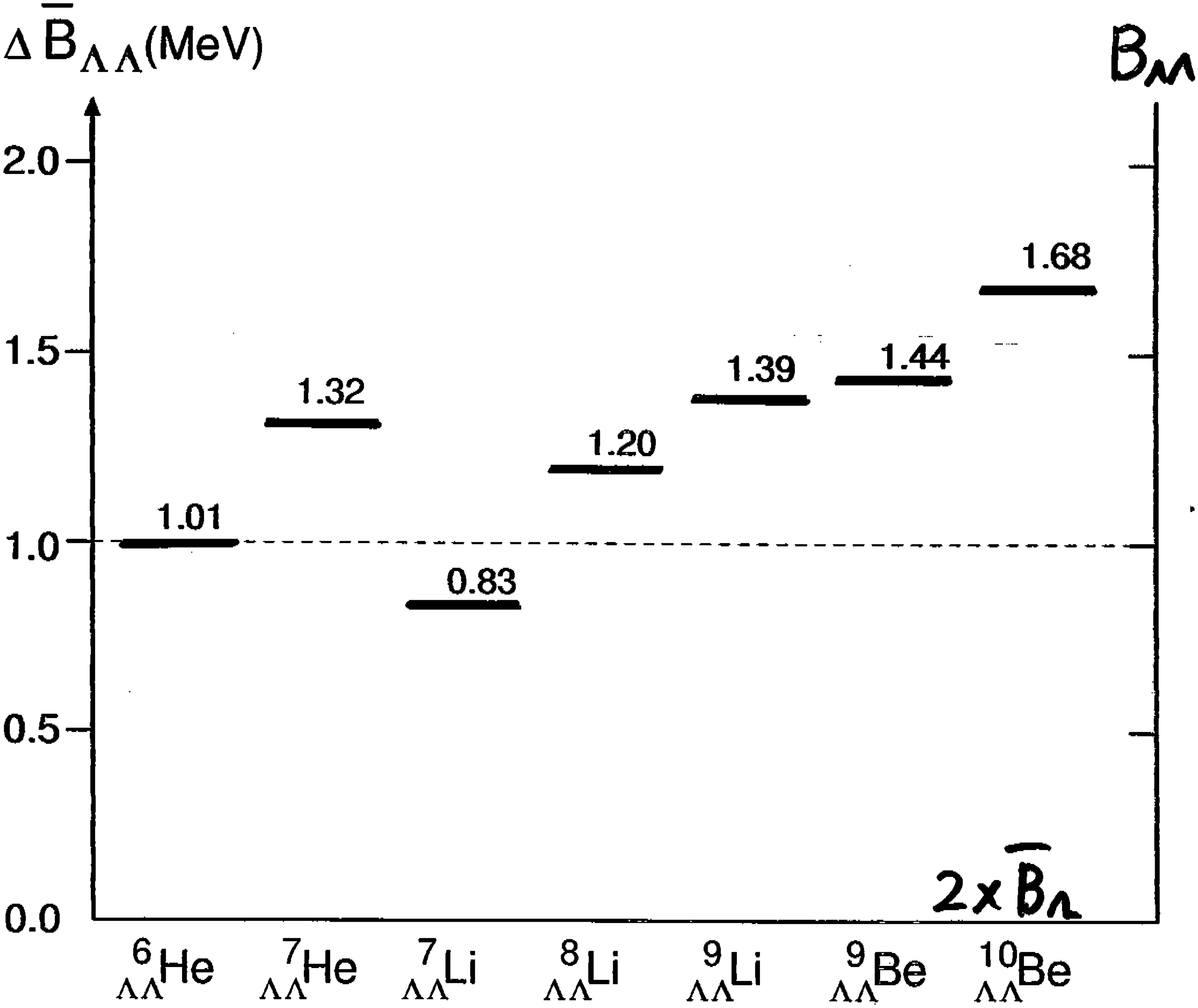
Λ hypernucleus

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2 \times B_\Lambda$$

$$\Downarrow$$

$$\Delta \bar{B}_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2 \times \underline{\bar{B}_\Lambda}$$

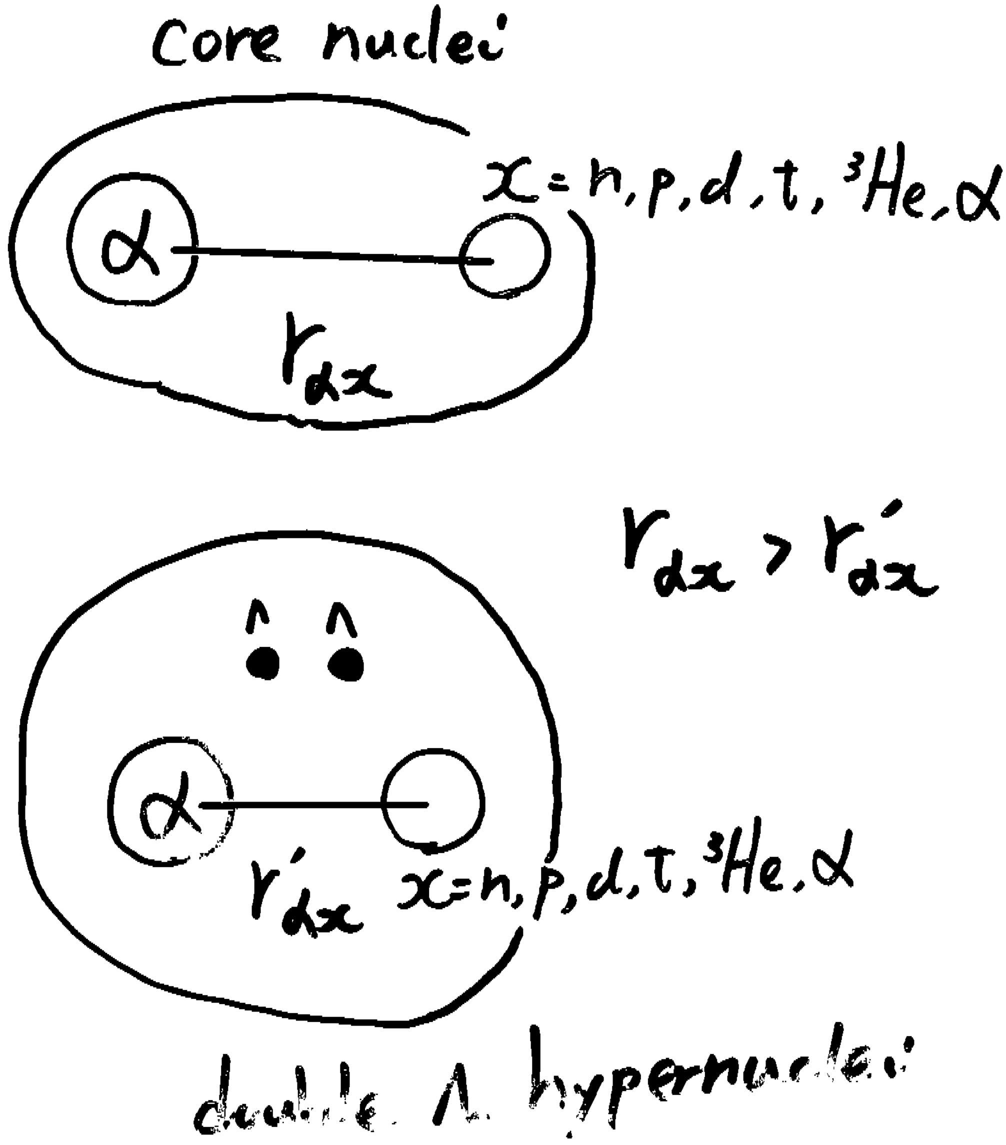
$$\Delta \bar{B}_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2 \times \bar{B}_{\Lambda}$$



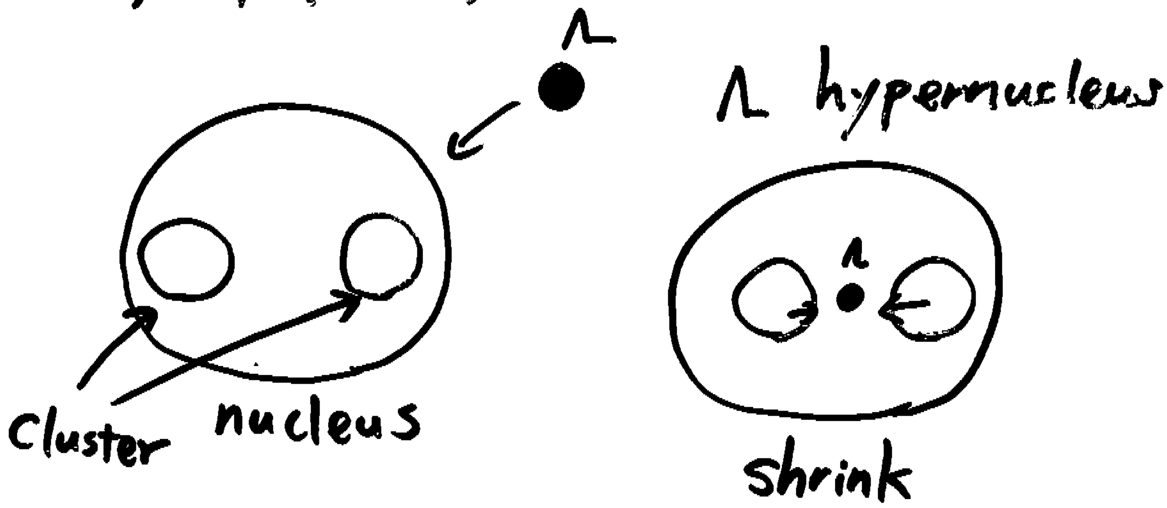
Although $\Delta \bar{B}_{\Lambda\Lambda}$ is free from the effect of the ΛN spin-spin interaction, its magnitude for $A = 7 \sim 10$ deviates significantly from $\Delta \bar{B}_{\Lambda\Lambda}$ of $^{6}_{\Lambda\Lambda}$He.

What does this deviation come from?

It comes from the effect of the dynamical change in the α+x core-nucleus structure, namely, effect of shrinkage [redacted] between α and x clusters by the addition of 2 Λ particles.



T. Motoba, H. Bandō and K. Ikeda, Prog. Theor. Phys. 70, 189 (1983)



· E. Hiyama, M. Kamimura, K. Miyazaki and T. Motoba, Phys. Rev. C59, 2351 (1999)

21 % shrinkage in size in $^{7}_{\Lambda}$Li

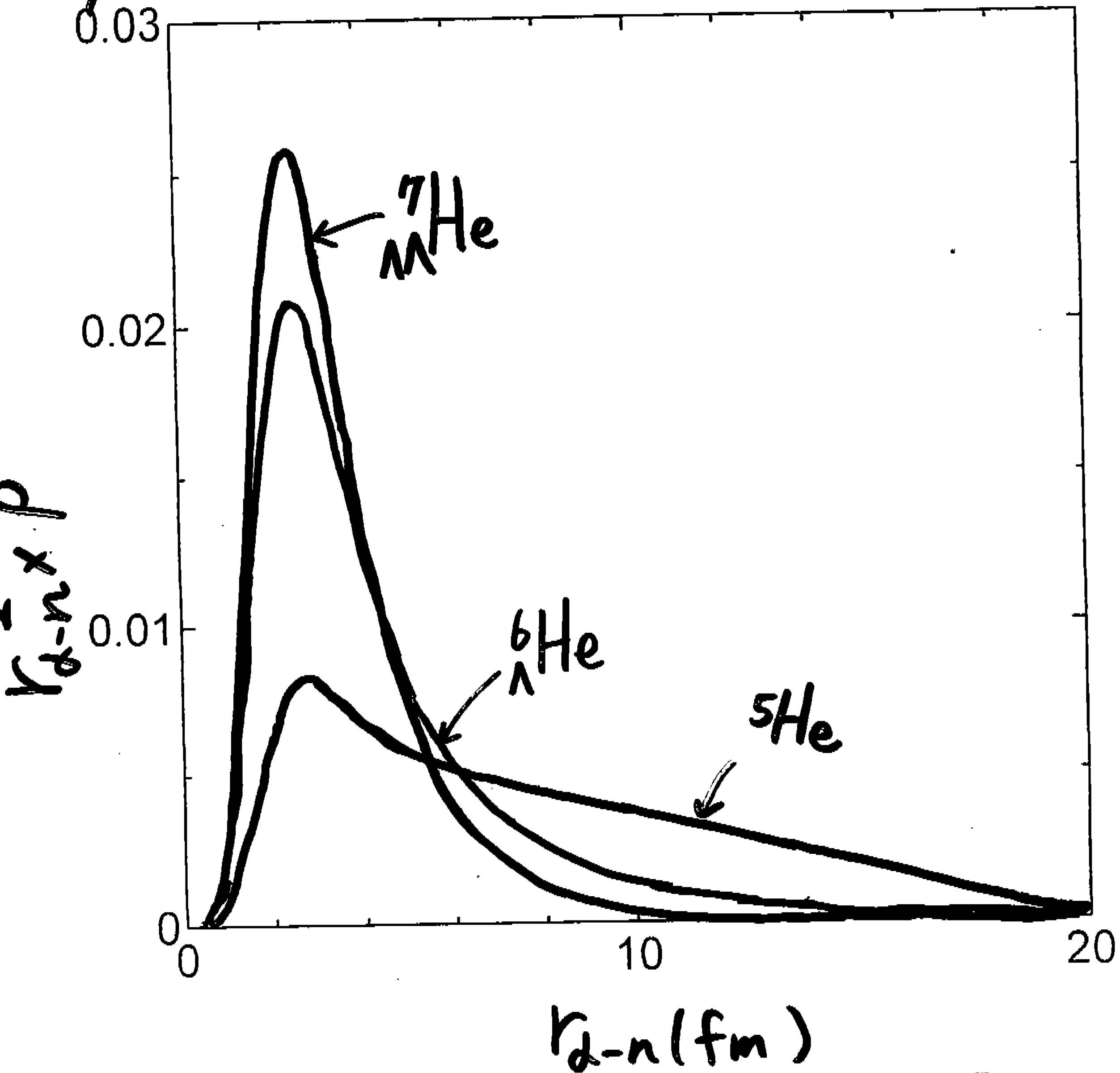
· K. Tanida et al., Phys. Rev. Lett. 86, 1982 (2001)

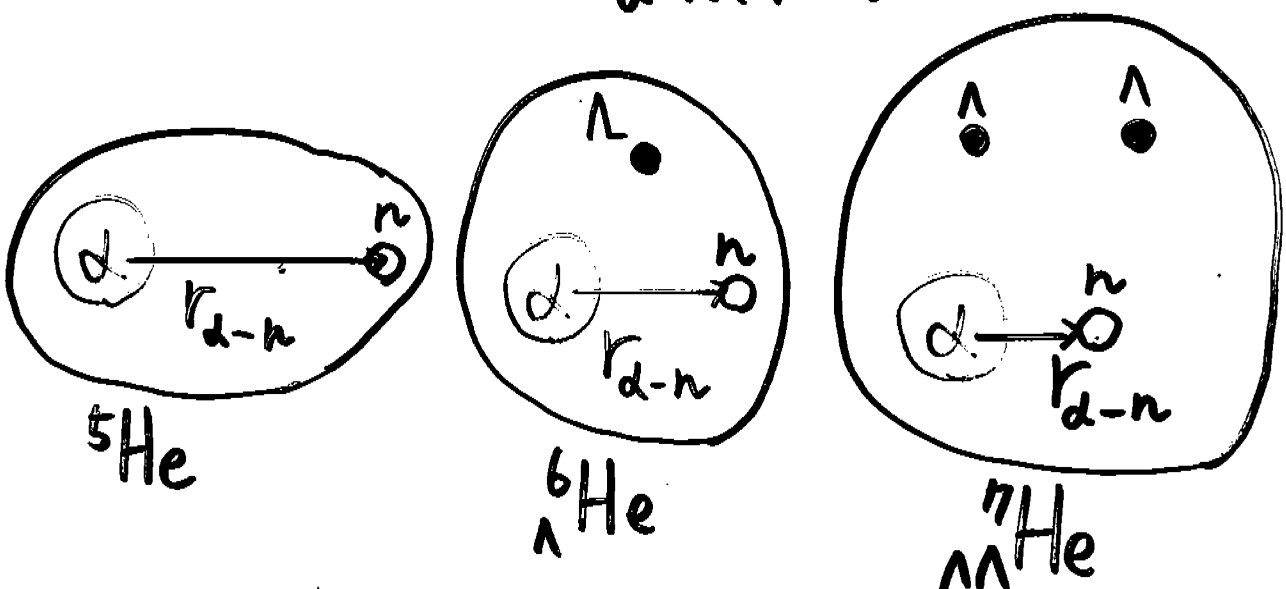
shrinkage effect was confirmed quantitatively by the KEK-E419 experiment.

It is quite reasonable that in a double Λ hypernucle further shrinkage effect of the nuclear core can be induce by the participation of one more Λ particle.

Shrinkage of the core nucleus density by the addition of Λ particle (31)

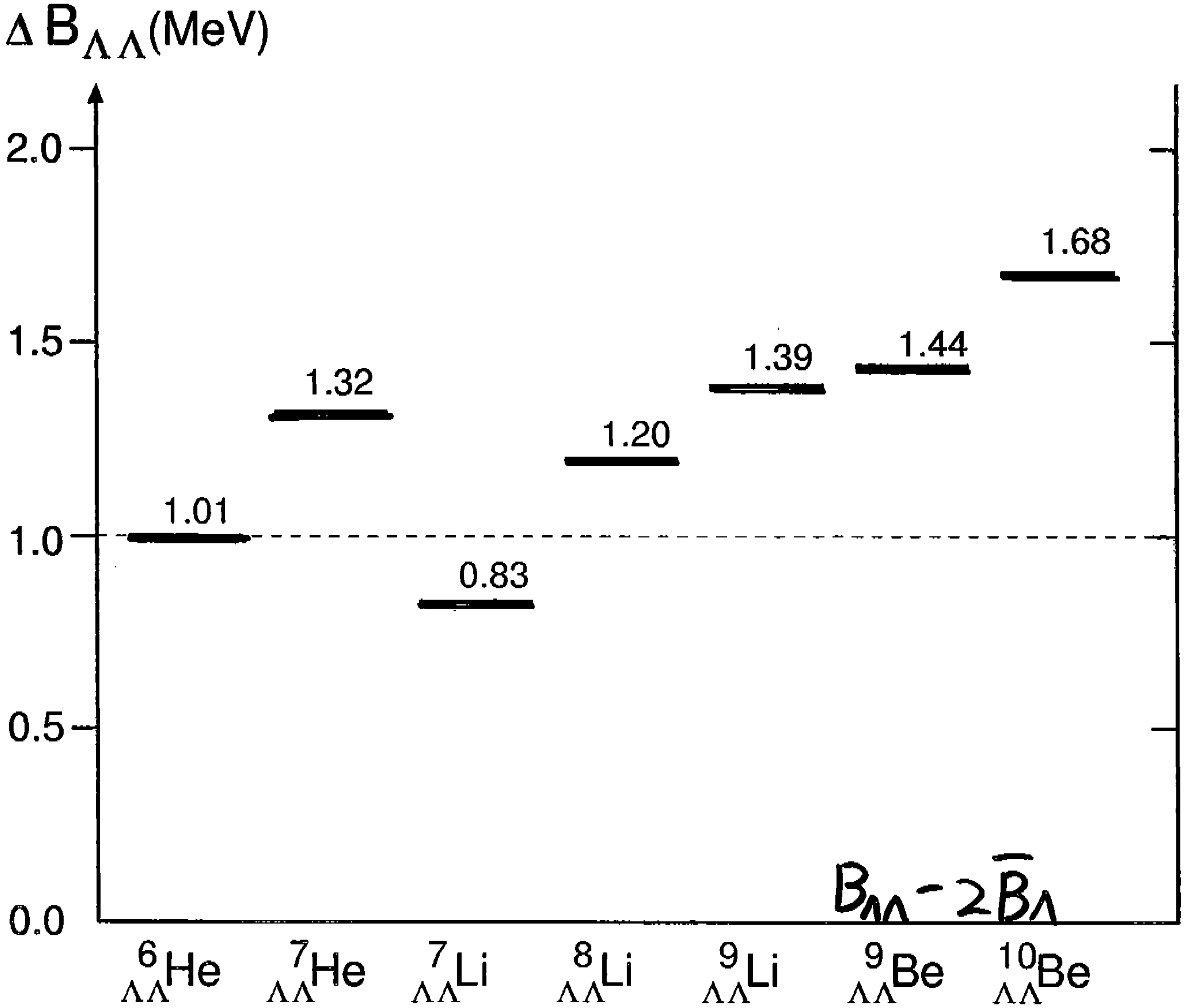
23





$r_{\alpha-n}(^{5}He) > r_{\alpha-n}(^{6}_{\Lambda}He) > r_{\alpha-n}(^{7}_{\Lambda\Lambda}He)$

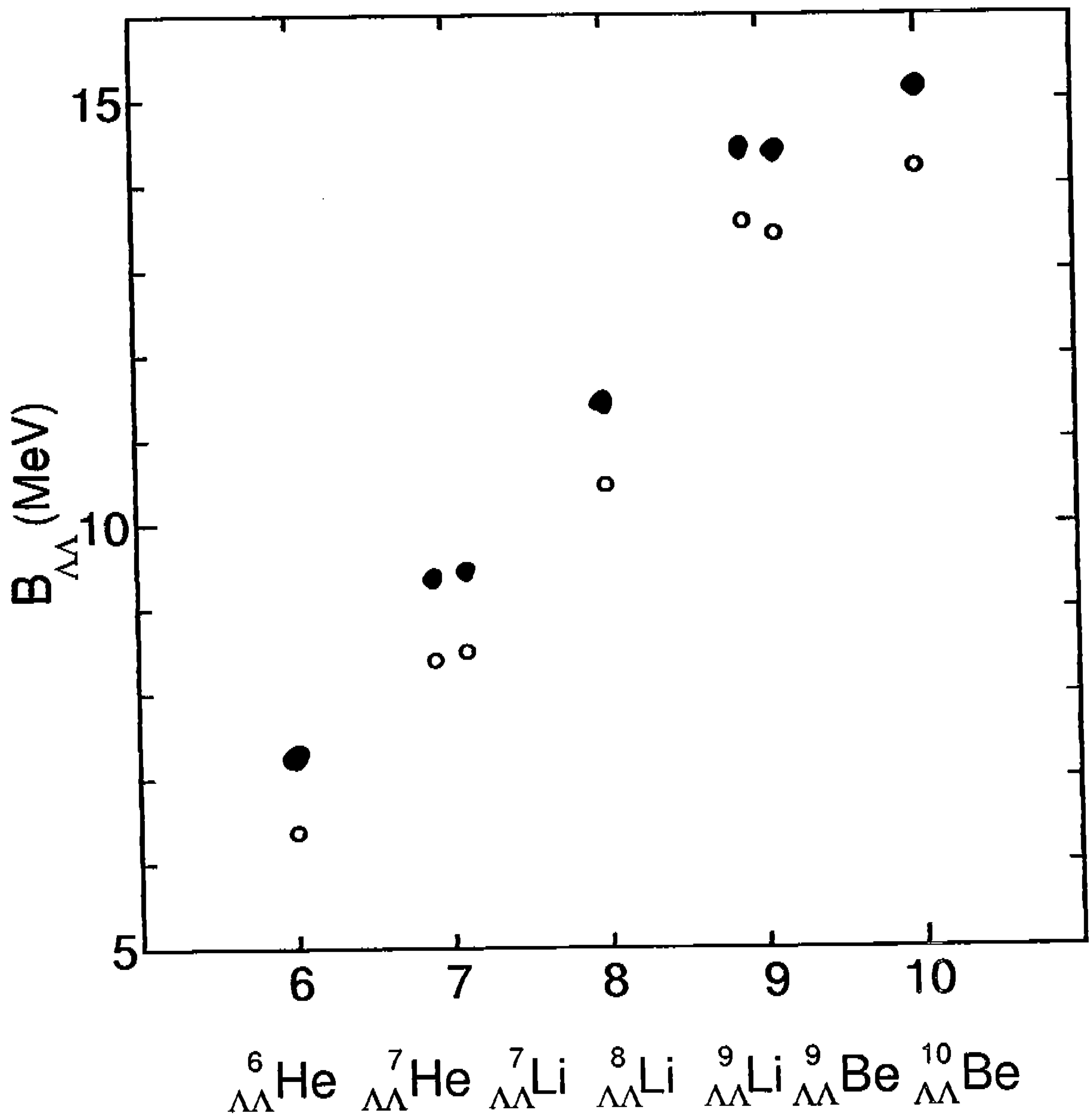
$$\Delta\bar{B}_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2\bar{B}_{\Lambda}$$



$\Delta\bar{B}_{\Lambda\Lambda}$ scatters in a range of a factor of 2.

Even if you employs $\Delta\bar{B}_{\Lambda\Lambda}$, it is impossible to extract any constant value of the $\Lambda\Lambda$ bond energy from this figure.

In the future, if the double Λ hypernuclei with A:7~9 are observed, please provide us $B_{\Lambda\Lambda}$ rather than $\Delta B_{\Lambda\Lambda}$.

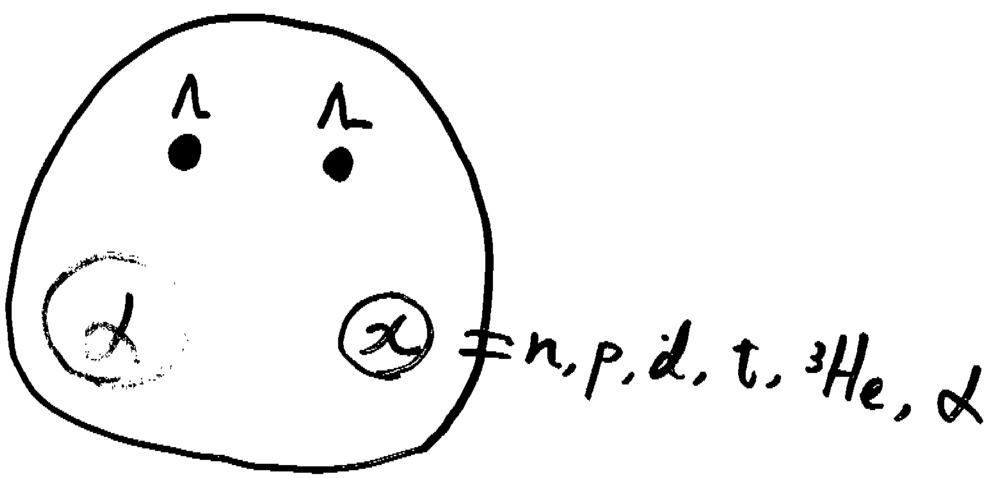


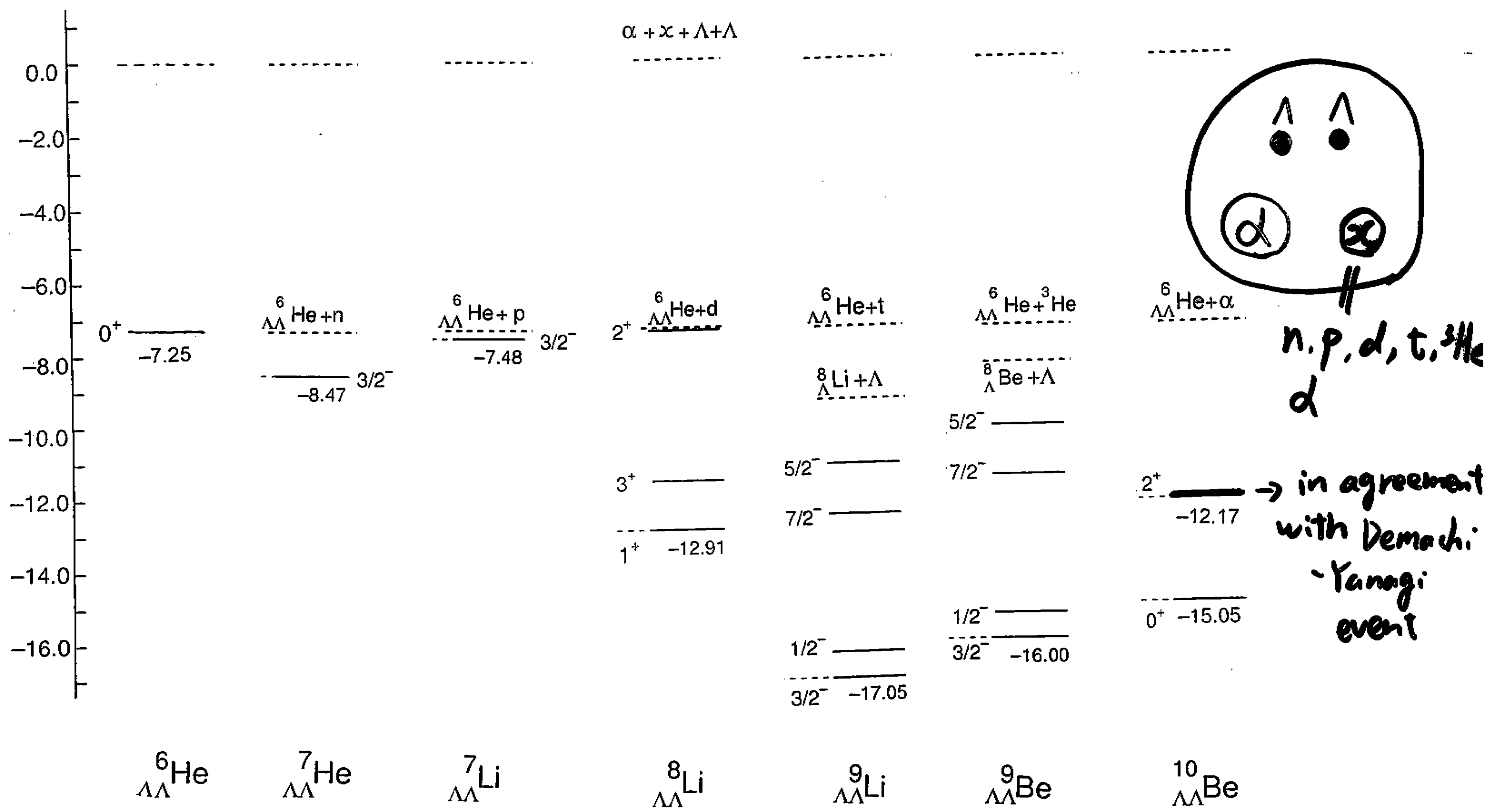
There is linear relation between $B_{\Lambda\Lambda}$ and mass number of double Λ hypernuclei.

Summary

I carried out structure calculation for A=7～10 double Λ hypernuclei within the framework of an α+x+Λ+Λ model.

2-body interaction of α+x, Λ+x and Λ+Λ well reproduce the observed binding energies of Λ+x systems and α+x+Λ and α+Λ+Λ systems.





Spectroscopy of ΛΛ hypernuclei at JPARC

E. Hiyama et al.,
PRC 66 024007 (2002)