Semiclassical distorted wave model analysis of (π, K) and (γ, K) inclusive spectra for studying the Σ hyperon in nuclei

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- ☐ Σ s.p. potential has not been established.

  current understanding: repulsive
  - Nijmegen model F predicts repulsive potential. Other Nijmegen models attractive.
  - Prediction of SU_6 quark model potential: real part of U_Σ : $+10 \sim +40$ MeV

  M. Kohno *et al.*, Nucl. Phys. A674 (2000) 229.

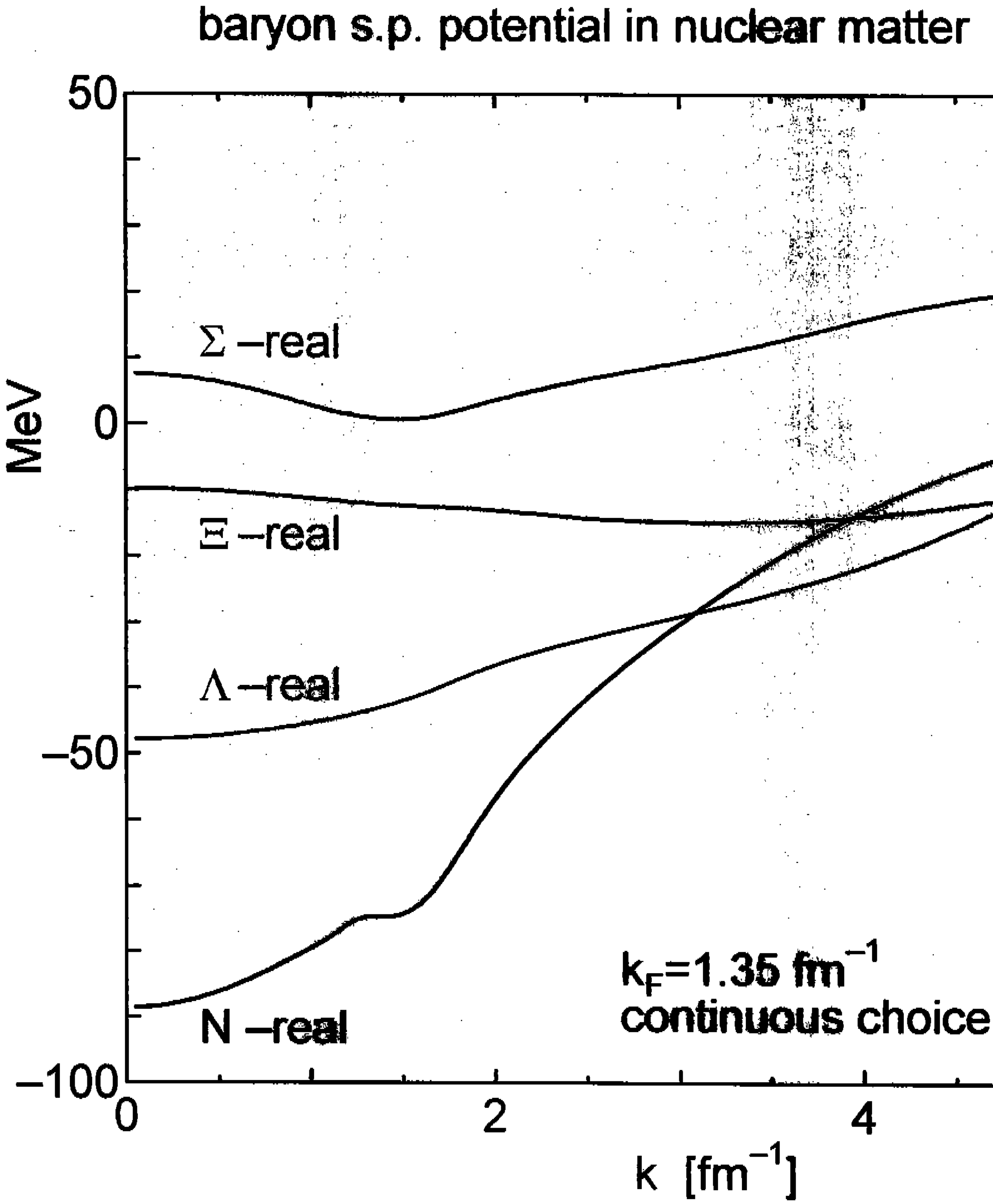
- ☐ Recent (π^-, K^+) inclusive spectra taken at KEK suggested Σ s.p. potential is strongly repulsive.
  - $\Rightarrow$ real part of U_Σ > +90 MeV ?

  H. Noumi *et al.*, Phys. Rev. Lett. 89, 072301 (2002).

- ☐ The knowledge of ΣN interaction is important for understanding the octet baryon world.

- ☐ SCDW method (originally developed for intermediate energy (p,p'x) and (p,nx) reactions) is applied to describe (π^-, K^+) inclusive spectra.

- ☐ Extension to (γ, K^+) processes is straightforward.



baryon s.p. potential in nuclear matter
50
0
−50
−100
MeV
0
2
4
k [fm^{-1}]
Σ −real
Ξ −real
Λ −real
N −real
k_F=1.35 fm^{-1}
continuous choice

Double differential cross section

$$\frac{d^2\sigma}{dW d\Omega} = \frac{\mu_i \mu_f}{(2\pi)^2}\frac{p_f}{p_i}\int\int d\boldsymbol{r} d\boldsymbol{r}' \, \frac{1}{4\omega_i\omega_f} \, \chi_f^{(-)*}(\boldsymbol{r})v(\boldsymbol{r})\chi_i^{(+)}(\boldsymbol{r}) \times \chi_f^{(-)}(\boldsymbol{r}')v(\boldsymbol{r}')\chi_i^{(+)*}(\boldsymbol{r}') \sum_{p,h} \phi_p^*(\boldsymbol{r})\phi_h(\boldsymbol{r})\phi_p(\boldsymbol{r}')\phi_h^*(\boldsymbol{r}') \times \delta(W - \epsilon_p + \epsilon_h)\theta(\epsilon_F - \epsilon_h).$$

p : outgoing particle, ϵ_F : Fermi energy up to which nucleons are occupied, $v(\boldsymbol{r})$: transition potential.

$$\boldsymbol{R} = \frac{\boldsymbol{r}' + \boldsymbol{r}}{2} \quad \text{and} \quad \boldsymbol{s} = \boldsymbol{r}' - \boldsymbol{r},$$

semiclassical approximation means

$$\chi_f^{(-)}(\boldsymbol{r}') = \chi_f^{(-)}(\boldsymbol{R} + \frac{1}{2}\boldsymbol{s}) = \mathrm{e}^{\frac{1}{2}\boldsymbol{s}\cdot\nabla_R}\chi_f^{(-)}(\boldsymbol{R}) \simeq \mathrm{e}^{i\frac{1}{2}\boldsymbol{s}\cdot\boldsymbol{k}_f(\boldsymbol{R})}\chi_f^{(-)}(\boldsymbol{R})$$

$$\chi_f^{(-)}(\boldsymbol{r}) = \chi_f^{(-)}(\boldsymbol{R} - \frac{1}{2}\boldsymbol{s}) = \mathrm{e}^{-\frac{1}{2}\boldsymbol{s}\cdot\nabla_R}\chi_f^{(-)}(\boldsymbol{R}) \simeq \mathrm{e}^{-i\frac{1}{2}\boldsymbol{s}\cdot\boldsymbol{k}_f(\boldsymbol{R})}\chi_f^{(-)}(\boldsymbol{R})$$

$$\chi_i^{(+)}(\boldsymbol{r}') = \chi_i^{(+)}(\boldsymbol{R} + \frac{1}{2}\boldsymbol{s}) \simeq \mathrm{e}^{i\frac{1}{2}\boldsymbol{s}\cdot\boldsymbol{k}_i(\boldsymbol{R})}\chi_i^{(+)}(\boldsymbol{R})$$

$$\chi_i^{(+)}(\boldsymbol{r}) = \chi_i^{(+)}(\boldsymbol{R} - \frac{1}{2}\boldsymbol{s}) \simeq \mathrm{e}^{-i\frac{1}{2}\boldsymbol{s}\cdot\boldsymbol{k}_i(\boldsymbol{R})}\chi_i^{(+)}(\boldsymbol{R})$$

$$\chi_f^{(-)*}(\boldsymbol{r})\chi_i^{(+)}(\boldsymbol{r})\chi_f^{(-)}(\boldsymbol{r}')\chi_i^{(+)*}(\boldsymbol{r}') \to |\chi_f^{(-)}(\boldsymbol{R})|^2|\chi_i^{(+)}(\boldsymbol{R})|^2\mathrm{e}^{i\boldsymbol{s}\cdot(\boldsymbol{k}_f(\boldsymbol{R})-\boldsymbol{k}_i(\boldsymbol{R}))}$$

The sum over hole states

$$\sum_h \phi_h(\boldsymbol{r})\phi_h^*(\boldsymbol{r}')\delta(W - \epsilon_p + \epsilon_h)\theta(\epsilon_F - \epsilon_h)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dp \, \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dq \, \frac{1}{q} \, \mathrm{e}^{i(W-\epsilon_p)p+\epsilon_F q}\sum_h \phi_h(\boldsymbol{r})\phi_h^*(\boldsymbol{r}')\mathrm{e}^{-(q-ip)\epsilon_h}$$

Thomas-Fermi approx. for the Bloch density $\sum_i \phi_i(\boldsymbol{r})\phi_i(\boldsymbol{r}')\mathrm{e}^{-\beta\epsilon_i}$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dp \, \mathrm{e}^{i(W-\epsilon_p)p}\frac{1}{(2\pi)^3}\int_{K<k_F(\boldsymbol{R})} d\boldsymbol{K} \, \mathrm{e}^{-ip(\frac{\hbar^2}{2m}K^2+V(\boldsymbol{R}))}\mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{s}}$$

$$= \frac{1}{(2\pi)^3}\int_{K<k_F(\boldsymbol{R})} d\boldsymbol{K} \, \delta(W - \epsilon_p + \frac{\hbar^2}{2m}K^2 + V(\boldsymbol{R})) \, \mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{s}}$$

The Fermi energy ϵ_F is determined by the local density:

$$\rho_\tau(\boldsymbol{R}) = \rho_p(\boldsymbol{R}) \ \text{ or } \ \rho_n(\boldsymbol{R}).$$

Namely, the local Fermi momentum is defined by

$$k_F(\boldsymbol{R}) = [3\pi^2 \rho_\tau(\boldsymbol{R})]^{1/3}.$$

$$\phi_p^*(\boldsymbol{r})\phi_p(\boldsymbol{r}') = \phi_p^*(\boldsymbol{R} + \frac{1}{2}\boldsymbol{s})\phi_p(\boldsymbol{R} - \frac{1}{2}\boldsymbol{s}) = |\phi_p(\boldsymbol{R})|^2 \mathrm{e}^{-i\boldsymbol{k}_p(\boldsymbol{R})\cdot\boldsymbol{s}}$$

The double differential cross section becomes

$$\begin{aligned}
\frac{d^2\sigma}{dW d\Omega} &= \frac{\mu_i \mu_f}{(2\pi)^2} \frac{p_f}{p_i} \int\int d\boldsymbol{R} d\boldsymbol{s} \, |\chi_f^{(-)}(\boldsymbol{R}_0)|^2 \chi_i^{(-)}(\boldsymbol{R}_0)|^2 \\
&\times \frac{1}{(2\pi)^3} \int_{K<k_F(\boldsymbol{R})} d\boldsymbol{K} \, \frac{1}{4\omega_i \omega_f} \, \frac{1}{(2\pi)^3} \sum_{spin} \int d\boldsymbol{p} \, |\phi_p(\boldsymbol{R})|^2 |v(\boldsymbol{K}, \boldsymbol{k}_i)|^2 \\
&\times e^{i\boldsymbol{s}\cdot(\boldsymbol{k}_f(\boldsymbol{R}) - \boldsymbol{k}_i(\boldsymbol{R}) - \boldsymbol{K} - \boldsymbol{k}_p(\boldsymbol{R}))} \, \delta(W - \epsilon_p + \frac{\hbar^2}{2m} K^2 + V(\boldsymbol{R})) \\
&= \frac{\mu_i \mu_f}{(2\pi)^5} \frac{p_f}{p_i} \int d\boldsymbol{R} \int_{K<k_F(\boldsymbol{R})} d\boldsymbol{K} \sum_{spin} \int d\boldsymbol{p} \, \frac{1}{4\omega_i \omega_f} \, |\chi_f^{(-)}(\boldsymbol{R})|^2 \\
&\times |\chi_i^{(+)}(\boldsymbol{R})|^2 |\phi_p(\boldsymbol{R})|^2 |v_0(\boldsymbol{K}, \boldsymbol{k}_i)|^2 \delta(\boldsymbol{K} + \boldsymbol{k}_i(\boldsymbol{R}) - \boldsymbol{k}_f(\boldsymbol{R}) - \boldsymbol{k}_p(\boldsymbol{R})) \\
&\times \delta(W - \epsilon_p + \frac{\hbar^2}{2m} K^2 + V(\boldsymbol{R}))
\end{aligned}$$

- **transition strength** $|v|^2$

$$\frac{d^2\sigma(\pi^- p \to K^+ \Sigma^-)}{dE d\Omega} = \frac{1}{(4\pi)^2} \frac{p_K}{p_\pi} \frac{\omega_\pi \omega_K}{s} \, |v|^2$$

- **Distorted waves $\chi^{\pm}$ for π and K.**

$$\{\nabla^2 + k^2 + U_c^2(r) - 2\omega U_c(r) - 2\omega U_N(r)\}\chi(q, \boldsymbol{r}) = 0.$$

$$-2\omega U_N(r) = b_0 k^2 \rho(r) + b_1 \vec{\nabla}\rho(r)\vec{\nabla}.$$

$$\textbf{Im } b_0^{eff} = \frac{1}{k}\{< \sigma_{tot} >_{av} - < \sigma_{\ell=1} >\}, \ \textbf{Im } b_1 = \frac{1}{k}\{< \sigma_{\ell=1} >$$

parameters used		$< \sigma_{tot} >_{av}$	**Im b_0**
(Re b_0=0, $b_1 = 0$)	π^-	**35 mb**	**0.58 fm^3**
	K$^+$	**12.5 mb**	**0.41 fm^3**

- **To define the classical momentum $\boldsymbol{k_c}$ at $\boldsymbol{r}$, the following momentum $\boldsymbol{k_q}$ is first calculated by**

$$\boldsymbol{k_q(r)} = \frac{1}{2}\frac{\chi^{\pm *}(\boldsymbol{r})(-i)\nabla\chi^{\pm}(\boldsymbol{r}) - (-i)(\nabla\chi^{\pm *}(\boldsymbol{r}))\chi^{\pm}(\boldsymbol{r})}{|\chi^{\pm}(\boldsymbol{r})|^2}$$

and then the magnitude is renormalized by the energy-momentum relation $\frac{\hbar^2}{2\mu}\boldsymbol{k_c^2} + \boldsymbol{U(r)} = \boldsymbol{E}$ as

$$\boldsymbol{k_c(r)} = \boldsymbol{k_q(r)}\frac{\boldsymbol{k_c}}{\sqrt{\boldsymbol{k_q^2(r)}}}.$$

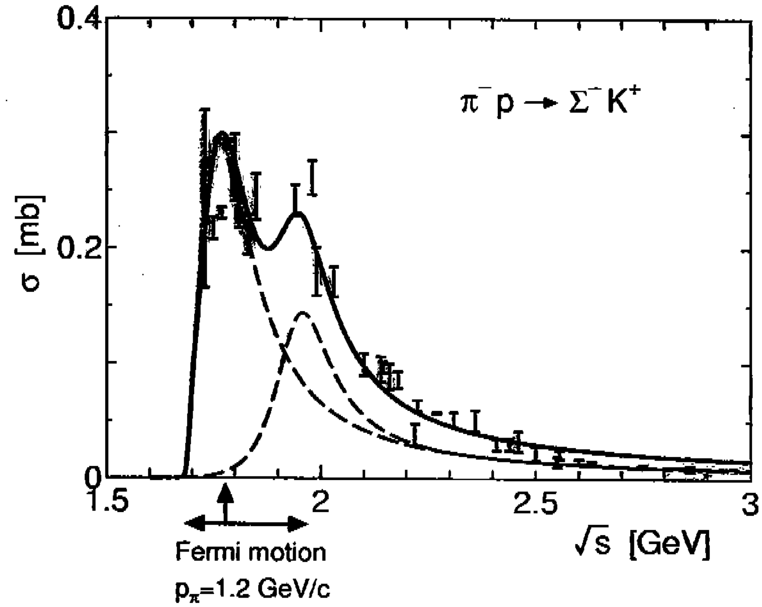
- **Instead of using a complex Σ potential and Green's function description,** use a real potential and smear the spectrum by hand with a Lorentz weight factor

$$\boldsymbol{f(\omega; \omega', \Delta)} = \frac{\Delta}{\pi}\frac{1}{(\omega - \omega')^2 + \Delta^2},$$

where the width Δ is set to be 15 MeV.

Parameterization of total cross section $\sigma(\pi^- p \to \Sigma^- K^+)$
by Tsushima, Huang and Faessler [Phys. Lett. B337 (1994) 245]

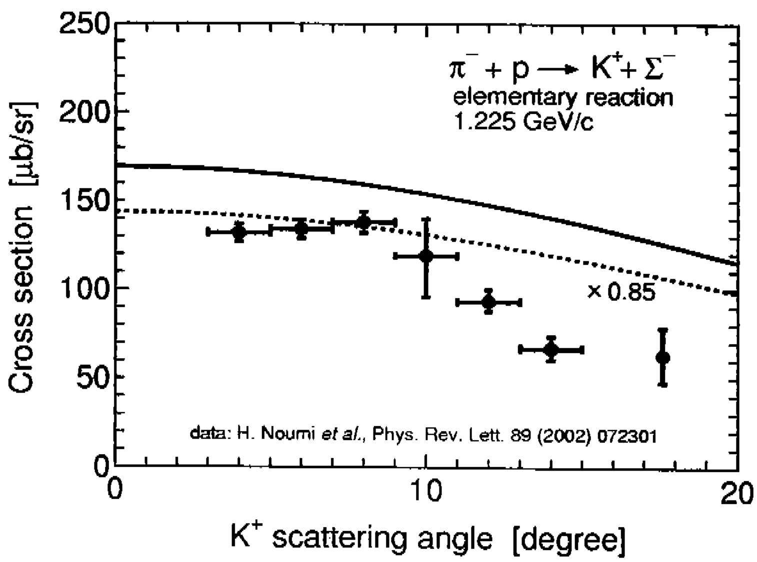
$$\sigma(\pi^- p \to \Sigma^- K^+)$$
$$= \frac{0.009803(\sqrt{s}-1.688)^{0.6021}}{(\sqrt{s}-1.742)^2+0.006583} + \frac{0.006521(\sqrt{s}-1.688)^{1.4728}}{(\sqrt{s}-1.940)^2+0.006248}$$

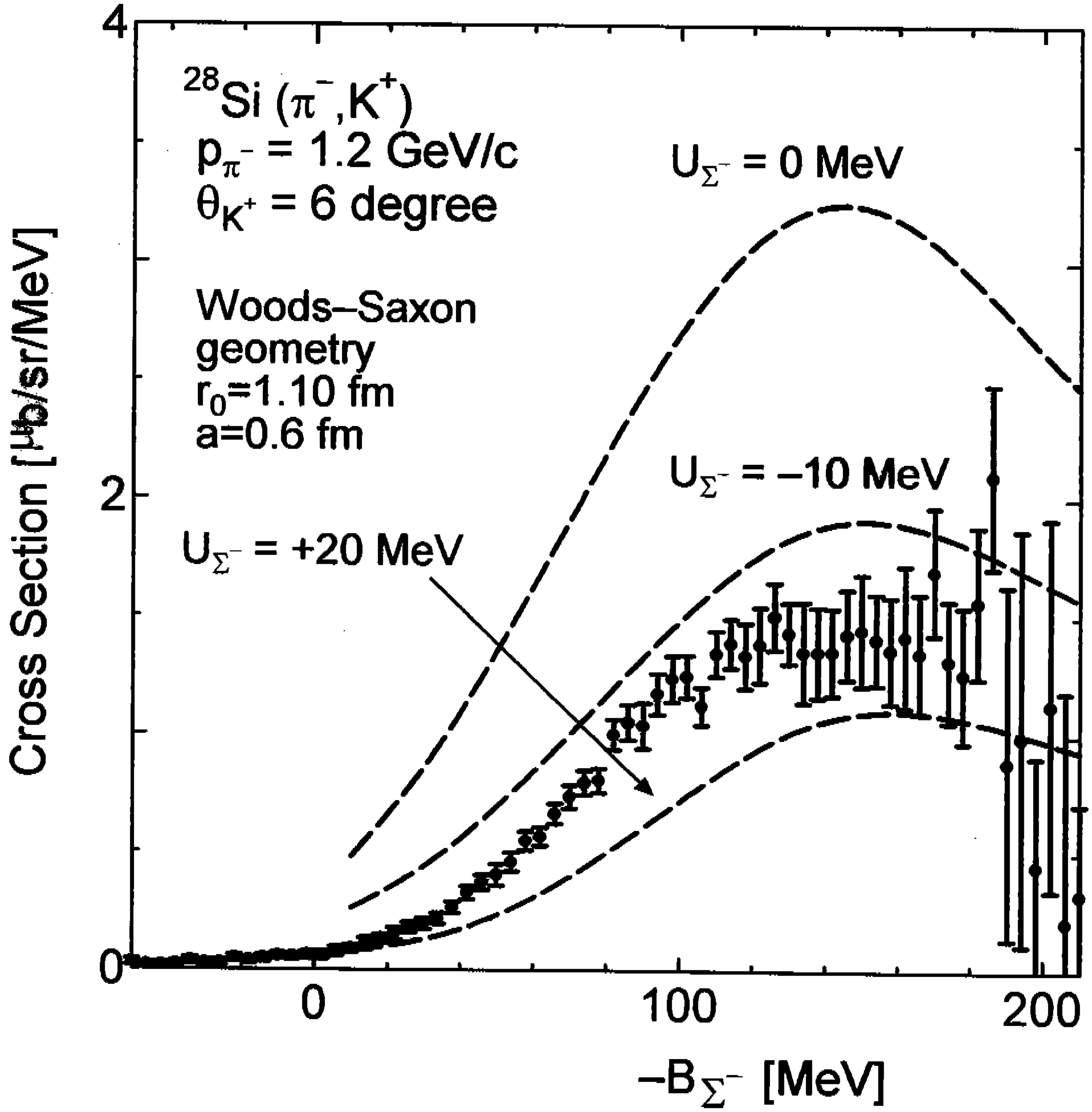


assumption of s-wave dominance $\Rightarrow$ amplitude t

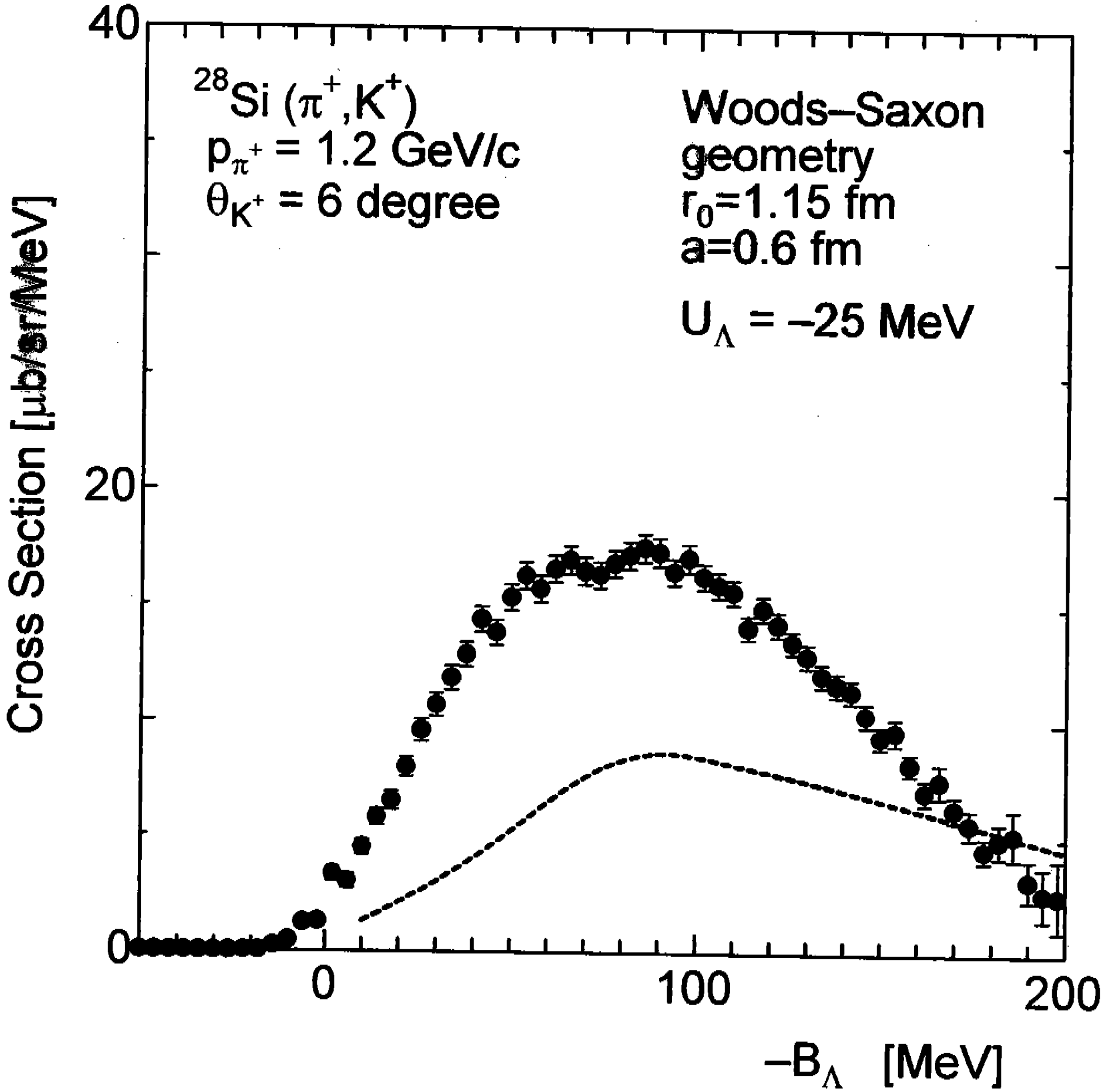
$$t_{\pi^- p \to K^+\Sigma^-}(\boldsymbol{r}_\pi, \boldsymbol{r}_p, \boldsymbol{r}_\Sigma, \boldsymbol{r}_K) = t(s)\delta(\boldsymbol{r}_\pi - \boldsymbol{r}_p)\delta(\boldsymbol{r}_p - \boldsymbol{r}_\Sigma)\delta(\boldsymbol{r}_\Sigma - \boldsymbol{r}_K)$$

$$t(s) = \sqrt{\frac{4\pi p_\pi s \sigma}{p_K E_N E_\Sigma(s)}}$$

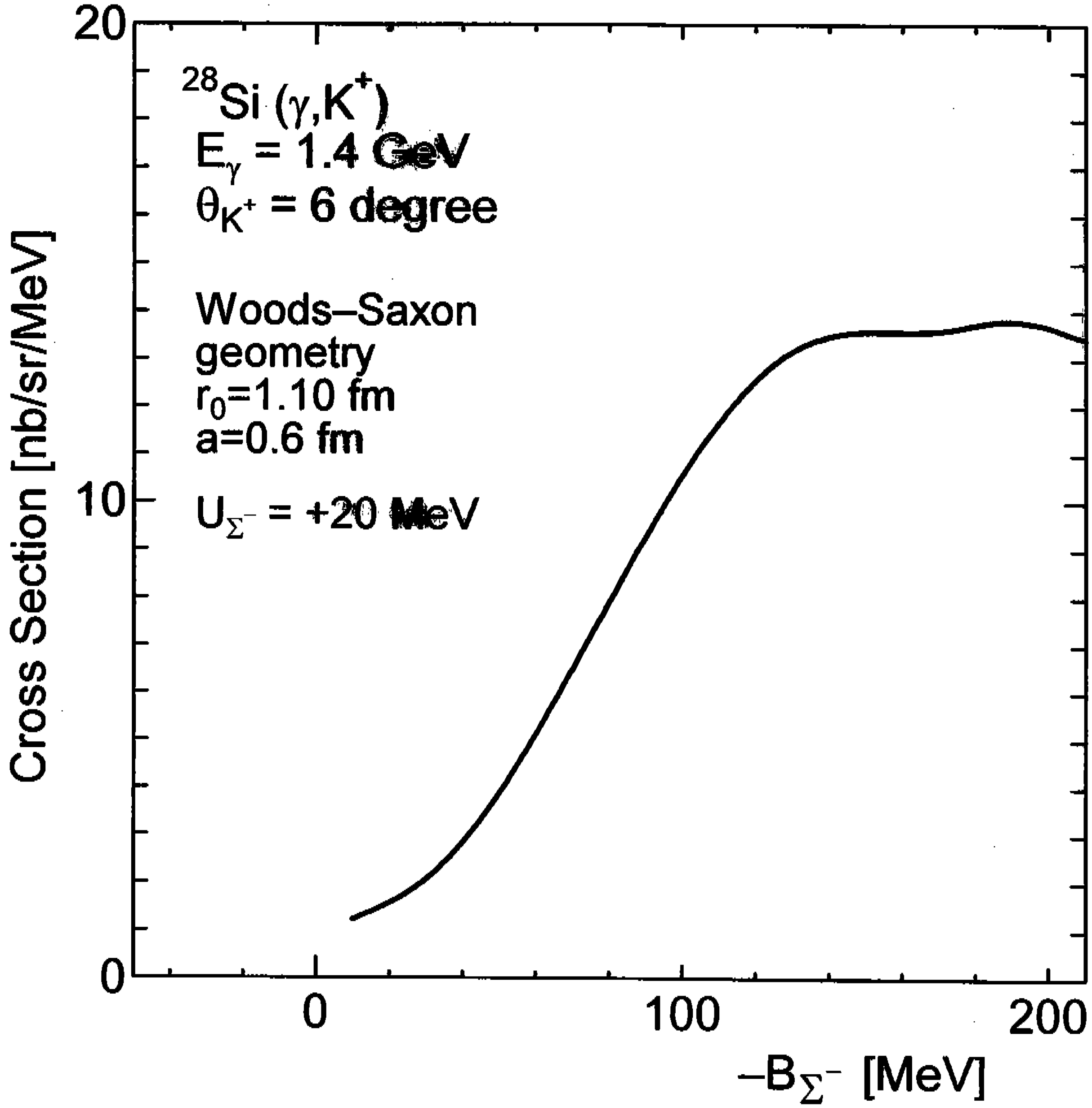




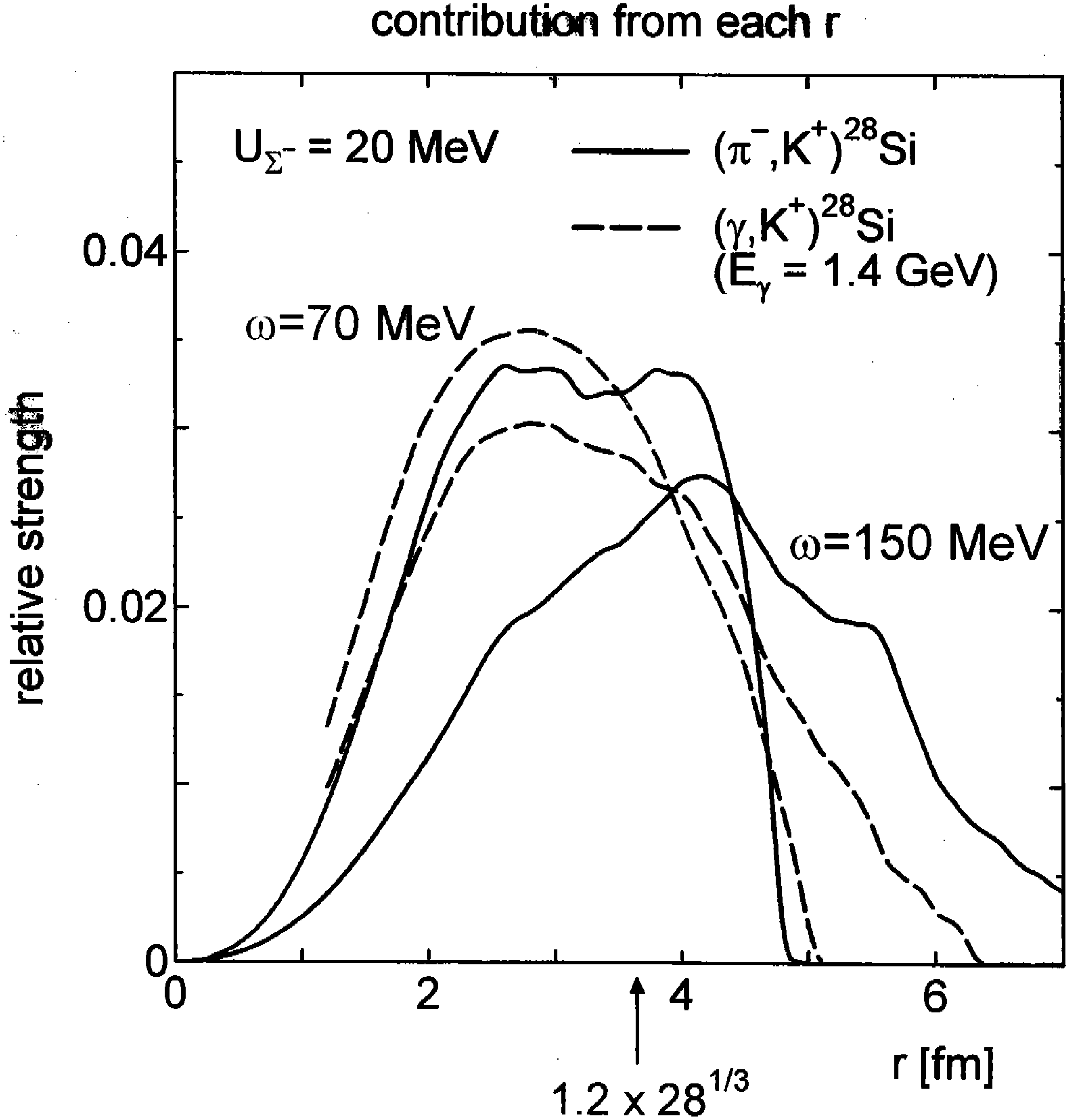
^{28}Si (π^-,K$^+$)
p_{π^-} = 1.2 GeV/c
θ_{K^+} = 6 degree
Woods–Saxon
geometry
r_0=1.10 fm
a=0.6 fm
U_{Σ^-} = 0 MeV
U_{Σ^-} = −10 MeV
U_{Σ^-} = +20 MeV
Cross Section [μb/sr/MeV]
$-B_{\Sigma^-}$ [MeV]
0
2
4
0
100
200



^{28}Si (π^+,K$^+$)
p_{π^+} = 1.2 GeV/c
θ_{K^+} = 6 degree
Woods–Saxon
geometry
r_0=1.15 fm
a=0.6 fm
U_Λ = –25 MeV
Cross Section [μb/sr/MeV]
$-B_\Lambda$ [MeV]
40
20
0
0
100
200



^{28}Si (γ,K$^+$)
E_γ = 1.4 GeV
θ_{K^+} = 6 degree
Woods–Saxon
geometry
r_0=1.10 fm
a=0.6 fm
U_{Σ^-} = +20 MeV
Cross Section [nb/sr/MeV]
$-B_{\Sigma^-}$ [MeV]
20
10
0
0
100
200



contribution from each r
U_{Σ^-} = 20 MeV
$(\pi^-,K^+)^{28}$Si
$(\gamma,K^+)^{28}$Si
(E_γ = 1.4 GeV)
ω=70 MeV
ω=150 MeV
relative strength
0.04
0.02
0
0
2
4
6
r [fm]
1.2 x $28^{1/3}$

Summary

- ☐ SCDW model was developed for describing (π^-, K^+) inclusive spectra.

  no free parameter

- ☐ KEK data of $^{28}Si\ (\pi^-, K^+)$ can be reproduced by a weakly repulsive Σ potential.

  Peak position and shape of the spectrum are well reproduced.

- ☐ Before drawing final conclusions, we have to improve the model in several points.

  π and K distorted waves,

  Green's function description for Σ with complex pot.,

  Estimation of multi–step contributions,

  refinement of the description of elementary processes.

- ☐ Extension to include multi–step processes is feasible in the SCDW.

- ☐ SCDW method will be used to investigate medium effects on elementary processes.

  (π, K), $(K, \pi$ [illegible] $K)$