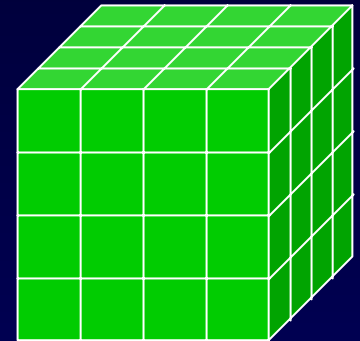


Baryon resonances and pentaquarks on the lattice

Frank X. Lee

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- Introduction
- Baryon Resonances and QCD
 - The nature of the Roper resonance
 - Level ordering of low-lying baryons
 - The η' ghost in the chiral region in quenched QCD
- Pentaquarks
 - Is there evidence on the lattice?



Quantum Chromodynamics (QCD)

--- the fundamental theory of the strong interaction
(in terms of quarks and gluons)

$$L_{QCD} = \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \bar{q}(\gamma^\mu D_\mu + m_q)q$$

Field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$

Covariant derivative: $D_\mu = \partial_\mu + gA_\mu$

QCD quarks: $m_u \sim 3 \text{ MeV}$, $m_d \sim 6 \text{ MeV}$, $m_s \sim 100 \text{ MeV}$

- **Chiral symmetry** and its spontaneous breaking
- At high energy, perturbative (**asymptotic freedom**)
- At low energy, non-perturbative (**confinement**)

CQM quarks vs. QCD quarks

- Constituent quarks
 - $m_u = m_d \sim 340 \text{ MeV}$,
 $m_s \sim 500 \text{ MeV}$
- Most of the proton mass comes from the quark masses.
- The mass splittings are provided by residual pair-wise interactions.

- QCD quarks
 - $m_u \sim 3 \text{ MeV}$, $m_d \sim 6 \text{ MeV}$,
 $m_s \sim 100 \text{ MeV}$
- Most of the mass and splittings come from interactions, to all orders of interaction.
- The quark masses only contribute to about 1% of the proton mass.

Current spectrum of baryon resonances:

01 0001-56

Baryon Summary Table

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the main Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the short table are not established as baryons. The names with masses are of baryons that decay strongly. For N , Δ , and Ξ resonances, the partial wave is indicated by the symbol $L_{J,2J}$, where L is the orbital angular momentum (S, P, D, \dots), I is the isospin, and J is the total angular momentum. For Λ and Σ resonances, the symbol is $L_{I,2J}$.

p	P_{11}	****	$\Delta(1232)$	P_{33}	****	Λ	P_{01}	****	Σ^+	P_{11}	****	Ξ^0, Ξ^-	P_{11}	****
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	$\Lambda(1405)$	S_{01}	****	Σ^0	P_{11}	****	$\Xi(1530)$	P_{13}	****
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	$\Lambda(1520)$	D_{03}	****	Σ^-	P_{11}	****	$\Xi(1620)$		*
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Lambda(1600)$	P_{01}	***	$\Sigma(1385)$	P_{13}	****	$\Xi(1690)$		***
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Lambda(1670)$	S_{01}	****	$\Sigma(1480)$		*	$\Xi(1820)$	D_{13}	***
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Lambda(1690)$	D_{03}	****	$\Sigma(1560)$		**	$\Xi(1950)$		***
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Lambda(1800)$	S_{01}	***	$\Sigma(1580)$	D_{13}	**	$\Xi(2030)$		***
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Lambda(1810)$	P_{01}	***	$\Sigma(1620)$	S_{11}	**	$\Xi(2120)$		*
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Lambda(1820)$	F_{05}	****	$\Sigma(1660)$	P_{11}	***	$\Xi(2250)$		**
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Lambda(1830)$	D_{05}	****	$\Sigma(1670)$	D_{13}	****	$\Xi(2370)$		**
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Lambda(1890)$	P_{03}	****	$\Sigma(1690)$		**	$\Xi(2500)$		*
$N(1900)$	P_{13}	**	$\Delta(1950)$	F_{37}	****	$\Lambda(2000)$	*	*	$\Sigma(1750)$	S_{11}	***			*
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Lambda(2020)$	F_{07}	*	$\Sigma(1770)$	P_{11}	*	Ω^-		****
$N(2000)$	F_{15}	**	$\Delta(2150)$	S_{31}	*	$\Lambda(2100)$	G_{07}	****	$\Sigma(1775)$	D_{15}	****	$\Omega(2250)^-$		***
$N(2080)$	D_{13}	**	$\Delta(2200)$	G_{37}	*	$\Lambda(2110)$	F_{05}	***	$\Sigma(1840)$	P_{13}	*	$\Omega(2380)^-$		**
$N(2090)$	S_{11}	*	$\Delta(2300)$	H_{39}	**	$\Lambda(2325)$	D_{03}	*	$\Sigma(1880)$	P_{11}	**	$\Omega(2470)^-$		**
$N(2100)$	P_{11}	*	$\Delta(2350)$	D_{35}	*	$\Lambda(2350)$	H_{09}	***	$\Sigma(1915)$	F_{15}	****			*
$N(2190)$	G_{17}	****	$\Delta(2390)$	F_{37}	*	$\Lambda(2585)$	**	**	$\Sigma(1940)$	D_{13}	***	Λ_c^+		****
$N(2200)$	D_{15}	**	$\Delta(2400)$	G_{39}	**				$\Sigma(2000)$	S_{11}	*	$\Lambda_c(2593)^+$		***
$N(2220)$	H_{19}	****	$\Delta(2420)$	$H_{3,11}$	****				$\Sigma(2030)$	F_{17}	****	$\Lambda_c(2625)^+$		***
$N(2250)$	G_{19}	****	$\Delta(2750)$	$h_{3,13}$	**				$\Sigma(2070)$	F_{15}	*	$\Lambda_c(2765)^+$		*
$N(2600)$	$h_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**				$\Sigma(2080)$	P_{13}	**	$\Lambda_c(2880)^+$		**
$N(2700)$	$K_{1,13}$	**							$\Sigma(2100)$	G_{17}	*	$\Sigma_c(2455)$		****
									$\Sigma(2250)$		***	$\Sigma_c(2520)$		***
									$\Sigma(2455)$		**	Ξ_c^+, Ξ_c^0		***
									$\Sigma(2620)$		**	Ξ_c^{*+}, Ξ_c^{*0}		***
									$\Sigma(3000)$		*	$\Xi_c(2645)$		***
									$\Sigma(3170)$		*	$\Xi_c(2790)$		***
												$\Xi_c(2815)$		***
												Ω_c^0		***
												Λ_b^0		***
												Ξ_b^0, Ξ_b^-		*

21 N^* : $P_{11}(1440) - K_{1,13}(2700)$

22 Δ^* : $P_{33}(1232) - K_{3,15}(2950)$

17 Λ^*

21 Σ^*

10 Ξ^*

3 Ω^*

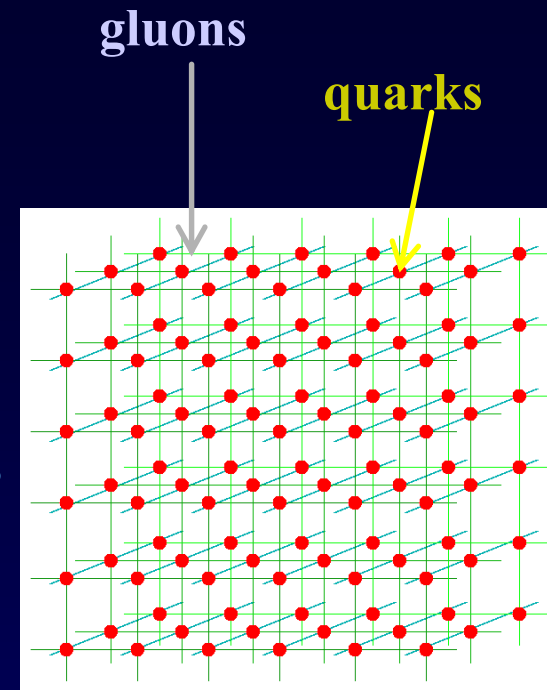
6 charmed $\Lambda^*, \Sigma^*, \Xi^*$

100 baryon resonances!

...but many of them are poorly known...

Key ideas of lattice QCD

- First principles approach: no small coupling expansion needed!
- Discretization of 4-dimensional space-time by a lattice of spacing a , size $V=N_x \times N_y \times N_z \times N_t$
 - e.g., $16^3 \times 28$ at $a=0.2$ fm, so the physical volume is $(3.2 \text{ fm})^3 \times 5.6 \text{ fm}$
 - Finite degrees of freedom
 - Quarks reside on sites, gluons reside on links

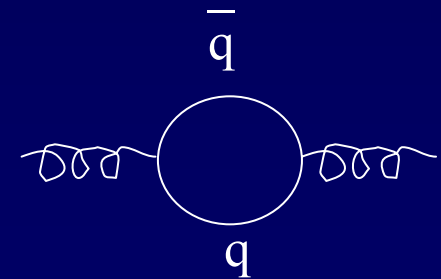


Quark Propagator in QCD

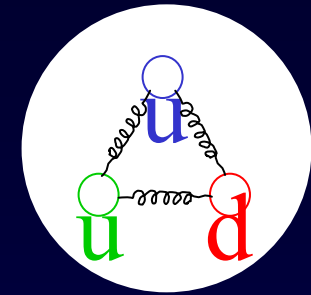
$$M^{-1}(x,0) = \langle vac | T[q(x)\bar{q}(0) | vac \rangle = \left\langle \frac{1}{\mathcal{D} + m_q} \right\rangle$$

$$\equiv \frac{\int DADqD\bar{q}[\bar{q}(x)q(0)]e^{-S_{QCD}}}{\int DADqD\bar{q}e^{-S_{QCD}}} = \frac{\int DA M^{-1}\det M e^{-S_G}}{\int DA \det M e^{-S_G}}$$

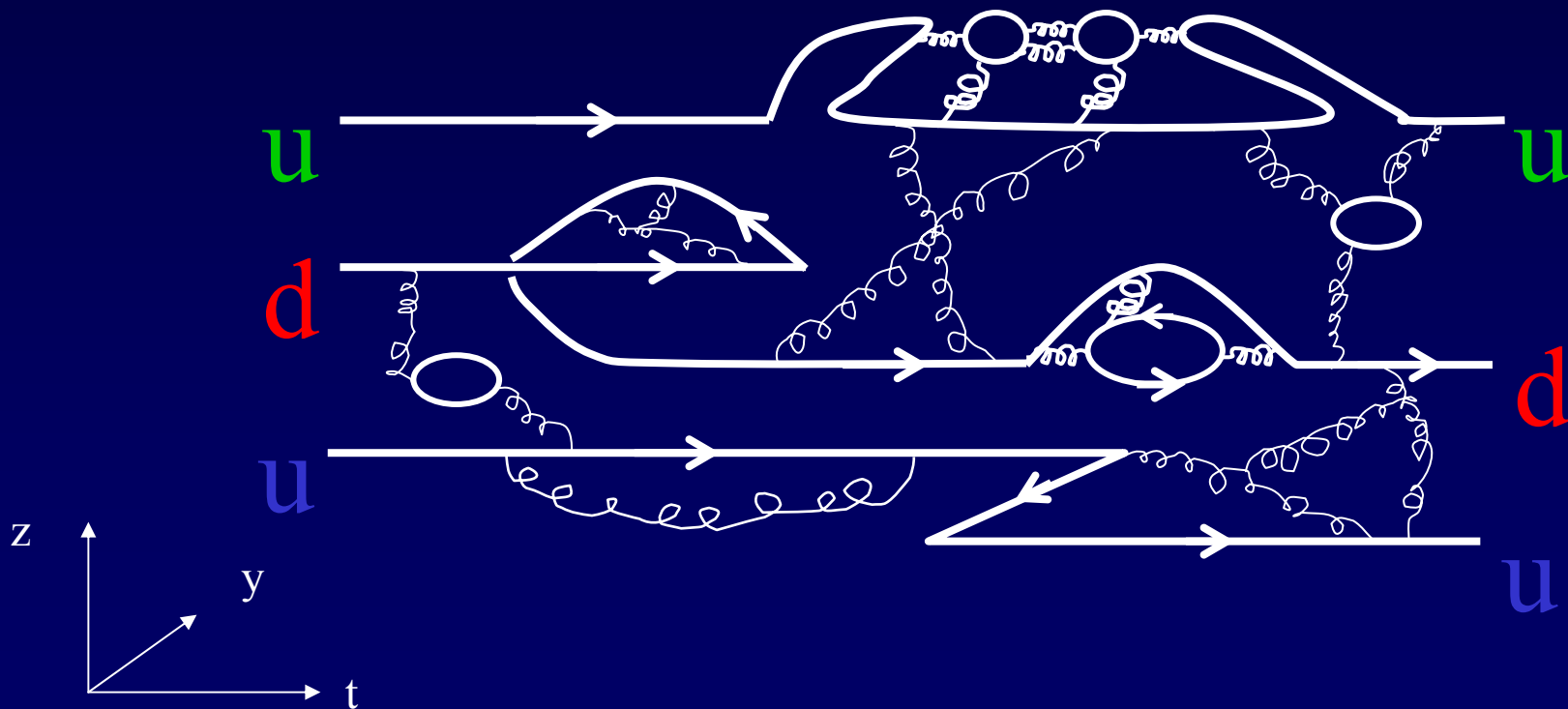
- Quark matrix $M = \mathcal{D} + m_q$
 - Inversion of large and sparse matrix in a gauge background
- Path integrals
 - Evaluated numerically on a space-time lattice
 - Monte-Carlo sampling method
- Quenched approximation (set $\det M = 1$)
 - Physically, suppress quark-antiquark bubbles in the vacuum



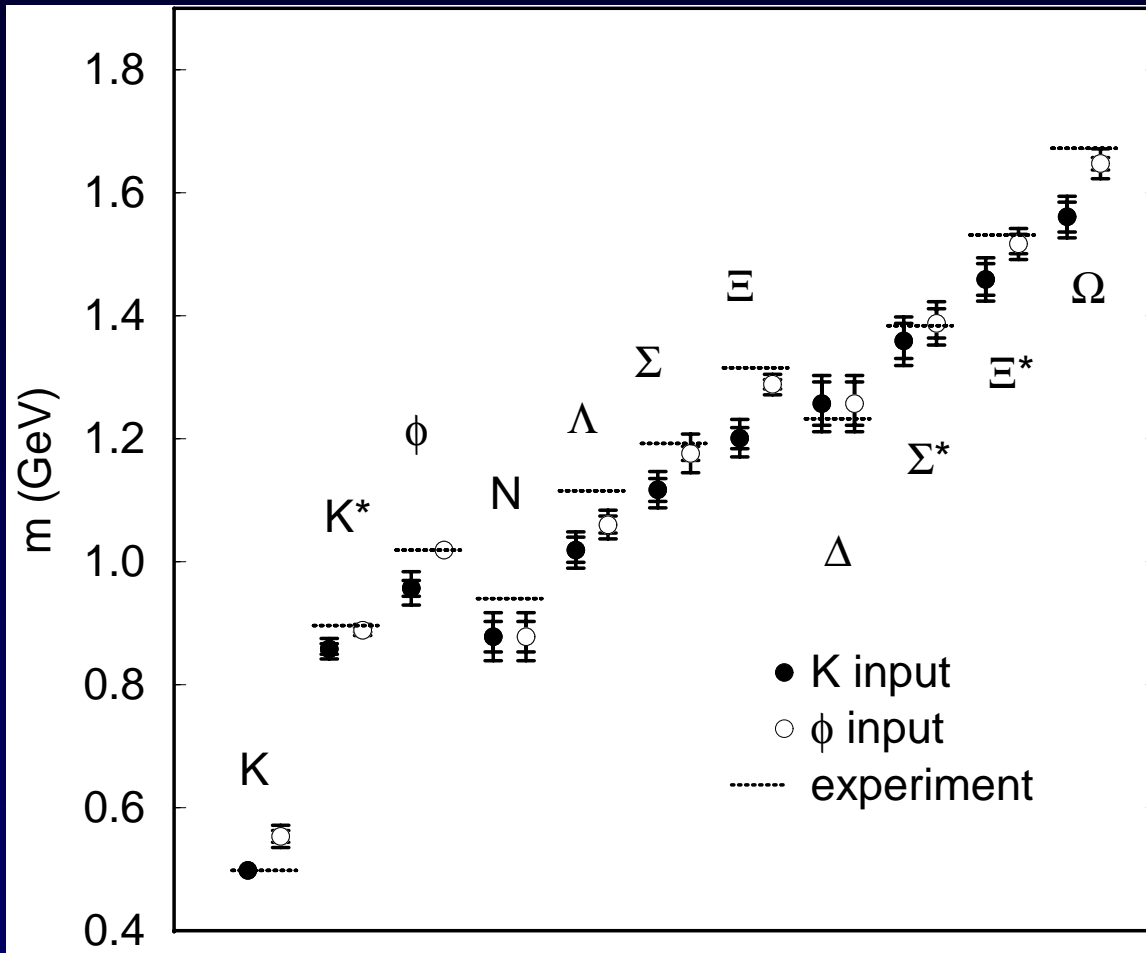
The proton in the quark model:



The proton in QCD:



How good is the quenched approximation?

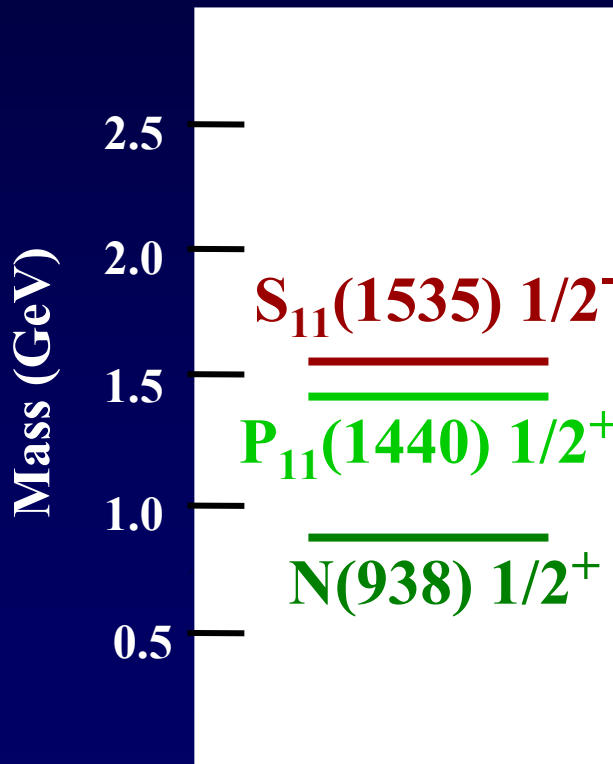


- Light hadron spectrum from CP-PACS, hep-lat/0206090.
- Lattices: $32^3 \times 56$ to $64^3 \times 128$
- Spacing 0.1 fm to 0.05 fm
- M_π / M_ρ is 0.75 to 0.4
- 1 to 3 % statistical error
- 2% systematic error
- Took more than a year of running on a dedicated computer sustaining 300 Gflops.

The computed quenched light hadron spectrum is within 10% of the experiment. The remaining discrepancy is attributed to the quenched approximation.

Comparison with lattice results

1. What is the nature of the Roper ($P_{11}(1440) 1/2^+$) resonance?



Naïve quark model gives the wrong ordering

————— $N(1440)1/2^+$

$\hbar \omega$

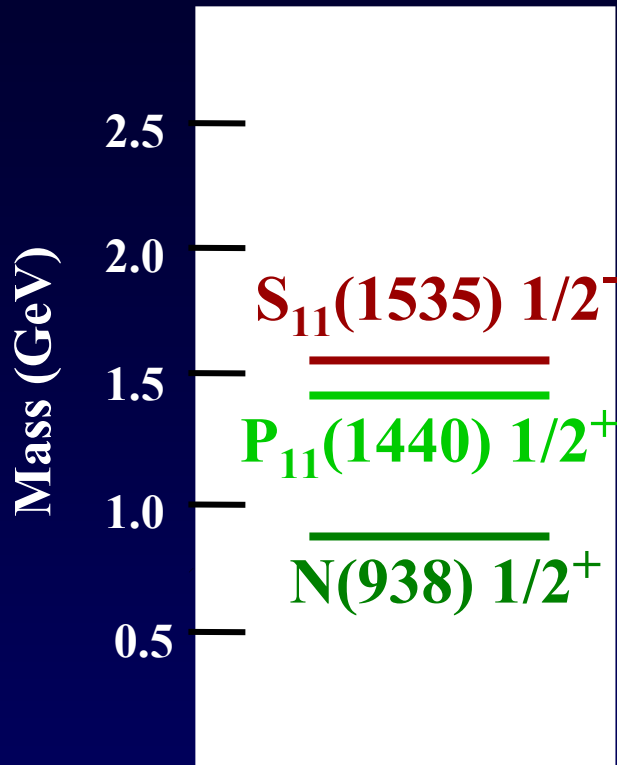
————— $N(1535)1/2^-$

$\hbar \omega$

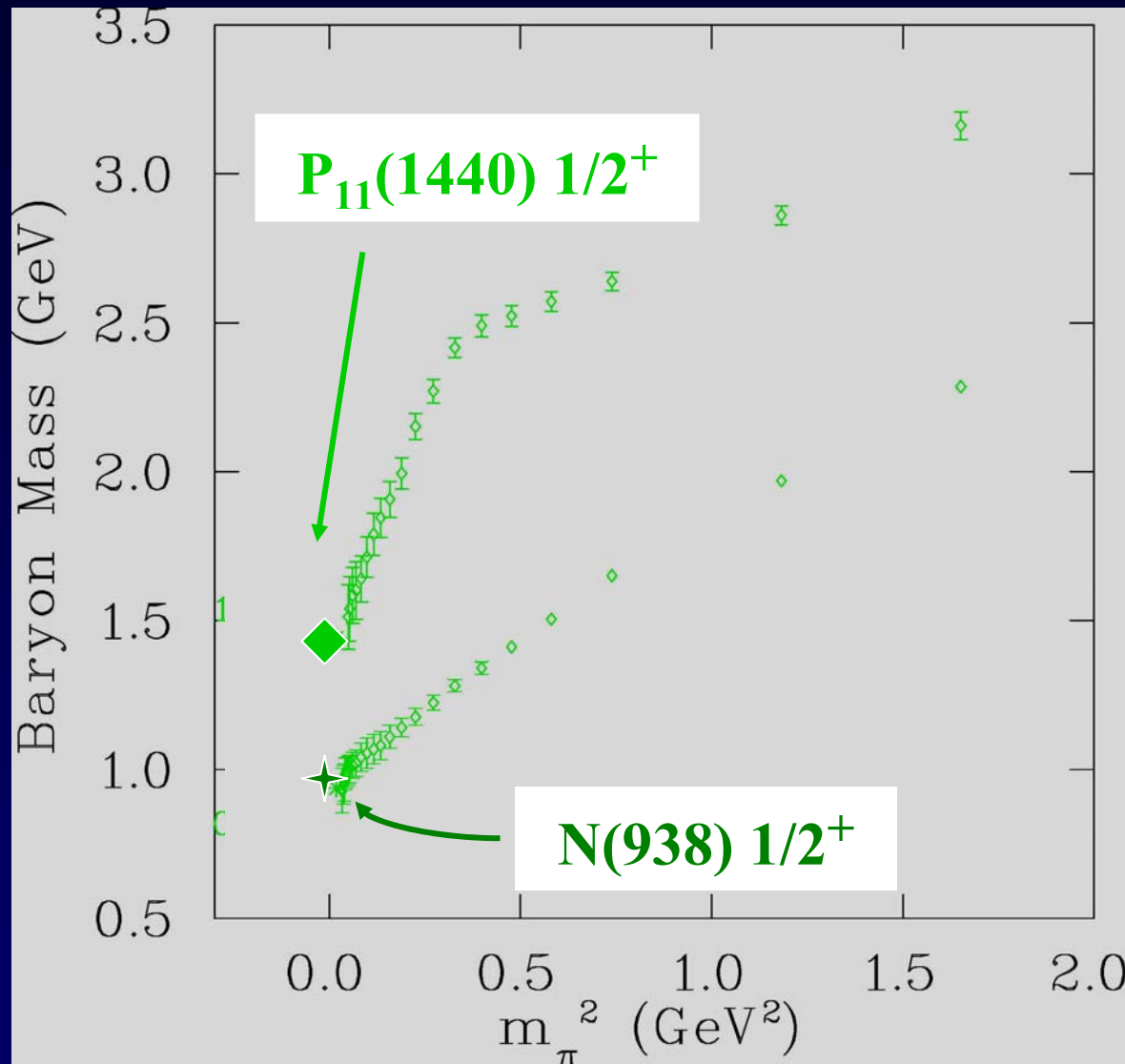
————— $N(938)1/2^+$

- Hybrid state ($qqqg$)?
- Dynamical meson-baryon state?

1. What is the nature of the Roper ($P_{11}(1440) 1/2^+$) resonance?

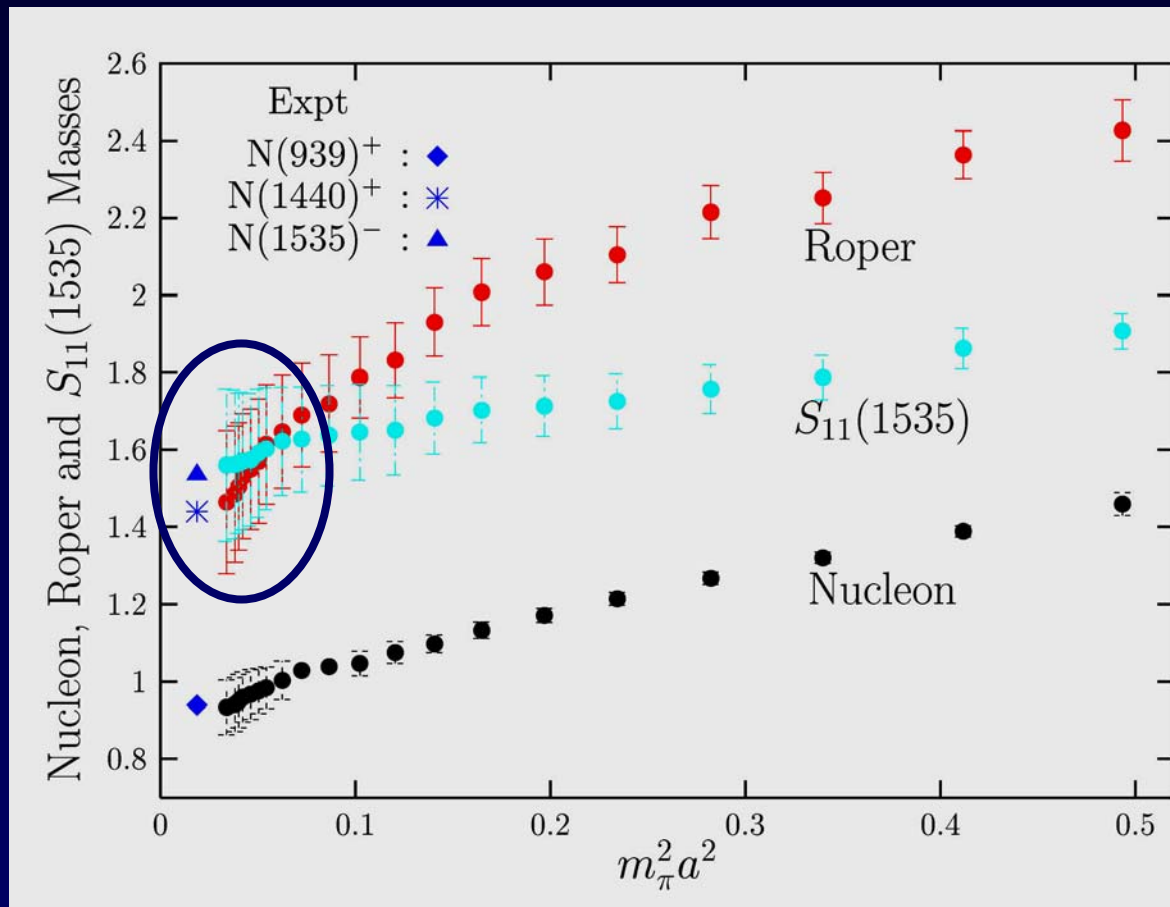
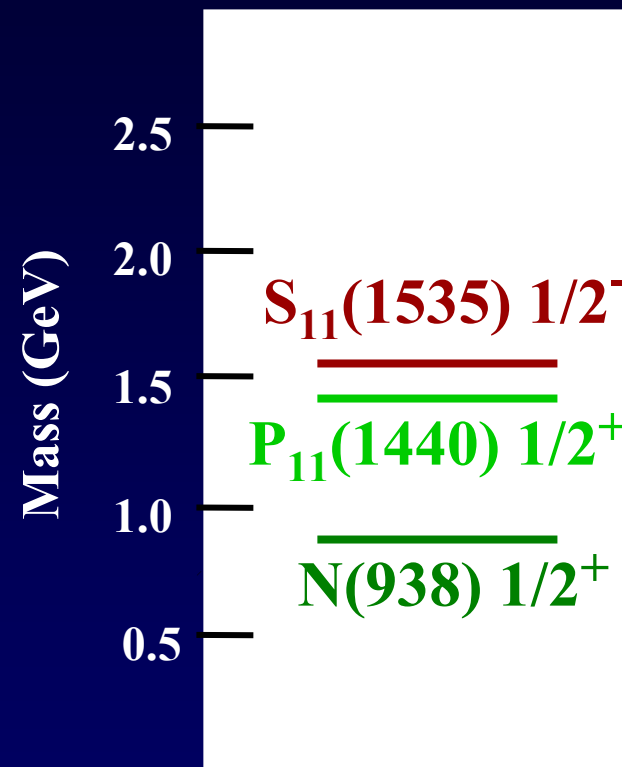


$m_\pi^2 \propto m_q$ by chiral symmetry



Answer: The Roper ($P_{11}(1440) 1/2^+$) is just a regular 3-quark state!

2. Can we understand the level ordering?

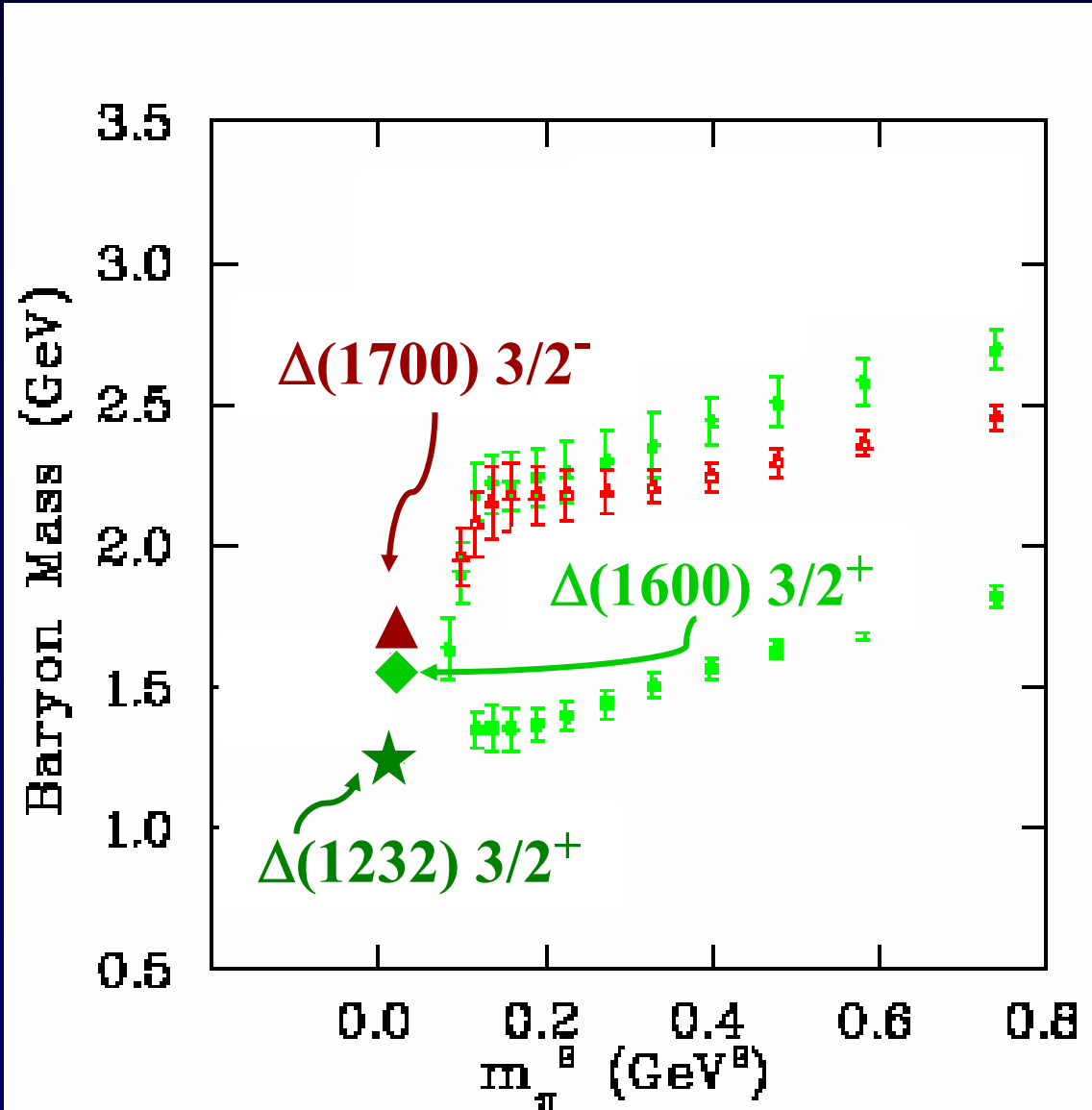
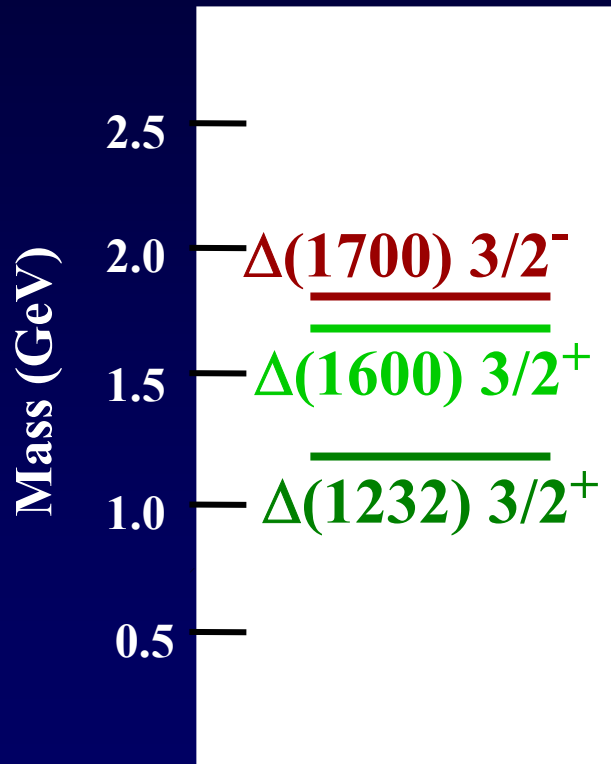


hep-ph/0306199

Cross over occurs very close to chiral limit!

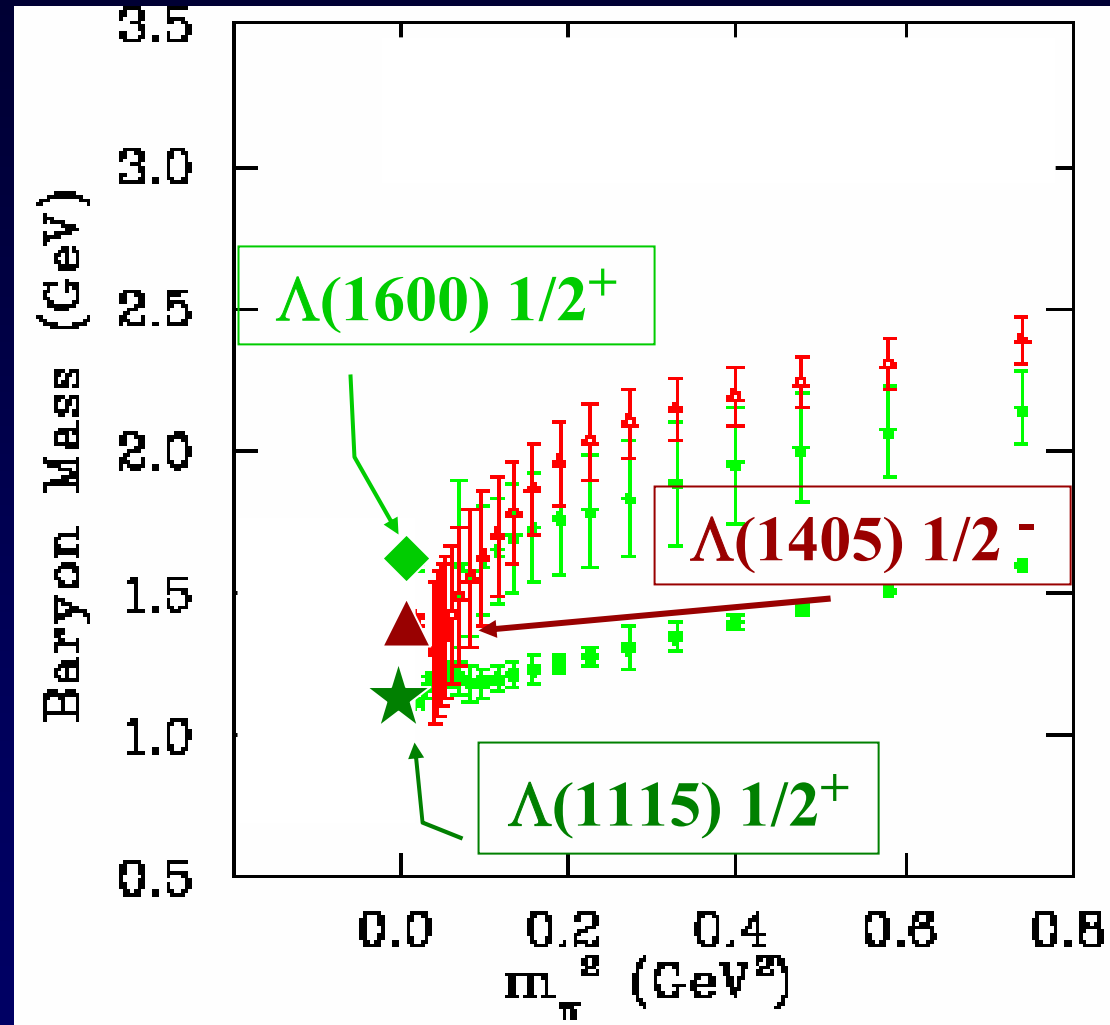
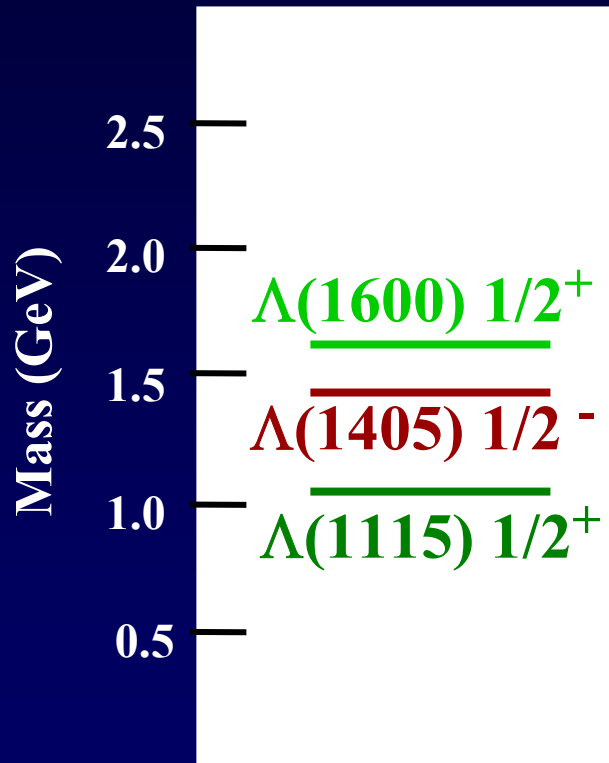
3. What about the Deltas?

...same story!!

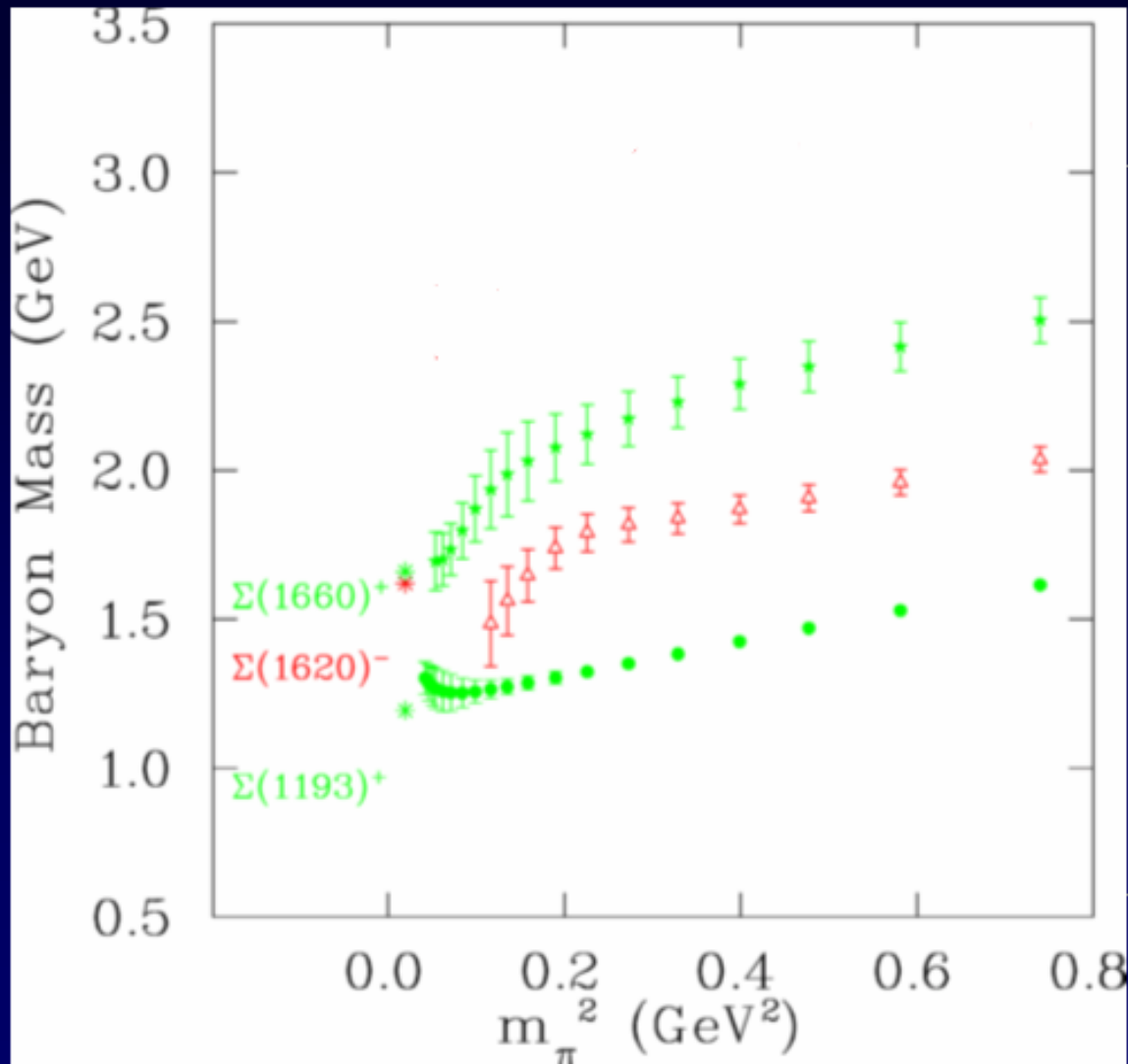


4. What about Hyperons? The $\Lambda(1405)$?

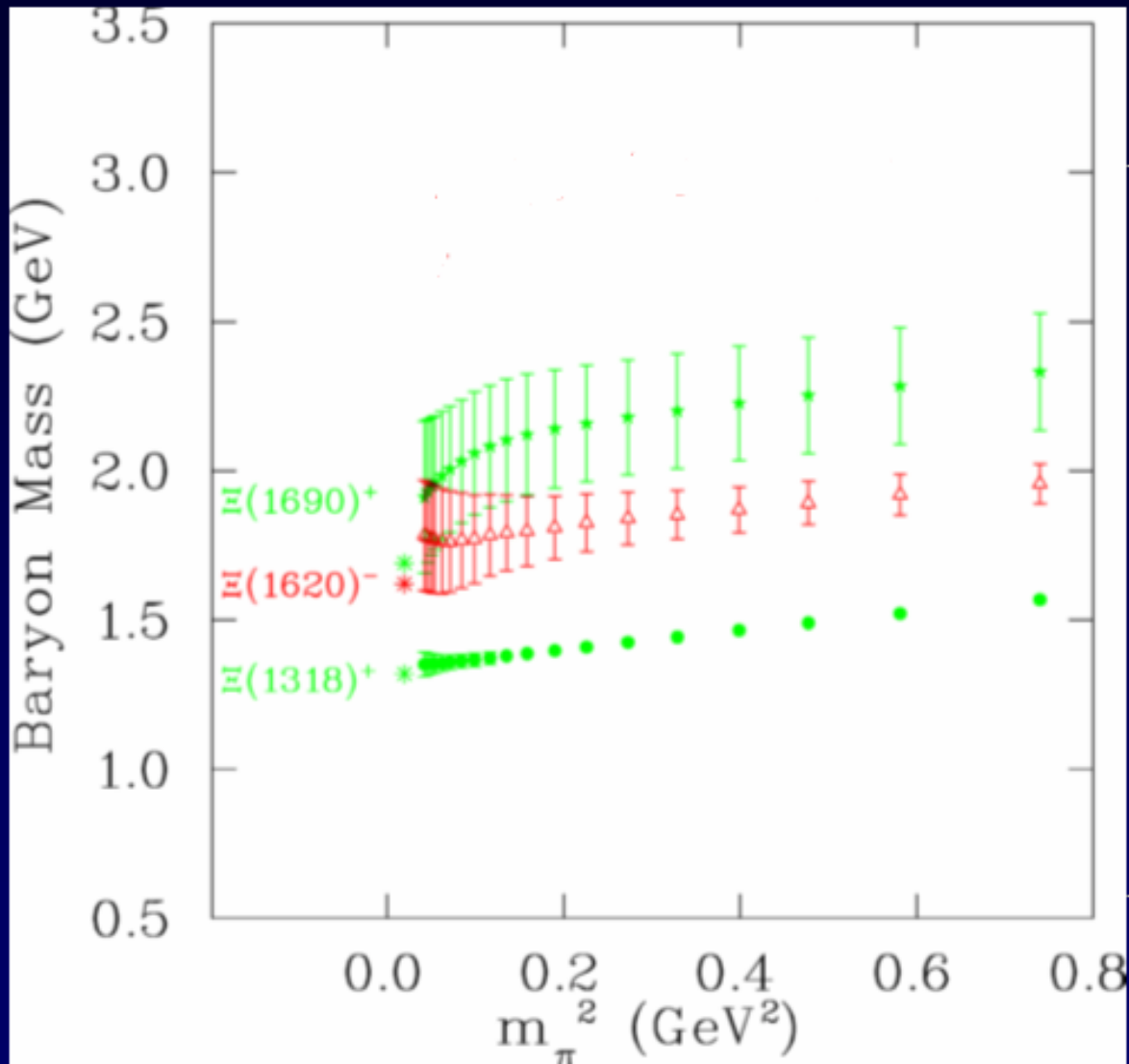
...*different* story!!



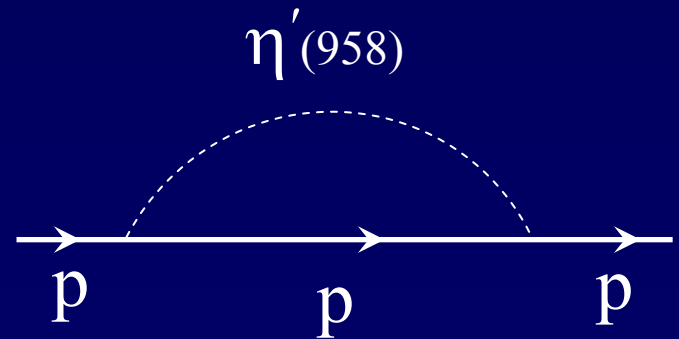
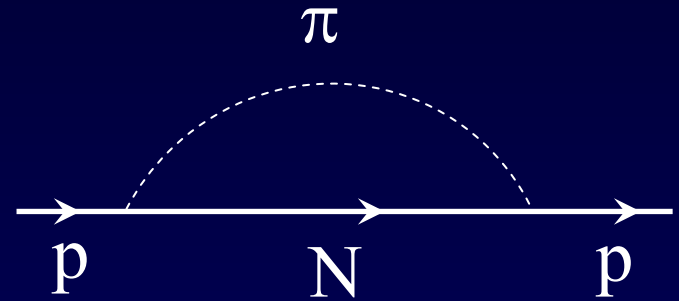
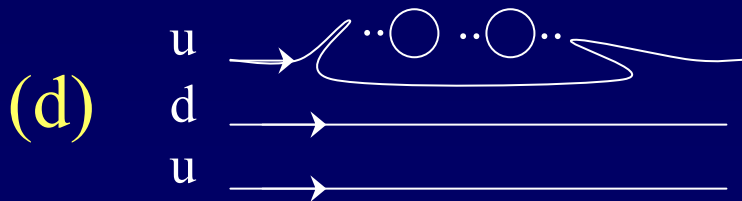
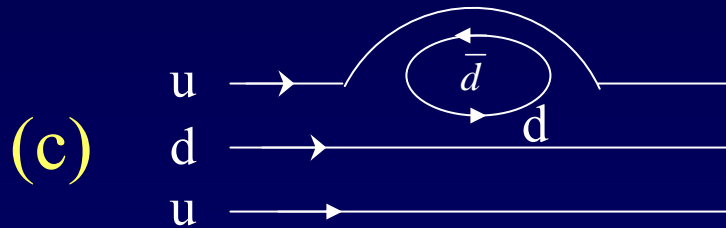
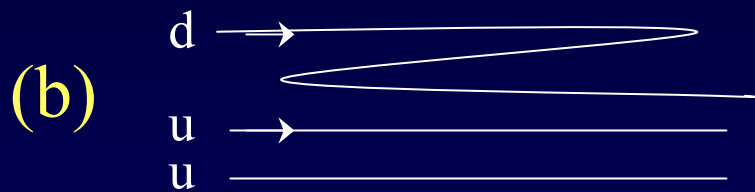
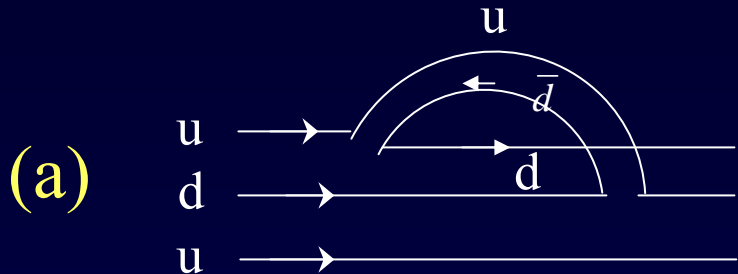
Level ordering in the $\Sigma(1/2)$ channel



Level ordering in the $\Xi(1/2)$ channel

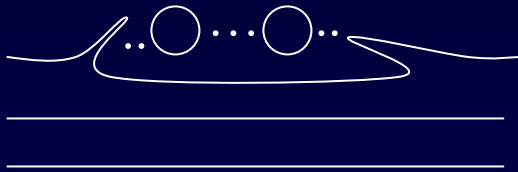


Chiral Dynamics: meson cloud in QCD

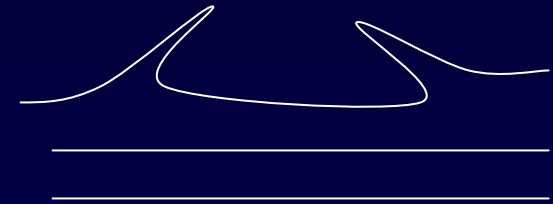


The η' ghost in quenched QCD

Full QCD



Quenched QCD



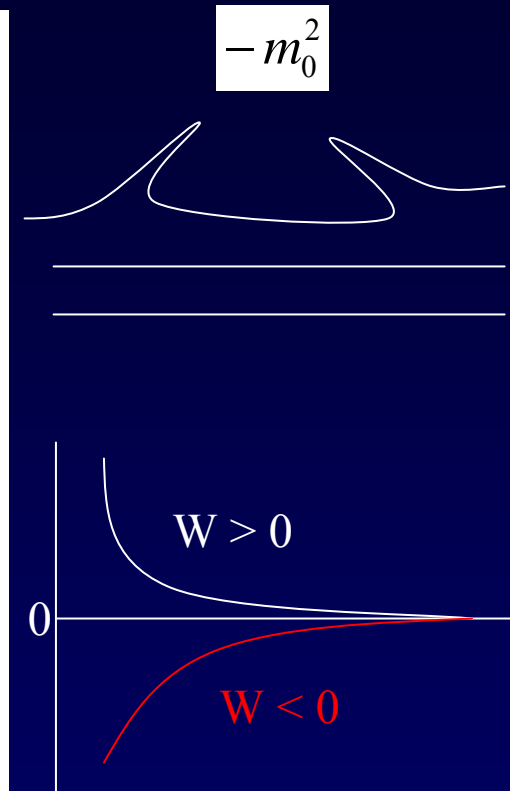
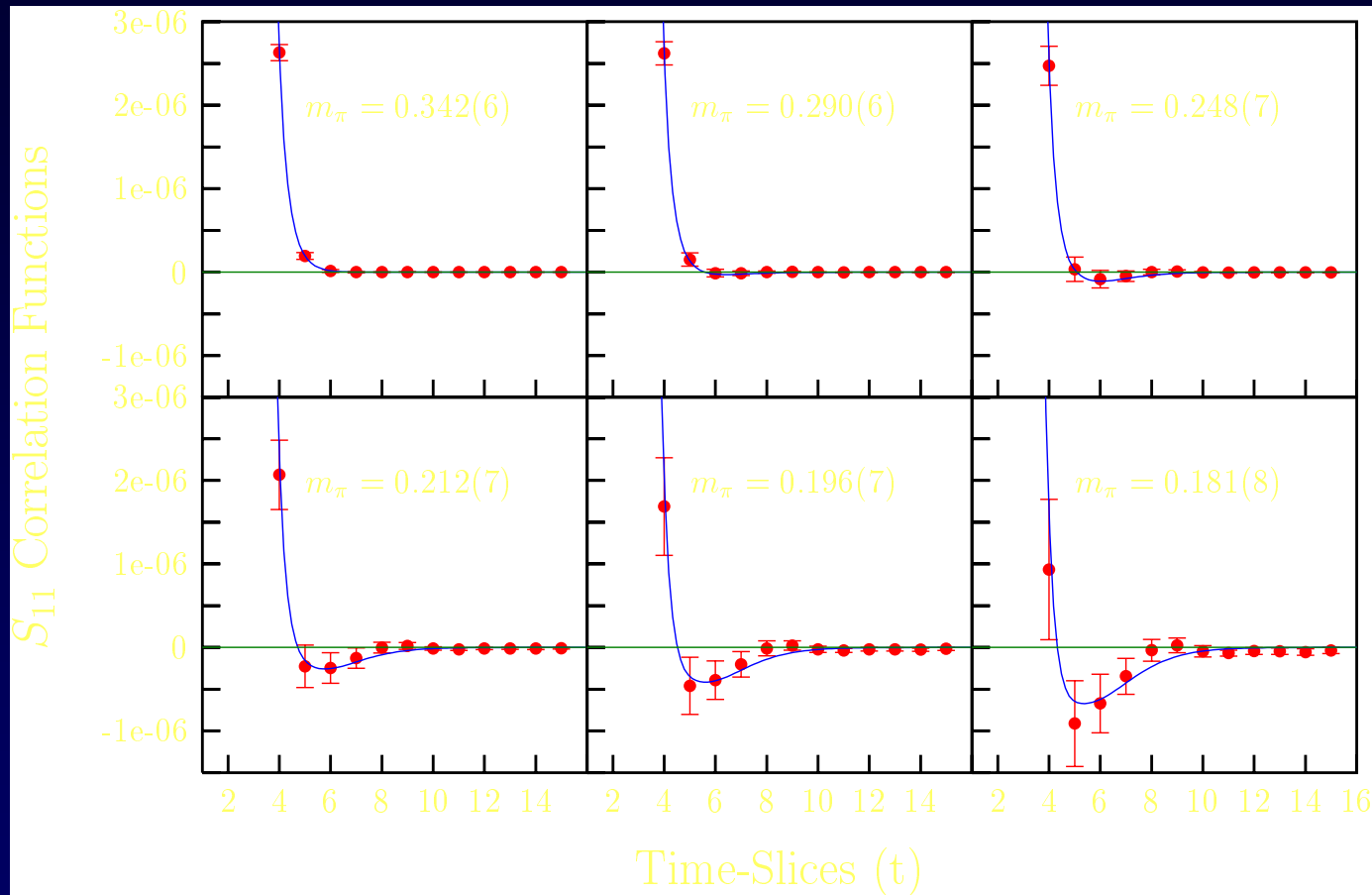
(hairpin)

- It becomes a light degree of freedom
 - with a mass degenerate with the pion mass.
- It is present in all hadron correlators $G(t)$.
- It gives a negative contribution to the $G(t)$
 - It is unphysical (thus the name ghost)
 - A pathology of quenched approximation

Modeled as part of $G(t)$ as: $w(1 + E_{\pi}t)e^{-E_{\eta'N}t}$

- weight w is negative
- prefactor $(1 + E_{\pi}t)$ preserves the double-pole structure of the hairpin diagram
- $E_{\eta'N}$ is treated as fit parameter to account for interactions between η' and N

Evidence of η' N ghost state in S_{11}

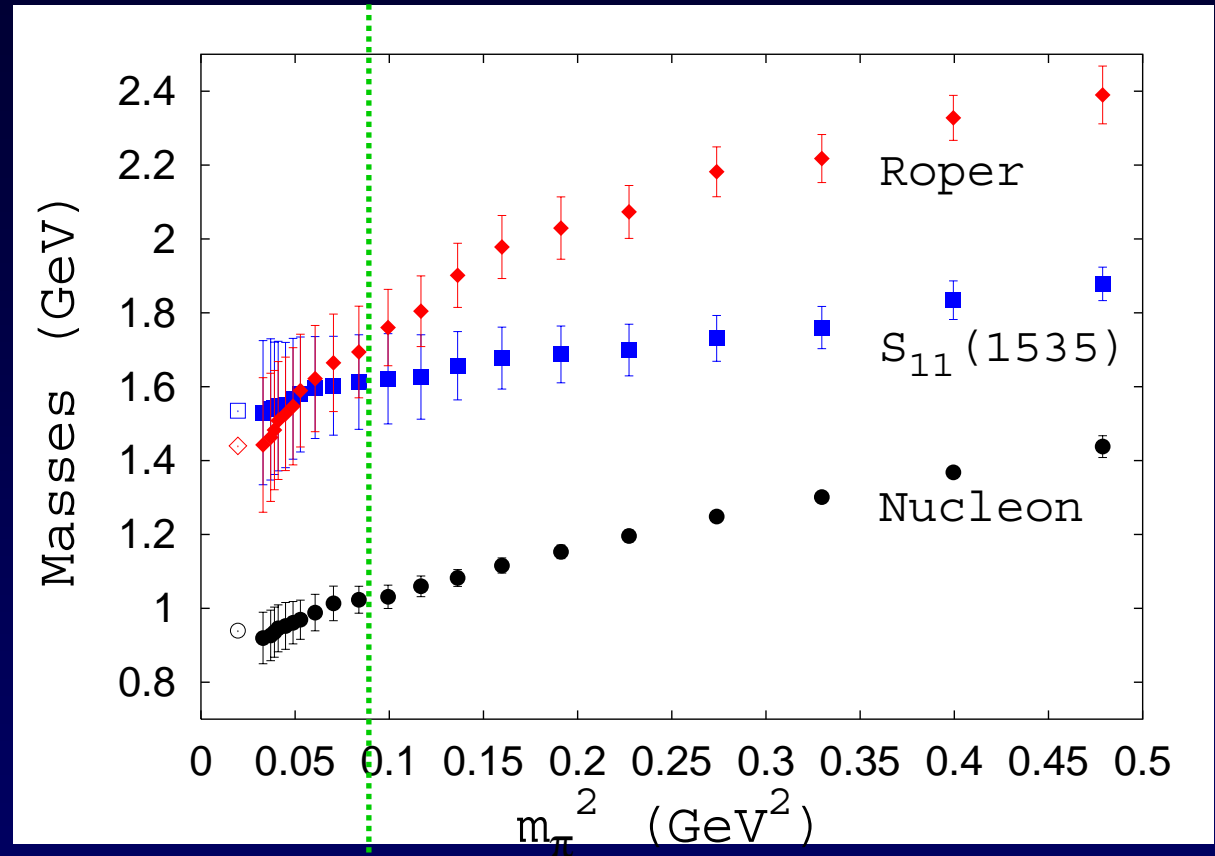
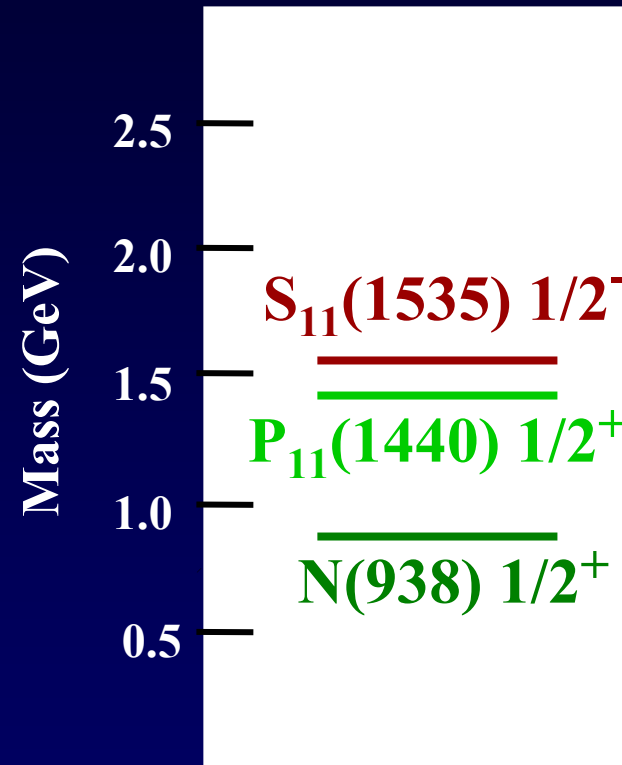


The effect of the ghost state decreases as pion mass increases.

First time observed in a baryon channel.

It was seen in a_0 meson channel by Bardeen et al, PRD65, 014509 (2001).

Nucleon, Roper and S_{11} : final results



Cross-over around $m_\pi \approx 300$ MeV.

Our smallest $m_\pi \approx 180$ MeV.

Transition from spin-color to flavor-color

[hep-ph/0306199](https://arxiv.org/abs/hep-ph/0306199)

Summary of N^* Physics

- Baryon Resonances like the Roper and $\Lambda(1405)$ can be reproduced on the lattice with 3 QCD quarks (not constituent quarks)
- We observed a cross-over of Roper and S_{11} in the region around $m_\pi \sim 300$ MeV where chiral dynamics dominates. This gives the correct ordering of low-lying states.
- The effects of η' ghost must be dealt with in the chiral region (below $m_\pi \sim 300$ MeV) in all hadron channels in quenched QCD.

Pentaquarks

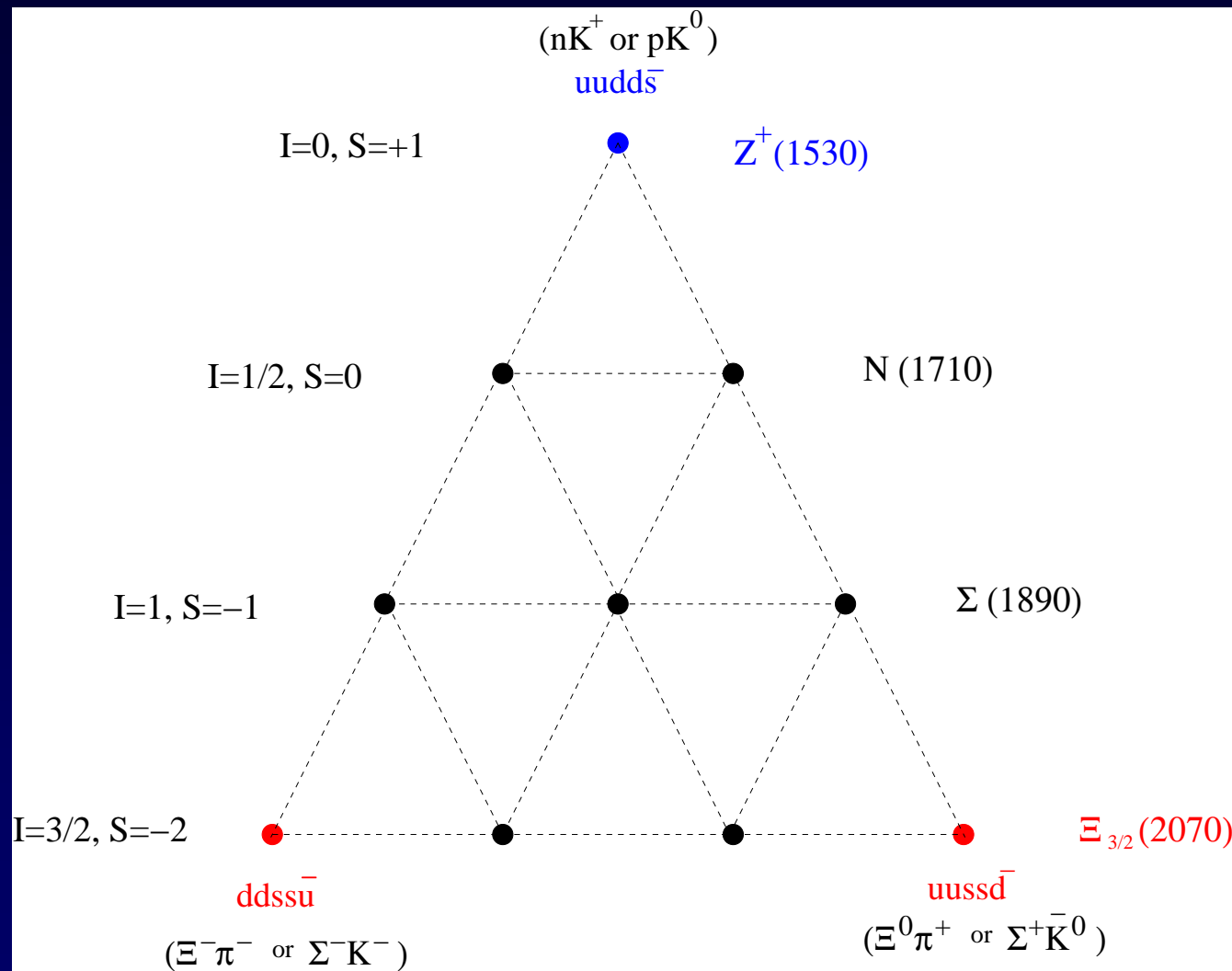
New Topic

What do the experiments say?

- Existence of a pentaquark named as Θ^+
 - quark content: $uudd\bar{s}$
 - mass around 1540 MeV
 - narrow width less than 20 MeV
 - decays into K^0p or K^+n
- What about quantum numbers ?
 - Positive strangeness ($S = +1$)
 - Isospin and spin-parity undetermined by the experiments. Need angular distribution analysis.
 - Based on its quark content alone: isospin could be $I = 0, 1, 2$
 - Spin could be $J = 1/2, 3/2, 5/2, \dots$
 - Parity could be either positive or negative.
- Most likely: $I(J^P)=0(1/2^+)$?

The anti-decuplet

Diakonov, Petrov, Polyakov, hep-ph/9703373



- 3rd rotational mode
- nonlinear pion interactions
- anti-decuplet
- corners are exotic
- lowest $I(J^P)=0(1/2^+)$
- strangeness +1
- mass 1530 MeV
- width <15 MeV

Breaking news:
 Observation of $\Xi_{3/2}^-(uuss\bar{d})$ at 1862 MeV at CERN in p-p collisions. hep-ex/0310014

Theoretical Studies

- A search on SPIRES would reveal more than 60 papers so far.
- Chiral Soliton and large N_c models
 - expects $J^P = 1/2^+$
- Constituent quark models
 - expects $J^P = 1/2^-$
- KN phase shifts
 - width smaller than 1 MeV
- QCD sum rules
 - $J^P = 1/2^-$

Pentaquarks in Lattice QCD

- “A search for pentaquarks on the lattice”
 - presented by F.X. Lee at Lattice2003 in July (final results to be released soon)
 - Conclusion: correlation function dominated by KN scattering states. No evidence for pentaquark of either parity
- “Pentaquark hadrons from lattice QCD”
 - F. Csikor, et al, hep-lat/0309090
 - Claim: observed Θ^+ of negative parity near 1540 MeV
- “Lattice study of exotic $S=+1$ baryon”
 - S. Sasaki, hep-lat/0310014
 - Claim: observed Θ^+ of negative parity near 1760 MeV

Pentaquark on the lattice: interpolating field

Consider interpolating field with $I=0$ and $J=1/2$.

Color structure is not unique, aside from being a color singlet.

Simplest choice: colorless meson and a colorless baryon.

$$\chi_{\{K^0_p\}}(x) = [\bar{s}(x)\gamma_5 d(x)] \epsilon^{abc} [u^{Ta}(x)C\gamma_5 d^b(x)] u^c(x),$$

$$\chi_{\{K^+_n\}}(x) = [\bar{s}(x)\gamma_5 u(x)] \epsilon^{abc} [d^{Ta}(x)C\gamma_5 u^b(x)] d^c(x).$$

$$\chi(x) = \chi_{\{K^0_p\}}(x) - \chi_{\{K^+_n\}}(x).$$

$$|I = 0, I_z = 0 \rangle = |I_z = +\frac{1}{2}, I_z = -\frac{1}{2} \rangle - |I_z = -\frac{1}{2}, I_z = +\frac{1}{2} \rangle.$$

Pentaquark Correlation Function

KN
scattering
state is part
of this
correlation
function

$$\langle \chi(x) \bar{\chi}(0) \rangle =$$

$$2\epsilon^{abc}\epsilon^{a'b'c'} \{ S_u^{aa'} C\gamma_5 S_d^{eb'T} S_s^{ef*} S_d^{bfT} C\gamma_5 S_u^{cc'}$$

$$+ S_u^{aa'} \text{Tr}(S_d^{eb'} C\gamma_5 S_d^{cc'T} C\gamma_5 S_d^{bf} S_s^{ef*T})$$

$$- \text{Tr}_{spin,color}(S_d S_s^\dagger) S_u^{aa'} C\gamma_5 S_d^{bb'T} C\gamma_5 S_u^{cc'}$$

$$- \text{Tr}_{spin,color}(S_d S_s^\dagger) S_u^{aa'} \text{Tr}(S_u^{bb'} C\gamma_5 S_d^{cc'T} C\gamma_5) \}$$

$$\mp 2\epsilon^{abc}\epsilon^{a'b'c'} \{ -S_d^{cf} S_s^{ef*T} S_u^{ec'} \text{Tr}(S_d^{aa'} C\gamma_5 S_u^{bb'T} C\gamma_5)$$

$$- S_d^{cb'} (C\gamma_5 S_u^{ba'T} C\gamma_5) S_d^{af} S_s^{ef*T} S_u^{ec'}$$

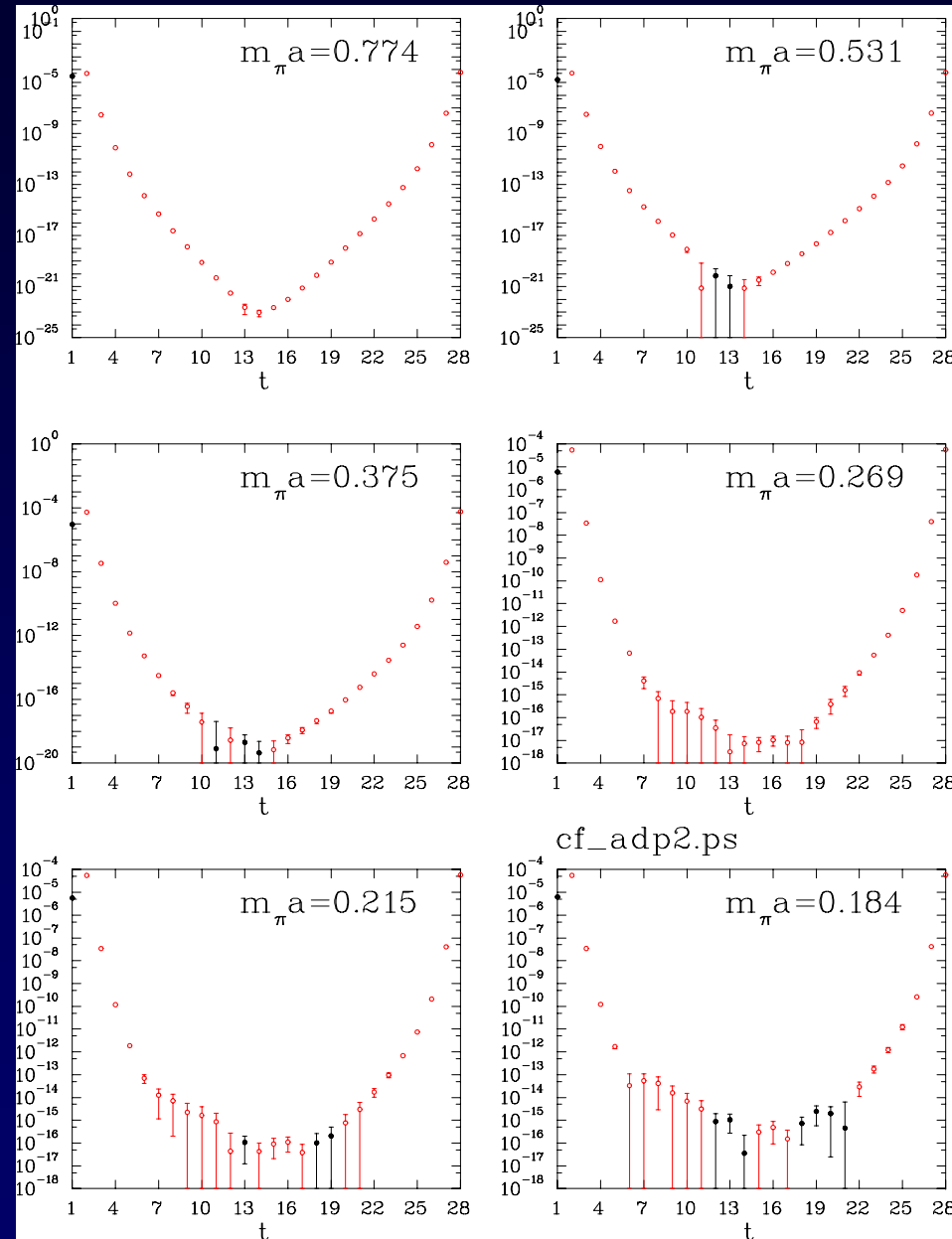
$$- S_d^{cf} S_s^{ef*T} S_u^{ea'} (C\gamma_5 S_d^{ab'T} C\gamma_5) S_u^{bc'}$$

$$- S_d^{cb'} C\gamma_5 S_u^{eaT} S_s^{ef*} S_d^{afT} C\gamma_5 S_u^{bc'} \}.$$

Pentaquark (uudds) Correlation Function

$$G_L(t) = A_+ e^{-m_+(t-t_0)} + A_- e^{-m_-(N_t+t_0-t)} + \dots$$

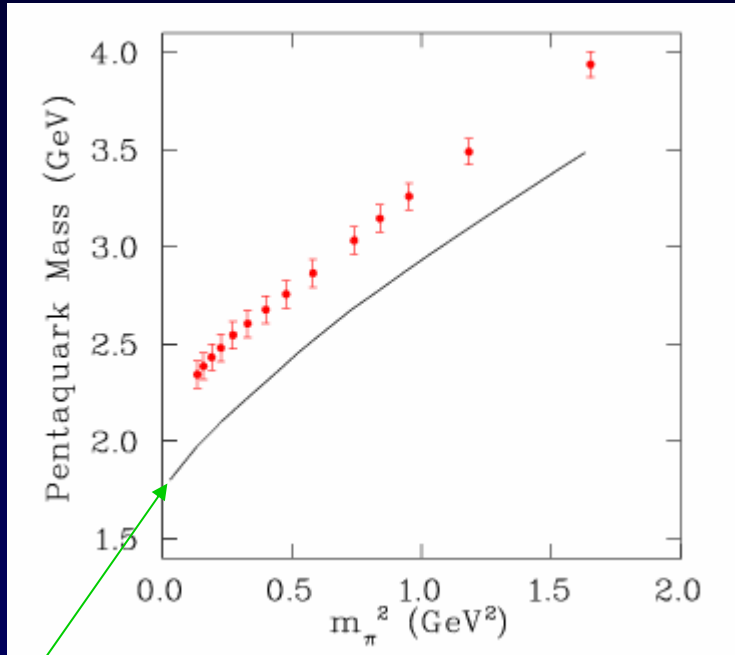
- Left half dominated by $\frac{1}{2}^+$
- Right half dominated by $\frac{1}{2}^-$
- Anti-periodic b.c. in t for quarks



Pentaquark $uudd\bar{s}$: $I=0, J=1/2$

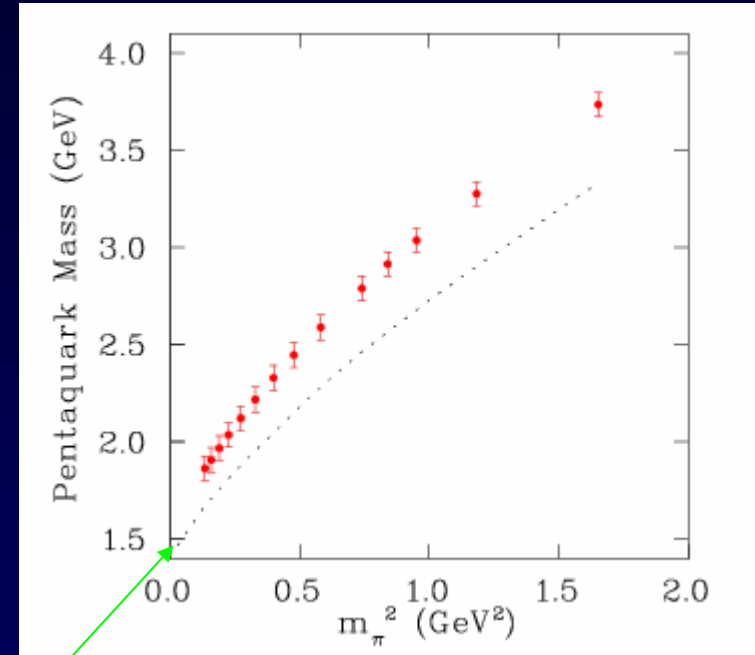
Positive Parity

Negative Parity



KN p-wave threshold

$$E_{KN} = \sqrt{m_K^2 + p^2} + \sqrt{m_N^2 + p^2}$$



KN s-wave threshold

$$E_{KN} = m_K + m_N = 1.43 \text{ GeV}$$

The minimum $p \sim 520 \text{ MeV}$ imposed by our lattice raises the threshold from 1.43 GeV to 1.79 GeV

Ξ -type pentaquark $uus\bar{s}\bar{d}$: $I=3/2, J=1/2$

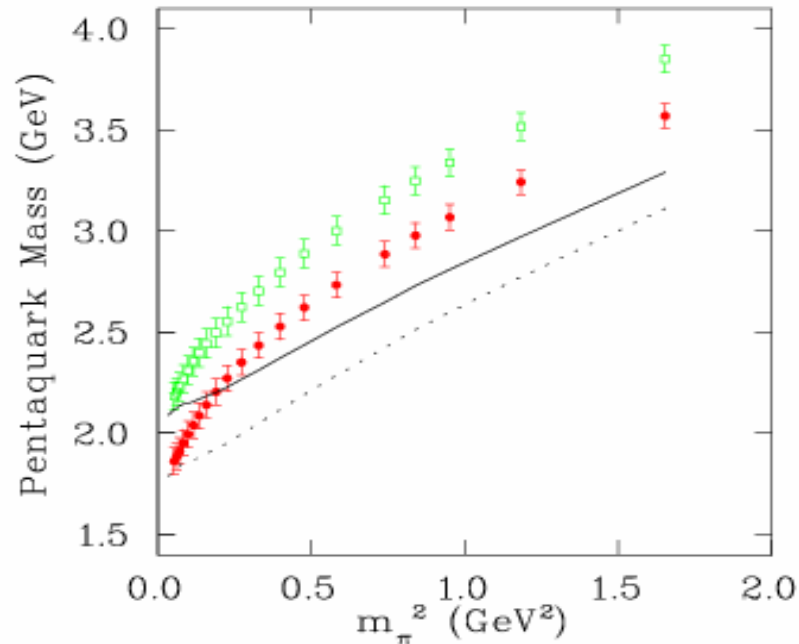
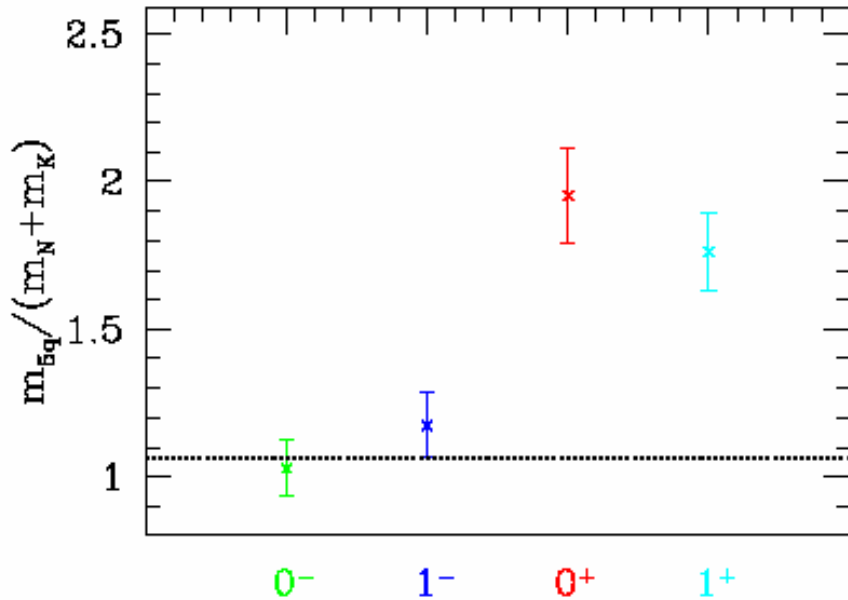


Figure 5. The computed mass of the Ξ -type pentaquark state ($uus\bar{s}\bar{d}$) with isospin $3/2$ and spin $1/2$ as a function of m_π^2 . The solid symbols are for negative parity, while the empty symbols are for positive parity. The solid line is the $K\Sigma$ p-wave energy and the dashed line the s-wave.

The results are consistent with $K\Sigma$ scattering states

Comment on hep-lat/0309090 (Csikor et al)



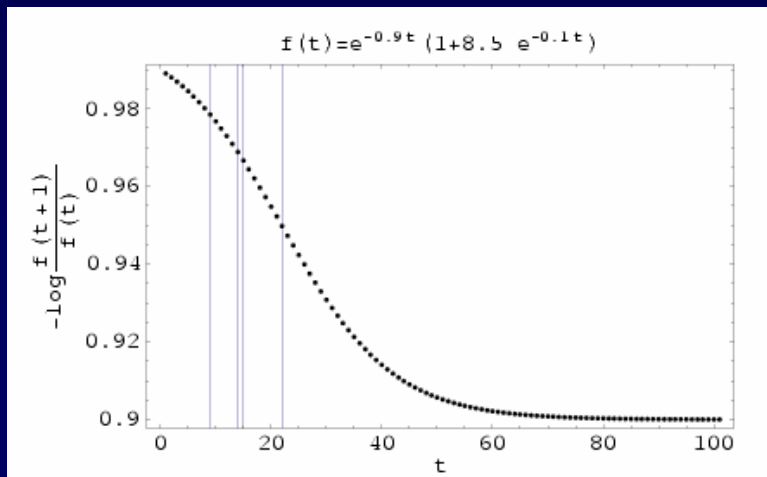
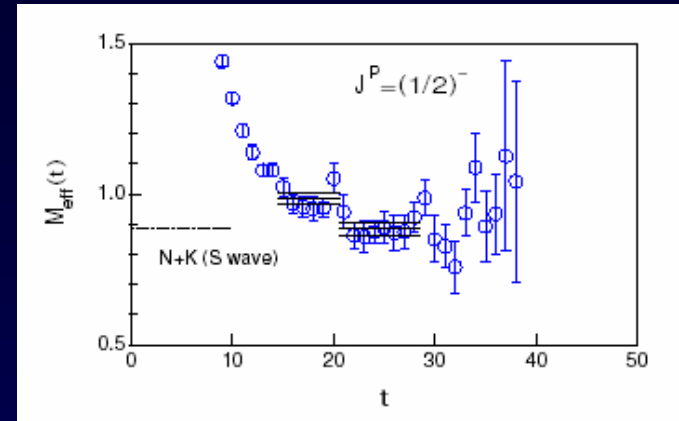
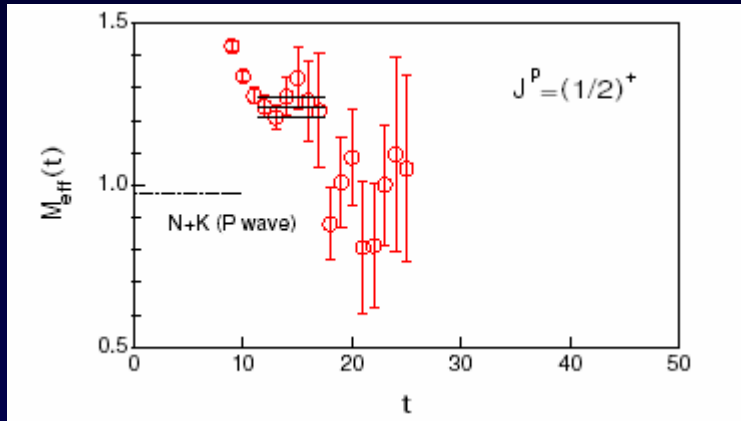
Initial claim: observed pentaquark in the $\frac{1}{2}^+$ channel at 1540 MeV, in perfect agreement with experiment.

Recent claim: parity was misidentified. Now it has negative parity.

Our critique:

- 1) It's very difficult to justify a bound pentaquark so close to the KN s-wave scattering threshold. It's mostly the KN s-wave.
- 2) Pion mass is too high (above 600 MeV) to detect significant volume dependence.

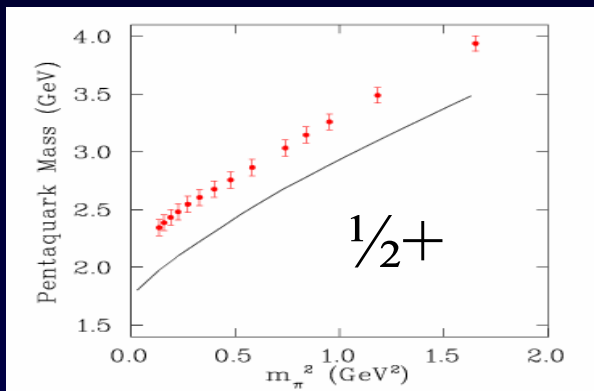
Comment on hep-lat/0310014 (Sasaki)



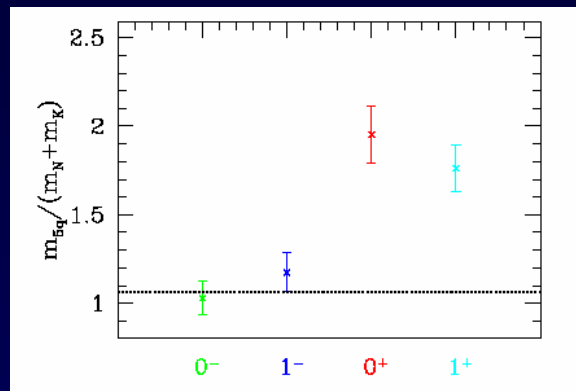
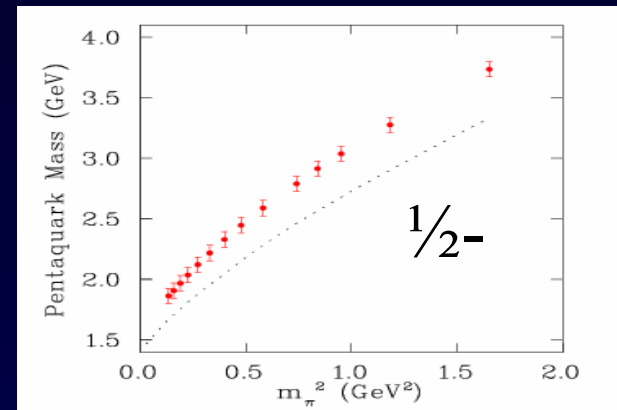
Claim: observed two closely-separated states in the $1/2^-$ channel. The 2nd is identified as the pentaquark.

Our critique: it is highly unlikely to separate two states with the parameters used in this work. It would require much better resolution on the lattice. It's most likely one state: the KN s-wave scattering state.

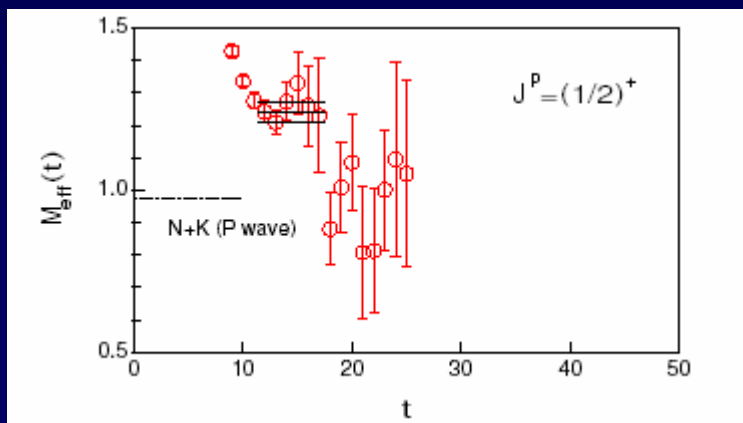
Comparison of the 3 calculations



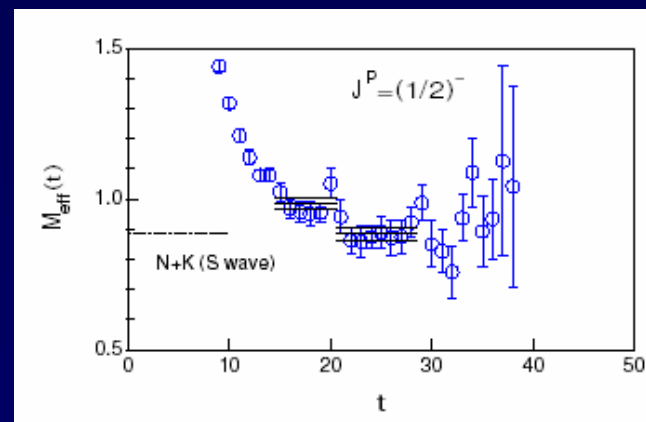
(Lee et al)



(Csikor et al)



(Sasaki)



They are consistent with: $\frac{1}{2}+$ near 2 GeV, $\frac{1}{2}-$ near 1.5 GeV

Conclusion on Pentaquarks on the lattice

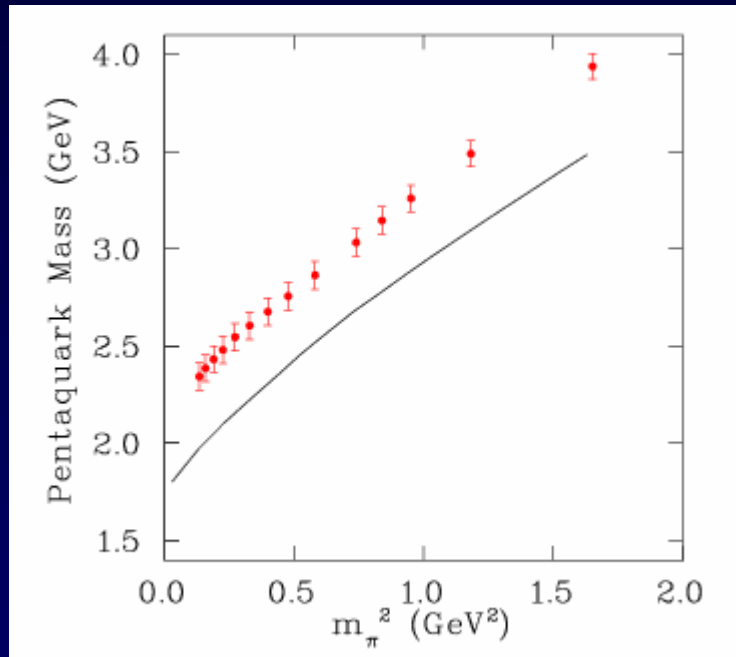
- Existing results reveal no evidence for a pentaquark of positive parity in the vicinity of 1540 MeV.
- Serious questions exist on the claim of seeing negative-parity pentaquarks near 1540 MeV in hep-lat/0309090 (Csikor et al) and hep-lat/0310014 (Sasaki).
 - Inadequate lattice resolution to separate two close states
 - Pion mass too high (above 600 MeV) to see volume dependence
 - Too close to KN s-wave threshold to clearly identify a pentaquark in $\frac{1}{2}^-$ channel
- In fact, all 3 calculations are consistent with domination by KN scattering s-wave and p-wave states near their thresholds:
 - A low mass near 1.5 GeV in the $\frac{1}{2}^-$ channel
 - A high mass near 2 GeV in the $\frac{1}{2}^+$ channel

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- Collaborators:
 - Keh-Fei Liu (University of Kentucky)
 - Nilmani Mathur (University of Kentucky)
 - Shao-Jing Dong (University of Kentucky)
 - Terry Draper (University of Kentucky)
 - Ivan Horvath (University of Kentucky)
 - Jianbo Zhang (University of Adelaide)
 - Cornelius Bennhold (George Washington University)
- U.S. Department of Energy
 - DE-FG05-84ER40154 and DE-FG02-95ER40807
- Computing Resources at NERSC and PSC and JLab

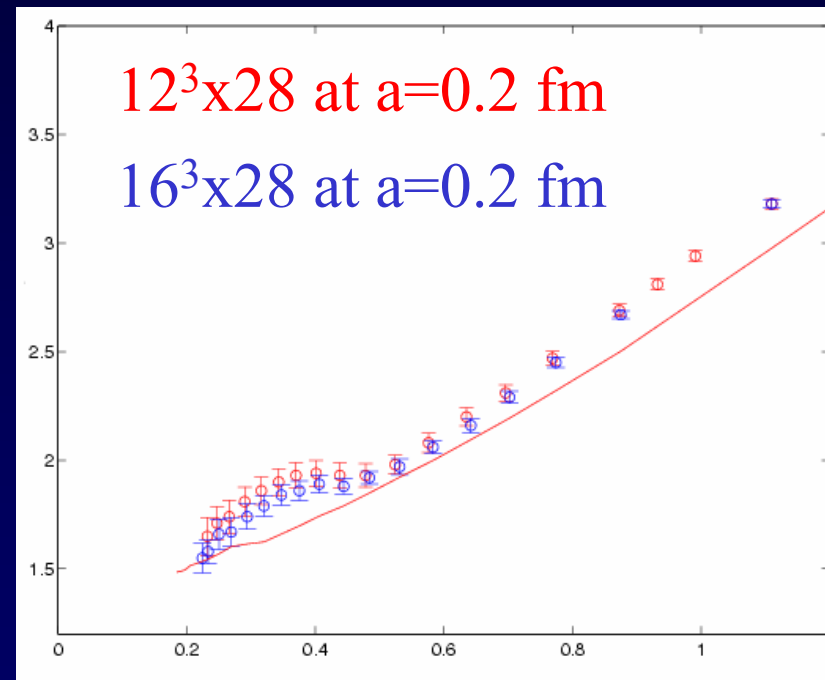
What would be needed to claim a genuine pentaquark near 1.54 GeV?

$1/2^+$: below threshold



There is no sign of that.

$1/2^-$: a bound state
+ volume insensitivity

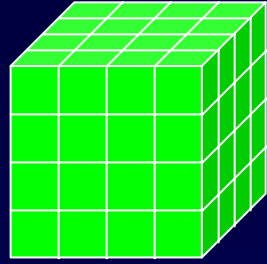


There is appreciable volume dependence, especially in the chiral region.

Reserve Slides

New Topic

What about errors in Lattice QCD?



- **Statistical errors**

$$\langle O \rangle \approx \frac{1}{N} \sum_{i=1}^N O(\{U\}_i)$$

- ← Typical error of a few percent with 100 to 300 configurations
- ← For the same error, the bigger lattice, the fewer configurations needed (volume average)

- **Systematic errors**

- ← Finite V
- ← Finite a
- ← Finite quark masses ($D+m_q$)
- ← Quenched approximation

- **Taking the continuum limit**

- ← Volume limit ($V \rightarrow \infty$)
- ← Spacing limit ($a \rightarrow 0$)
- ← Chiral limit ($m_u, m_d \rightarrow 0$)
- ← Un-quenching

Lattice QCD is an exact solution of QCD

- ← All the errors can be systematically removed with increasing computing power.

Baryon Interpolating Fields

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$:

$$\chi_1 = \varepsilon_{abc} (u^{aT} C \gamma_5 d^b) u^c$$

$$\chi_2 = \varepsilon_{abc} (u^{aT} C d^b) \gamma_5 u^c$$

Negative parity (multiply by γ_5): $\chi_1^- = \gamma_5 \chi_1$, $\chi_2^- = \gamma_5 \chi_2$

Non-relativistic limit:

$\chi_1 \rightarrow$ (big - big - big) $\rightarrow O(1)$ (couples to nucleon)

$\chi_2 \rightarrow$ (big - small - small) $\rightarrow O(p^2 / E^2)$ (couples to ?)

$\chi_1^- \rightarrow$ (big - big - small) $\rightarrow O(p / E)$ (couples to $\frac{1}{2}^-$ state)

$\chi_2^- \rightarrow$ (big - small - big) $\rightarrow O(p / E)$ (couples to $\frac{1}{2}^-$ state)

In the spectrum : $N^*(1535) \frac{1}{2}^-$ and $N^*(1620) \frac{1}{2}^-$.

Baryon Two-point Function

$$G(t) = \sum_{\vec{x}} \langle \text{vac} | T [\chi_1(x) \bar{\chi}_1(0)] | \text{vac} \rangle$$
$$= (1 + \gamma_4) [A_+ e^{-m_+(t-t_0)} + bA_- e^{-m_-(N_t+t_0-t)}] + (1 - \gamma_4) [bA_+ e^{-m_+(N_t+t_0-t)} + A_- e^{-m_-(t-t_0)}]$$

Fixed boundary condition (b=0)

$$G(t) = (1 + \gamma_4) A_+ e^{-m_+(t-t_0)} + (1 - \gamma_4) A_- e^{-m_-(t-t_0)}$$

$$\text{upper components: } G_U(t) = 2A_+ e^{-m_+(t-t_0)}$$

$$\text{lower components: } G_L(t) = 2A_- e^{-m_-(t-t_0)}$$

Anti-periodic boundary condition (b= -1)

$$G_U(t) = 2A_+ e^{-m_+(t-t_0)} - 2A_- e^{-m_-(N_t+t_0-t)}$$

$$G_L(t) = -2A_+ e^{-m_+(N_t+t_0-t)} + 2A_- e^{-m_-(t-t_0)}$$

Lattice studies of N^* spectrum

- **Standard Wilson + OPE**
 - Leinweber, PRD51, 6383 (1995)
- **Tadpole-improved, anisotropic actions ($D\chi_{34}, D234$)**
 - Lee, Leinweber, hep-lat/9809095, hep-lat/0011060, hep-lat/0110164
- **Doman-wall fermion**
 - Sasaki, Blum, Ohta, hep-lat/0004252, hep-lat/0011010, hep-lat/0102010
- **NP-improved clover**
 - Richards, hep-lat/0011025, Gockeler et al, hep-lat/0106022
- **Fat-link clover (FLIC)**
 - Adelaide group, hep-lat/0202022
- **Standard Wilson + Maximum Entropy**
 - Sasaki et al, hep-lat/0208070
- **Overlap fermions + Bayesian priors**
 - Kentucky collaboration, hep-ph/0306199
- **Chirally-improved fermion + variational method**
 - BGR collaboration, hep-ph/0307073

Quark mass coverage

No.	$m_0 a$	$m_\pi a$	$m_\rho a$	m_π/m_ρ	$m_\pi L$
1	1.20000	1.5766	1.9524	0.808	25.23
2	1.00000	1.4679	1.7807	0.824	23.49
3	0.80000	1.3047	1.5676	0.832	20.88
4	0.60000	1.1039	1.3511	0.817	17.66
5	0.40000	0.8731	1.1361	0.769	13.97
6	0.32200	0.7739	1.0517	0.736	12.38
7	0.26833	0.7013	0.9938	0.706	11.22
8	0.22633	0.6410	0.9501	0.675	10.26
9	0.18783	0.5820	0.9142	0.637	9.31
10	0.15633	0.5300	0.8843	0.599	8.48
11	0.12950	0.4823	0.8501	0.567	7.72
12	0.10850	0.4423	0.8407	0.526	7.08
13	0.08983	0.4040	0.8194	0.493	6.46
14	0.07583	0.3730	0.8070	0.462	5.97
15	0.06417	0.3460	0.7966	0.434	5.54
16	0.05367	0.3190	0.7945	0.402	5.10
17	0.04433	0.2940	0.7818	0.376	4.70
18	0.03617	0.2700	0.7749	0.348	4.32
19	0.03033	0.2520	0.7697	0.327	4.03
20	0.02567	0.2350	0.7637	0.308	3.76
21	0.02333	0.2270	0.7613	0.298	3.63
22	0.02100	0.2170	0.7591	0.286	3.47
23	0.01867	0.2080	0.7567	0.275	3.33
24	0.01750	0.2030	0.7561	0.268	3.25
25	0.01633	0.1980	0.7553	0.262	3.17
26	0.01400	0.1870	0.7534	0.248	2.99

- $16^3 \times 28$ lattice with $1/a=0.978$ GeV or $a=0.20$ fm from f_π
- Strange quark mass set at No. 6 from $\phi(1020)$ input.
- Smallest pion mass is about 181(8) MeV.
- Physical $m_\pi / m_\rho = 0.18$
- Box size is 3 times the smallest pion Compton wavelength

Fitting model

Positive-parity channel: $N + \eta' N (p=2\pi/L) + \text{Roper} + \dots$

$$G(t) = w_N e^{-m_N t} + w_{\eta' N} (1 + E_\pi t) e^{-m_{\eta' N} t} + w_{N'} e^{-m_{N'} t} + \dots$$

$w_{\eta' N}$ is constrained to be negative.

$$m_{\eta' N} \text{ is constrained near } E_\pi + E_N = \sqrt{m_\pi^2 + p^2} + \sqrt{m_N^2 + p^2}$$

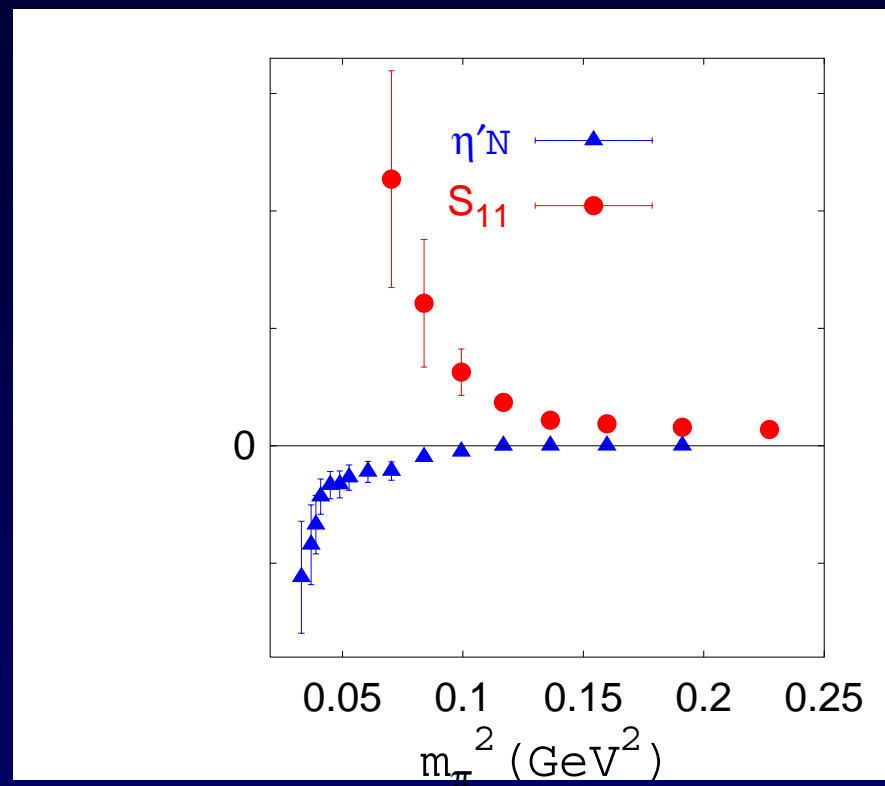
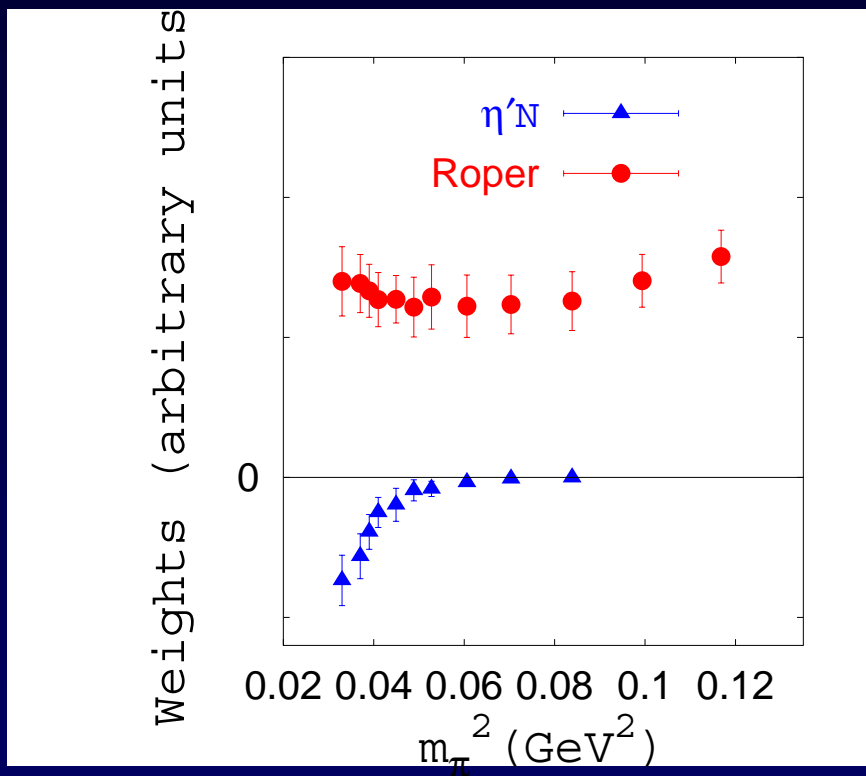
Negative-parity channel: $\eta' N (p=0) + \eta' N (p=2\pi/L) + S_{11} + \dots$

$$G(t) = w_1 (1 + m_\pi t) e^{-m_1 t} + w_2 (1 + E_\pi t) e^{-m_2 t} + w_{S_{11}} e^{-m_{S_{11}} t} + \dots$$

w_1 and w_2 are constrained to be negative.

m_1 and m_2 are constrained near $\eta' N (p=0)$ and $\eta' N (p=2\pi/L)$

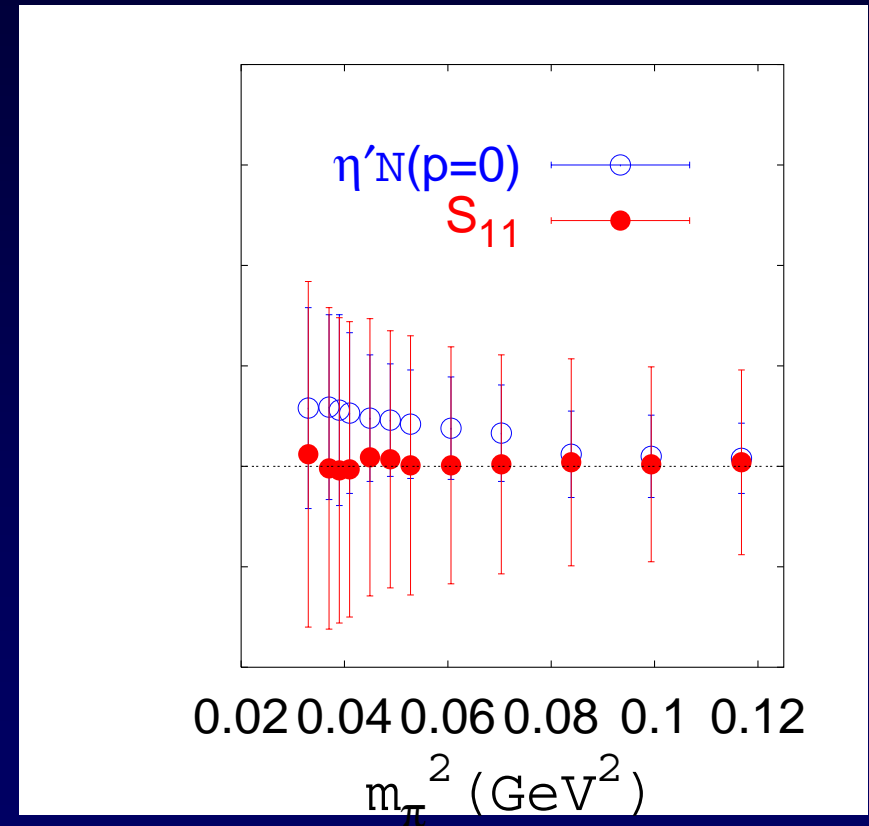
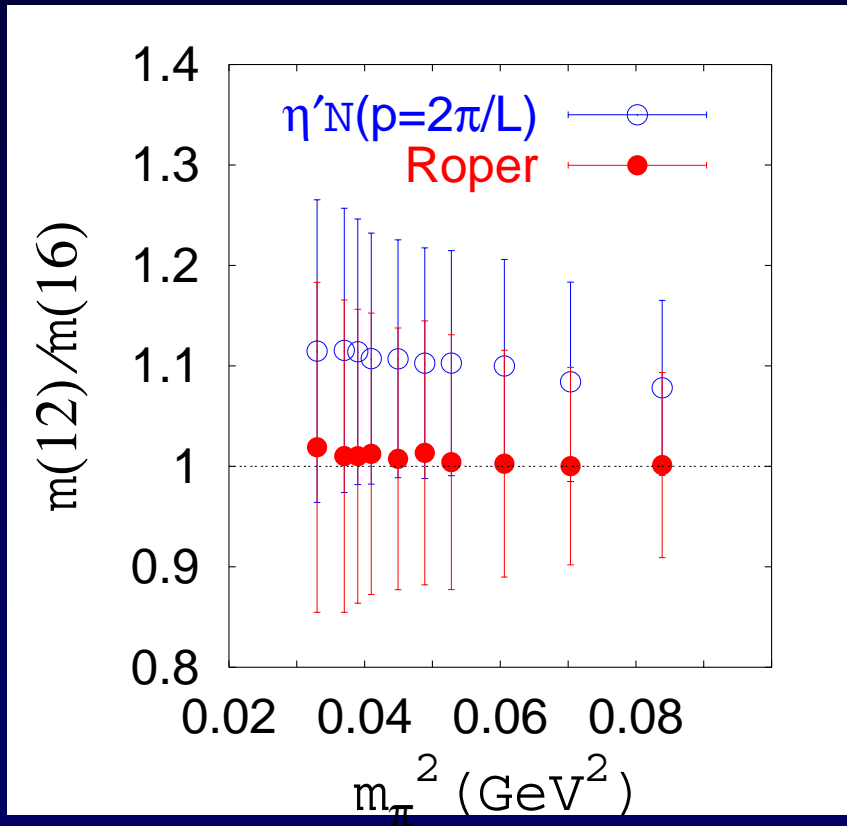
Decoupling of η' N ghost state



- The η' N ($p=2\pi/L$) ghost state decouples from the correlator above $m_\pi \approx 300$ to 400 MeV.

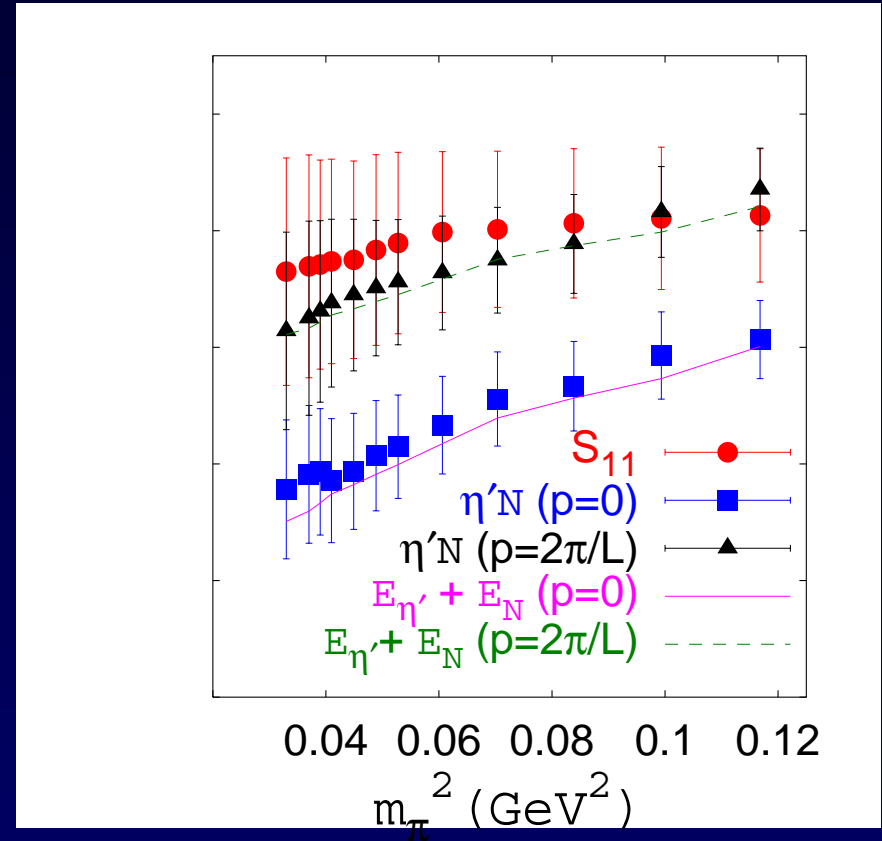
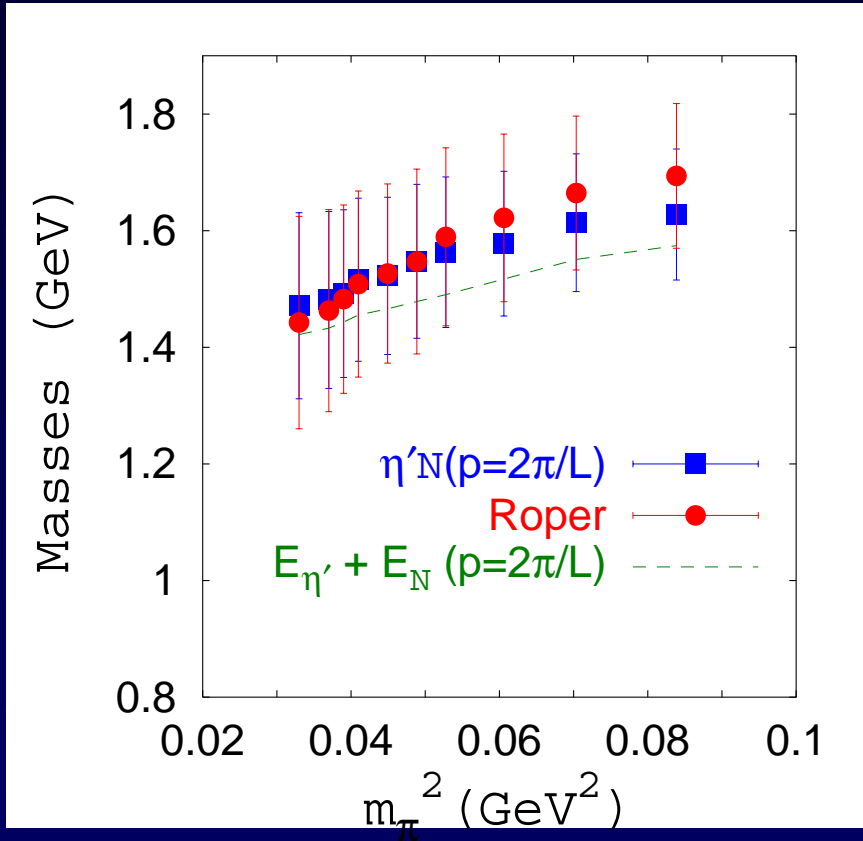
Volume dependence of $\eta' N$ ghost state

Two lattices: $12^3 \times 28$ and $16^3 \times 28$ at $a=0.2$ fm.



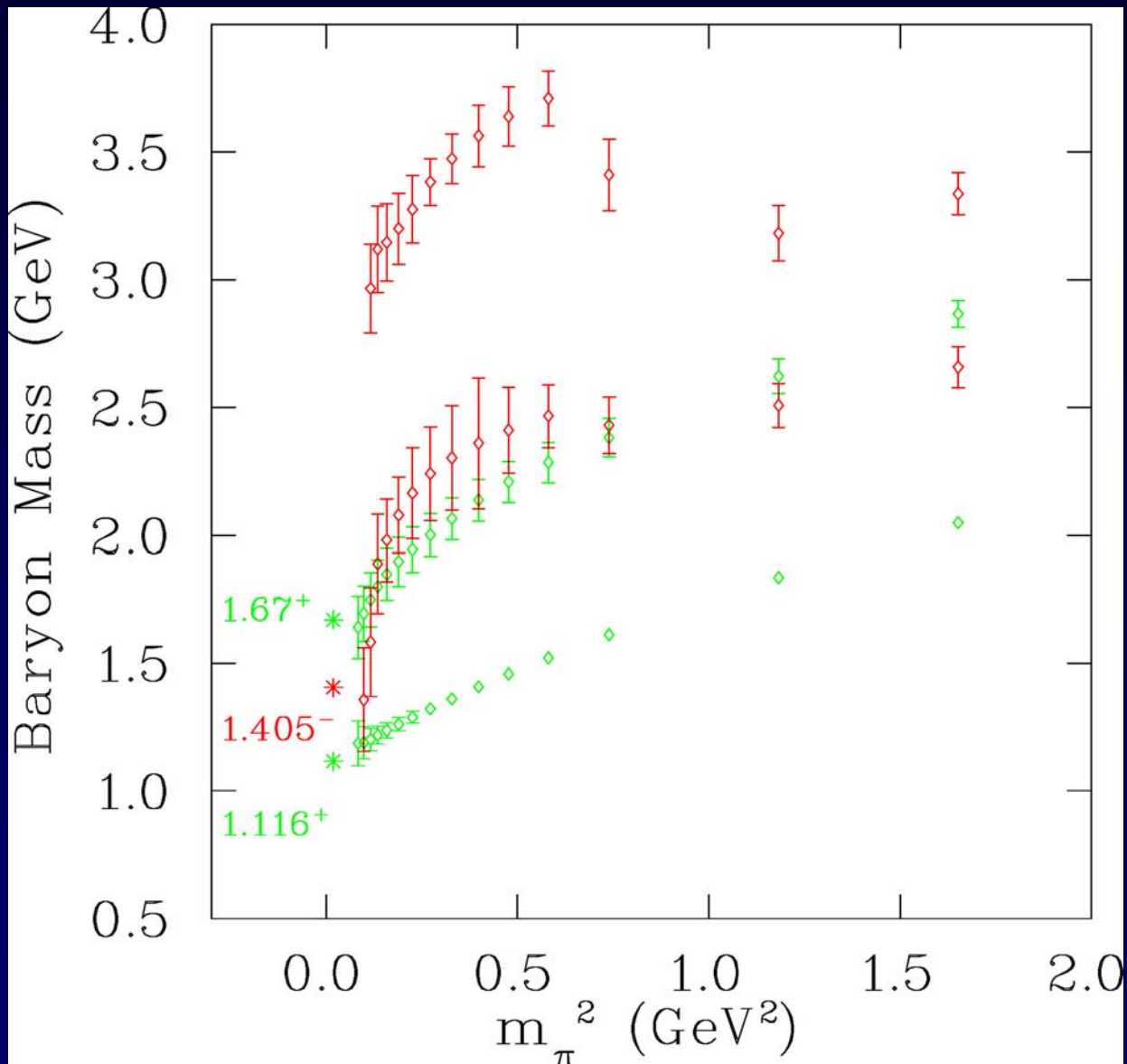
- $\eta' N$ is sensitive to volume
- But Roper and S_{11} are not

Roper, S_{11} and $\eta' N$ masses in chiral region



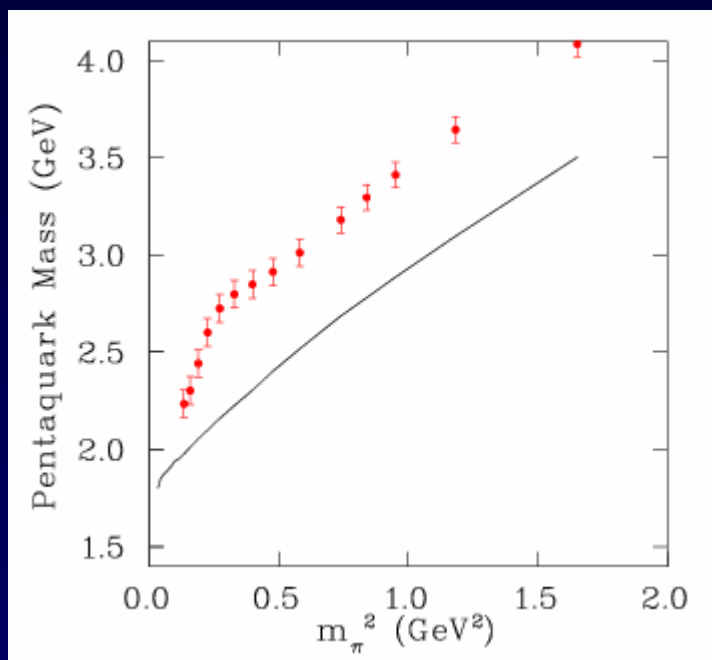
- $\eta' N$ and Roper are almost degenerate
- That $\eta' N$ has negative weight is crucial in its identification
- Slight interaction in the two-particle $\eta' N$ state

Level ordering in the Λ ($1/2$) channel



Pentaquark $uudd\bar{s}$: $I=1, J=1/2$

Positive Parity



Negative Parity

