The Weak Production of Λ Particles in Electron-Proton Scattering and the Contributing Form Factors

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Recently at TJNAF there has been an interest in observing the reaction

$$e^- + p \rightarrow \Lambda + \nu_e$$

over a range of energy from near threshold (~194 MeV) to large energy.

There was some interest in attempting a neutrino mass measurement but leaving this aside there are still some very valid reasons for looking at this reaction

(1) The form factors, particularly F_A and F_V could be obtained over a large energy range. It is known that SU(3) based form factors work well for Λ beta decay but should break down somewhere

(2) It might be possible to observe the F_E form factor. This in an $n \leftrightarrow p$ transition is forbidden by G parity but not here.

In this brief talk we shall consider the situation from near threshold to around 10 GeV

We will determine the size of the cross sections and the contributions of the form factors including interference terms.

The purpose is to see what the experimental prospects might be for observing this reaction

We start by writing the matrix element

$$\langle \nu_e \Lambda | H_w | e^- p \rangle = \frac{G}{\sqrt{2}} \sin\theta_c \, \bar{u}_\nu \gamma^\lambda (1-\gamma_5) u_e \times \langle \Lambda | J_\lambda^+(0) | p \rangle$$

where the current may be written as

$$J_\mu(0) = V_\mu(0) - A_\mu(0)$$

the current matrix elements may be written as:

$$\langle \Lambda | V_\mu^+(0) | p \rangle = \bar{u}_f \left[\gamma_\mu F_V(q^2) + i \frac{F_M(q^2) \sigma_{\mu\nu} q^\nu}{2m_p} - F_S(q^2) \frac{q_\mu}{2m_p} \right] u_i$$

$$\langle \Lambda | A_\mu^+(0) | p \rangle = \bar{u}_f \left[\gamma_\mu \gamma_5 F_A(q^2) + q_\mu \frac{\gamma_5 F_P(q^2)}{m_\pi} + i \frac{F_E(q^2) \sigma_{\mu\nu} q^\nu \gamma_5}{2m_p} \right] u_i$$

The structure of the particles and the transition is contained in the 6 form factors $F_V(q^2)$, $F_M(q^2)$, $F_S(q^2)$, $F_A(q^2)$, $F_P(q^2)$ and $F_E(q^2)$

Because this is an electron induced process all terms in the transition matrix squared which contain F_P or F_S are proportional to the electron mass squared, m_e^2, we may ignore these terms.

Because this is a strangeness changing process, SU(3) rather than SU(2) relations must be used to obtain unknown form factors.

We use SU(3) Relations to obtain the form factors

We use the well known relations

$$F_r^{ijk} = -i f^{ijk} \tilde{F}_r + d^{ijk} \tilde{D}_r$$

Here quantities with ~ are SU(3) functions and quantities without are the form factors. Here i = current octet number
k = initial baryon octet number
j = final baryon octet number

r stands for V, M, A, E or 1, 2, and 3

if an electromagnetic current is described

Using $V_\mu^{(3)} + \frac{1}{\sqrt{3}} V_\mu^{(8)}$ for the electromagnetic current for proton and then the neutron

$$\tilde{D}_V = 0, \quad \tilde{F}_V = F_1^p, \quad \tilde{D}_M = \frac{3}{2} F_2^n$$

$$\tilde{F}_M = -F_p^2 - \frac{1}{2} F_2^n$$

One has $F_r = -\frac{1}{\sqrt{6}} (3\tilde{F}_r + \tilde{D}_r)$

These lead to the results

$$F_V(q^2) = F_V(0)/(1 - q^2/M_V^2)^2$$

$$F_M(q^2) = F_M(0)/(1 - q^2/M_M^2)^2$$

with $F_V(0) = 1.2247$ $\quad M_V = .98\ \mathrm{GeV}/c^2$

$F_M(0) = 1.793/2m_p$ $\quad M_M = .71\ \mathrm{GeV}/c^2$

Thus the vector current matrix element is determined except for F_S

For the axial current, data from the β decay $\Lambda \to p + e^- + \bar{\nu}_e$ yields

$$\frac{F_A(0)}{F_V(0)} = 0.718 \pm 0.015$$

and these measurements yield a dipole fit $\quad F_A(q^2) = F_A(0)/(1 - q^2/M_A^2)^2$

$F_A(0) = .8793$ $\quad M_A = 1.25\ \mathrm{GeV}/c^2$

From a theoretical reference

$$F_E(0) \simeq .705/2m_p$$

and from other arguments

we expect

$$\frac{F_E(q^2)}{F_E(0)} \approx \frac{F_M(q^2)}{F_M(0)}$$

so that

$$F_E \approx F_E(0)/(1 - q^2/M_M^2)^2$$

we may now calculate

the matrix element squared

in the usual way and obtain

$$
\begin{aligned}
|M|^2 = \frac{1}{m_e m_\nu}\Bigg[& \frac{4|F_V|^2}{m_i m_f}[p_f\cdot\nu p_i\cdot e + p_f\cdot e p_i\cdot\nu - e\cdot\nu m_i m_f] + \\
& \frac{4|F_A|^2}{m_f m_i}[p_f\cdot\nu p_i\cdot e + p_f\cdot e p_i\cdot\nu + e\cdot\nu m_i m_f] + \\
& \frac{2|F_M|^2}{m_p^2 m_i m_f}[e\cdot\nu(p_i\cdot e p_f\cdot e + p_i\cdot\nu p_f\cdot\nu + e\cdot\nu m_i m_f)] + \\
& \frac{8F_V F_A}{m_i m_f}[p_f\cdot\nu p_i\cdot e - p_i\cdot\nu p_f\cdot e] + \\
& \frac{4F_M F_V}{m_p m_f}[e\cdot\nu(p_f\cdot\nu - p_f\cdot e)] + \\
& \frac{4F_M F_V}{m_p m_i}[e\cdot\nu(p_i\cdot e - p_i\cdot\nu)] + \\
& \frac{4F_A F_M}{m_p m_f}[e\cdot\nu(p_f\cdot e + p_f\cdot\nu)] + \\
& \frac{4F_A F_M}{m_p m_i}[e\cdot\nu(p_i\cdot e + p_i\cdot\nu)] + \\
& \frac{|F_E|^2}{m_p^2 m_i m_f}[e\cdot\nu(2(p_f\cdot e p_i\cdot e + p_f\cdot\nu p_i\cdot\nu) + q^2 p_f\cdot p_i] + \\
& \frac{4F_E F_V}{m_p m_i}[e\cdot\nu(p_i\cdot\nu + p_i\cdot e)] - \\
& \frac{4F_E F_V}{m_p m_f}[e\cdot\nu(p_f\cdot\nu + p_f\cdot e)] + \\
& \frac{4F_E F_A}{m_p m_f}[e\cdot\nu(p_f\cdot\nu - p_f\cdot e)] + \\
& \frac{4F_E F_A}{m_p m_i}[e\cdot\nu(p_i\cdot\nu - p_i\cdot e)]\Bigg]
\end{aligned}
\tag{10}
$$

We note that every term in $|M|^2$ is proportional to E_ν and E_e

The differential cross section may be written as:

$$\frac{d\sigma}{d\Omega} = \frac{m_e m_\nu G^2 \sin^2\theta_c M_f p_f |M|^2}{(2\pi)^2 8E\left[M_i + E - \frac{E E_f \cos\theta}{p_f}\right]}$$

There is an interesting kinematical effect in this problem. The outgoing Λ energy may be written as:

$$E_f = \frac{\rho\eta + |\vec{p}_e|\cos\theta\left[\eta^2 - 4m_f^2\rho^2 + 4|\vec{p}_e|^2 m_f^2\cos^2\theta\right]^{\frac{1}{2}}}{2\left[\rho^2 - |\vec{p}_e|^2\cos\theta\right]}$$

$\rho = m_i + E$, $\eta = \alpha + 2m_i E$ and

$\alpha = m_i^2 + m_f^2 + m_e^2$

here we make no approximations. This leads to a maximal outgoing angle for Λ

$$\sin^2(\theta_{max}) = 1 - \frac{2\delta}{m_f} - \frac{\delta m_e^2 E}{m_f^2 p^2} + \frac{m_e^2 \delta}{m_f p^2} - \frac{2\delta E}{p^2} + \frac{\delta^2}{m_f^2} + \frac{\delta^2}{p^2} - \frac{m_e^2 \delta^2}{2 p^2 m_f^2} + \frac{3 E \delta^2}{m_f p^2} - \frac{\delta^3 E}{m_f^2 p^2} - \frac{\delta^3}{m_f p^2} + \frac{\delta^4}{m_f^2 p^2} + \frac{m_e^4}{4 m_f^2 p^2} + \frac{E m_e^2}{m_f p^2}$$

$\delta = m_f - m_i$

At large E this becomes

$$\sin^2(\theta_{max}) = 1 - \frac{2\delta}{m_f} + \frac{\delta^2}{m_f^2}$$

If we look at the denominator of $\frac{d\sigma}{d\Omega}$ we find

$$\left| m_i + E - \frac{E_f E \cos(\theta)}{p_f} \right|$$

$$\approx \frac{\left[p^2 - E^2 \cos^2(\theta_{max})\right] 2 m_f \sin^2\theta_{max}\, \epsilon^{1/2}}{\eta \cos(\theta_{max})}$$

$\epsilon = \theta_{max} - \theta$

Thus there is a mild singularity at $\theta = \theta_{max}$. This can be removed by using a wave packet for the Λ

$$\frac{d\bar{\sigma}}{d\Omega_f} = N \int \chi(k_{f0}, p) \frac{d\sigma}{d\Omega_f} d^3p$$

A gaussian wave packet works very well.

There are a few kinematical points of interest

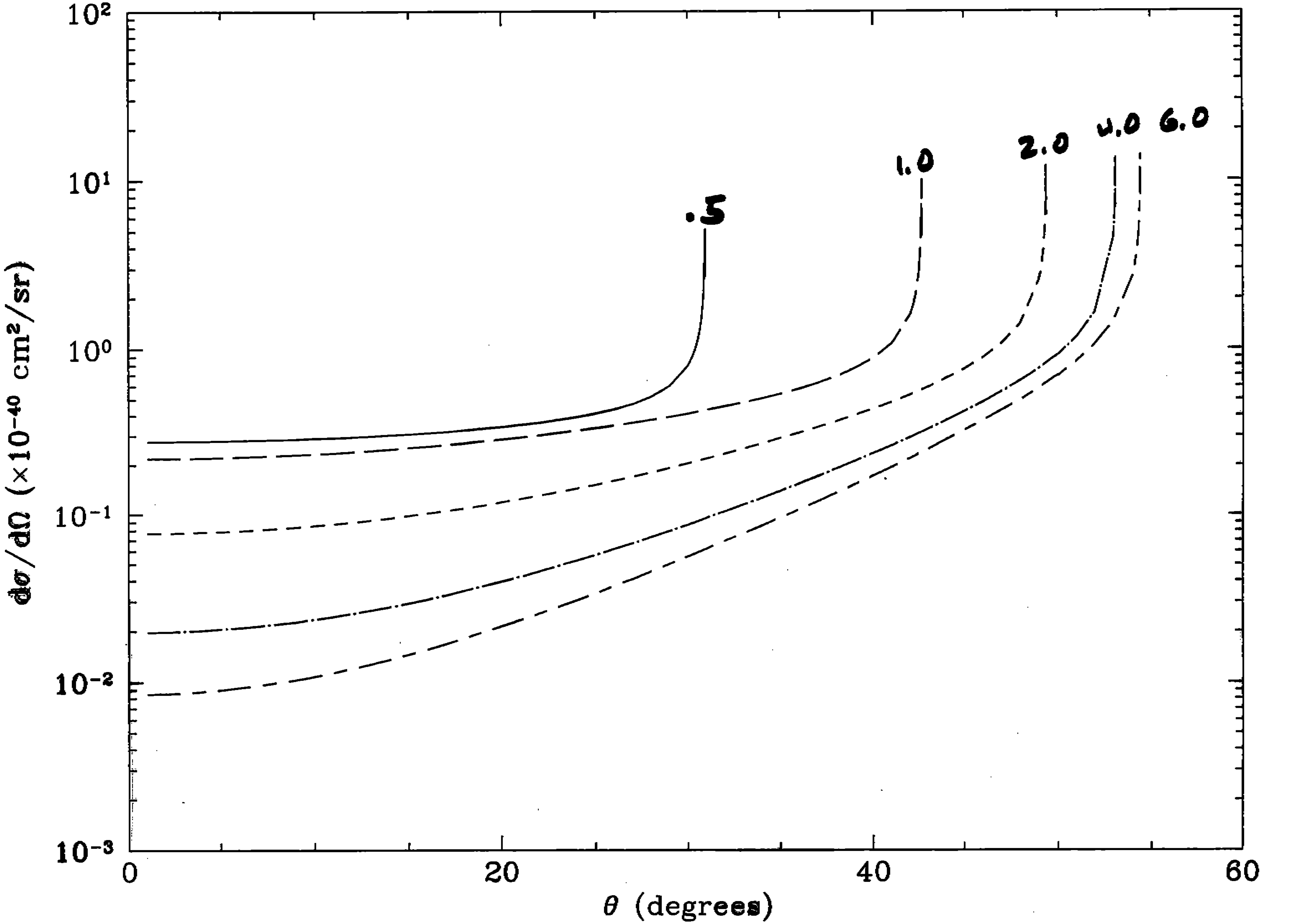
a) E in the denominator of $d\sigma/d\Omega$ is cancelled by E in $|M|^2$

b) The form factors are largest at small q^2 thus driving up $d\sigma/d\Omega$

c) At small q^2, E_ν is large also driving up $d\sigma/d\Omega$

Thus the differential cross section is largest where q^2 is small and increasing E drives up the peaks.

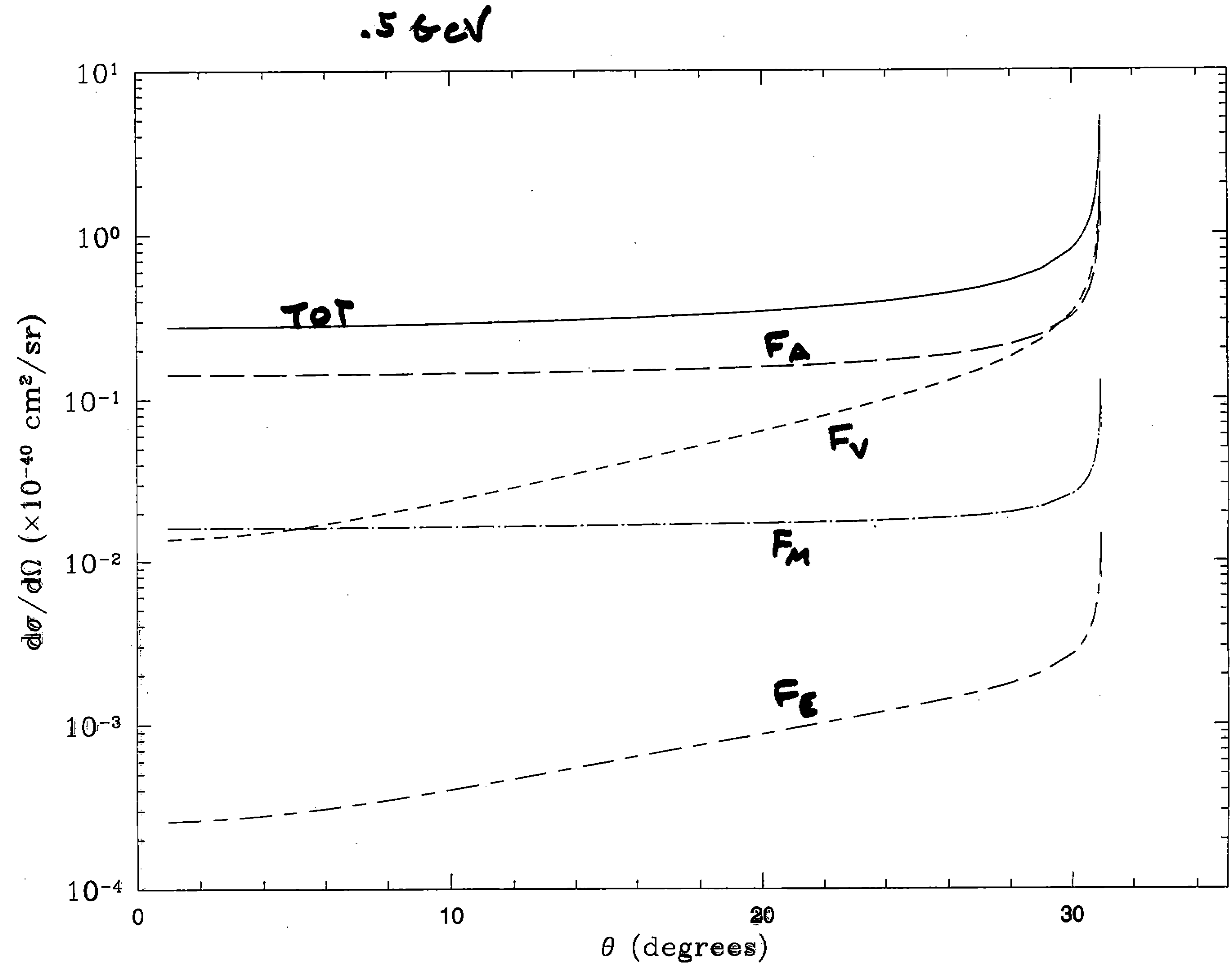
Diff. cross sections



Now we consider the contributions of the various form factors

(1) We note that at all energies, F_A and F_V are dominant where the cross sections are large

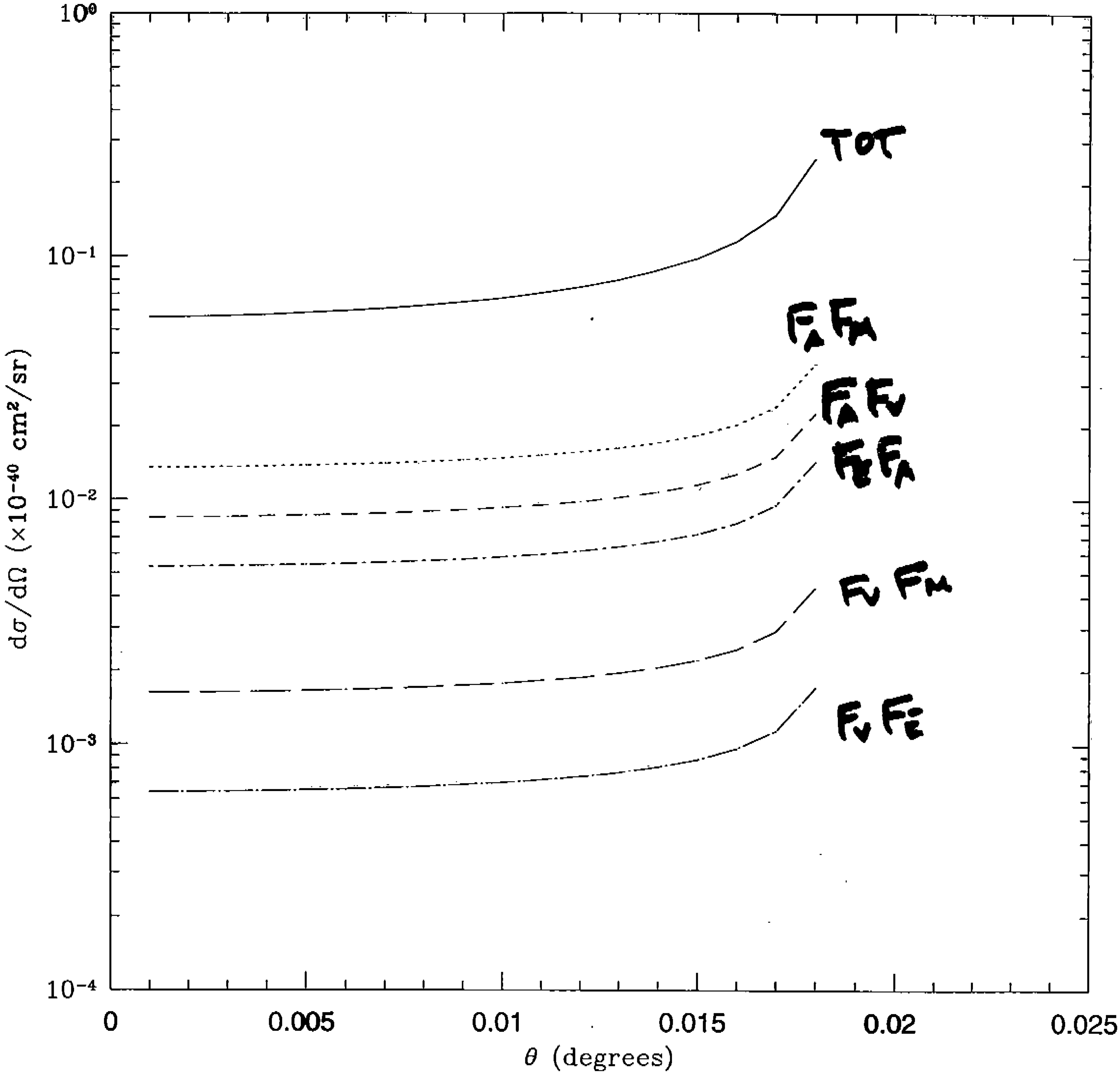
(2) F_M and F_E are relatively small and the hope for seeing them is in interference terms



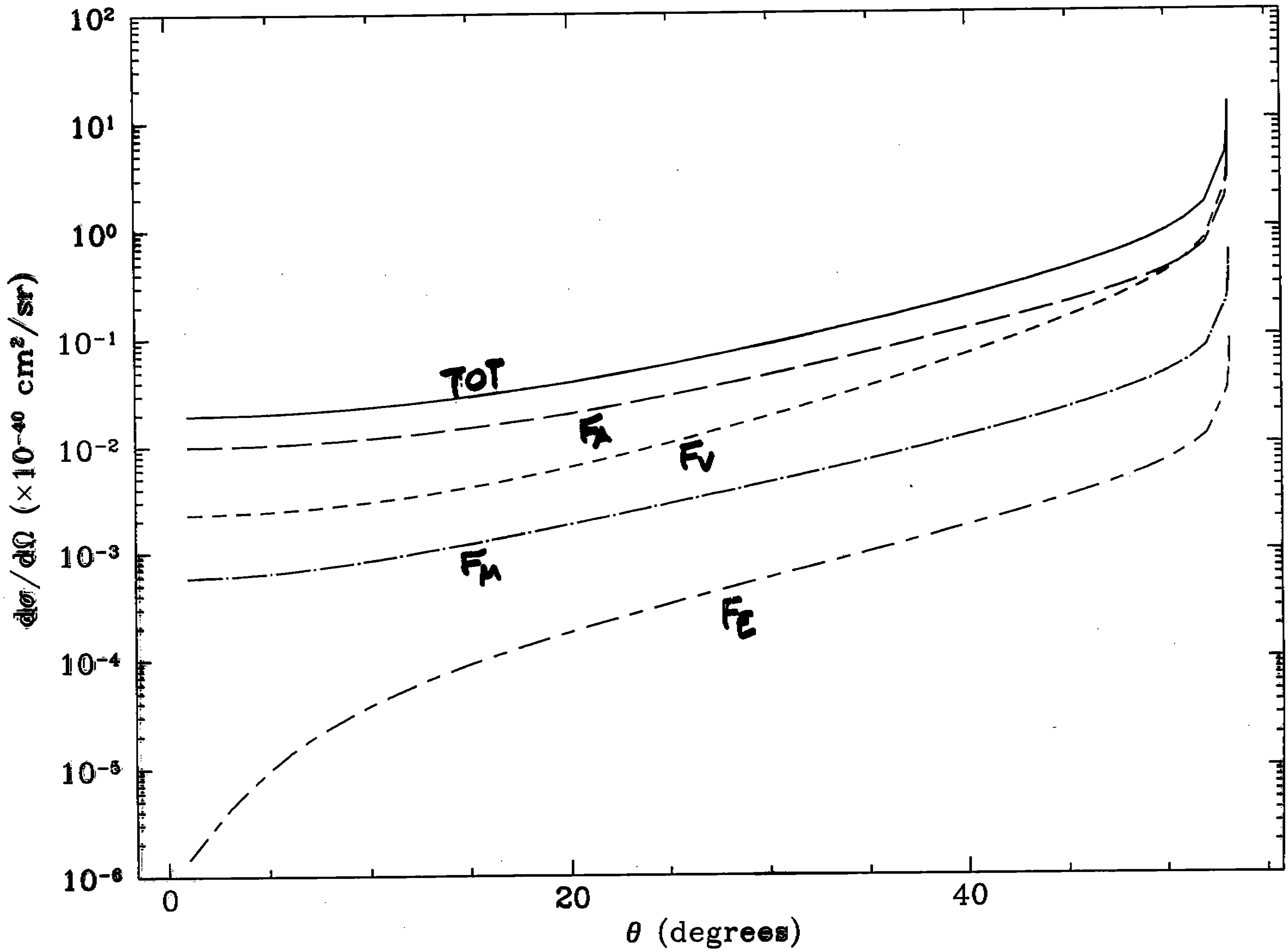
.5 GeV
TOT
F_A
F_V
F_M
F_E
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$d\sigma/d\Omega$ ($\times 10^{-40}$ cm^2/sr)
0
10
20
30
θ (degrees)

194.25 MeV

Sig 1



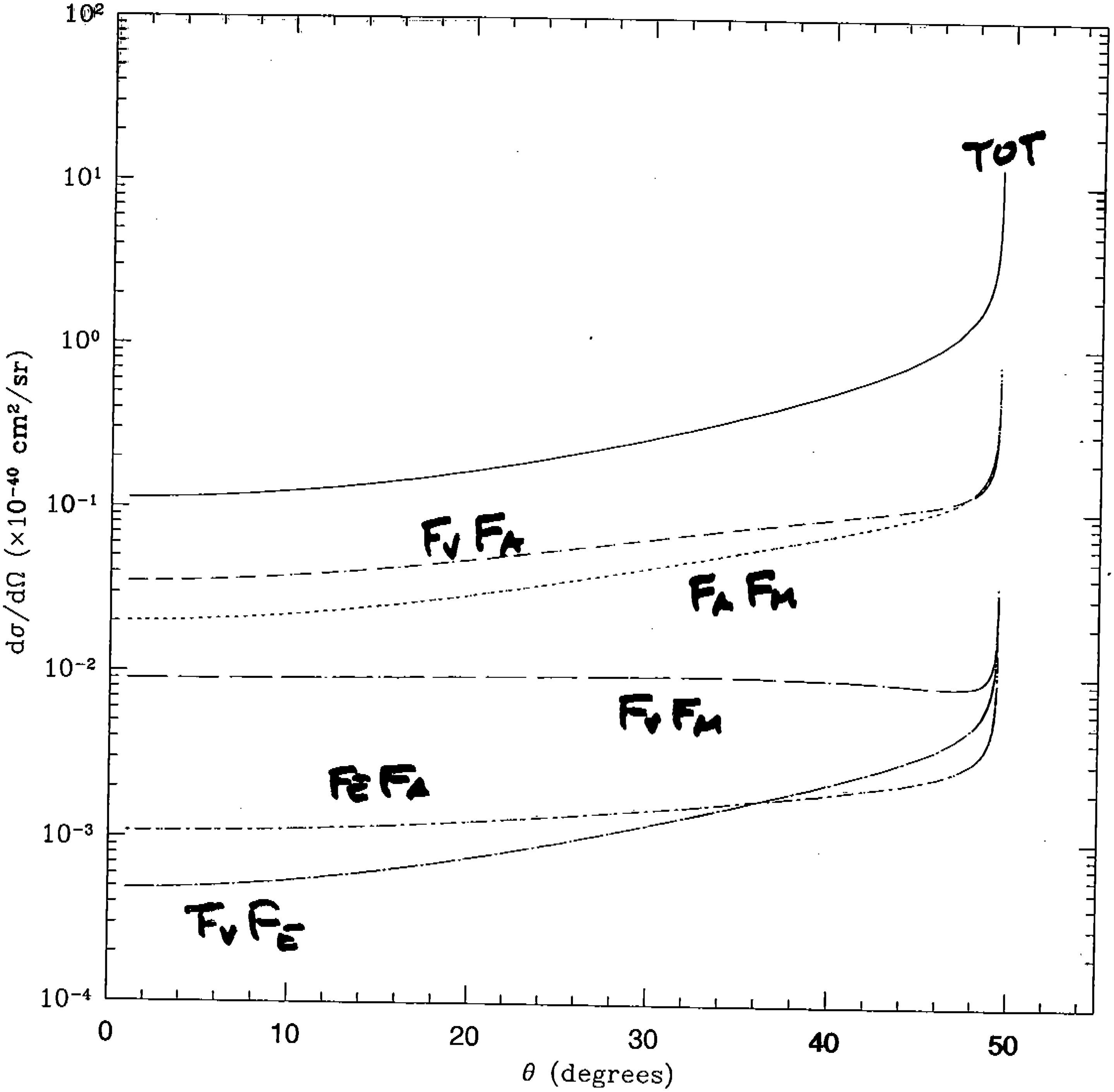
4 GeV



$d\sigma/d\Omega$ ($\times 10^{-40}$ cm^2/sr)
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
TOT
F_A
F_V
F_M
F_E
0
20
40
θ (degrees)

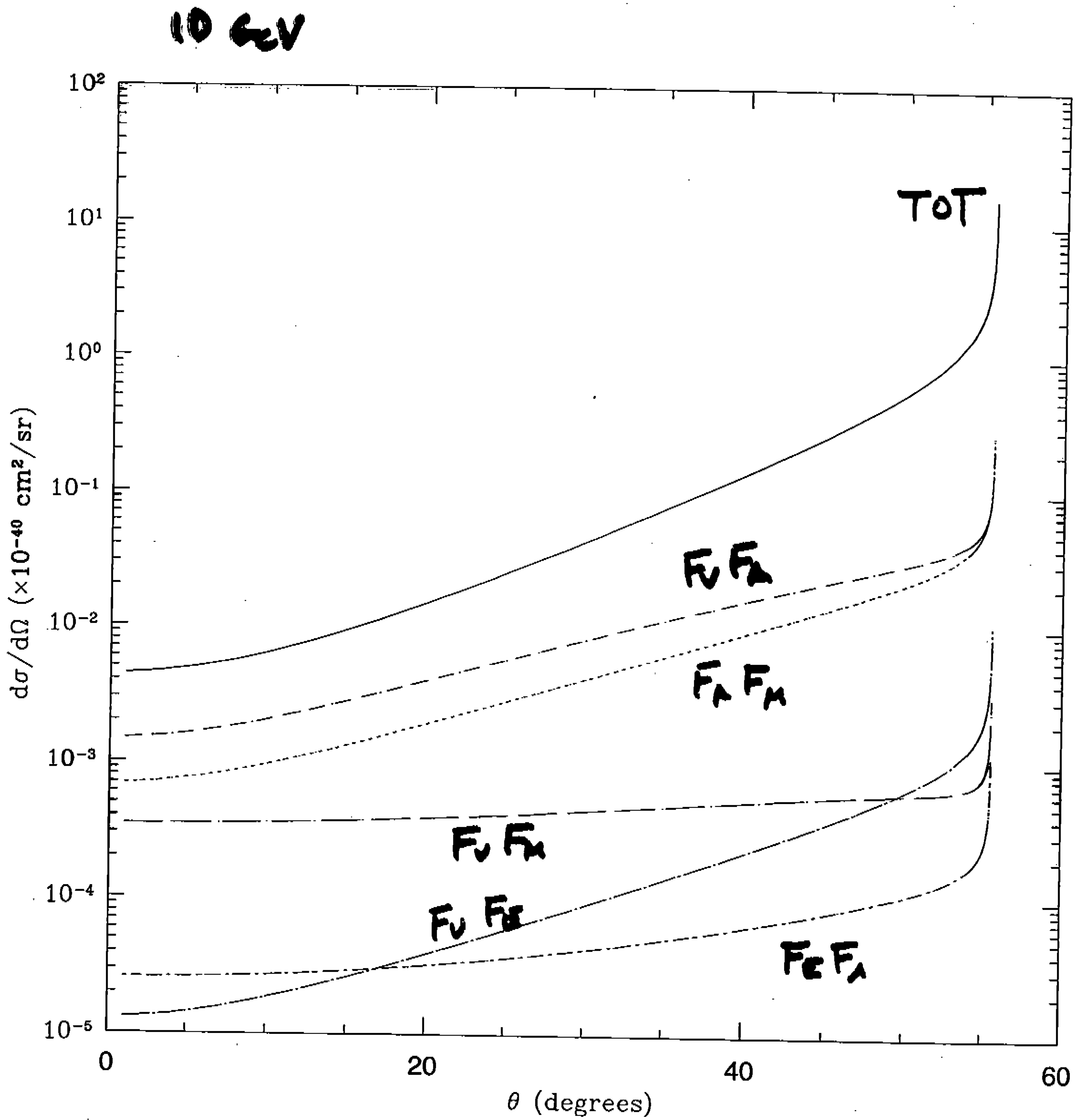
2 GeV

fig 2



TOT
$F_V F_A$
$F_A F_M$
$F_V F_M$
$F_E F_A$
$F_V F_E$
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$d\sigma/d\Omega$ ($\times 10^{-40}$ cm^2/sr)
0
10
20
30
40
50
θ (degrees)

fig 3


10 GeV
TOT
$F_V F_A$
$F_A F_M$
$F_V F_M$
$F_V F_E$
$F_E F_A$
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$d\sigma/d\Omega$ ($\times 10^{-40}$ cm^2/sr)
0
20
40
60
θ (degrees)

Conclusions

1. By keeping q^2 fixed but varying E and θ, it should be possible to determine F_A and F_V
2. It might be possible to determine F_M to the 25% level
3. It might be possible to put limits on F_E but not to accurately determine it.

This would allow us to see under what conditions the SU(3) results for the form factors are valid

For the Future

(1) We can consider heavy lepton beams where additional measurements such as asymmetry become possible and F_p and F_s are not suppressed

(2) Can consider using neutrino beams should any of the neutrino factories be built.