Faddeev-Yakubovsky calculations for A=4 hypernuclear systems

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- Motivation
- Yakubovsky equations for hypernuclei
- NN and YN force dependence
- YN force dependence of CSB
- Conclusions and outlook

(see PRL 88,172501)

Probing YN forces in hypernuclei

YN two-body system remains largely unexplored (very scarce data set)

How can we extend the existing NN force models to the YN system
How can we go on to S=-2 systems?

Here : How can we verify the approach chosen with the existing data?

Few-body hypernuclei are a good laboratory

- unfortunately there is no YN two-body bound state
- lightest bound hypernucleus $^{3}_{\Lambda}H$ is only weakly bound

  one spin state

  isospin singlet
- four-body systems $^{4}_{\Lambda}H$ and $^{4}_{\Lambda}He$ are stronger bound

  0^+ ground state and 1^+ exited state

  isospin doublet

The four-body system allow us to study spin dependence, ... of the YN interaction.

YN interactions

The YN interaction has a very interesting new feature compared to the NN force:

$\Lambda-\Sigma$ conversion

Λ-Σ conversion is extremely important to understand the ΛN interaction !

- OBE models of the nuclear force **predict $\Lambda\Sigma\pi$ coupling**
- **$\Lambda\Lambda\pi$ coupling is suppressed**
- strong s-wave contribution ($\neq \Delta$ contribution in NN system)
- mass difference of Λ and Σ is smaller than m_π ($\approx$ 80 MeV)

➡ virtual Σ accounts for the long range part of the ΛN interaction

$\Lambda-\Sigma$ conversion leads to strong medium dependence of the ΛN force:

$$\Lambda + \text{core} \longleftrightarrow \Sigma + \text{core}^*$$

Isospin conversion enforces in some cases an excitation of the "core" nucleus
if the overlap of excitation and ground state core is small, the conversion is suppressed (e.g. $^5_\Lambda He$)

Several OBE and quark cluster models of the YN interaction have been developed:

- Nijmegen SC89, SC97a-f **(Maessen et. al. PRC 40, 2226)** (Rijken et al. PRC 59, 21)
- Jülich A,B **(Holzenkamp et. al. NPA 500,485)**
- quark cluster models **(Fujiwara et al. PRC 64,054001, Ping et. al. NPA 657,95)** (not used here)

All these YN interactions take $\Lambda-\Sigma$ conversion into account

Yakubovsky equations for the 3N-Λ system

Ψ is antisymmetric under exchange of the 3 nucleons (here particles 123)

➡ Momentum space Yakubovsky equations reduce to a set of 5

$$
\begin{aligned}
\psi_{1A} &= G_0\, t_{12}\, P\,(\psi_{1A}+\psi_{1B}+\psi_{2A}) + (1+G_0\, t_{12})\, G_0\, V^{(3)}_{123}\, \Psi \\
\psi_{1B} &= G_0\, t_{12}\, [(1-P_{12})\,(1-P_{23})\psi_{1C} + P\, \psi_{2B}] \\
\psi_{1C} &= G_0\, t_{14}\, (\psi_{1B}+\psi_{1A}+\psi_{2A} - P_{12}\, \psi_{1C} + P_{12}\, P_{23}\, \psi_{1C} + P_{13}\, P_{23}\, \psi_{2B}) \\
\psi_{2A} &= G_0\, t_{12}\, [(P_{12}-1)\, P_{13}\, \psi_{1C} + \psi_{2B}] \\
\psi_{2B} &= G_0\, t_{34}\, (\psi_{1A}+\psi_{1B}+\psi_{2A}) \\
\Psi &= (1+P)\,(\psi_{1A}+\psi_{1B}+\psi_{2A}+\psi_{2B}) + (1-P_{12})\,(1+P)\,\psi_{1C}
\end{aligned}
$$

$$G_0\, t_{ij} = G_0\, V_{ij} + G_0\, V_{ij}\, G_0\, t_{ij}$$

- Λ–Σ conversion increases the number of possible isospin states (equations are unchanged)
- **we consider the mass differences of Σ's and N's and CSB of nuclear forces in** $G_0\, t_{ij}$ **(permutations / coordinate transformations use average mass of multiplets)**
- High dimensional (10^8) linear eigenvalue equation is solved by a Lanczos-like iteration method
- 3NF are considered; ΣNN forces are not considered

NN force dependence

^{3}H and ^{3}He are the "cores" of $^{4}_{\Lambda}H$ and $^{4}_{\Lambda}He$

their binding energies depend on the NN interaction

➡ The binding energies of $^{4}_{\Lambda}H$ and $^{4}_{\Lambda}He$ also depend on the NN force

Does the dependence cancel for the Λ-separation energies?

interactions	$E(^{3}He)$	$E(^{4}_{\Lambda}He)$	$E^{\Lambda}_{sep} = E(^{3}He) - E(^{4}_{\Lambda}He)$
SC97e+Nijm 93	-7.01	-8.55	1.54
SC97e+Bonn B	-7.26	-8.92	1.66
SC97e+Nijm 93 +TM	-7.76	-9.32	1.56 (for 0^{+} state)

➡ the separation energy depends only moderately on the NN force **(100 keV)**

Comparison to SC simulating interactions

Many hypernuclear calculations are based on Gaussian approximations to Nijmegen SC ??
Do those approximations reflect the properties of the original?

	$E^{\Lambda}_{sep}\ (0^+)$	$E^{\Lambda}_{sep}\ (1^+)$	$\Delta(0^+ - 1^+)$
SC97 f-sim	2.32	0.74	1.52
Akaishi et. al. PRL 84, 3539	2.18	0.70	1.48
Hiyama et. al. PRC 65, 011301	2.28	0.54	1.74
SC97 f-original	1.72	0.53	1.16

(for ${}^{4}_{\Lambda}He$)

- The Gaussian approximations can give insight into the mechanisms, which bind hypernuclei (e.g. contributions from Λ–Σ conversion)
- Quantitatively, these approximations fail

➡ Conclusions on the quality of realistic interactions based on their approximations are dangerous!

Importance of Λ–Σ conversion

YN interactions enter via their t-matrix into the Yakubovsky equations

➡
- use the complete interaction for the calculation of the t-matrix
- restrict the t-matrix to ΛN-ΛN matrix elements

➡
- new t-matrix is phase equivalent in the ΛN system
- new t-matrix is linked to an effective ΛN interaction without Λ–Σ conversion

interactions	$E^{\Lambda}_{sep}(0^+)$	$E^{\Lambda}_{sep}(1^+)$	$\Delta(0^+ - 1^+)$
SC97e (full)	1.54	0.72	0.79
SC97e (w/o Σ)	1.12	0.79	0.30 (for ${}^{4}_{\Lambda}He$ and Nijm 93)

- Σ contribution is important in the four-body system
- medium dependence shows up (higher order forces are necessary for effective interactions)
- Σ contribution is spin dependent (Gibson et al. PRC 37, 679)

➡ It is impossible to extract information on the spin dependence of ΛN cross section from hypernuclei without understanding the Λ–Σ conversion process beforehand

0^+-1^+ splitting

The spin dependence of $\Delta(0^+-1^+)=E^{\Lambda}_{sep}(0^+)-E^{\Lambda}_{sep}(1^+)$ is often considered as an indication for the spin dependence of the YN force

interactions	$E^{\Lambda}_{sep}(0^+)$	$E^{\Lambda}_{sep}(1^+)$	$\Delta(0^+-1^+)$	$a_s(\Lambda p)$	$a_t(\Lambda p)$	
SC97f+Nijm 93	1.72	0.53	1.16	-2.51	-1.73	
SC97e+Nijm 93	1.54	0.72	0.79	-2.10	-1.83	
SC97d+Nijm 93	1.29	0.80	0.47	-1.70	-1.93	
SC 89+Nijm 93	2.14	0.02	2.06	-2.59	-1.38	
Jülich Ã+Nijm 93	0.43	0.48	-0.05	-2.07	-1.33	
Expt.	2.39	1.24	1.15	-	-	(for ${}^{4}_{\Lambda}He$)

- SC89 describes the ground state binding energy almost correctly
- SC97f describes the splitting correctly
- triplet stronger than singlet does not mean that 1^+ is the ground state

Does SC 97f describe the spin dependence correctly ?
Can we conclude that it is the most realistic interaction?

3BF effects

3NF contribution to binding in the 3N system is roughly 600 keV

The nucleon separation energy is 8.5-2.2=6.5 MeV

We estimate a 3BF contribution to the Λ separation energy of

1/3 600 keV=200 keV

on top of the 3BF effect due to $\Lambda-\Sigma$ conversion

Because this effect will be spin dependent, one cannot exclude a 400 keV contribution to the 0^+-1^+ splitting, which is unrelated to the YN force

0^+-1^+ splitting might not be the clearest signal for failures or successes of the YN interaction

CSB of separation energies

- CSB of separation energies depends on strength of $\Lambda - \Sigma$ conversion
  - strong contribution from the mass difference within Σ multiplet
  - strong contribution from explicit CSB in the YN interaction
  - Coulomb contribution is less important
- **CSB is likely to be independent of YNN forces similar to 3NF's in the 3N system**

interactions	$E_{sep}^{\Lambda}\ ({}^{4}_{\Lambda}He)$	$E_{sep}^{\Lambda}\ ({}^{4}_{\Lambda}H)$	$\Delta(0^{+}-1^{+})$	$P(\Sigma)$
SC97e	1.54	1.47	0.07	1 %
SC89	2.14	1.80	0.34	4 %
Expt	2.39	2.04	0.35	(for 0^+ state and Nijm 93)

- SC 89 (which does not bind the excited state sufficiently) does a good job for CSB
- **SC97e fails for this observable** (the same is true for the excited state)
- CSB is related to the Σ component of the wave function
- **CSB results support a sizable Σ component**

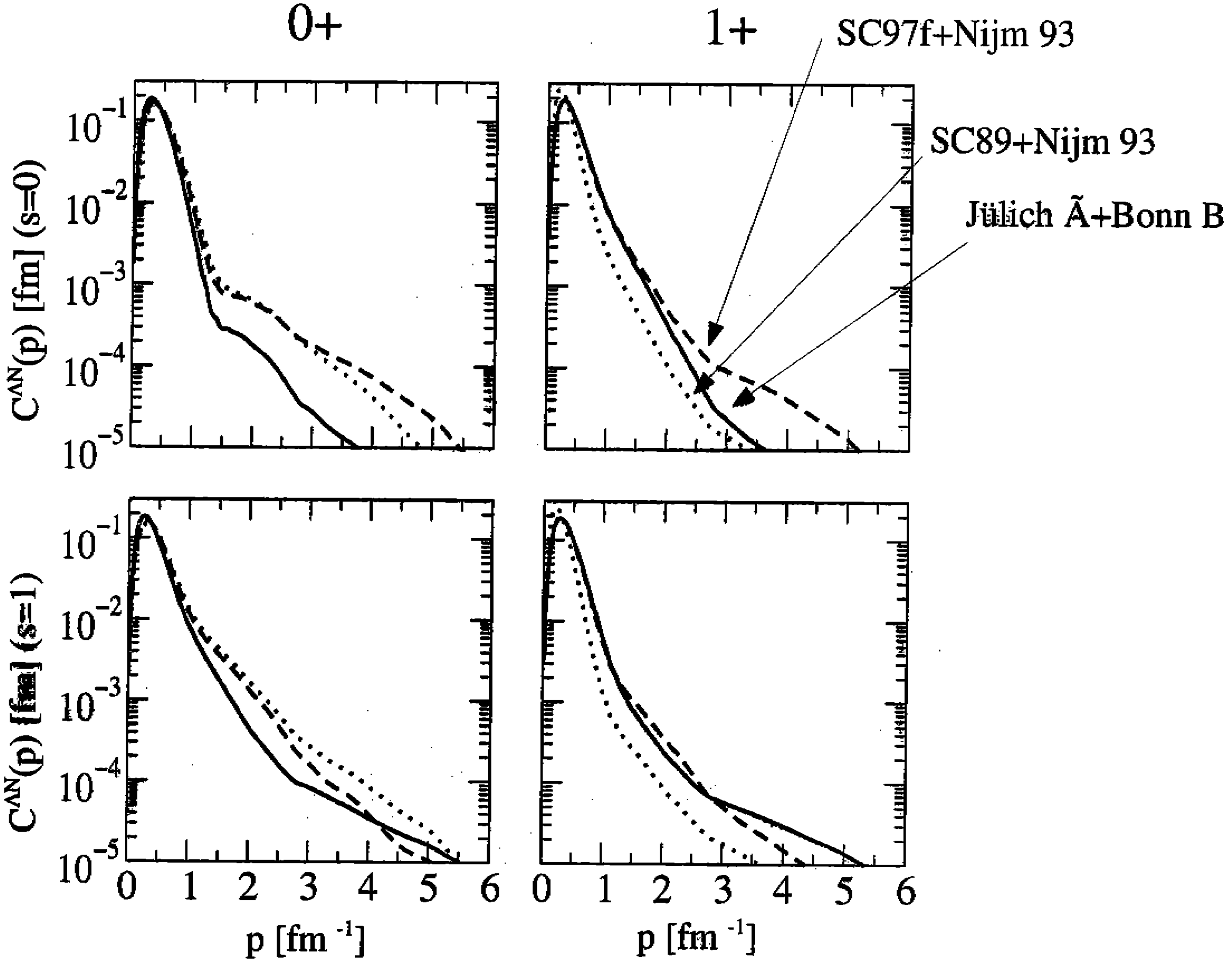
  estimation based on π^+ decay of ${}^{4}_{\Lambda}He$ indicates the same **(Gibson et al. NPA 628, 417)**

Contributions to CSB

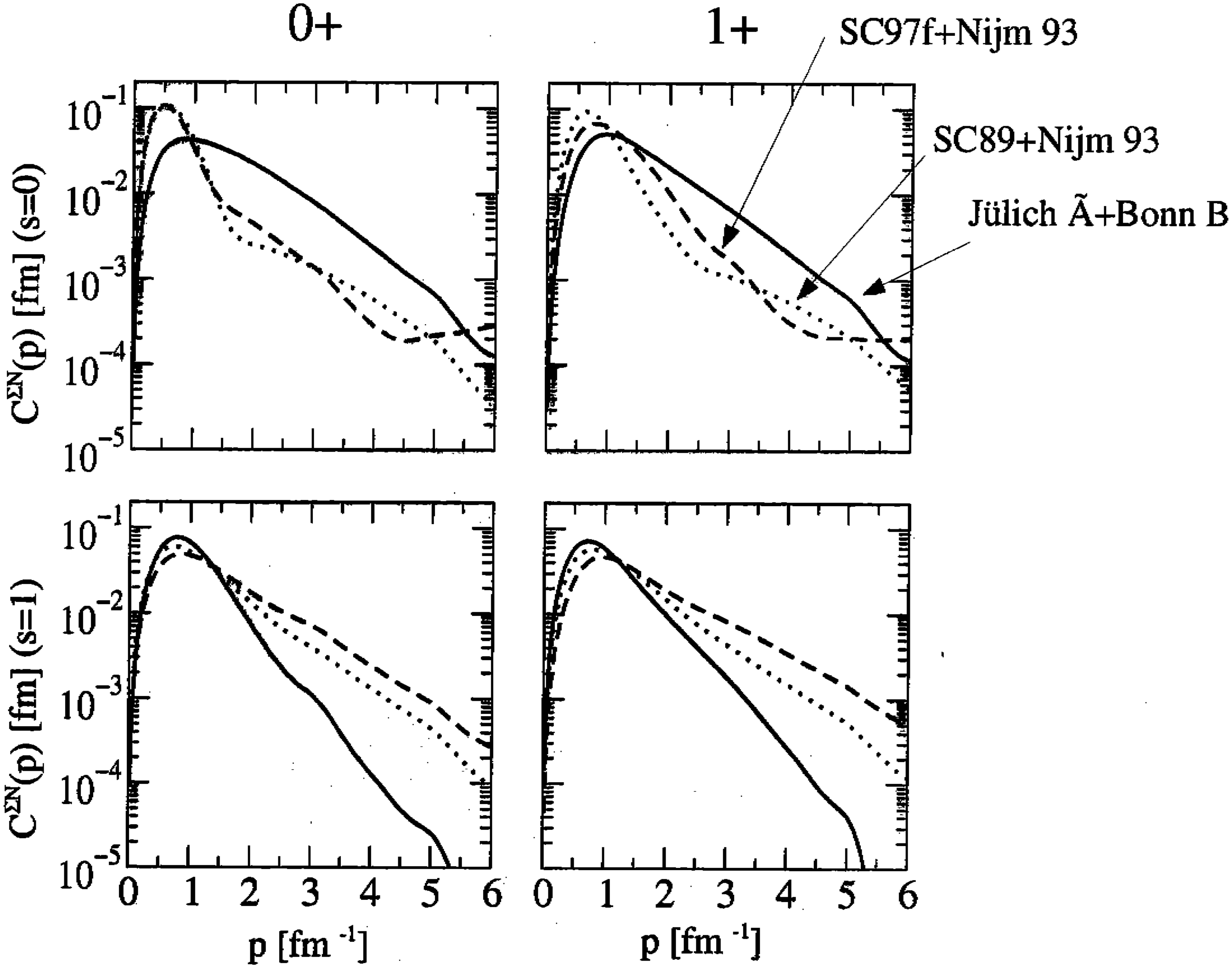
interactions	SC89	SC97e	
$\Delta\ T^{CSB}$	132	47	
$\Delta\ V^{CSB}_{NN,Coul}$	-9	-9	
$\Delta\ V^{CSB}_{YN,nucl}$	255	44	
$\Delta\ V^{CSB}_{YN,Coul}$	-27	-7	
Total	351	75	
non-pert	340	70	
Expt.	350	350	(for 0^+ state and Nijm 93)

interactions	P_{Σ^+}	P_{Σ^0}	P_{Σ^-}	
SC97f+Nijm 93	1.15	0.59	0.02	$\Delta\ T^{CSB}_{mass} = (P_{\Sigma^+} - P_{\Sigma^-})\,(m_{\Sigma^-} - m_{\Sigma^+})$
SC97e+Nijm 93	1.02	0.52	0.03	
SC97d+Nijm 93	0.95	0.50	0.04	
SC89+Nijm 93	2.67	1.36	0.05	
Jülich Ã+Bonn B	0.99	1.10	1.20	(for ${}^4_\Lambda He$)

ΛN momentum correlations



ΣN momentum correlations



Conclusions and Outlook

- Yakubovsky calculations are a reliable method to solve the hypernuclear 4-body bound state problem for complete, realistic interactions

- None of the Nijmegen SC interactions and Jülich Ã can describe all 4 body states simutaneously

- 0^+-1^+ splitting is sensitive to the YN interaction, but might also be influenced by 3BF's
- CSB is probably not sensitive to 3BF's – it depends on the Σ component of [illegible]

- New interactions: quark-cluster, ESC need to be studied
- other observables, which depend on the Σ component of the wave function (**π^+ decay of $^{4}_{\Lambda}He$**)
- photoproduction of 4-body hypernuclei