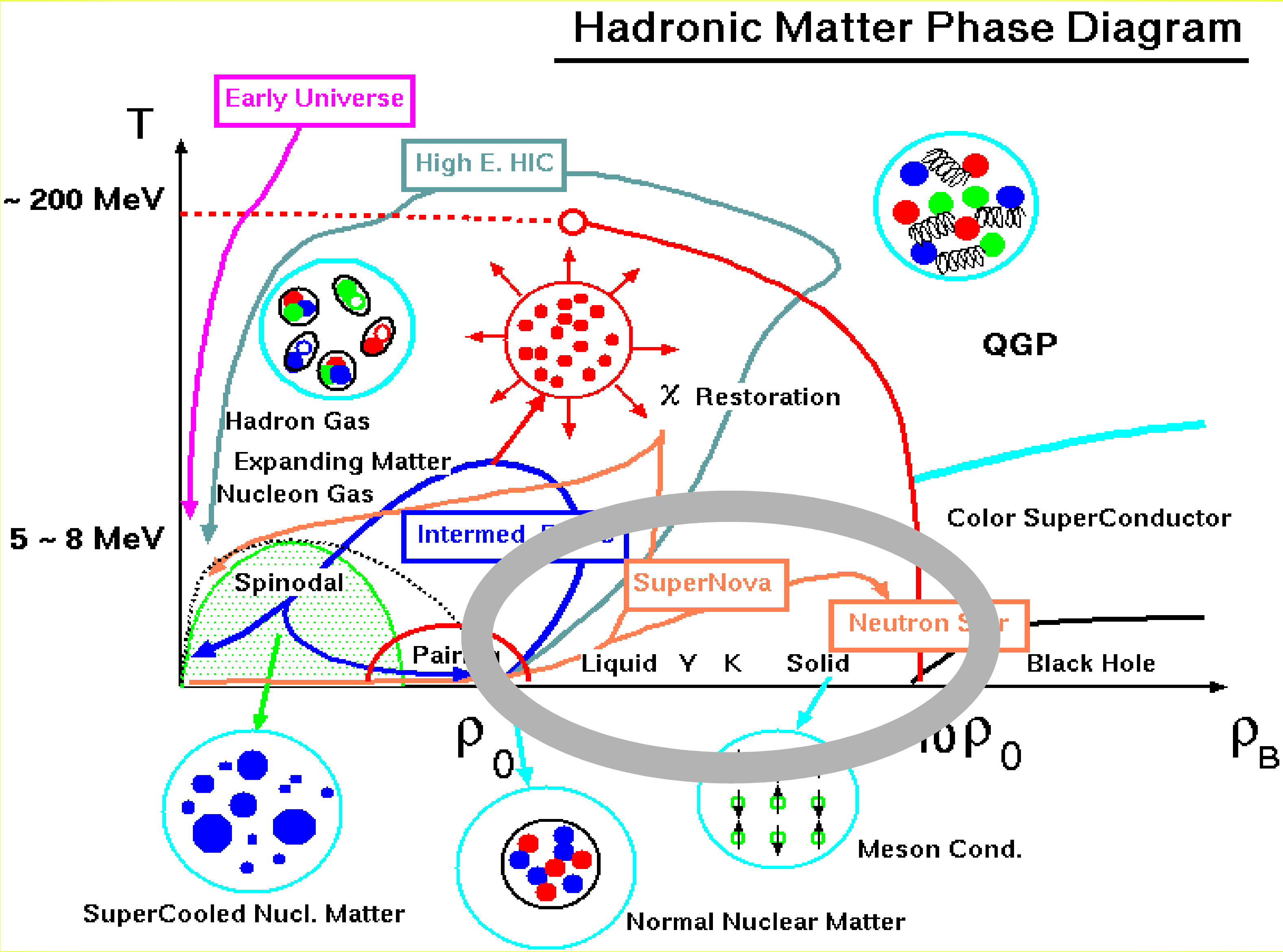
Hyp2003, J-Lab, Oct. 14-18, 2003

SU(3) chiral linear σ model for positive and negative parity baryons in dense matter

A. Ohnishi, K. Naito (Hokkaido Univ.)

- **Introduction: Hyperons in Dense Matter**
- **SU(3) Chiral sigma model with baryons**
- **Application to Symmetric Nuclear Matter**
- **Summary**

Hadronic Matter Phase Diagram



Hyperons in Dense Matter

Hyperons in Neutron Star (cf Talk by Bombaci, Vidana)

Tsuruta-Cameron (66), Langer-Rosen (70), Pand-haripande (71), Itoh(75), Glendenning, Weber-Weigel, Sugahara-Toki, Schaffner-Mishustin, Balberg-Gal, Baldo et al., Vidana et al., Nishizaki-Yamamoto-Takatsuka, Kohno-Fujiwara et al., ...

Hyperons during Supernova Explosion

- Supernova explode in pure 1D hydro,
  but with ν transport shock stalls.
- 3 %increase of ν flux revive shock wave (Janka et al.)
- Hyperons increase explosion energy by around 4 %
  (Ishizuka, AO, Sumiyoshi,Yamada, in preparation)

Hyperons play crutial roles in dense matter,
such as in neutron stars and supernova explosion.

Hyperon Potentials at High Densities

- **Hyperon Potentials at around ρ_0**

  $U(\Lambda) \sim -30$ MeV

  $U(\Xi) \sim -(14-16)$ MeV **(KEK-E224, BNL-E885, BNL-E906)**

  $U(\Sigma) \sim (-30 \sim +150)$ **MeV** **(Noumi / Kohno)**

- **Hyperon Potentials at high densities (V. Koch's talk)**

  Exp't Info. : Hyperon flow, K^+/π^+ enhancement, ...

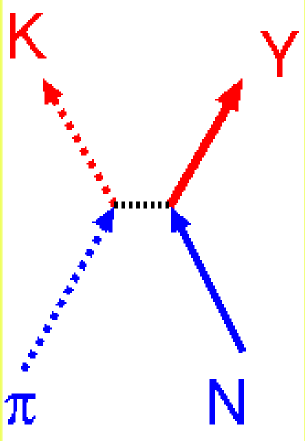
  Theor. Prediction: ***Strongly depends on the model***

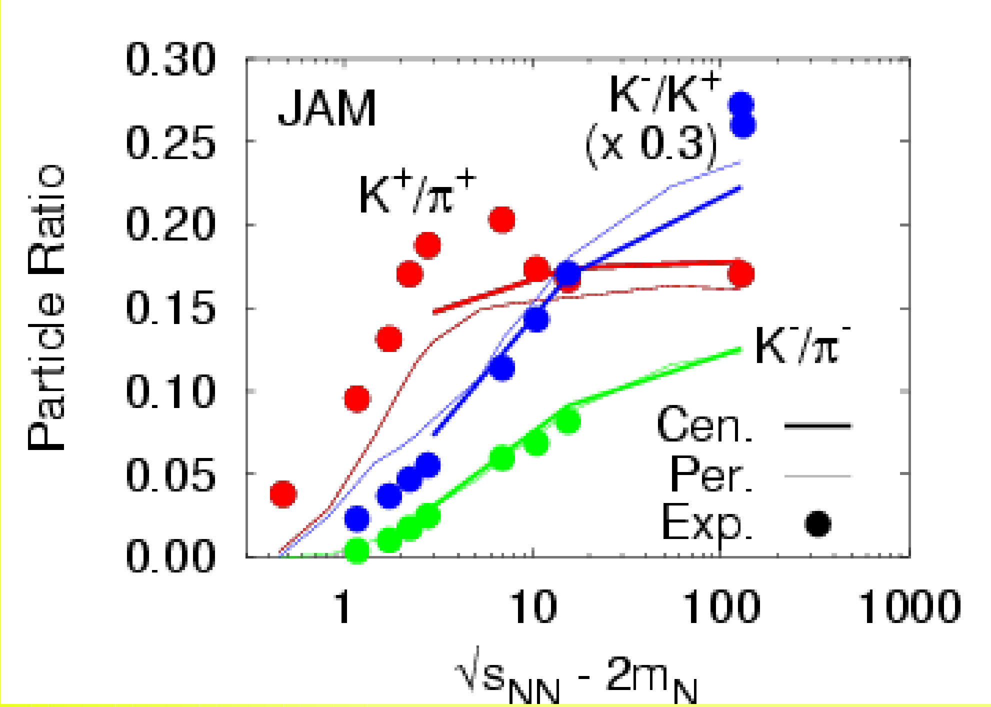
  (Shinmura's Talk)

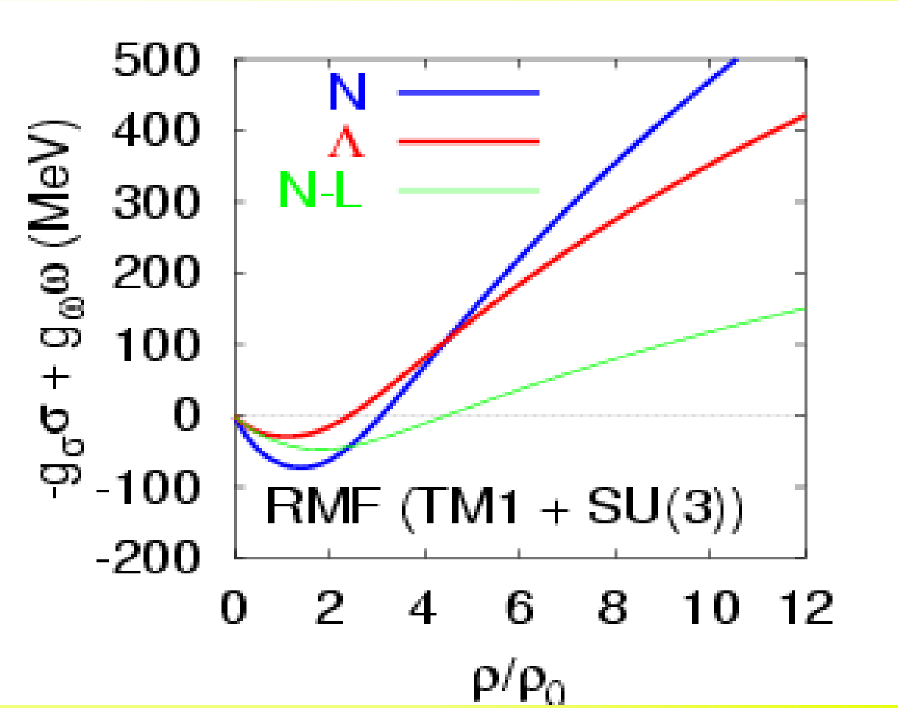
We need reliable models
with smaller number of free parameters
and/or derived from the first principle.
→ Chiral Symmetry

Does Hyperon Potential Help It ?

- Rescattering of Resonances/Strings (RQMD)
- Baryon Rich QGP Formation
- High Baryon Density Effect (Associated Prod. of Y)







At $\rho > 4\rho_0$, Hyperon Feels More Attractive Potential than N

Nuclear Matter in SU(2) Chiral Linear σ Model

☆ Chiral Linear σ Model

Good model in describing hadron properties.
Dynamical change of σ condensate
→ suitable for nuclear matter study

☆ Problems in Nuclear Matter

Naive model leads to sudden change of condensate, $\sigma \sim f_\pi \to 0$
→ Dynamical generation of ω meson mass (σω coupling)
(J. Boguta, PLB120,34/PLB128,19)

Equation of State is too stiff.
→ Loop Effects (vacuum renormalization)
(N.K. Gledenning, NPA480,597,
M. Prakash and T. L. Ainsworth, PRC36, 346)
Higher order terms ($\boldsymbol{\sigma}^6$, $\boldsymbol{\sigma}^8$)
(P.K. Sahu and AO, PTP104,1163)

Can we soften the EOS with Hyperons ?

SU(3) Chiral Linear σ Model with Baryons

BBM coupling in SU(2) chiral linear σ model

⭑ Hadron transformation

Baryons: fundamental repr.

$$N = \begin{pmatrix} p \\ n \end{pmatrix} , \quad N_L \to L\, N_L , \quad N_R \to R\, N_R$$

Mesons: Adjoint repr.

$$M = \Sigma + i\,\Pi \to L\, M\, R^{+}$$

⭑ Chiral Invariant Coupling

$$L_{BBM} = g\,(N_L^{+} M\, N_R + N_R^{+} M^{+} N_L)$$
$$\to g\,(N_L^{+} L^{+} L\, M\, R^{+} R\, N_R + c.c.)$$

How about in SU(3) ?

Mesons and Baryons in SU(3)

Meson Matrix

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{s} & s\bar{d} & s\bar{s} \end{pmatrix} \qquad M_{PS} = \Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} \end{pmatrix}$$

Baryon Matrix

$$\Psi = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

BBM Coupling in SU(3) Chiral Linear sigma Model

- **Mesons: Transforms as in SU(2)**

$$M = \Sigma + i\,\Pi \rightarrow L\,M\,R^{+}$$

- **Baryons: Two kind of transformations are proposed**
  - **Type 1: Stokes-Rijken '97, Gasiolowiz-Geffen 60's**

$$\Psi_L^{(1)} \rightarrow L\,\Psi_L^{(1)} L^{+} \quad , \quad \Psi_R^{(1)} \rightarrow R\,\Psi_R^{(1)} R^{+}$$

Lowest order chiral invariant coupling = BMBM

$$Tr\ \left(\overline{\Psi}_L^{(1)} M\,\Psi_R^{(1)} M^{+}\right) \rightarrow Tr\ \left(L\,\overline{\Psi}_L^{(1)} L^{+} L\,M\,R^{+} R\,\Psi_R^{(1)} R^{+} R\,M^{+} L^{+}\right)$$

  - **Type 2:** Papazoglou et., 98

$$\Psi_L^{(2)} \rightarrow L\,\Psi_L^{(2)} R^{+} \quad , \quad \Psi_R^{(2)} \rightarrow R\,\Psi_R^{(2)} L^{+}$$

Lowest order chiral invariant coupling = D-type coupling

$$Det'\ \left(\overline{\Psi}_R^{(2)}, M, \Psi_L^{(2)}\right) \equiv \epsilon_{ijk}\,\epsilon_{lmn}\,\overline{\Psi}_{R\,il}^{(2)} M_{jm}\,\Psi_{kn}^{(2)} \rightarrow |R|\,|L|\,Det'\ \left(\overline{\Psi}_R^{(2)}, M, \Psi_L^{(2)}\right)$$

Relation to Quark Field

- **Spin half 3 quark field**

Positive Parity, Having NR Limit (8 states)

$$B^{(1)}_{dc} = N^{-3}\left(q^T_{i,a} C \gamma_5 q_{j,b}\right) q_{k,d}\, \epsilon_{ijk}\, \epsilon_{abc}$$

Negative Parity, No NR Limit (9 states)

$$B^{(2)}_{dc} = N^{-3}\left(q^T_{i,a} C\, q_{j,b}\right) q_{k,d}\, \epsilon_{ijk}\, \epsilon_{abc}$$

(ijk: color, abcd: flavor)

- **Transformation properties**

$$\Psi^{(1)} \equiv \left(B^{(1)} + \gamma_5 B^{(2)}\right)/\sqrt{2} \quad , \quad \Psi^{(2)} \equiv \left(B^{(1)} - \gamma_5 B^{(2)}\right)/\sqrt{2}$$

$$\Psi^{(1)}_L \to L\, \Psi^{(1)}_L\, L^+ \quad , \quad \Psi^{(1)}_R \to R\, \Psi^{(1)}_R\, R^+$$

$$\Psi^{(2)}_L \to L\, \Psi^{(2)}_L\, R^+ \quad , \quad \Psi^{(2)}_R \to R\, \Psi^{(2)}_R\, L^+$$

SU(3) Chiral Invariant BBM Coupling

- **Trace type coupling** (G.A. Christos, Phys. Rev. D35 (1987), 330).

Another Type: $Tr\left(\overline{\Psi}_L^{(1)} M \Psi_R^{(2)}\right) \to Tr\left(L \overline{\Psi}_L^{(1)} L^+ L M R^+ R \Psi_R^{(2)} L^+\right)$

$$-L_{BM}^{Tr} = \frac{g_{tr}}{\sqrt{2}} Tr\left(\overline{\Psi}_L^{(1)} M \Psi_R^{(2)} + \overline{\Psi}_R^{(1)} M^+ \Psi_L^{(2)}\right) + h.c.$$

$$= g_{tr}\left(d_{abc} + i f_{abc}\right)\left(\overline{\psi}^{1a} m^b \psi^{2c} + h.c.\right)$$

- **Determinant type Coupling** (Papazoglou et. al. PRC57 (1998) 2576)

$$-L_{BM}^{Det} = \sqrt{2} g^{det}\left(Det'\left(\overline{\Psi}_R^{(2)}, M, \Psi_L^{(2)}\right) + h.c.\right)$$

$$= 2 g_{det} d'_{abc} \overline{\psi}^{2a} m^{+b} \psi^{2c}$$

$$\left(m^a \equiv \sigma^a + i \gamma_5 \pi^a\right)$$

In order to have both of D and F type BBM Coupling,
We need two types of baryons !

Baryon Masses

Explicit Breaking term

* **Mean Field Approx. + Diagonalization**

$$-L_{BM}^{MF} = \sum_i \left(\bar{\psi}_i^1 \quad \bar{\psi}_i^2\right)\begin{pmatrix} 0 & g_{tr}\sigma_i + n_i^s m_s \\ g_{tr}\sigma_i + n_i^s m_s & -g_{det}\sigma'_i \end{pmatrix}\begin{pmatrix} \psi_i^1 \\ \psi_i^2 \end{pmatrix}$$

$$= \sum_i \left(\bar{\psi}_i^{[+]} \quad -\bar{\psi}^{[-]}{}_i\right)\begin{pmatrix} M_i^{[+]} & 0 \\ 0 & -M_i^{[-]} \end{pmatrix}\begin{pmatrix} \psi_i^{[+]} \\ \psi_i^{[-]} \end{pmatrix}$$

$$\sigma_i \equiv \sqrt{\frac{2}{3}}(\sigma_0 + a_i\sigma_8) \quad , \quad \sigma'_i \equiv \sqrt{\frac{2}{3}}(\sigma_0 + b_i\sigma_8)$$

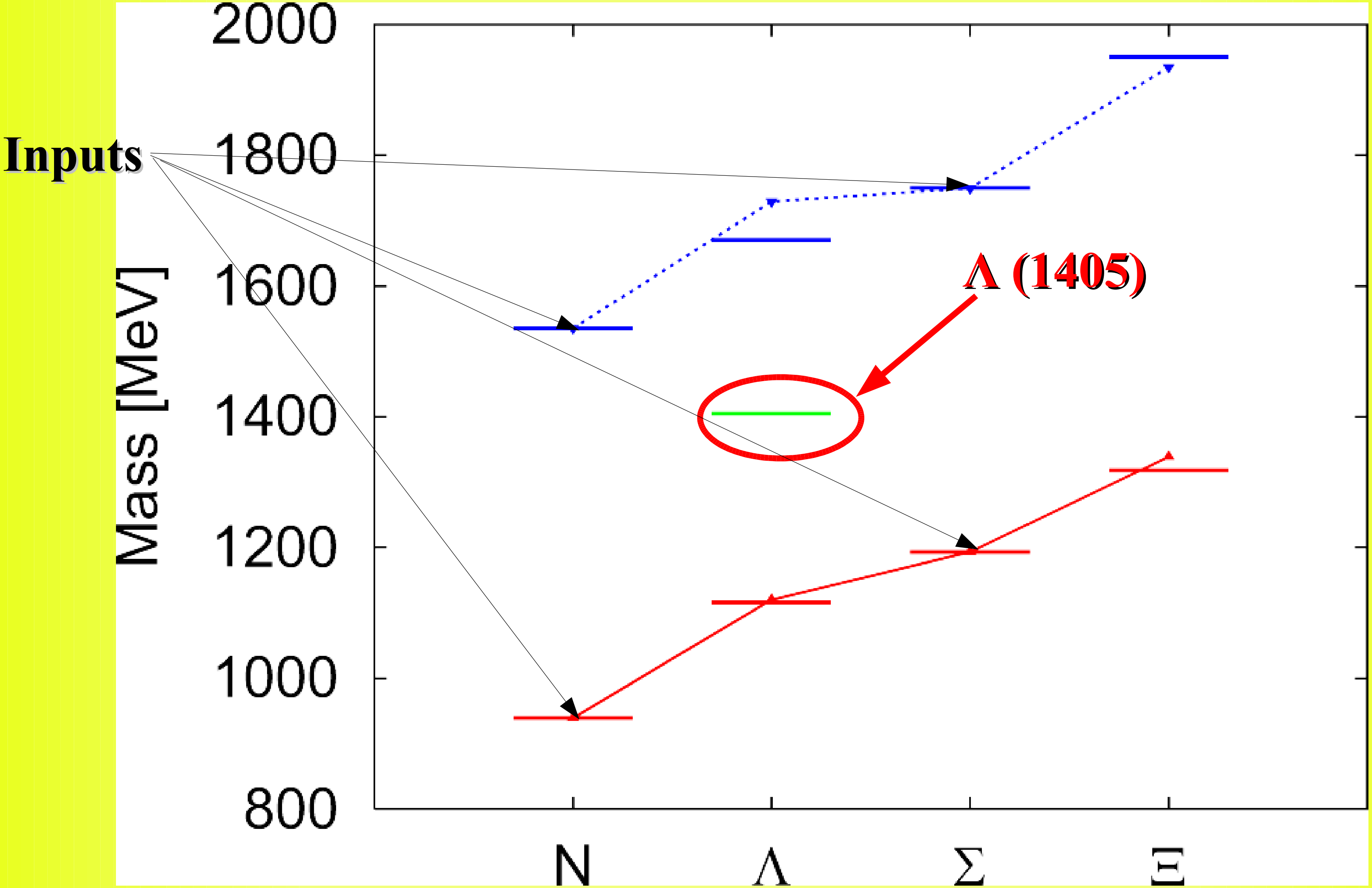
$$\begin{pmatrix} \psi_i^1 \\ \psi_i^2 \end{pmatrix} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix}\begin{pmatrix} \psi_i^{[+]} \\ \gamma_5\psi_i^{[-]} \end{pmatrix}$$

* **Positive and Negative Parity Octet Baryon Masses**

$$M_i^{[\pm]} = \sqrt{|g_{tr}\sigma_i + n_i^s m_s|^2 + (g_{det}\sigma'_i/2)^2} \pm g_{det}\sigma'_i/2$$

Four Free Parameters < Numbers of Baryon Masses to be fitted

Baryon Mass Spectrum



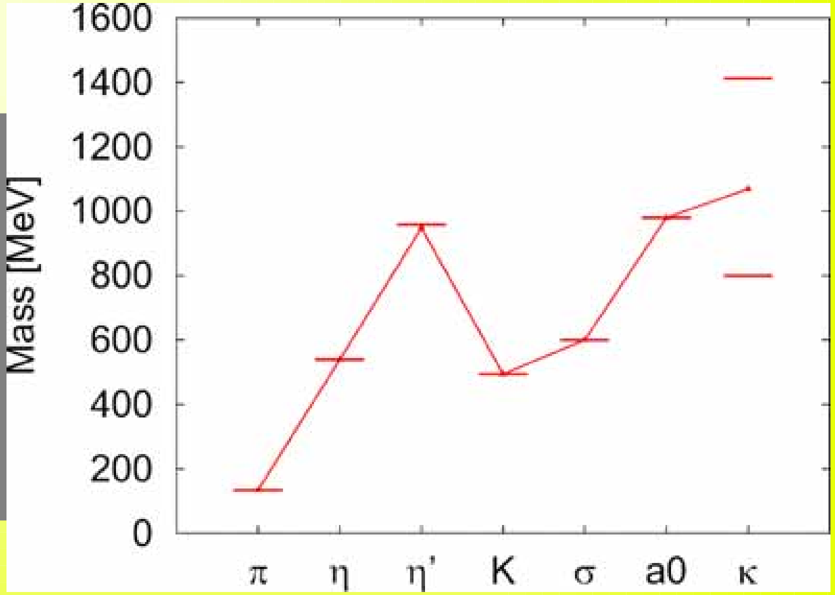
Positive and Negative Spin-half Baryon Masses are well reproduced except for Λ (1405)

Meson Lagrangian: Standard

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}[\partial_\mu M \partial^\mu M^\dagger] - \frac{1}{2}\mu^2 \mathrm{tr}[M M^\dagger] - \lambda \mathrm{tr}[M M^\dagger M M^\dagger] - \lambda' \mathrm{tr}[M M^\dagger]^2 + c(\det M + \det M^\dagger) + \frac{\sqrt{3}D}{4}\left\{\mathrm{tr}[M M^\dagger \lambda^8] + \mathrm{tr}[M \lambda^8 M^\dagger]\right\} + c_\sigma \sigma + c_\zeta \zeta$$

Kinetic+ Second order + Fourth order + Det. Int. + Explicit breaking

Seven Free Parameters
Fitting 2 Decay constants (π and K)
and 4 Meson masses (π , K, η , a_0)
→ One parameter = σ Mass



Application to Symmetric Nuclear Matter

Mean Field Lagrangian

$$L^{MF} = \frac{\mu^2}{2}(\sigma^2+\zeta^2) - \frac{\lambda}{4}(\sigma^2+\zeta^2)^2 - \frac{\lambda'}{4}(\sigma^4+2\zeta^4) + c\,\sigma^2\zeta$$
$$-\nu\zeta^2 + H_\sigma \sigma + H_\zeta \zeta$$
$$+\sum_i \left(\sum_{k=1,2} \overline{\psi}_i^k i\,\partial_\mu \gamma^\mu \psi_i^k\right)$$
$$+\sum_i \left[g_{tr}(\overline{\psi}_i^1 \psi_i^2 + \overline{\psi}_i^2 + \psi_i^1)(\sigma_i + n_i^s m_s) - g_{det}\overline{\psi}_i^2 \psi_i^2 \sigma'_i\right]$$

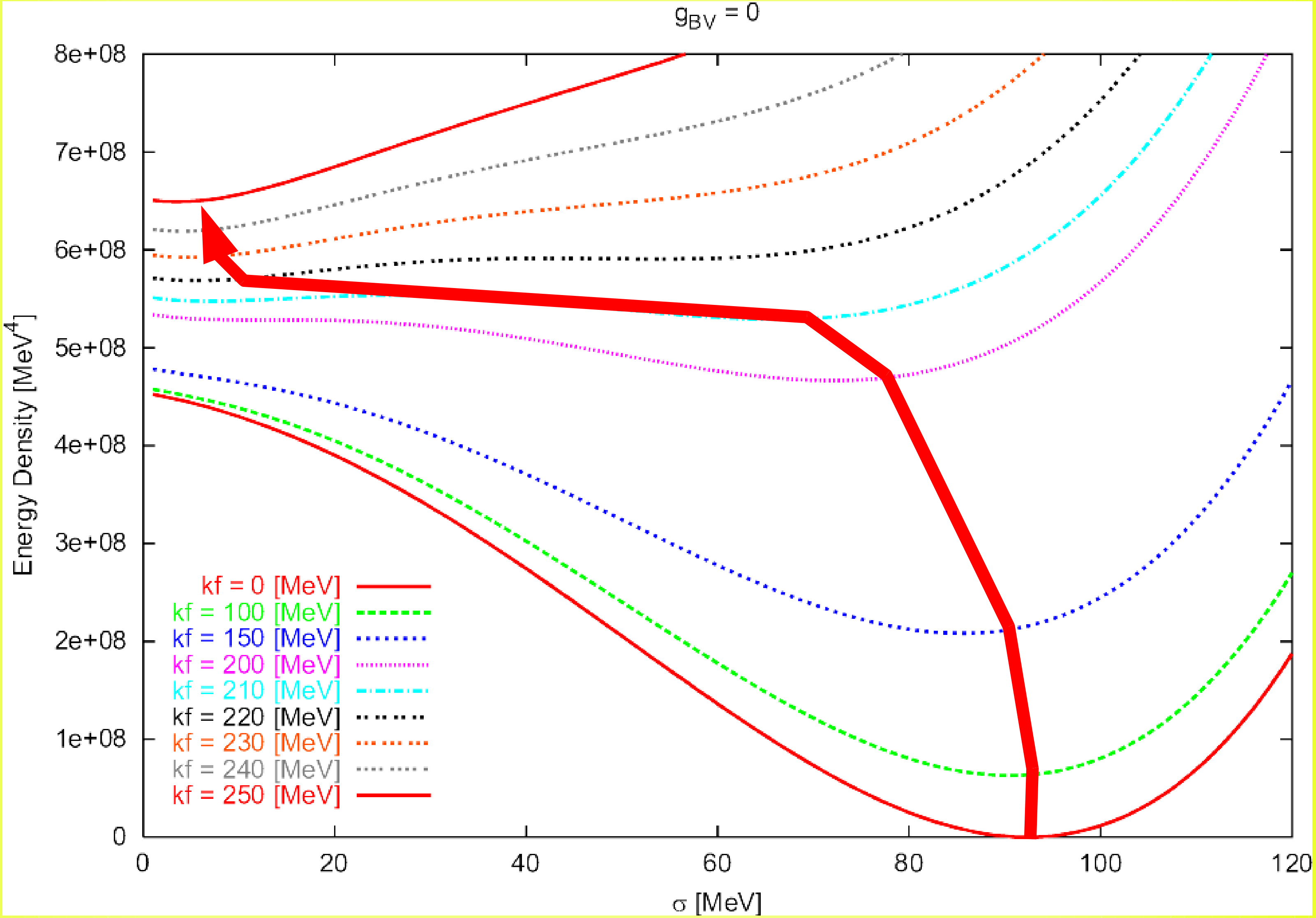
- **Equation of Motion**

$$\frac{\partial L}{\partial \sigma} = \mu^2\sigma - \lambda(\sigma^2+\zeta^2)\sigma - \lambda'\sigma^3 + 2c\,\sigma\zeta + H_\sigma - g_\sigma^{[+]}\rho_s \qquad = 0$$

In Symmetric Nuclear Matter,

$$g_\sigma^{[+]} = 2g_{tr}\sin\theta_N \cos\theta_N - g_{det}\cos\theta_N^2$$

Free Energy (1): without σ ω Coupling



Sudden Chage of σ Value → Chiral Phase Transition below ρ_0

Mean Field Lagrangian

$$L^{MF} = \frac{\mu^2}{2}(\sigma^2+\zeta^2) - \frac{\lambda}{4}(\sigma^2+\zeta^2)^2 - \frac{\lambda'}{4}(\sigma^4+2\zeta^4) + c\,\sigma^2\zeta$$
$$-\nu\zeta^2 + H_\sigma\sigma + H_\zeta\zeta$$
$$+\sum_i\left(\sum_{k=1,2}\overline{\psi}_i^k\, i\,\partial_\mu\gamma^\mu\psi_i^k\right)$$
$$+\sum_i\left[g_{tr}\left(\overline{\psi}_i^1\psi_i^2+\overline{\psi}_i^2+\psi_i^1\right)\left(\sigma_i+n_i^s m_s\right) - g_{det}\overline{\psi}_i^2\psi_i^2\sigma'_i\right]$$

$$-g_{VB}\,\omega\,\rho_B + \lambda_{VS}\,\sigma^2\omega^2/2$$

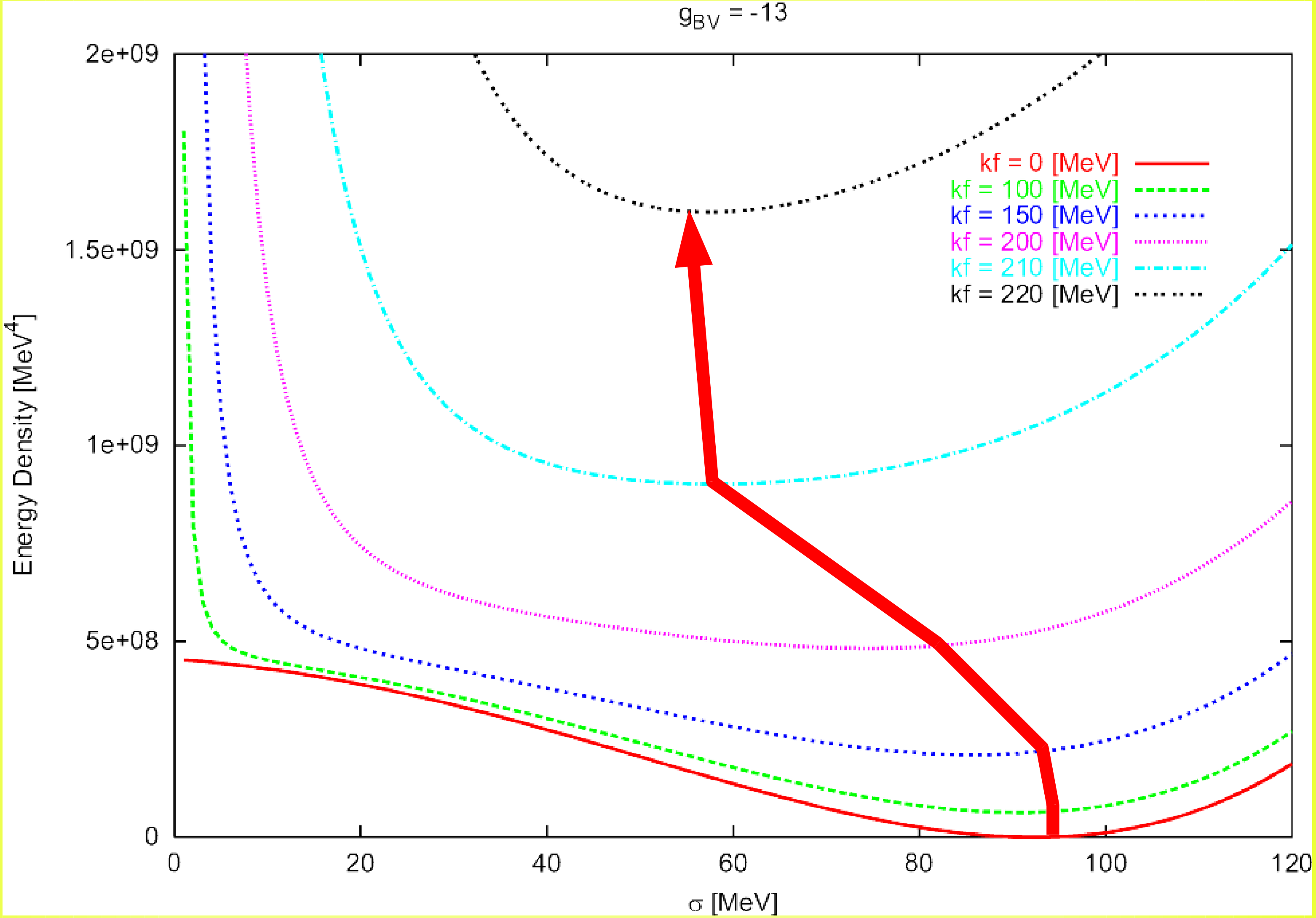
- Equation of Motion

$$\frac{\partial L}{\partial\sigma} = \mu^2\sigma - \lambda(\sigma^2+\zeta^2)\sigma - \lambda'\sigma^3 + 2c\,\sigma\zeta + H_\sigma - g_\sigma^{[+]}\rho_s + \lambda_{VS}\,\sigma\,\omega^2 = 0$$

$$\frac{\partial L}{\partial\omega} = -g_{VB}\,\rho_B + \lambda_{VS}\,\sigma^2\omega = 0 \quad\rightarrow\quad \omega = g_{VB}\,\rho_B/\lambda_{VS}\,\sigma^2$$

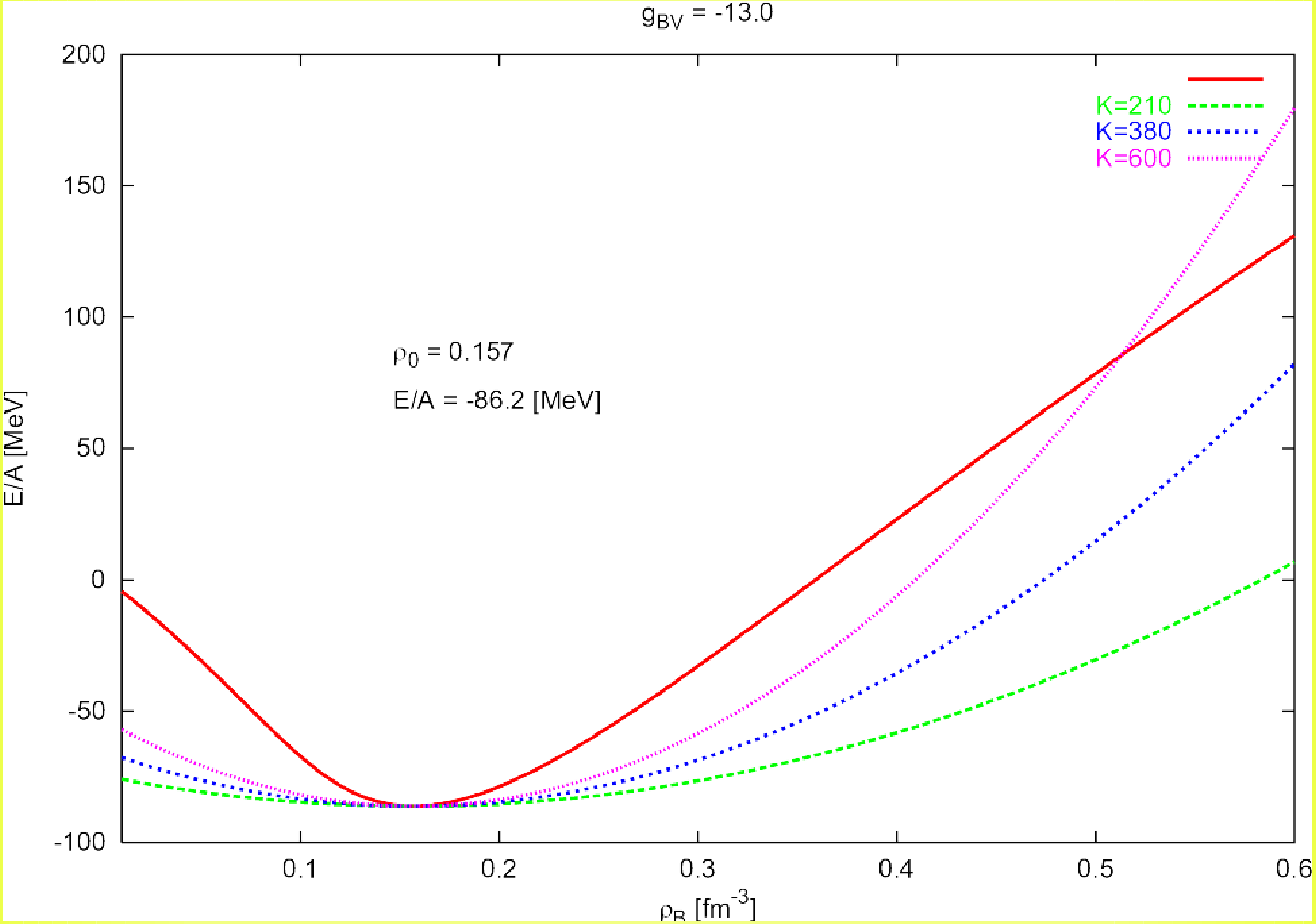
$$m_\omega^2 = \lambda_{VS}\,\sigma^2 = 782\,MeV$$ **(Boguta)**

Free Energy (2): with σ ω Coupling

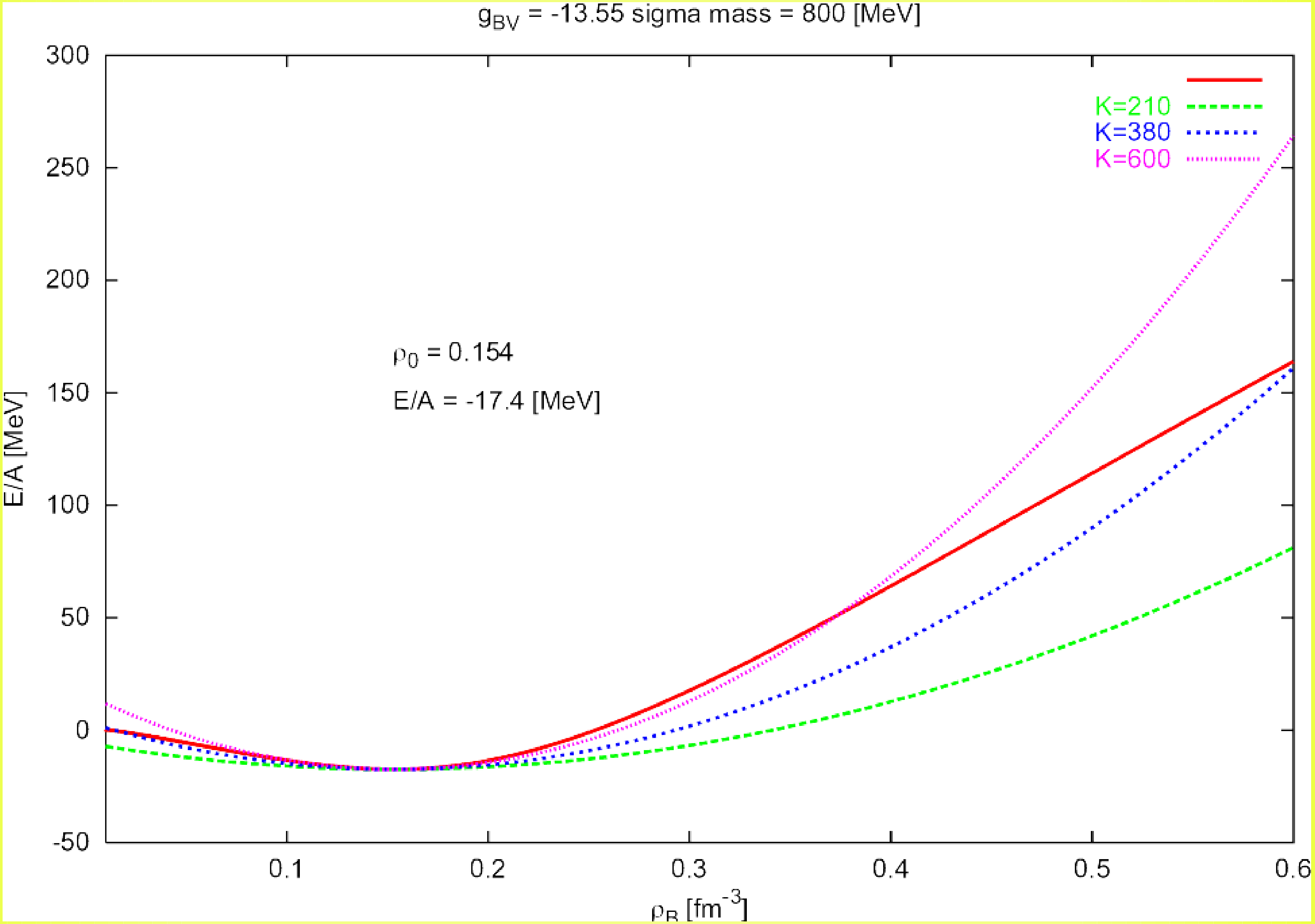


Smooth Chage of σ Value / Only One Local Minima → Stability of Normal Vacuum

Equation of State (1): M_σ = 600 MeV



Equation of State (2): M_σ = 800 MeV



We can fit ρ_0 and E/ A by adjusting g_{BV} and M_σ but EOS becomes too stiff.

Summary

- **An SU(3) chiral sigma model with baryons is presented.**

  Two types of transformation, B(1) and B(2)   (Christos)
  Two types of Lowest order BBM coupling
  (Christos / Papazoglou et. al.)
  Explicit breaking term (Strange quark mass) → Octet Baryon Mass
  Meson Lagrangian : Standard

> ***Positive and Negative parity baryons are necessarily couple in constructing the lowest order chirally invariant Lagrangian having D as well as F coupling.***

- **This model is applied to symmetric nuclear matter.**

  Coupling of σ ω :
  Dynamical generation of vector meson mass (Boguta)
  BV coupling:
  Repulsive NN interaction
  EOS = Too Stiff !
  One problem in SU(2) model is not solved yet !

Problems and Future Directions

- **How can we make EOS softer ?**

  "Classical" Interaction BMBM
  Loop (Gledenning / Prakash-Ainsworth)
  Higher order terms (Sahu-AO)
  Dilatation Field (Papazoglou et. al.)
  Vector Realization (Sasaki-Harada)
  Non-Linear Realization

  ....

- **How does the model predict Hyperon Potentials in Dense Matter ?**
- **Behavior of Negative Parity Baryons in Nuclear Matter**
- **F/D Ratio in Pseudoscalar BB coupling = 1.6 !**

  (← emprically 0.5-0.7)

- **Perturbative contribution to condensate in baryon and meson sectors.**

		Transf.	Repr., (L,R)
Quarks	q_L, q_R	Lq_L, Rq_R	$(3,1),(1,3)$
Mesons	$M = \lambda_a(\bar{q}_R\lambda_a q_L)$	$LMR^\dagger$	$(3, 3^*)$
Baryons	Ψ^1_L, Ψ^1_R	$L\Psi^1_L L^\dagger, R\Psi^1_R R^\dagger$	$(8,1),(1,8)$
	Ψ^2_L, Ψ^2_R	$L\Psi^2_L R^\dagger, R\Psi^2_R L^\dagger$	$(3,3^*),(3^*,3)$

	$\Psi^1_L(8,1)$	$\Psi^1_R(1,8)$	$\Psi^2_L(3,3^*)$	$\Psi^2_R(3^*,3)$
$\bar{\Psi}^1_L(8,1)$	—	—	0	$\mathrm{Tr}\left[\bar{\Psi}^1_L M \Psi^2_R\right]$
$\bar{\Psi}^1_R(1,8)$	—	—	$\mathrm{Tr}\left[\bar{\Psi}^1_R M^\dagger \Psi^2_L\right]$	0
$\bar{\Psi}^2_L(3^*,3)$	0	$\mathrm{Tr}\left[\bar{\Psi}^2_L M \Psi^1_R\right]$	—	$\mathrm{Det'}\left[\bar{\Psi}^2_L, M^\dagger, \Psi^2_R\right]$
$\bar{\Psi}^2_R(3,3^*)$	$\mathrm{Tr}\left[\bar{\Psi}^2_R M^\dagger \Psi^1_L\right]$	0	$\mathrm{Det'}\left[\bar{\Psi}^2_R, M, \Psi^2_L\right]$	—

$$B^1_{lk} = N^{-3}(q^T_{a,i} C\gamma_5 q_{b,j})\, q_{c,l}\epsilon_{abc}\epsilon_{ijk}\ , \qquad B^2_{lk} = N^{-3}(q^T_{a,i} C q_{b,j})\, q_{c,l}\epsilon_{abc}\epsilon_{ijk}\ ,$$

$$\Psi^1 = \left(B^1 + \gamma_5 B^2\right)/\sqrt{2}\ :\ \Psi^1_L \to L\Psi^1_L L^\dagger\ ,\quad \Psi^1_R \to R\Psi^1_R R^\dagger\ ,$$

$$\Psi^2 = \left(B^1 - \gamma_5 B^2\right)/\sqrt{2}\ :\ \Psi^2_L \to L\Psi^2_L R^\dagger\ ,\quad \Psi^2_R \to R\Psi^2_R L^\dagger\ ,$$

$$\begin{aligned} d'_{abc} &\equiv \frac{1}{4}\epsilon_{ijk}\,\epsilon_{lmn}\lambda^a_{il}\lambda^b_{jm}\lambda^c_{kn} \\ &= d_{abc} - \frac{\sqrt{6}}{2}\left[\delta_{a0}\delta_{bc} + \delta_{b0}\delta_{ca} + \delta_{c0}\delta_{ab} - 3\delta_{a0}\delta_{b0}\delta_{c0}\right] \end{aligned}$$

Isoscalar Vector Meson ω

Dynamical generation of ω mass: σ ω Coupling

$$m_\omega^2 = \lambda_{VS}\sigma^2$$ **782 MeV**

Coupling to Baryon: Repulsive BB interaction

$$g_{BV_1}\left\{\mathrm{tr}\left[\overline{\Psi}^1_L l_\mu \gamma^\mu \Psi^1_L\right] + \mathrm{tr}\left[\overline{\Psi}^1_R r_\mu \gamma^\mu \Psi^1_R\right]\right\}$$
$$+ g_{BV_2}\left\{\mathrm{tr}\left[\overline{\Psi}^2_L l_\mu \gamma^\mu \Psi^2_L\right] + \mathrm{tr}\left[\overline{\Psi}^2_R r_\mu \gamma^\mu \Psi^2_R\right]\right\}$$

$$\lambda\omega_0\sigma^2 + \frac{g_{BV_1}\cos^2\theta_N + g_{BV_2}\sin^2\theta_N}{2}\left(\langle p^\dagger p\rangle_F + \langle n^\dagger n\rangle_F\right)$$
$$= \quad 0$$

$$g_{BV} \equiv g_{BV1}\cos^2\theta_N + g_{BV2}\sin^2\theta_N$$ ***: Free Parameter !***