UNITARIZED CHIRAL DYNAMICS : SU(3) AND RESONANCES

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E. O.

- Meson baryon interaction
- Unitarized chiral perturbation theory
- Dynamically generated resonances
  - 2 octets of DGR
    - The two $\Lambda(1405)$ states
- experimental perspectives

Unitarized Chiral Perturbation Theory of Hadrons

• Chiral Lagrangians are an efficient way to account for the dynamics of QCD at low energies

• They are effective Lagrangians accounting for the symmetries of QCD, among them chiral symmetry in the limit $m_q \to 0$

- Building blocks of QCD q, g

$$\text{Chiral Lagrangians} \to \begin{cases} \text{Octet of } 0^- \text{ mesons } (\pi, K, \eta) \\ \text{Octet of } \frac{1}{2}^+ \text{ baryons } (N, \Sigma, \Lambda, \Xi) \end{cases}$$

• Chiral Perturbation Theory (χPT) provides a systematic method to use the chiral Lagrangians making an expansion in powers of the momenta of the particles.

Chiral Lagrangians

(Gasser, Leutwyler, Me[illegible] Kaiser, Bernard, Ecker, Pich [illegible]

Meson Meson Interaction

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots$$

$$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu U^\dagger \partial^\mu U + M(U + U^\dagger) \rangle$$

$$U(\Phi) = \exp\left(\frac{i\sqrt{2}\Phi}{f}\right)$$

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

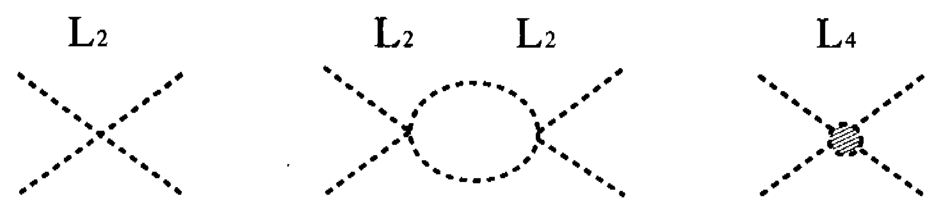
Meson-Baryon interaction

$$\mathcal{L}_1^{(B)} = \langle \overline{B} i\gamma^\mu \nabla_\mu B \rangle - M_B \langle \overline{B} B \rangle + $$
$$+ \frac{D}{2} \langle \overline{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{F}{2} \langle \overline{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle$$

$$u_\mu = i u^\dagger \partial_\mu U u^\dagger \quad ; u^2 = U$$

$$B(x) \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}$$

χPT: (mesons)



Successful at low energies

Problems → Limited energy range of applicability; Cannot deal with resonances

Unitarized Chiral Perturbation Theory

Skillful combination of the information of the Chiral Lagrangians and unitarity in coupled channels.

- Pioneering work of *Kaiser, Siegel, Waas, Weise 95-97* using Lipmann-Schwinger eq. and input from Chiral Lagrangians as potential.

- Subsequent work

  - Inverse Amplitude Method (IAM)→ {

  - (N/D) method→ {

  – Bethe-Salpeter eq.→ {

  –Applications→ {

General scheme *Oller Meissner PLB 01* (meson baryon as exemple)

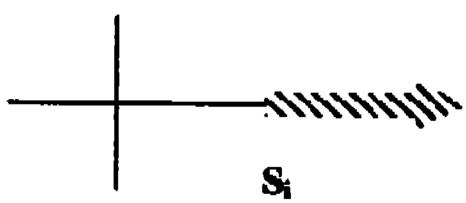
- **Unitarity** in coupled channels $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$, $\eta\Lambda$, $K\Xi$ in $S=-1$

$$
\begin{aligned}
\mathrm{Im} T_{ij} &= T_{il}\sigma_{ll}T^*_{lj} \\
\sigma_l &\equiv \sigma_{ll} \equiv \frac{2Mq_l}{8\pi\sqrt{s}} \\
\sigma &= -\mathrm{Im} T^{-1}
\end{aligned}
$$

- Dispersion relation

$$
\begin{aligned}
T^{-1}_{ij} &= -\delta_{ij}\left\{\hat{a}_i(s_0) + \frac{s-s_0}{\pi}\int_{s_i}^{\infty} ds' \frac{\sigma(s')_i}{(s-s')(s'-s_0)}\right\} + \\
&+ V^{-1}_{ij} \equiv -g(s)_i\delta_{ij} + V^{-1}_{ij}
\end{aligned}
$$

g(s) accounts for the right hand cut



V accounts for local terms, pole terms and crossed dynamics. V is determined by matching the general result to the χPT expressions (usually at one loop level)

$$
\begin{aligned}
g(s) &= \frac{2M_i}{16\pi^2}\left\{a_i(\mu) + \log\frac{m_i^2}{\mu^2} + \frac{M_i^2 - m_i^2 + s}{2s}\log\frac{M_i^2}{m_i^2} + \right. \\
&+ \left. \frac{q_i}{\sqrt{s}}\log\frac{m_i^2 + M_i^2 - s - 2q_i\sqrt{s}}{m_i^2 + M_i^2 - s + 2q_i\sqrt{s}}\right\}
\end{aligned}
$$

μ regularization mass

a_i subtraction constant

Inverting T^{-1}:

$$T = [1 - Vg]^{-1}V$$

Example 1: Take V $\equiv$ lowest order chiral amplitude

In meson-baryon S-wave

$$[1 - V\,g]\,T = V \rightarrow T = V + V\,g\,T$$

Bethe Salpeter eqn. with kernel V

This is the method of [illegible] using cut off to regularize the loops

[illegible] show equivalence of methods with

$$a_i(\mu) \simeq -2\ln\left[1 - \sqrt{1 + \frac{m_i^2}{\mu^2}}\right];$$

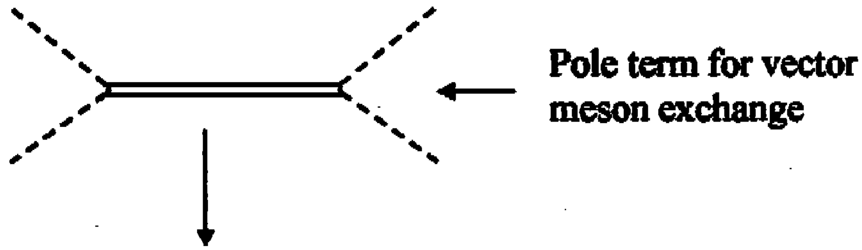
μ cut off

$a_i \simeq -2 \rightarrow \mu \simeq 630\,\text{MeV in } K^-$ [illegible]

If higher order Lagrangians not well determined then fit a_i to the data

- **Example 2: Meson Meson**

$\mathbf{V} = t_2 +$



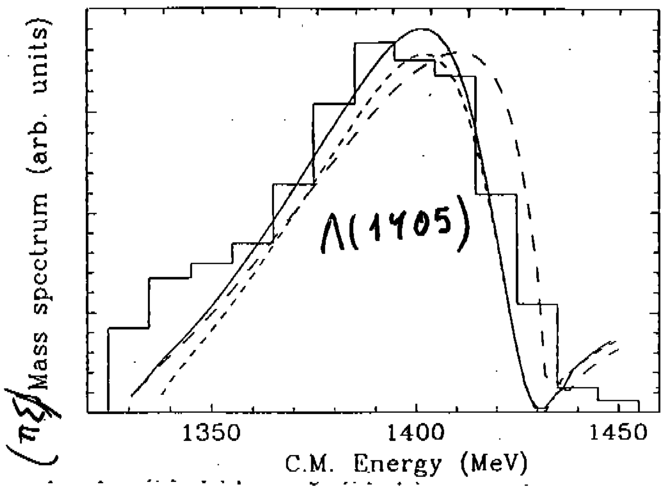
Efficient way to account for the $\mathcal{L}_4$ Lagrangian

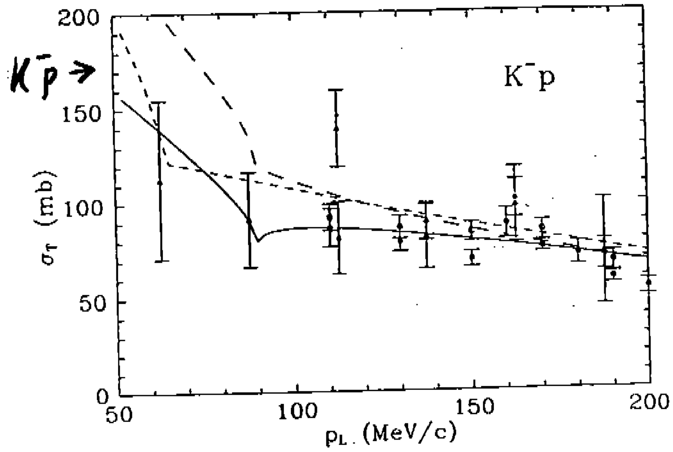
(Resonance saturation hypothesis; Ecker Gasser [illegible])

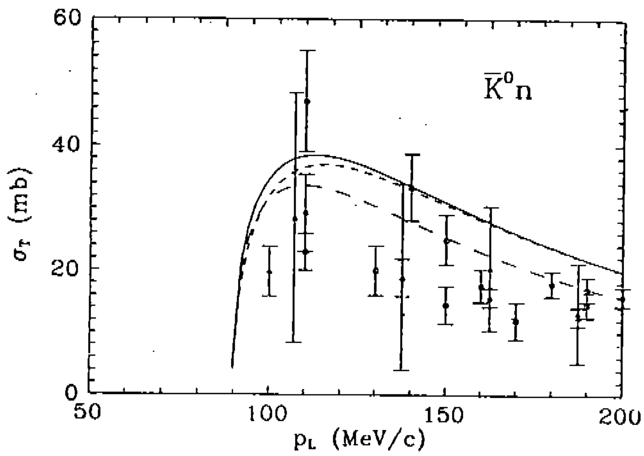
This leads to good reproduction of meson meson data up to 1.2 GeV. Method can be used to evaluate π, K form factors (Palomar [illegible]

- **Dynamically generated resonances:**

Without bare resonance poles one gets $\sigma(500)$, $f_0(980)$, $a_0(980)$, $\kappa(900)$ in $L = 0$, but not [illegible] and [illegible] $\rightarrow$ We call σ, f_0, a_0, κ dynamically generated by the multiple scattering. ρ, K^* are [illegible]







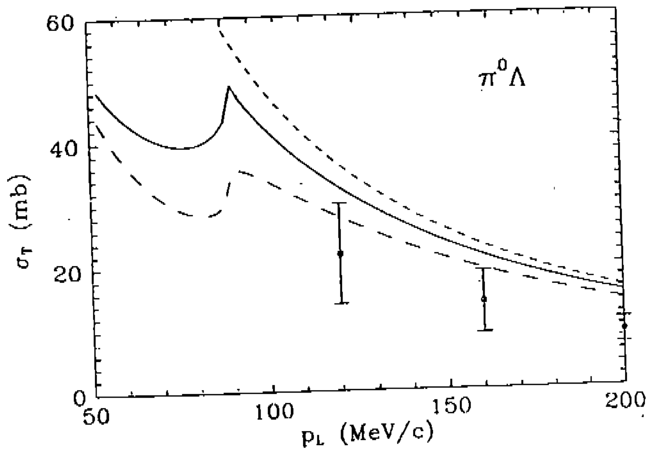
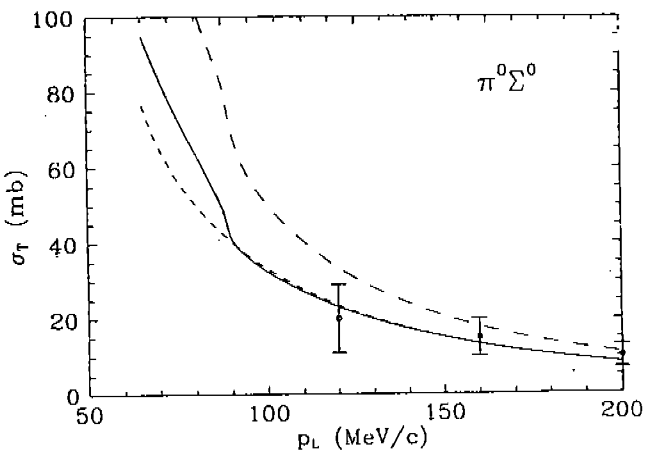
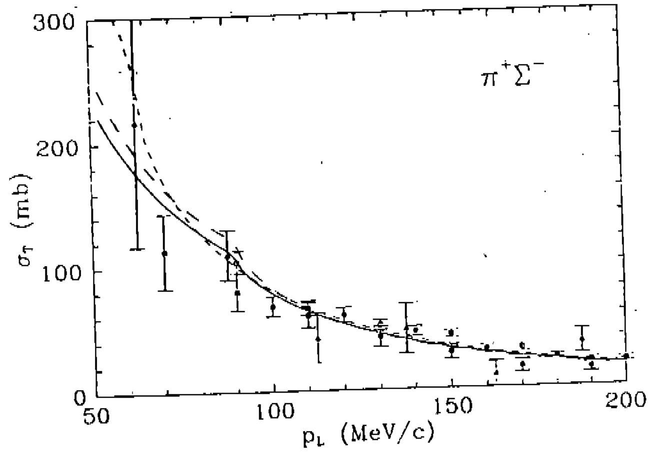
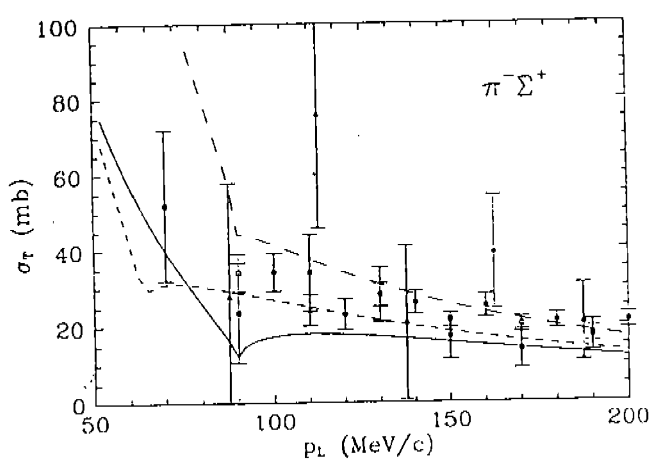
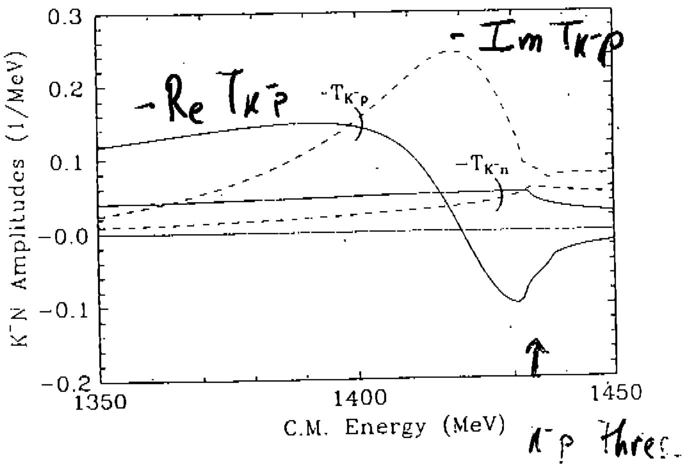


Fig. 5. Same as Fig. 3 for $K^- p \to \pi^0 \Lambda$.









- T in complex plane: close to a pole

$$T_{ij} \approx \frac{g_i g_j}{z - z_R} \qquad : \qquad \sqrt{s} \rightarrow \text{z complex}$$

$z_R \approx M_R + i\frac{\Gamma}{2}$ (2^{nd} Riemann sheet)

g_i: coupling of resonance to i channel

$g_i \rightarrow \Gamma_i$ partial decay width in channel i

- **Search for a $S = -2$ resonance:** Ramos, Bennhold, E.O. PRL 02

Pole found around $z_R = (1605 + i\,65)\,MeV$ with natural size values of a_i

- Experimental candidates:

$\Xi(1620)$ $*$ $I = 1/2$ $(J^\pi =?)$ $\Gamma_i =?$
$\Xi(1690)$ $***$ $I = 1/2$ $(J^\pi =?)$ $\Gamma_i = known$

Freedom with a_i (within natural size) changes z_R a bit

But Γ_i for resonance found disagree in factor 20-30 from Γ_i of $\Xi(1690)$

$\Rightarrow$ Ξ^* should be $\Xi(1620)$

$\Rightarrow$ determines theoretically J^π of this resonance as $1/2$

- S=0

$N^*(1535)$ generated in

- Further work

$SU(3)$ $\quad 8 \otimes 8 \rightarrow 1 + 8^S + 8^A + 10 + \overline{10} + 27$

M B

One should be getting two octets of dynamically generated mesons in $L = 0$
So far $\Lambda(1405)$ $(I = 0)$ seen in

Weise [illegible]
[illegible]
Oller [illegible]

$$\left.\begin{array}{l}\Lambda(1670)(I=0)\\ \Sigma(1620)(I=1)\end{array}\right\}\text{seen in}$$

$\Sigma(1620)$ not visible in amplitudes. Must be searched as a pole in the 2^{nd} Riemann sheet of the complex plane

- *What about the other octet?*

Hints in *Oller, Meissner* ...

Recent work: Jido, Oller, Ramos, E.O., U.G. Meissner
Nucl. Phys. A725 (2003) 181

Take SU(3) limits $m_{B_i} = \overline{m}_B,\ m_{M_i} = \overline{m}_M$

For S=-1 one obtains:

$$\left\{\begin{array}{lll} \text{1 singlet state,} & I = 0, & \Lambda \\ \text{1 octet state,} & I = 0, & \Lambda \\ \text{1 octet state,} & I = 1, & \Sigma \end{array}\right.$$

As one gradually makes the masses go to their physical values the octet states split apart

$$\left\{\begin{array}{l} \Lambda_0 \\ \Lambda_1, \Lambda_2 \rightarrow \text{moves to } \Lambda(1670) \\ \Sigma_1, \Sigma_2 \rightarrow \text{moves to } \Sigma(1620) \end{array}\right.$$

- Λ_1 moves close to Λ_0

$$\begin{cases} \Lambda_0, z_r = (1390 + i\,66)\,MeV, & \text{couples strongly to } \pi\Sigma \\ \Lambda_1, z_r = (1426 + i\,16)\,MeV, & \text{couples strongly to } \bar{K}N \end{cases}$$

What one sees in $\pi\Lambda \to K^+\pi\Sigma$ in the $\pi\Sigma$ invariant mass spectrum (the official $\Lambda(1405)$) is a mixture of Λ_0, Λ_1 dominated by Λ_0

Experimental challenge:

Can one devise other reactions which weight more the Λ_1, shifted in mass and narrower?

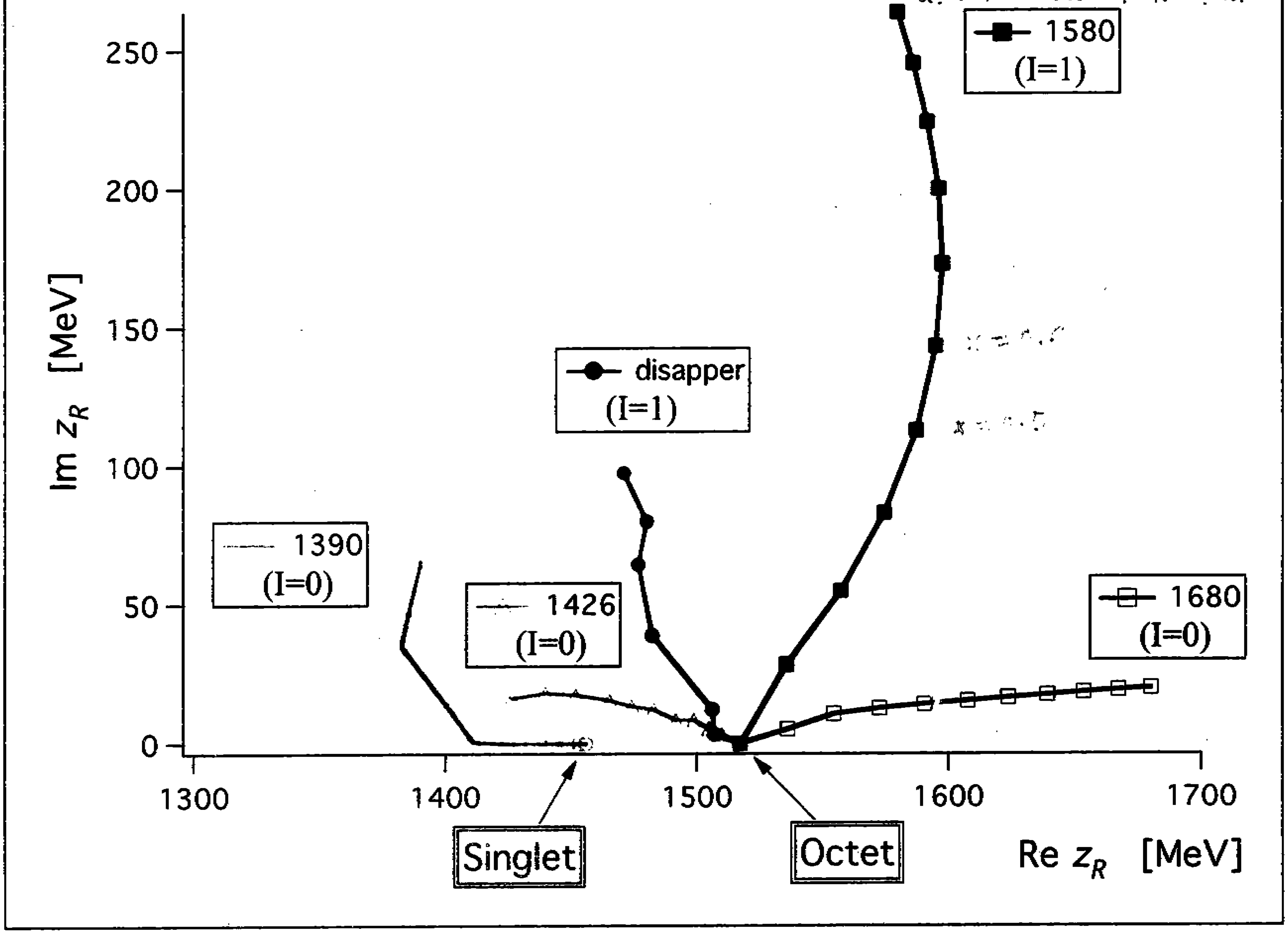
$$\begin{cases} \Lambda(1405) \text{ photoproduction} \\ K^- p \to \Lambda(1405)\gamma \end{cases}$$

D. Jido

$M_i(x) = M_0 + x(M_i - M_0)$

$m_i^2(x) = m_0^2 + x(m_i^2 - m_0^2)$

$a_i(x) = a_0 + x(a_i - a_0)$



TWO Λ(1405) STATES

D. Jido, J.A. Oller, E. Oset, A. Ramos and U.G. Meissner, nucl-th/0303062

Pole positions and couplings to $I = 0$ physical states

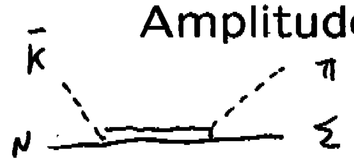
z_R ($I=0$)	$1390 + 66i$		$1426 + 16i$		$1680 + 20i$	
	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$
$\pi\Sigma$	$-2.5 - 1.5i$	2.9	$0.42 - 1.4i$	1.5	$-0.003 - 0.27i$	0.27
$\bar{K}N$	$1.2 + 1.7i$	2.1	$-2.5 + 0.94i$	2.7	$0.30 + 0.71i$	0.77
$\eta\Lambda$	$0.010 + 0.77i$	0.77	$-1.4 + 0.21i$	1.4	$-1.1 - 0.12i$	1.1
$K\Xi$	$-0.45 - 0.41i$	0.61	$0.11 - 0.33i$	0.35	$3.4 + 0.14i$	3.5

–SU(3) decomposition:Couplings of the $I = 0$ bound states to the meson–baryon SU(3) basis states

z_R	$1390 + 56i$ (evolved singlet)		$1426 + 16i$ (evolved octet 8_s)		$1680 + 20i$ (evolved octet 8_a)	
	g_γ	$\lvert g_\gamma \rvert$	g_γ	$\lvert g_\gamma \rvert$	g_γ	$\lvert g_\gamma \rvert$
1	$2.3 + 2.3i$	3.3	$-2.1 + 1.6i$	2.6	$-1.9 + 0.42i$	2.0
8_s	$-1.4 - 0.14i$	1.4	$-1.1 - 0.62i$	1.3	$-1.5 - 0.066i$	1.5
8_a	$0.53 + 0.94i$	1.1	$-1.7 + 0.43i$	1.8	$2.6 + 0.59i$	2.7
27	$0.25 - 0.031i$	0.25	$0.18 + 0.092i$	0.21	$-0.36 + 0.28i$	0.4

–Influence of the poles on the physical observables

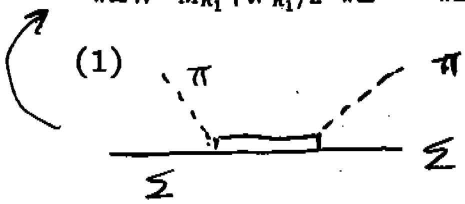
Amplitudes for $\bar{K}N \to \pi\Sigma$ and $\pi\Sigma \to \pi\Sigma$



$$g^{R_1}_{\bar{K}N}\frac{1}{W - M_{R_1} + i\Gamma_{R_1}/2}g^{R_1}_{\pi\Sigma} + g^{R_2}_{\bar{K}N}\frac{1}{W - M_{R_2} + i\Gamma_{R_2}/2}g^{R_2}_{\pi\Sigma} ,$$

$$g^{R_1}_{\pi\Sigma}\frac{1}{W-M_{R_1}+i\Gamma_{R_1}/2}g^{R_1}_{\pi\Sigma} + g^{R_2}_{\pi\Sigma}\frac{1}{W-M_{R_2}+i\Gamma_{R_2}/2}g^{R_2}_{\pi\Sigma} .$$

(1)



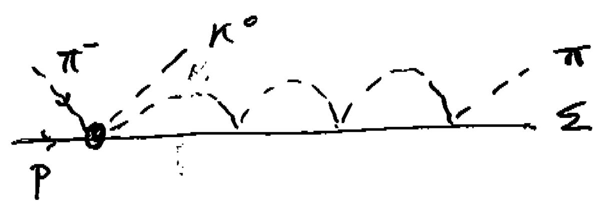
Normally for the description of the Λ(1405) one looks at the $\pi\Sigma$ invariant mass and assumes

$$\frac{d\sigma}{dM_i} = C|T_{\pi\Sigma\to\pi\Sigma}|^2 q_{\text{c.m.}} \ , \qquad (2)$$

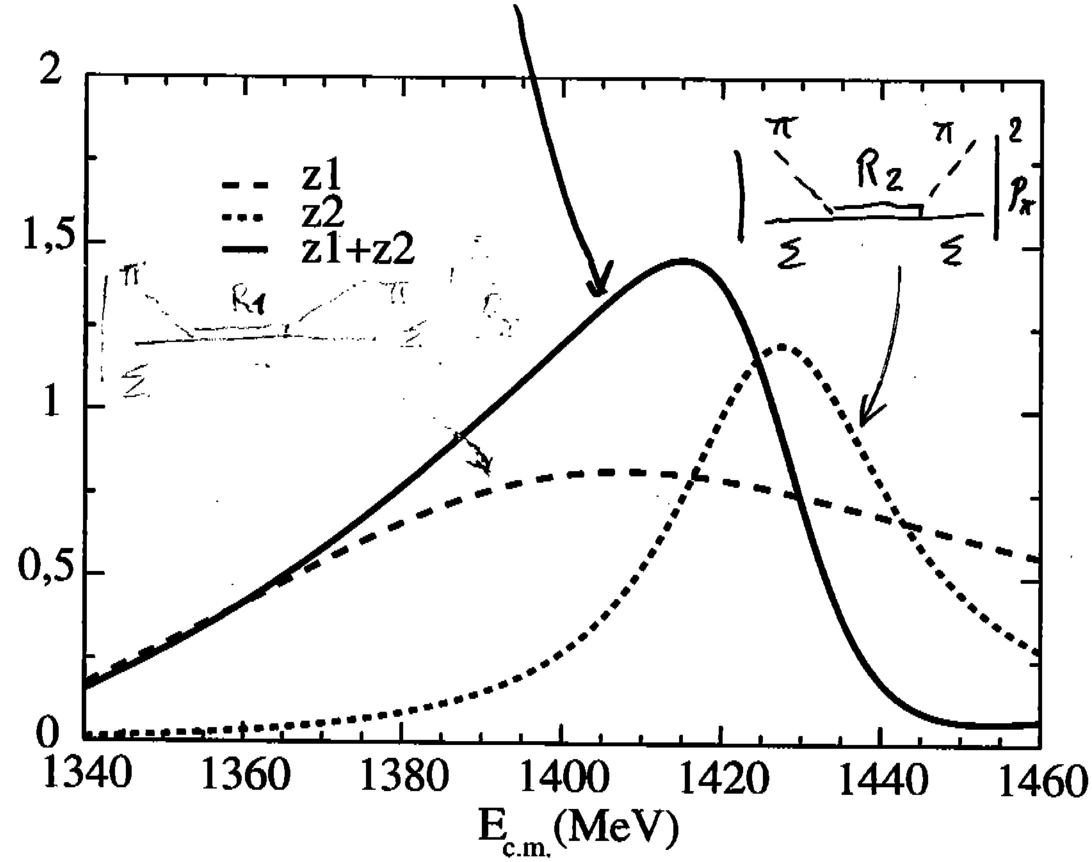
In the presence of the two Λ(1405) states this is not justified. One has

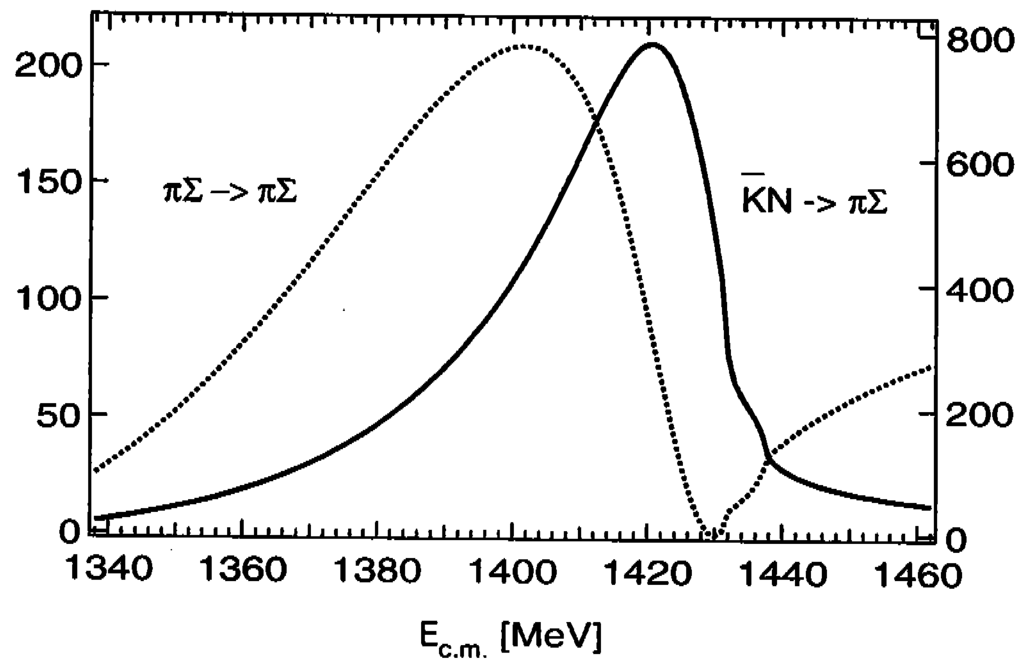
$$\frac{d\sigma}{dM_i} = \Big|\sum_i C_i T_{i\to\pi\Sigma}\Big|^2 q_{\text{c.m.}} \ , \qquad (3)$$

Example
$\pi^- p \to K^0 \pi \Sigma$

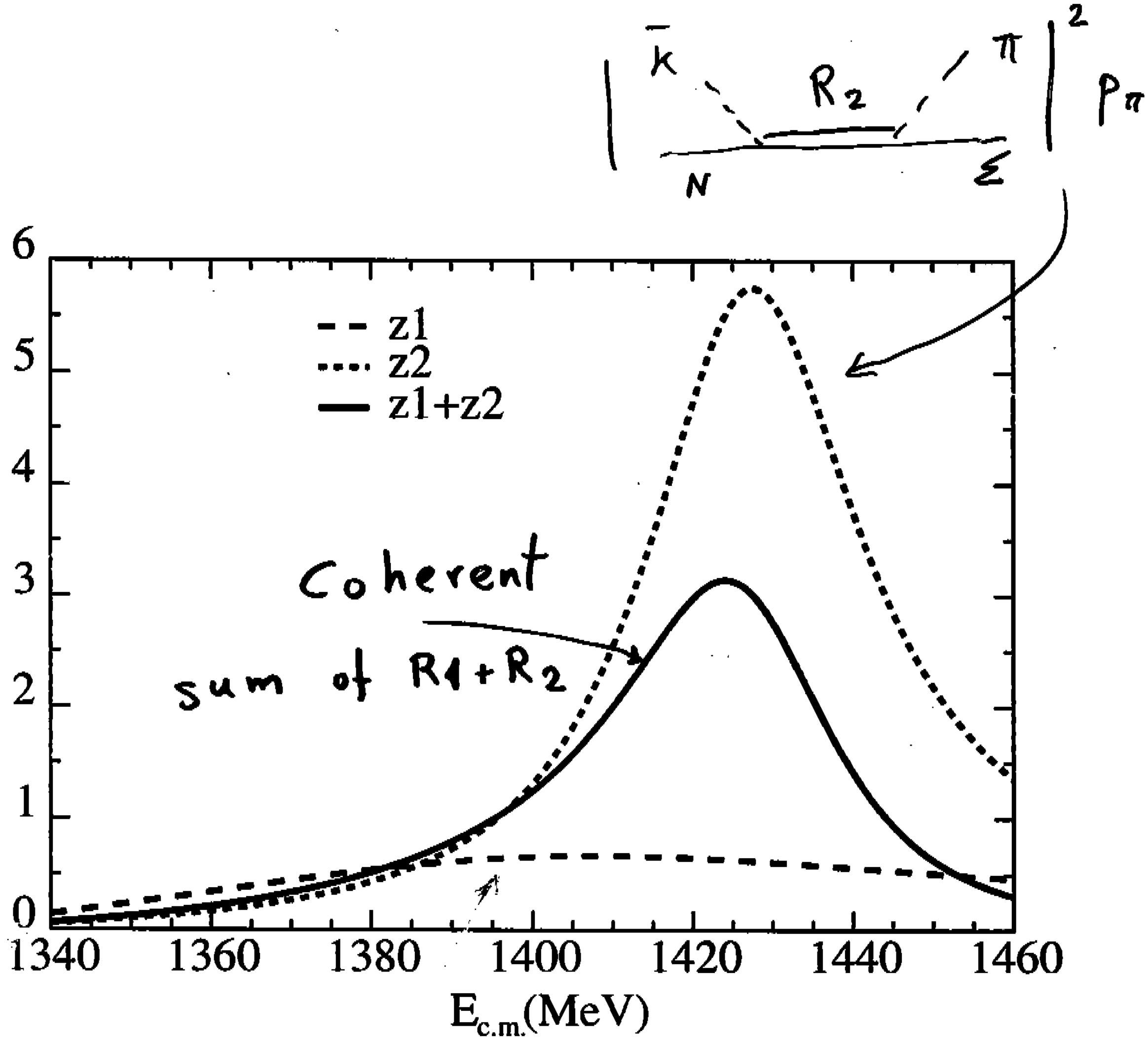


Coherent sum of $R_1 + R_2$

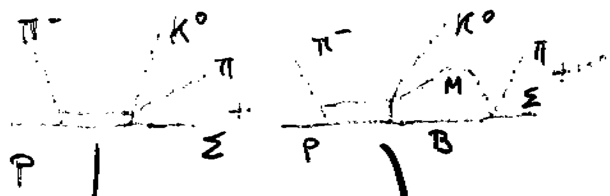




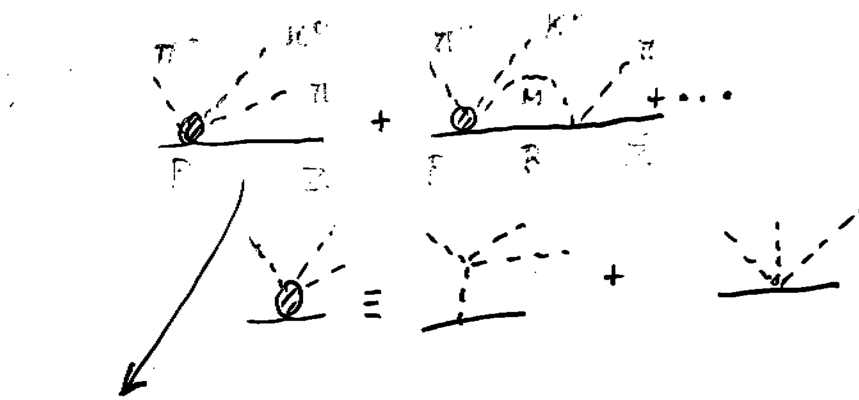
$|t_i|^2 p_\pi$ from E.O., A. Ramos Phys Lett.[5]



K̄
R₂
π
N
Σ
p_π
6
5
4
3
2
1
0
z1
z2
z1+z2
Coherent
sum of R1+R2
1340
1360
1380
1400
1420
1440
1460
E_c.m.(MeV)

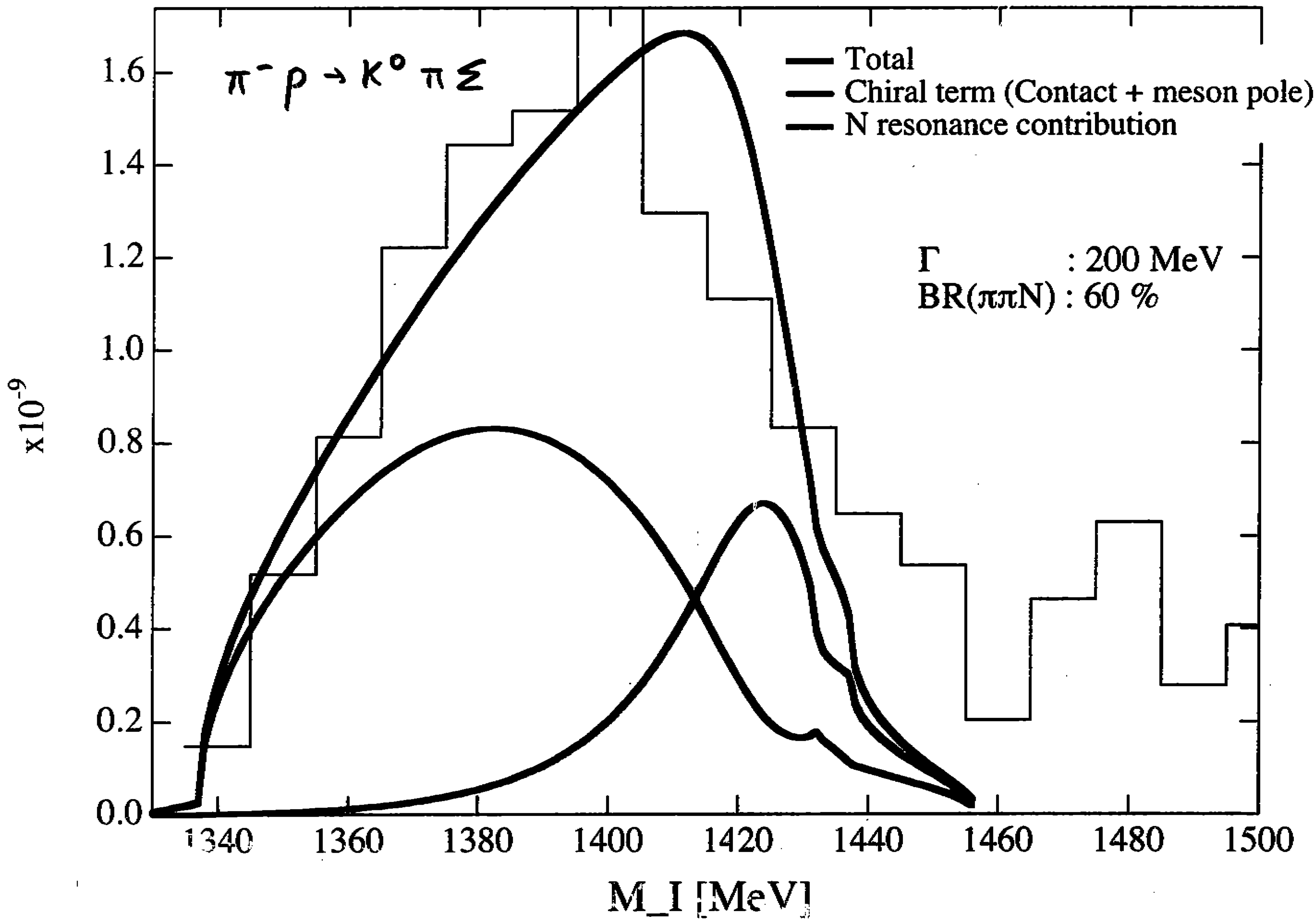


$N^*(1710)\ J^P = 1/2^+$



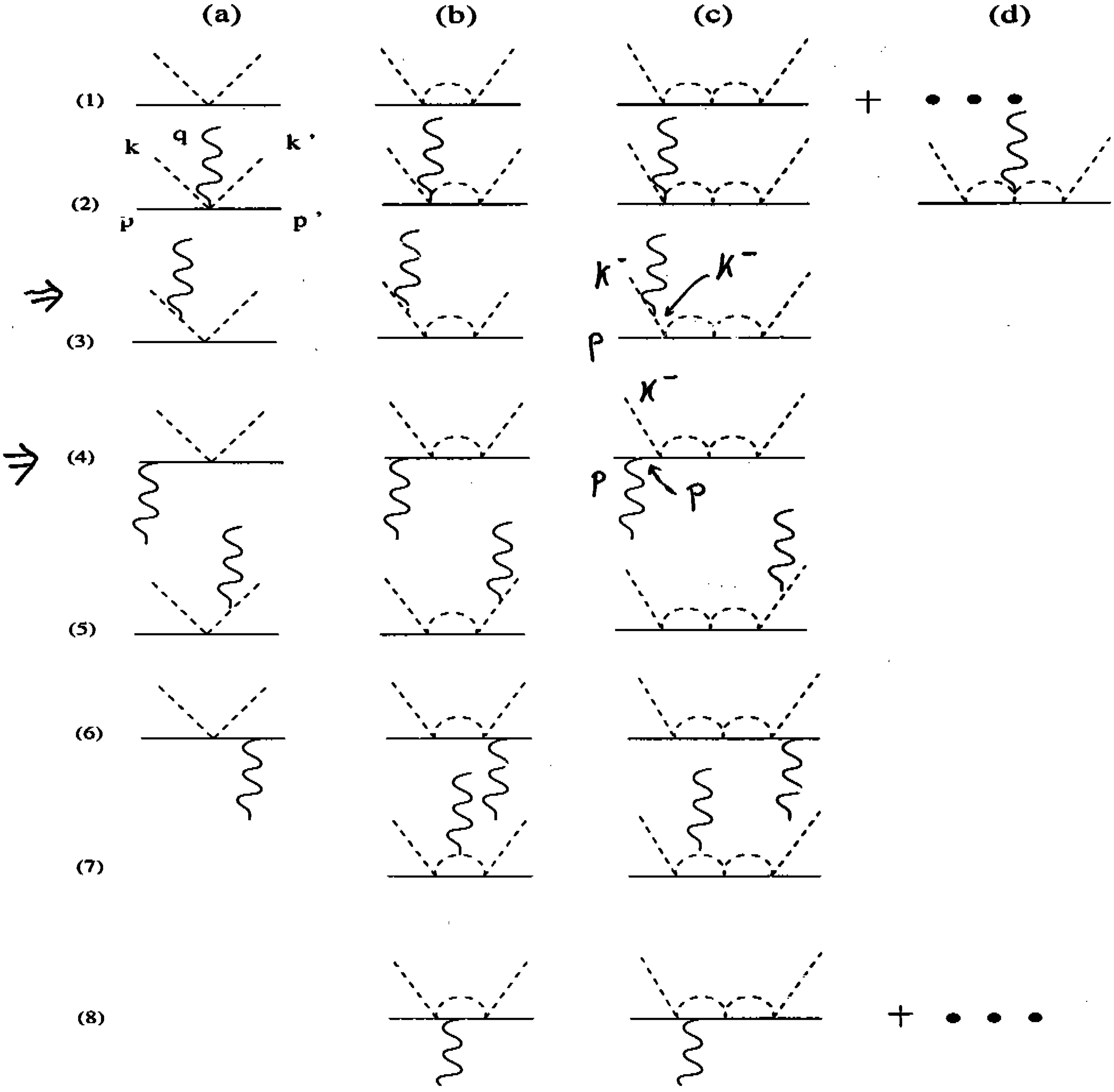
πΣ invariant mass distribution

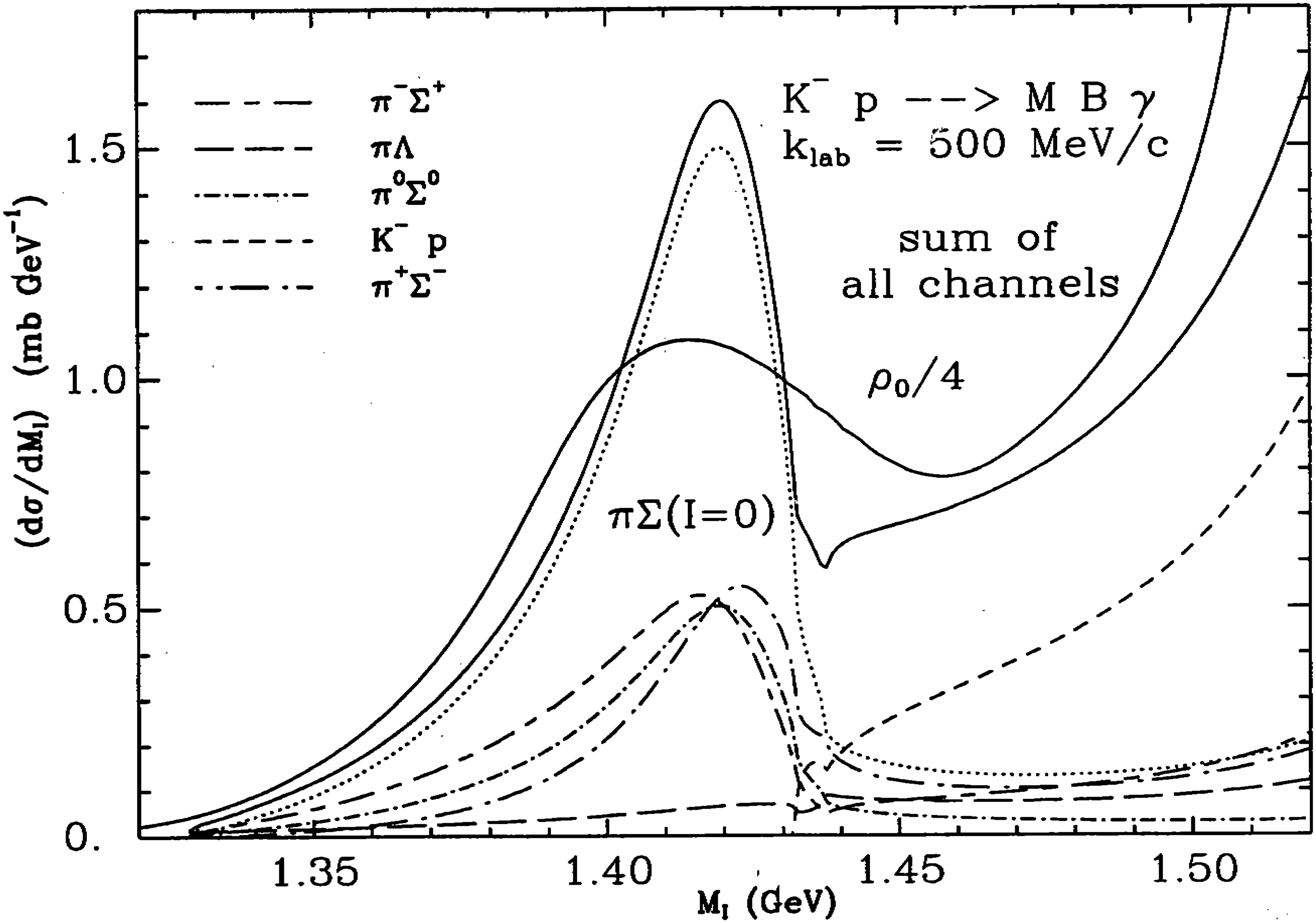
Hyodo, Hosaka, E. O, Ramos, Vicente Vacas
PRC (2003)



$K^- p \rightarrow \Lambda(1405)\gamma$

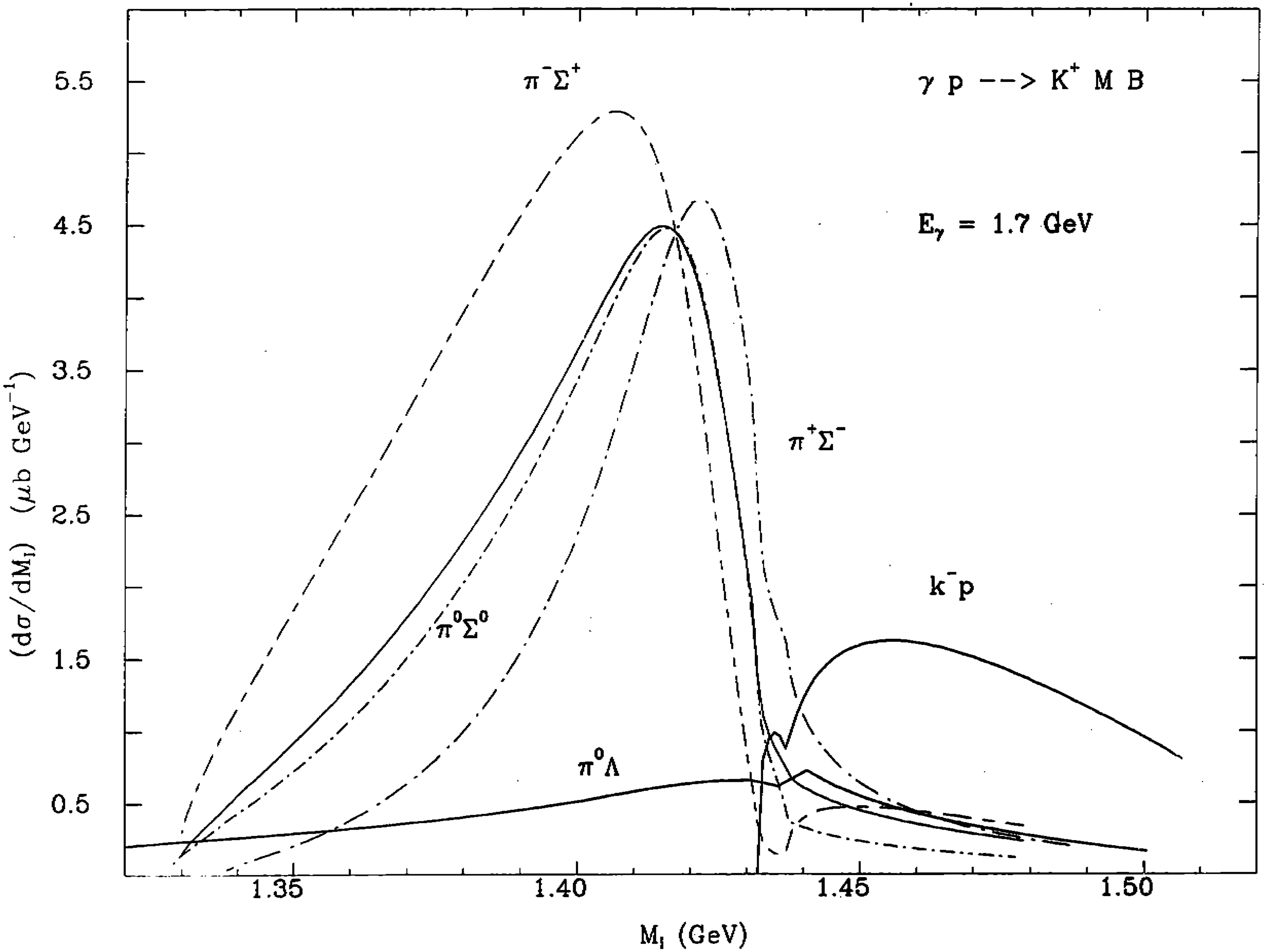
Nacher, E. ?., Toki, ?.
PLB 4?? (199?)





$\pi^-\Sigma^+$
$\pi\Lambda$
$\pi^0\Sigma^0$
K^- p
$\pi^+\Sigma^-$
K^- p --> M B γ
k_{lab} = 500 MeV/c
sum of all channels
$\rho_0/4$
$\pi\Sigma(I=0)$
$(d\sigma/dM_I)$ (mb GeV^{-1})
M_I (GeV)
0.
0.5
1.0
1.5
1.35
1.40
1.45
1.50

Nacher, E. O., H. Toki, Ramos
PLB 455 (99)



– New isopin I=1 state at 1400 MeV?

It is interesting to see the different shapes of the three $\pi\Sigma$ channels. This can be understood in terms of the isospin decomposition of the states

$$|\pi^+\Sigma^-\rangle = -\frac{1}{\sqrt{6}}|2,0\rangle - \frac{1}{\sqrt{2}}|1,0\rangle - \frac{1}{\sqrt{3}}|0,0\rangle \quad (4)$$

$$|\pi^-\Sigma^+\rangle = -\frac{1}{\sqrt{6}}|2,0\rangle + \frac{1}{\sqrt{2}}|1,0\rangle - \frac{1}{\sqrt{3}}|0,0\rangle \quad (5)$$

$$|\pi^0\Sigma^0\rangle = \sqrt{\frac{2}{3}}|2,0\rangle - \frac{1}{\sqrt{3}}|0,0\rangle \quad (6)$$

Disregarding the I = 2 contribution which is negligible, the cross sections for the three channels go as:

$$\frac{1}{2}|T^{(1)}|^2 + \frac{1}{3}|T^{(0)}|^2 + \frac{2}{\sqrt{6}} Re(T^{(0)}T^{(1)*})\,; \quad \pi^+\Sigma^- \quad (7)$$

$$\frac{1}{2}|T^{(1)}|^2 + \frac{1}{3}|T^{(0)}|^2 - \frac{2}{\sqrt{6}} Re(T^{(0)}T^{(1)*})\,; \quad \pi^-\Sigma^+ \quad (8)$$

$$\frac{1}{3}|T^{(0)}|^2\,; \quad \pi^0\Sigma^0 \quad (9)$$

The crossed term $T^{(0)}T^{(1)*}$ is what makes these cross sections different. We can also see that

$$3\frac{d\sigma}{dM_I}(\pi^0\Sigma^0) \simeq \frac{d\sigma}{dM_I}(I=0)$$

$$\frac{d\sigma}{dM_I}(\pi^0\Sigma^0)+\frac{d\sigma}{dM_I}(\pi^+\Sigma^-)+\frac{d\sigma}{dM_I}(\pi^-\Sigma^+) \simeq \frac{d\sigma}{dM_I}(I=0)+\frac{d\sigma}{dM_I}(I=1) \tag{10}$$

CONCLUSIONS

- Mounting evidence about two $\Lambda(1405)$ states
- Different reactions can show different shapes for the $\pi\Sigma$ invariant mass distribution
- Study new reactions experimentally
- Theoretical studies of reactions to suggest new ones

  Try to explain present experiments where the $\Lambda(1405)$ is seen
- Is there a $S=-1$, $I=1$ state around 1400 MeV?

  Spring 8 has the key