The tower structure of $L = 1$ excited baryons and their strong decays.

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Talk at the HYP2003

October 16, 2003

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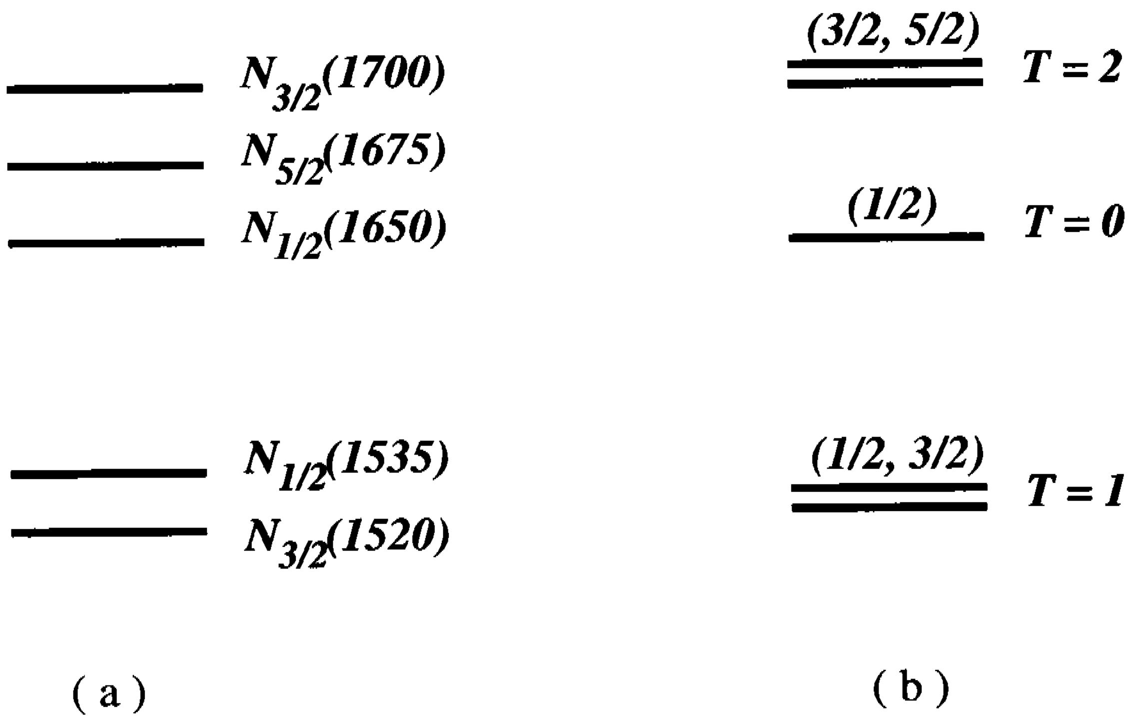
PRD 67, 096009 (2003)
hep-ph/0301187 , hep-ph/0308125

The Spectrum

- The physical spectrum and the $N_c \to \infty$ towers



- The $N_c \to \infty$ QCD prediction are 3 infinite towers labeled by $T = 0, 1, 2$ so that $|J - I| \leq T$.

- The towers are irreducible representations of a contracted symmetry $SU(4)_c$, that follows from $N^* \pi$ scattering consistency relations [D. Pirjol, T.M. Yan, PRD57, 1449 (1998), PRD57, 5434 (1998)].

Question: Is this tower structure still visible in the $N_c = 3$ limit ? What is the size of the $1/N_c$ corrections ?

The quark operator picture

J. L. Goity, Phys. Lett. B **414**, 140 (1997);
C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, PRD59, 114008 (1999);
T. D. Cohen and R. F. Lebed, PRD 67, 096008 (2003), hep-ph/0301167 ;
D. Pirjol and C. Schat, PRD 67, 096009 (2003) .

Nucleons ($I = \frac{1}{2}$)

- Three quarks of spin $\frac{1}{2} \longrightarrow S = \frac{1}{2}, \frac{3}{2}$.
- Orbital angular momentum

$$\mathbf{L=1} \quad \to J = \mathbf{L} + S = \begin{cases} \frac{1}{2} \quad , \quad \frac{3}{2} \\ \frac{1}{2} \quad , \quad \frac{3}{2} \quad , \quad \frac{5}{2} \end{cases}$$

Mass operator:

$$\hat{M} \quad = \quad \sum_k C_k \mathcal{O}_k$$

- Natural size for the coefficients: $C_k \sim 500$ MeV.
- Basic building blocks for constructing the operators $\mathcal{O}_k$:
  The generators of $SU(4) \otimes O(3)$:

s^i, t^a, g^{ia}	excited quark
$S_c^i, T_c^a, G_c^{ia} = \sum_{\alpha=1}^{N_c-1} s^i_{(\alpha)} t^a_{(\alpha)}$	symmetric core
and l^i	orbital degrees of freedom

The $SU(4)$ algebra

$$
\begin{aligned}
[S_i, S_j] &= i\epsilon_{ijk}S_k \\
[T_a, T_b] &= i\epsilon_{abc}T_c \\
[S_i, G_{ja}] &= i\epsilon_{ijk}G_{ka} \\
[T_a, G_{ib}] &= i\epsilon_{abc}G_{ic} \\
[G_{ia}, G_{jb}] &= \frac{i}{4}\delta_{ij}\epsilon_{abc}T_c + \frac{i}{4}\epsilon_{ijk}\delta_{ab}S_k
\end{aligned}
$$

The contracted $SU(4)_c$ algebra

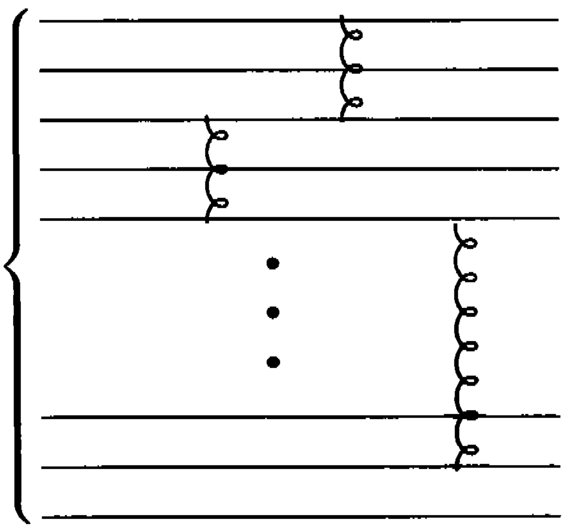
$$
X_{ia}^0 \equiv \lim_{N_c\to\infty} \frac{G_{ia}}{N_c}
$$

$$
\begin{aligned}
[S_i, S_j] &= i\epsilon_{ijk}S_k \\
[T_a, T_b] &= i\epsilon_{abc}T_c \\
\left[S_i, X_{ja}^0\right] &= i\epsilon_{ijk}X_{ka}^0 \\
\left[T_a, X_{ib}^0\right] &= i\epsilon_{abc}X_{ic}^0 \\
\left[X_{ia}^0, X_{jb}^0\right] &= 0
\end{aligned}
$$

Large N_c QCD

- Large N_c counting rules: $g \propto \frac{1}{\sqrt{N_c}}$ and $\langle G_c^{ia} \rangle \propto N_c$

nucleon



m-body operator

$\propto \frac{1}{N_c^{m-1}}$

The order N_c^0 operators are:

$$\mathcal{O}_1 = N_c \mathbf{1}\,, \qquad \mathcal{O}_2 = l^i s^i\,, \qquad \mathcal{O}_3 = \frac{3}{N_c} l^{(2)ij} g^{ia} G_c^{ja}\,.$$

$l^{(2)ij} = \frac{1}{2}\{l^i, l^j\} - \frac{1}{3} l^2 \delta^{ij}$

Mixing matrices

$$\mathbf{M}_{N_{1/2}} = \begin{pmatrix} C_1 N_c - \frac{2}{3} C_2 & -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 \\ -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 & C_1 N_c - \frac{5}{6} C_2 - \frac{5}{16} C_3 \end{pmatrix} ,$$

$$\mathbf{M}_{N_{3/2}} = \begin{pmatrix} C_1 N_c + \frac{1}{3} C_2 & -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 \\ -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 & c_1 N_c - \frac{1}{3} C_2 + \frac{1}{4} C_3 \end{pmatrix} ,$$

$$\mathbf{M}_{N_{5/2}} = C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3 \ .$$

Eigenvalues:

$$M_0 \equiv C_1 N_c - C_2 - \frac{5}{8} C_3 \ ,$$

$$M_1 \equiv C_1 N_c - \frac{1}{2} C_2 + \frac{5}{16} C_3 \ ,$$

$$M_2 \equiv C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3 \ .$$

Mixing angles

$$\begin{pmatrix} N(1535) \\ N(1650) \end{pmatrix} = \begin{pmatrix} \cos\theta_{N1} & \sin\theta_{N1} \\ -\sin\theta_{N1} & \cos\theta_{N1} \end{pmatrix} \begin{pmatrix} N_{1/2} \\ N'_{1/2} \end{pmatrix} ,$$

$$\begin{pmatrix} N(1520) \\ N(1700) \end{pmatrix} = \begin{pmatrix} \cos\theta_{N3} & \sin\theta_{N3} \\ -\sin\theta_{N3} & \cos\theta_{N3} \end{pmatrix} \begin{pmatrix} N_{3/2} \\ N'_{3/2} \end{pmatrix} .$$

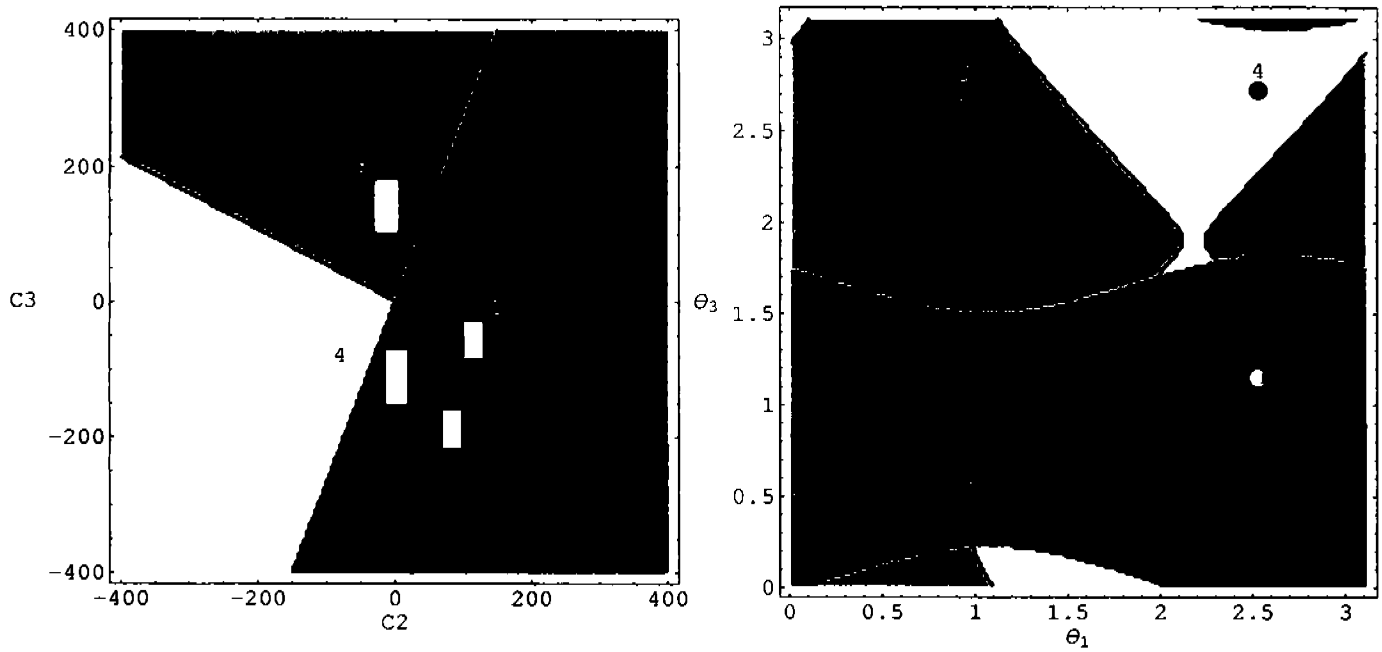
$$|T=0\,, J=\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}N_{1/2} + \sqrt{\frac{2}{3}}N'_{1/2}$$

$$|T=1\,, J=\frac{1}{2}\rangle = -\sqrt{\frac{2}{3}}N_{1/2} + \frac{1}{\sqrt{3}}N'_{1/2}$$

$$|T=1\,, J=\frac{3}{2}\rangle = \frac{1}{\sqrt{6}}N_{3/2} + \sqrt{\frac{5}{6}}N'_{3/2}$$

$$|T=2\,, J=\frac{3}{2}\rangle = -\sqrt{\frac{5}{6}}N_{3/2} + \frac{1}{\sqrt{6}}N'_{3/2}$$

The 4 assignments



ordering of towers:

$$\#1: \quad \{M_0, M_2\} > M_1,$$
$$\#2: \quad M_2 > M_1 > M_0,$$
$$\#3: \quad M_1 > \{M_0, M_2\},$$
$$\#4: \quad M_0 > M_1 > M_2.$$

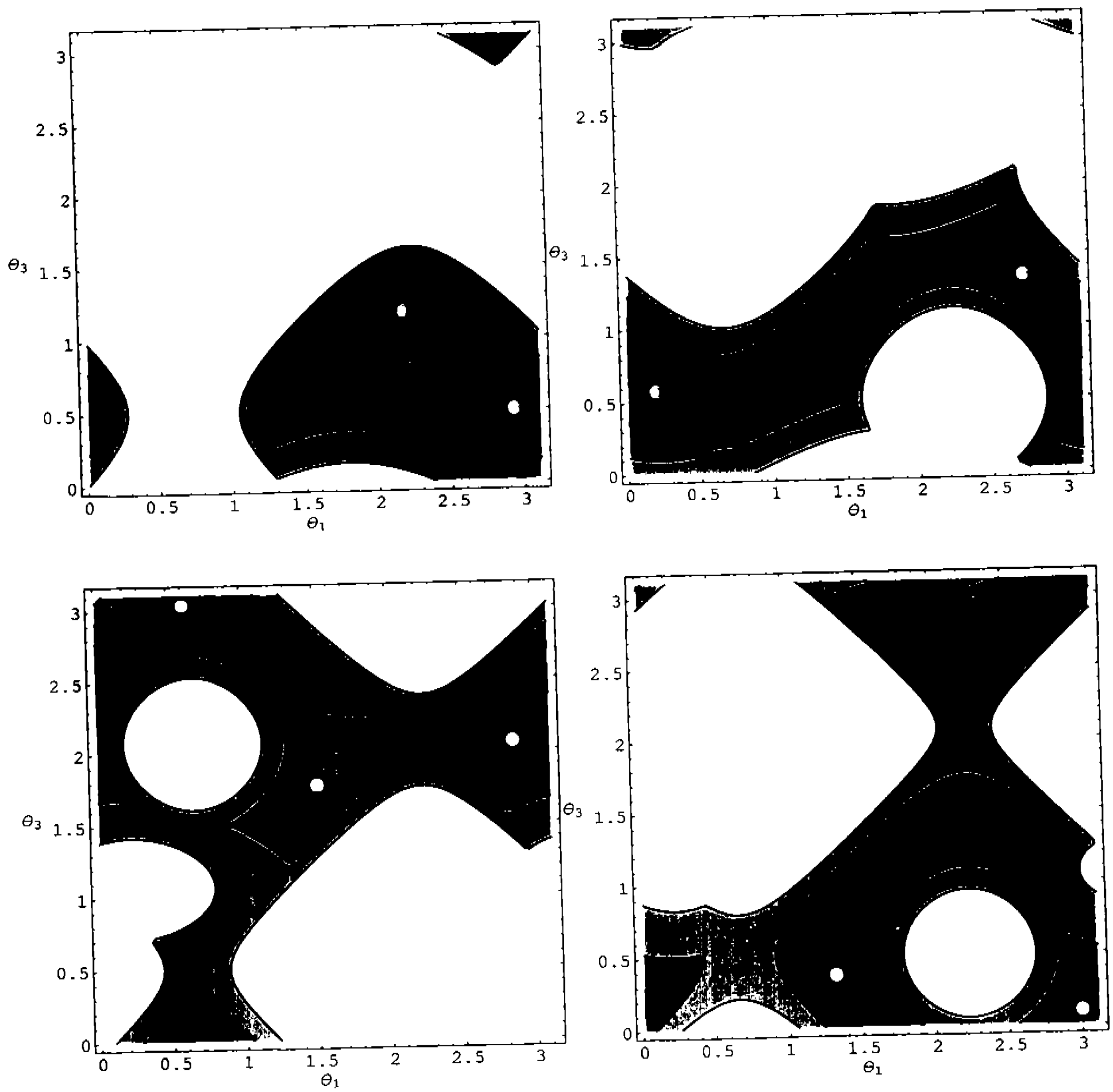
Leading order fit:

	$N_c C_1^{(0)}$	$C_2^{(0)}$	$C_3^{(0)}$	χ^2/d.o.f.	M_0	M_1	M_2	θ_{N1}	θ_{N3}
# 1	1625	83	−188	0.38	1660	1525	1679	2.53	1.1
# 2	1617	115	−57	20.3	1538	1542	1679	0.96	1.1
# 3	1615	−12	142	94.1	1538	1666	1601	0.96	2.7
# 4	1592	2	−111	98.5	1660	1557	1601	2.53	2.7

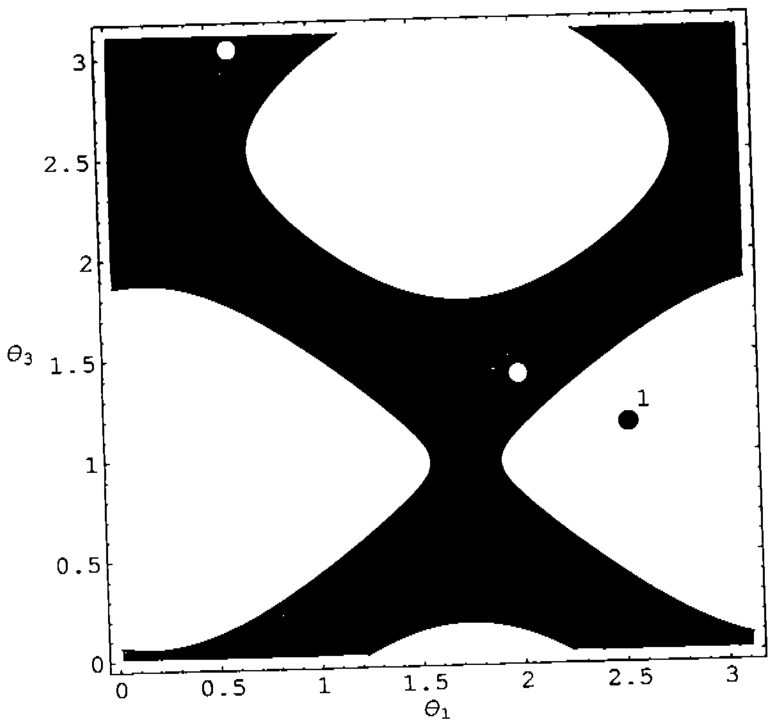
The $1/N_c$ corrections

$$\mathcal{O}_4 = ls + \frac{4}{N_c+1} ltG_c\,, \qquad \mathcal{O}_5 = \frac{1}{N_c} lS_c\,, \qquad \mathcal{O}_6 = \frac{1}{N_c} S_cS_c\,,$$

$$\mathcal{O}_7 = \frac{1}{N_c} sS_c\,, \qquad \mathcal{O}_8 = \frac{1}{N_c} l^{(2)} sS_c\,.$$



	θ_{N1}	θ_{N3}	$N_cC_1^{\rm eff}$	C_2	C_3	C_4	C_5	$C_6^{\rm eff}$	C_8
#1*a*	2.18	1.18	1694	83	−187	−5	28	−114	−120
#1*b*	2.93	0.50	1446	84	−188	−14	−97	333	−272
#2*a*	0.19	0.58	1466	116	−58	−29	−138	297	−46
#2*b*	2.72	1.34	1670	115	−58	−125	−56	−71	205
#3*a*	1.51	1.77	1758	−11	143	41	211	−230	360
#3*b*	2.86	2.06	1611	−13	142	−111	109	35	451
QM	0.61	3.04	1410	−35	123	85	84	398	49
#4*a*	1.32	0.37	1508	2	−110	186	97	222	−360
#4*b*	3.00	0.10	1379	2	−110	22	0	454	−245



$$\Delta^*_{1/2} = N_cC_1 + \frac{1}{3}C_2 - \frac{4}{9}C_5 + \frac{2}{3}C_6 - \frac{1}{3}C_7 \, ,$$

$$\Delta^*_{3/2} = N_cC_1 - \frac{1}{6}C_2 + \frac{2}{9}C_5 + \frac{2}{3}C_6 - \frac{1}{3}C_7 \, .$$

$$T=2: \quad \Gamma(N_{3/2} \to [N\pi]_D) + \Gamma(N_{3/2} \to [\Delta\pi]_D) = \Gamma(N_{5/2} \to [N\pi]_D) + \Gamma(N_{5/2} \to [\Delta\pi]_D) \, .$$

These relations are broken by $1/N_c$ terms in the expansion of the $N^* \to N$ axial current, and by kinematical phase space effects.

$T=0$	$T=1$	$T=2$
$(N_{\frac{1}{2}} \to [N\pi]_S) = 0$	$(N_{\frac{1}{2}} \to [N\pi]_S) = \sqrt{2}c_S$	$(N_{\frac{3}{2}} \to [\Delta\pi]_S) = 0$
	$(N_{\frac{3}{2}} \to [\Delta\pi]_S) = c_S$	
$(N_{\frac{1}{2}} \to [\Delta\pi]_D) = 0$	$(N_{\frac{1}{2}} \to [\Delta\pi]_D) = c_{D1}$	$(N_{\frac{3}{2}} \to [N\pi]_D) = c_{D2}$
	$(N_{\frac{3}{2}} \to [N\pi]_D) = -2c_{D1}$	$(N_{\frac{3}{2}} \to [\Delta\pi]_D) = -\frac{1}{2}c_{D2}$
	$(N_{\frac{3}{2}} \to [\Delta\pi]_D) = -c_{D1}$	$(N_{\frac{5}{2}} \to [N\pi]_D) = \sqrt{\frac{2}{3}}c_{D2}$
		$(N_{\frac{5}{2}} \to [\Delta\pi]_D) = \frac{1}{2}\sqrt{\frac{7}{3}}c_{D2}$

Note that these predictions depend crucially on the T assignment of the excited baryons. In particular, the strong couplings of the $T=0$ states are suppressed by $1/N_c$. Also, the $J=3/2$, $T=2$ state is predicted to decay in a pure $D-$wave. Therefore one expects these predictions to be useful for distinguishing among the possible assignments.

$$\Gamma(N_{\frac{1}{2}} \to [N\pi]_S) : \Gamma(N_{\frac{3}{2}} \to [\Delta\pi]_S) = \quad \mathbf{1 : 1} \quad (T=1)$$

$$\Gamma(N_{\frac{1}{2}} \to [\Delta\pi]_D) : \Gamma(N_{\frac{3}{2}} \to [N\pi]_D) : \Gamma(N_{\frac{3}{2}} \to [\Delta\pi]_D) = \quad \mathbf{2 : 1 : 1}$$

$$\Gamma(N_{\frac{3}{2}} \to [N\pi]_D) : \Gamma(N_{\frac{3}{2}} \to [\Delta\pi]_D) : \Gamma(N_{\frac{5}{2}} \to [N\pi]_D) : \Gamma(N_{\frac{5}{2}} \to [\Delta\pi]_D)$$

$$= \quad \mathbf{\tfrac{1}{2} : \tfrac{1}{2} : \tfrac{2}{9} : \tfrac{7}{9}} \quad (T=2).$$

Conclusions

◇ We have shown explicitly how the predictions of the $SU(4)_c$ symmetry for non-strange excited baryons (the tower structure) follow from the quark mass operator approach in the large N_c limit.

◇ The $SU(4)_c$ symmetry also predicts a specific pattern for the strong decays.

◇ This is a non-trivial prediction of QCD. In the case of the ground state baryons we would just recover $SU(4)_{spin-flavor}$.

◇ The main aim of our work was to answer the two (related) questions:

1. are the predictions of large N_c $SU(4)_c$ symmetry still visible in the observed mass spectrum of the excited baryons?
2. What are the values of the coefficients of the various operators that enter in the mass operator?

◇ Two possiblities: Assignment #3 requires large $O(N_c^{-1})$ coefficients, while assignment #1 has small $O(N_c^{-1})$ corrections.

◇ The expressions for the decays to order $O(N_c^{-1})$ are needed would provide further constraints (work in progress with N.N. Scoccola and J.L. Goity). Many more applications possible.