MASSES OF EXCITED BARYONS IN THE 1/N$_c$ EXPANSION OF QCD

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QCD has no obvious expansion parameter. However, in 1974 t'Hooft realized that if one extends the QCD color group from SU(3) to SU(N_c), where N_c is an arbitrary (odd) large number, then $1/N_c$ may treated as the relevant expansion parameter of QCD.

Baryons formed by N_c "valence" quarks with $M_b \sim \vartheta(N_c^1)$ and $A_{bb} \sim \vartheta(N_c^1)$. Moreover, $A_{bm} \sim \vartheta(N_c^0)$. Moreover, it was realized that for ground state baryons a dynamical spin-flavor symmetry emerges for Large N_c (Gervais-Sakita, Dashen-Jenkins-Manohar).

For N_f =3 this allows to write any operator in terms of a power expansion of products of the 35 SU(6) generators

$$S_i = \sum s_i^{(n)},\ T_a = \sum t_a^{(n)},\ G_{ia} = \sum s_i^{(n)} t_a^{(n)}$$

Based on the existence of this Large N_c spin-flavor symmetry the operator analysis corresponds to expanding any operator associated with some observable as

$$\hat{O} = \sum_{n,i} c_{n,i}\, \hat{O}_{n,i}$$

where $c_{n,i}$ are unkown real coefficients to be determined by fitting to empirical accesible quantities. These coefficients encode the non-perturbative QCD dynamics which cannot be constrained by symmetries. Calculating these coefficients would be equivalent to solving QCD in the baryon sector.

On the other hand, $\hat{O}_{n,i}$ is n-body operator obtained as the product of color singlet (but, in general, spin-flavor non-singlet !) 1-body operators.

Rules to determine the N_c order of a given n-body operator are:

1. Note that n-body operators need at least n quarks exchanging *(n-1)* gluons. Thus, according to previous rules it carries a suppresion factor $1/N_c^{n-1}$.

2. Some operators may act as coherent : i.e. they may have matrix elements of $\vartheta(N_c)$

In this work we analyzed the masses of orbitally excited [70, 1$^-$]-plet and [56, 2$^+$]-plet baryons [illegible] that the spin-[illegible] [illegible] is still a [illegible] [illegible] states in the L[illegible] [illegible]. Thus, apart from the generators of SU(6) listed above for the core and excited particle, the orbital momentum of the excited quark l_i can be used to construct the effective operators.

The [70,1$^-$]-plet states

SU(6) irrep	SU(3)$_f$ irrep	J^P	S = 0	S= - 1 I = 0	I = 1	S = - 2	S = - 3
70^-(l=1)	$^2 8$	$3/2^-$	N(1520)	Λ(1690)	Σ(1580)**	Ξ(1820)	
		$1/2^-$	N(1535)	Λ(1670)	Σ(1620)**		
	$^4 8$	$1/2^-$	N(1650)	Λ(1800)	Σ(1750)		
		$5/2^-$	N(1675)	Λ(1830)	Σ(1775)		
		$3/2^-$	N(1700)	?	Σ(1670)		
	$^2 10$	$1/2^-$	Δ(1620)				
		$3/2^-$	Δ(1700)				
	$^2 1$	$1/2^-$		Λ(1405)			
		$3/2^-$		Λ(1520)			

Following the N_c counting rules discussed above and making use of various reduction formulae we find that up to $\vartheta(1/N_c)$ there are eleven independent operators: 1 operator of $\vartheta(N_c)$, 3 operators of $\vartheta(N_c^{\,0})$ and 7 operators of $\vartheta(1/N_c)$.

Note the existence of operators of $\vartheta(N_c^{\,0})$. For excited baryons spin-flavor symmetry might be broken even for $N_c \to \infty$, unless corresponding coefficients turn out to be dynamically suppressed.

The breaking of SU(3) flavor symmetry is driven by the mass difference between the s quark mass and that of the u,d quarks. A measure of the breaking is given by the ratio

$$\varepsilon = \left(M_K^2 - M_\pi^2\right)/\Lambda^2$$

where Λ is some light hadronic mass, e.g. the rho meson mass. We have included the SU(3) breaking at order ε. Note that for N_c=3, ε and $1/N_c$ are of similar size. Thus, corrections of order ε/N_c are neglected. [illegible]

The most general mass operator to this order is a linear combination of the 15 basis operators. Fortunately, the experimental data available in the case of the 70-plet is enough to obtain the unknown constants by performing a fit.

Inputs are:

- 17 masses of negative parity baryons with 3 or more stars in PDG.
- The two leading order mixing angles $\theta_1=0.61\pm0.09$ and $\theta_3=3.04\pm0.15$ (from strong decays of non-strange baryons in the multiplet. See Isgur-Karl, Carone et al. ,Manohar –Jenkins [illegible]

[illegible] 17 [illegible] these 19 [illegible].

Before proceeding it is important to establish relations among observables that hold to the order of the present analysis. It can be checked that, to order ϵ, there are 35 independent observables, namely 21 masses + 14 mixing angles. Since we have only 15 free parameters, there should be 20 relations. Seven of them involve mixing angles. The other 13 do not. They are

- 5 GMO relations (one per each octet)
- 4 equal spacing rules (two per decuplet)
- 4 novel relations which involve splittings across multiplets.

Not enough experimental input to [illegible] all these relations— [illegible]

What about the fitting parameters ?

N_c^{1}	$O_1 = N_c\, 1$	$c_1 = 449 \pm 2$
N_c^{0}	$O_2 = l_i\, s_i$ $O_3 = \frac{3}{N_c} l_{ij}^{(2)}\, g_{ia}\, G_{ja}^c$ $O_4 = \frac{4}{N_c+1} l_i\, t_a\, G_{ja}^c$	$c_2 = 52 \pm 15$ $c_3 = 116 \pm 44$ $c_4 = 110 \pm 16$
N_c^{-1}	$O_5 = \frac{1}{N_c} l_i\, S_i^c$ $O_6 = \frac{1}{N_c} S_i^c\, S_i^c$ $O_7 = \frac{1}{N_c} s_i\, S_i^c$ $O_8 = \frac{2}{N_c} l_{ij}^{(2)}\, s_i\, S_j^c$ $O_9 = \frac{3}{N_c^2} l_i\, g_{ja} \left\{S_j^c, G_{ia}^c\right\}$ $O_{10} = \frac{2}{N_c^2} t_a \left\{S_i^c, G_{ia}^c\right\}$ $O_{11} = \frac{3}{N_c^2} l_i\, g_{ia} \left\{S_j^c, G_{ja}^c\right\}$	$c_5 = 74 \pm 30$ $c_6 = 480 \pm 15$ $c_7 = -159 \pm 50$ $c_8 = 3 \pm 55$ $c_9 = 71 \pm 51$ $c_{10} = -84 \pm 28$ $c_{11} = -44 \pm 43$
εN_c^{0}	$\overline{B}_1 = t_8 - \frac{1}{2\sqrt{3}\, N_c} O_1$ $\overline{B}_2 = T_8^c - \frac{N_c - 1}{2\sqrt{3}\, N_c} O_1$ $\overline{B}_3 = \frac{10}{N_c} d_{8ab}\, g_{ia}\, G_{ib}^c + \frac{5(N_c^2-9)}{8\sqrt{3}N_c^2(N_c-1)} O_1 + $ $+ \frac{5}{2\sqrt{3}(N_c-1)} O_6 + \frac{5}{6\sqrt{3}} O_7$ $\overline{B}_4 = 3\, l_i\, g_{i8} - \frac{\sqrt{3}}{2} O_2$	$d_1 = -81 \pm 36$ $d_2 = -194 \pm 17$ $d_3 = -15 \pm 30$ $d_4 = -27 \pm 19$

Natural order: For singlets ~ 500 MeV.
For SU(3) breaking ε.500 MeV ~150-200 MeV

We observe that:

- Although spin-flavor symmetry is broken at $\vartheta(N_c^0)$, the corresponding operators are dynamically suppressed. Their coefficients are substantially smaller than the natural size.

- The chief contribution to spin-flavor breaking stems form the $\vartheta(1/N_c)$ hyperfine operator $O_6 = \frac{1}{N_c} S_i^c S_i^c$ which is purely a core operator. In particular the singlet Λ's are not affected by O_6 (they have $S^c = 0$) while the other states are move upwards, explaining in a transparent way the lightness of these two states.

- The long standing problem in NRQM of reconciling the $\Lambda(1520) - \Lambda(1405)$ splitting with those of other spin orbit partners in the 70-plet is resolved. The Λ's receive contributions of O_1 and $l.s$ only, while for other states several others also do. Particularly important $O_4 = \frac{4}{N_c + 1} l_i t_a G_{ja}^c$ (OBE).

The [56,2$^+$]-plet states

SU(6) irrep	$SU(3)_f$ irrep	J^P	S = 0	S= - 1, I = 0	S= - 1, I = 1	S = - 2	S = - 3
$56^+(l=2)$	$^2 8$	$3/2^+$	N(1720)	Λ(1890)	?		
		$5/2^+$	N(1680)	Λ(1820)	Σ(1915)		
	$^4 10$	$1/2^+$	Δ(1910)				
		$3/2^+$	Δ(1920)		Σ(2080)**		
		$5/2^+$	Δ(1905)				
		$7/2^+$	Δ(1950)		Σ(2030)		

Proceeding as before we obtain up to order $1/N_c$ and εN_c^0

N_c^1	$O_1 = N_c\, 1$	$c_1 = 541 \pm 4$
N_c^{-1}	$O_2 = \frac{1}{N_c} l_i S_i^c$ $O_3 = \frac{1}{N_c} S_i^c S_i^c$	$c_2 = 18 \pm 16$ $c_3 = 241 \pm 14$
εN_c^0	$\overline{B_1} = -S$ $\overline{B_2} = \frac{1}{N_c} l_i G_{i8}^c - \frac{1}{2\sqrt{3}} O_2$ $\overline{B_3} = \frac{1}{N_c} S_i G_{i8}^c - \frac{1}{2\sqrt{3}} O_2$	$d_1 = -206 \pm 18$ $d_2 = -104 \pm 64$ $d_3 = 223 \pm 68$

We observe

- No SU(3) singlet operator at $\vartheta(N_c^0)$
- Breaking of spin-flavor symmetry from spin-orbit is small.
- SU(3) breaking dominated by $\overline{B_1}$ which gives 200 MeV per unit of strangeness. $\overline{B_{2,3}}$ provide e.g. $\Lambda - \Sigma$ splittings in the octet

Basis contains 6 operators and there are 24 masses we get 18 mass relations. Apart from GMO and EQS relations there are 8 relations. They are well satisfied by data when available (4 relations if one and 2 stars resonances excluded, 7 if included).

SUMMARY AND CONCLUSIONS

- The $1/N_c$ expansion for excited baryons has been implemented under the assumption that there is an approximate spin-flavor symmetry in the Large N_c limit Since our analysis shows that 0^{th} order corrections, when they exist, have a magnitude smaller that the natural size and, on the hand there are no unnaturally large corrections, the scheme seems to be consistent.

- A good fit to the known spectrum is obtained for both the **[70,1⁻]** and the **[56,2⁺]**-plets. At the same time, the approach has predictivity. Motivation to experimentally establish a few more states of both multiplets.

- In the **[70,1⁻]**-plet, the singlet Λ's appear naturally as the lightest particles in the multiplet and spin-orbit partners. The spin-orbit puzzle of NRQM is resolved: Λ(1520)- Λ(1405) splitting is due to [illegible]. The wrong ordering of Δ(1645) and Δ(1720) implied by this operator is compensated by various operators, most prominently the flavor exchange operator [illegible]

- Lattice QCD calculations could be very helpful in understanding the non-perturbative origin of the values obtained for the coefficients c_i and d_i.

- Present study should be complemented by a fully consistent Large N_c analysis of the strong decays of the states in the multiplets to the ground state baryons.