ΛΛ-ΞN-ΣΣ Coupling in $A = 6$ Nucleus

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Introduction

- **Nagara event : ${}^{6}_{\Lambda\Lambda}$He**

$$\Delta B^{exp}_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) - 2 \times B_{\Lambda}({}^{5}_{\Lambda}\text{He})$$
$$= 1.01 \pm 0.20^{+0.18}_{-0.11} \text{ MeV}$$

(Takahashi et al., PRL 87, 212502 (2001))

Case	ΛΛ int.	ΛΛ-ΞN-ΣΣ effect
o (1)	weakly attractive	small
(2)	zero ∼ weakly repulsive	rather large
(3)	repulsive	large
o (4)	H dibaryon	large

Real world ? : No body knows.

- **Before "Nagara"**

Hyperon mixing in ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{13}_{\Lambda\Lambda}$B

1) [core+Λ+Λ]+[core+Ξ+N]+[core+Σ+Σ] model

2) core+($3q$)+($3q$) model

OBEP : ND, Ehime ⇒ $\Delta B_{\Lambda\Lambda} \sim 4$ MeV

QCM : RGM-F (H-dibaryon) ⇒ $\Delta B_{\Lambda\Lambda} \sim 11$ MeV

FSS ⇒ $B_{\Lambda\Lambda} \sim 2.5$ MeV

Yamada and Nakamoto, PRC 62, 034319 (2000)

After "Nagara"

Structure studies of ${}^{6}_{\Lambda\Lambda}$He, ${}^{4}_{\Lambda\Lambda}$H etc.

- Filikhin and Gal: $\alpha + \Lambda + \Lambda$, $NN\Lambda\Lambda$ model
  PRC 65, 041001 (2002); NPA 707, 491 (2002);
  PRL 89, 172502 (2002); nucl-th/0303028, (2003).
- Afnan and Gibson: PRC 67, 017001 (2003).
- Myint, Shimura and Akaishi: JEPA 16, 2 (2003).
- Nemura, Akaishi and Myint
  PRC 67, 051001 (2003). $NN\Lambda\Lambda$ model
- Nemura et al.: ${}^{6}_{\Lambda\Lambda}$He=6-body cal.
- Hiyama et al.: ${}^{4}_{\Lambda\Lambda}$H$-{}^{4}_{\Xi}$H

YN&YY int. ($S = -2$) used above

- **phenomenological $\Lambda\Lambda$ pot.**
- **phase-shift-equivalent pot. to NSC97, ND**
  (simulated pot.: $V_{\Lambda\Lambda}$, $V_{\Lambda\Lambda-\Xi N}$, $V_{\Xi N}$)
  a_s(**NSC97a-f**)$=-0.27 \sim -0.53$ **fm**
  vs. $a_s \sim -0.7$ **fm** ($\alpha + \Lambda + \Lambda$ **model**)

Purpose

Structure of $^{6}_{\Lambda\Lambda}$He and hyperon mixing

$[\alpha+\Lambda+\Lambda]+[\alpha+\Xi+N]+[\alpha+\Sigma+\Sigma]$ model

$\Lambda\Lambda$-ΞN-$\Sigma\Sigma$ parts: NSC97e and NSC97f

α-Y and α-N parts: phenomenological pot.

$\rightarrow$ **characteristics of $\Lambda\Lambda$-ΞN-$\Sigma\Sigma$ coupling**

[α+Λ+Λ] + [α+Ξ+N] + [α+Σ+Σ] model, where the α particle is assumed as a frozen core. Six different versions of the Nigmegen NSC97 potential [6], NSC97a through NSC97f, are directly applied to the valence baryon-baryon interactions. Phenonomenological potentials are used for the $\alpha - B$ parts (B=N, Λ, Ξ and Σ). The formnulation is almost the same as one in our previous paper [11], where structure of light $S = -2$ nuclei and hyperon mixing were discussed. The Pauli blocking effect of the valence nucleon in the α+Ξ+N is taken into account properly. We will compare the calculated enegies and coupled-channel effects in ${}^{6}_{\Lambda\Lambda}$He for the six versions of the NSC97 potential and also discuss the characteristics of the potential.

II. FORMULATION

The total wave function of ${}^{6}_{\Lambda\Lambda}$He with total angular momentum J is given as

$$\Phi = \Phi_{\Lambda\Lambda} + \Phi_{\Xi N} + \Phi_{\Sigma\Sigma}, \tag{1}$$

$$\Phi_{\Lambda\Lambda} = \sum_{\beta=1}^{2} \sum_{LS} \mathcal{A}_{\Lambda\Lambda} \left[\Phi_L^{(\beta)}(\boldsymbol{r}_\beta, \boldsymbol{R}_\beta) \left[\chi_{1/2}(\Lambda)\chi_{1/2}(\Lambda)\right]_S \right]_J , \tag{2}$$

$$\Phi_{\Xi N} = \sum_{\beta=1}^{3} \sum_{LS} \left[\Phi_L^{(\beta)}(\boldsymbol{r}_\beta, \boldsymbol{R}_\beta) \left[\chi_{1/2}(\Xi)\chi_{1/2}(N)\right]_S \right]_J , \tag{3}$$

$$\Phi_{\Sigma\Sigma} = \sum_{\beta=1}^{2} \sum_{LS} \mathcal{A}_{\Sigma\Sigma} \left[\Phi_L^{(\beta)}(\boldsymbol{r}_\beta, \boldsymbol{R}_\beta) \left[\chi_{1/2}(\Sigma)\chi_{1/2}(\Sigma)\right]_S \right]_J , \tag{4}$$

where β denotes the Jacobian coordinate system (see Fig. @), and $\Phi_L^{(\beta)}$ and χ's represent, respectively, the wave function of the spatial part with total orbital angular momentum L and the spin functions for the valence baryons coupled to total spin S. In the α+Λ+Λ (α+Σ+Σ) channel, the antisymmetrization operator $\mathcal{A}_{\Lambda\Lambda}$ ($\mathcal{A}_{\Sigma\Sigma}$) is needed for the two Λ (Σ) particles. Therefore, it is enough to take the two Jacobi coordinate systems for the α+Λ+Λ (α+Σ+Σ) channel.

The wave function of the spatial part $\Phi_L(\boldsymbol{r}, \boldsymbol{R})$ in Eqs. (2), (3) and (4) is expanded in terms of the Gaussian basis, which is known to be suited for describing both short-range correlation and long-range tail behavior [12],

$$\Phi_{LM}(\boldsymbol{r}, \boldsymbol{R}) = \sum_{\ell_r, \ell_R} \sum_{n_r, n_R} C_L(n_r\ell_r, n_R\ell_R) \left[\varphi_{\ell_r}(\boldsymbol{r}, \nu_{n_r})\varphi_{\ell_R}(\boldsymbol{R}, \nu_{n_R})\right]_{LM} , \tag{5}$$

$$\varphi_{\ell m}(\boldsymbol{r}, \nu) = N_\ell(\nu) r^\ell \exp(-\nu r^2) Y_{\ell m}(\hat{\boldsymbol{r}}), \tag{6}$$

where $N_\ell(\nu)$ is the normalization factor. The Gaussian parameter ν is taken to be of geometrical progression,

$$\nu_n = 1/b_n^2, \qquad b_n = b_{min}a^{n-1}, \qquad n = 1 \sim n_{\max}. \tag{7}$$

It is noted that the prescription is found to be very useful in optimizing the ranges with a small number of free parameters together with high accuracy [12].

The total Hamiltonian within the framework of the $[\alpha+\Lambda+\Lambda]$ + $[\alpha+\Xi+N]$ + $[\alpha+\Sigma+\Sigma]$ model is given as

$$H = \delta_{cc'}\left[T_c + V_{\alpha B_1}(\boldsymbol{r}_1) + V_{\alpha B_2}(\boldsymbol{r}_2) + \Delta M_c\right] + v_{cc'}(\boldsymbol{r}_3) + \delta_{c2} V_{\rm Pauli}, \tag{8}$$

where c denotes the channel; $c = 1$ for $\alpha + \Lambda + \Lambda$, $c = 2$ for $\alpha + \Xi + N$ and $c = 3$ for $\alpha + \Sigma + \Sigma$. T_c and $V_{\alpha B}$ present, respectively, the kinetic energy operator and potential between the α particle and valence baryon, and $v_{cc'}$ denotes the interaction between the two valence baryons. In the present study, the baryon-channel coupling is assumed to come only from $v_{cc'}$. The mass difference matrix (diagonal and constant) ΔM_c is introduced to give the threshold-energy differences among the three channels, ΔM_1=0 MeV, ΔM_2=28 MeV and ΔM_3=160 MeV. The Pauli principle between the α cluster and valence nucleon in the α+Ξ+N channel is taken into account with the orthogonality condition model (OCM) [13]. The OCM projection operator $V_{\rm Pauli}$ is represented as

$$V_{\rm Pauli} = \lim_{\lambda\to\infty} \lambda \mid \varphi_{0s}(\boldsymbol{r}_{\alpha N})\rangle\langle\varphi_{0s}(\boldsymbol{r}'_{\alpha N}) \mid, \tag{9}$$

which removes the Pauli forbidden state φ_{0s} between the α cluster and valence nucleon in the core+Ξ+N three-body system [14]. The configuration of α cluster is assumed here to be of simple $(0s)^4$-shell-model type.

The potential bwetween the α cluster and valence baryon (B), $V_{\alpha B}$ is obtained by folding the effective baryon-nucleon (BN) interaction with the density of the α particle and adjusting their strength so as to reproduce the experimental binding energy for the ground state of the α+B system with use of the α+B potential model, where we have to take into account the OCM operator in Eq. (9) for the α+N system. As for the effective hyperon-nucleon (YN) and nucleon-nucleon (NN) interactions, we use the YNG-ND [15] interaction and HNY [16] interaction, respectively. It is known that the YNG-ND ΛN interaction reproduce nicely the Λ binding energy of ${}^{5}_{\Lambda}$He as well as other light Λ hypernuclei, and the ΞN interaction is consistent with the recent experimental data on ${}^{12}_{\Xi}$B obtained by the ^{12}C(K^-, K^+) reaction [17]. The YNG-ND ΣN interaction is also consistent with the experimental data of ${}^{4}_{\Sigma}$He. Concerning the density distribution for the α particle, we use the harmonic-oscillator-type one obtained by the election scattering experiment [18].

The interaction between the two valence baryons v_{BB} in Eq. (8) is given as

$$v(r) = v^{(0)}(r) + v^{(\sigma)}(r)(\sigma_1 \cdot \sigma_2) + v^{(ten)}(r)S_{12} + v^{(LS)}(r)\boldsymbol{L}\cdot\boldsymbol{S} + v^{(ALS)}(r)\boldsymbol{L}\cdot\boldsymbol{S}^{-} + v^{(QLS)}(r)Q_{12} - \left[\nabla^2\phi(r) + \phi(r)\nabla^2\right], \quad (10)$$

where the notation is self-explanatory. In the present paper, we use the six different versions of the Nijmegen NSC97 potential, NSC97a through NSC97f, for the interaction between the two valence baryons.

The equation of motion is derived from the Rayleigh-Ritz variational method,

$$\boxed{\delta\left[\langle\Phi|E - H|\Phi\rangle\right] = 0.} \quad (11)$$

Solving the equation numerically, we obtain the eigenenergies of the Hamiltonian given in Eq. (8) and expansion coefficients of the wave function C's in Eq. (5).

III. RESULTS AND DISCUSSION

The calculated results of ${}^{6}_{\Lambda\Lambda}$He are shown in Table ??, for various coupled-channel cases. First let us see the results of only the $\Lambda\Lambda$ channel, switched off the coupling with the ΞN and $\Sigma\Sigma$ channels. The calcuated $\Delta B_{\Lambda\Lambda}$ is as small as $0.2 \sim 0.4$ MeV for NSC97a through NSC97f, the value of which lies very close to the new experimental datum [1]. The small $\Delta B^{cal}_{\Lambda\Lambda}$'s value means that the $\Lambda\Lambda$ potential of the NSC97 model is rather weakly attractive.

Results

$$\Delta B^{exp}_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\text{He}) = 1.01 \pm 0.20^{+0.18}_{-0.11}\ \textbf{MeV}$$

channel		NSC97e	NSC97f
ΛΛ	$\Delta B_{\Lambda\Lambda}$ (MeV)	0.33	0.09
ΛΛ-ΞN	$\Delta B_{\Lambda\Lambda}$ (MeV)	1.33 (1.45) [no Pauli]	0.55
	$P_{\Lambda\Lambda}$ (%)	99.49	99.81
	$P_{\Xi N}$ (%)	0.51	0.19
ΛΛ-ΞN-ΣΣ	$\Delta B_{\Lambda\Lambda}$ (MeV)	0.56 (0.76) [no Pauli]	0.35
	$P_{\Lambda\Lambda}$ (%)	99.79	99.83
	$P_{\Xi N}$ (%)	0.19	0.16
	$P_{\Sigma\Sigma}$ (%)	0.01	0.01

$\Delta E_{\Lambda\Lambda-\Xi N-\Sigma\Sigma} = -0.2 \sim -0.3$ **MeV**

P_{Ξ} ~**0.2 %**, $P_{\Sigma\Sigma}$ ~**0.01%**

Importance of ΣΣ channel

G-matrix cal. in ${}^{\ 6}_{\Lambda\Lambda}$He with NSC97e

I. Vidaña, A. Ramos, A. Polls, nucl-th/0307096, 2003.

A complete treatment of Pauli blocking ~~reduces~~ gives the repulsive effect on $\Delta B_{\Lambda\Lambda}$. ⇐ However, it is as small as 0.2 MeV.

There should exist other reasons.

We study effective YY potentials with $S = -2$ in SU$_f$(3) irresps.

General form of OBEP

$$V = \{V_c(r) + V_\sigma(r)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_T(r)S_{12} + V_{SO}(r)\boldsymbol{L}\cdot\boldsymbol{S}$$
$$+V_Q(r)Q_{12} + V_{ASO}(r)\frac{1}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\boldsymbol{L}$$
$$-\left[\boldsymbol{\nabla}^2\phi(r) + \phi(r)\boldsymbol{\nabla}^2\right]\}P$$
$$= V_0 - [\nabla^2\phi(r) + \phi(r)\nabla^2]P$$

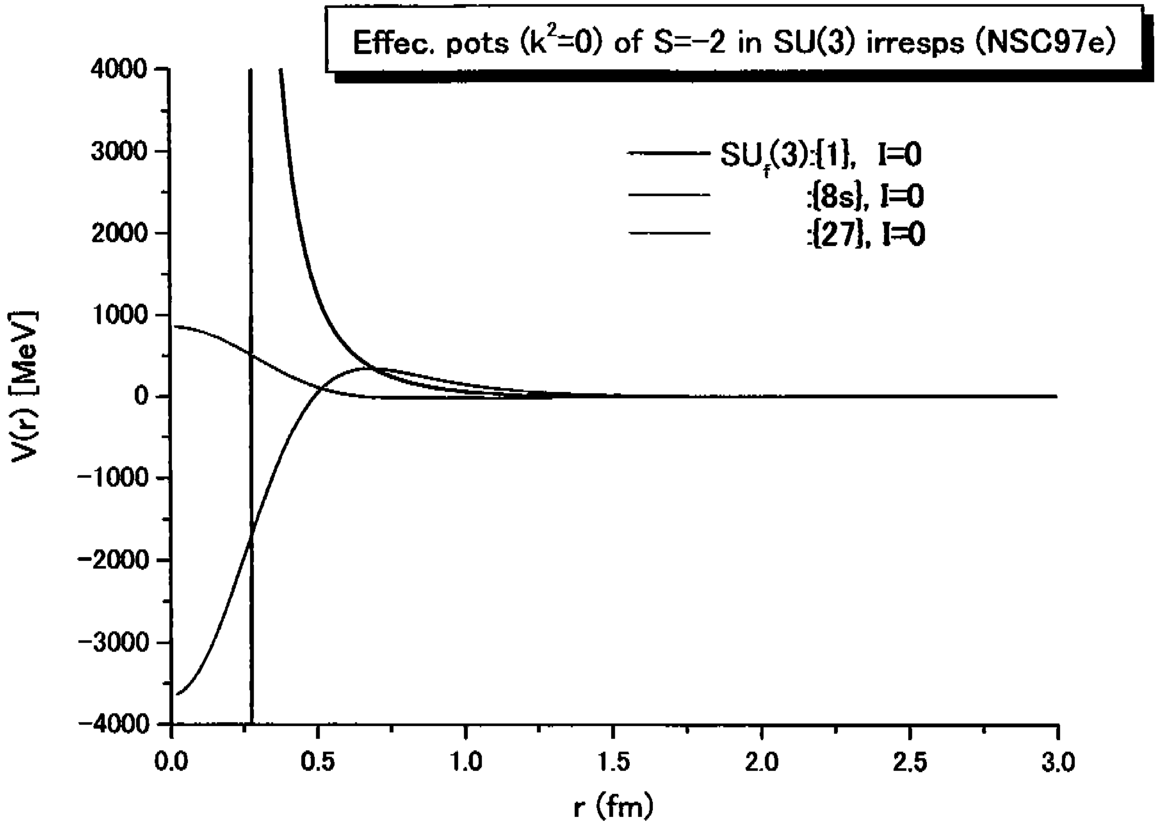
Two-body problem

- **Schrödinger equation:**

$$[-\frac{1}{2\mu}\boldsymbol{\nabla}^2 + V_0 - \left(\boldsymbol{\nabla}^2\phi(r) + \phi(r)\boldsymbol{\nabla}^2\right)]\psi = E\psi$$

- **effective potential:**

$$\frac{d^2 v_\ell}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \left(k^2 - 2\mu V_{eff}\right) = 0,$$
$$V_{eff} = \frac{V_0}{1+4\mu\phi} - \frac{1}{8\mu}\left(\frac{4\mu\phi'}{1+4\mu\phi}\right)^2 + \frac{k^2}{2\mu}\frac{4\mu\phi}{1+4\mu\phi},$$



- **V({1}), V({8s}) : deeply attractive in short range.**

One deeply bound state exists, respectively, in {1} and {8s} states.

(In case of $\Lambda\Lambda$-ΞN system, no bound states exist.)

Physical states should be orthogonal to the states.

Then, relative *B*-*B* wave functions have one node in short range.

$\Rightarrow$ **Repulsive effect ! Characteristics of NSC97(e-f).**

$$|\{1\}, I=0\rangle = \sqrt{\frac{1}{8}}\,|\Lambda\Lambda\rangle + \sqrt{\frac{4}{8}}\,|\Xi N\rangle - \sqrt{\frac{3}{8}}\,|\Sigma\Sigma\rangle,$$

$$|\{8s\}, I=0\rangle = \sqrt{\frac{1}{5}}\,|\Lambda\Lambda\rangle + \sqrt{\frac{1}{5}}\,|\Xi N\rangle + \sqrt{\frac{3}{5}}\,|\Sigma\Sigma\rangle,$$

$$|\{27\}, I=0\rangle = \sqrt{\frac{27}{40}}\,|\Lambda\Lambda\rangle - \sqrt{\frac{12}{40}}\,|\Xi N\rangle - \sqrt{\frac{1}{40}}\,|\Sigma\Sigma\rangle,$$

Pseudo Bound States

	NSC97e $B_{\Lambda\Lambda}$	NSC97f $B_{\Lambda\Lambda}$
{1}	7070.1 MeV	8509.1 MeV
{8s}	1474.8 MeV	1623.8 MeV (larger)

The position of the nodal point of wave function in NSC97f is nearer the origin ($r=0$) than in NSC97e. Thus, the repulsive effect in NSC97f is weaker than NSC97e.

Summary

- $\Lambda\Lambda$-ΞN-$\Sigma\Sigma$ coupling in ${}^{6}_{\Lambda\Lambda}$He

  [α+Λ+Λ]+[α+Ξ+N] +[α+Σ+Σ] model

  $\Lambda\Lambda$-ΞN-$\Sigma\Sigma$: NSC97e, NSC97f

  α-Y, α-N : phenomenological pot.

  - Calculated results

    $\Delta B^{cal}_{\Lambda\Lambda} = 0.4 \sim 0.6$ MeV ($\Delta B^{exp}_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV)

    $P_{\Lambda\Lambda}$=99.8 %, $P_{\Xi N}$=0.2 %, $P_{\Sigma\Sigma}$=0.01%

  - Characteristics of NSC97e and NSC97f

Future plans

- Improvment of α-Y part: ΛN-ΣN coupling

  α=$3N + N$ model: $E_x < 20$ MeV

  RGM (Furutani, Horiuchi, Tamagaki), OCM (Yamada et al)

  ${}^{5}_{\Lambda}$He=[(3N+N)+Λ]+[(3N+N)+Σ] model

  ${}^{6}_{\Lambda\Lambda}$He=[(3N + N) + Λ + Λ]+[(3N + N) + Ξ+N]

  +[(3N + N) + Σ + Σ] model

- Λ-Σ coupling in p-shell nuclei with $S = -1$

  structure of p-shell nuclei with $S = -2$

ΛN-ΣN coupling

ΛN-ΣN coupling in $^{5}_{\Lambda}$He

Nemura, Akaishi and Suzuki, PRL 89, 142504 (2002)

$P_{\Sigma}(^{5}_{\Lambda}\text{He}) = 1.6 \sim 2.0$ %, $B_{\Lambda}^{cal} = 2.10 \sim 3.17$ MeV

$\Delta E_c = (\langle T_c \rangle + \langle V_{NN} \rangle)_{^{5}_{\Lambda}\text{He}} - (\langle T_c \rangle + \langle V_{NN} \rangle)_{^{4}\text{He}} \sim 4.7$ MeV

However, $\sqrt{\langle r^2 \rangle_\alpha}$ is not changed.

($\Delta E_{\Lambda-\Sigma}$: attractive)

⇓ ⇓ Brueckner theory

The effect in $^{6}_{\Lambda\Lambda}$He ? : M. Kohno, Y. Fujiwara, Y. Akaishi, PRC 68, 034302 (2003)

$P_{\Sigma\Sigma} \to$ large ($2 \sim 4$ % ?)

$\Delta E_c(^{6}_{\Lambda\Lambda}\text{He})$: repulsive

enhancement of $\Delta E_{\Lambda\Lambda-\Xi N-\Sigma\Sigma}$: attractive

Then, $\Delta E_c + \Delta E_{\Lambda-\Sigma} + \Delta E_{\Lambda\Lambda-\Xi N-\Sigma\Sigma} \Rightarrow$ canceling (?)