Hadron Polarizabilities from Lattice QCD

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- QCD
- Compute Proton’s Mass in QCD
- Hadron Polarizability
- Some Results
- Summary

Quantum Chromodynamics

The fundamental theory of the strong interaction

$$L_{QCD} = \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \overline{q}(\gamma^{\mu}D_{\mu} + m_{q})q$$

Field strength tensor: $F_{\mu\nu} = \partial A_{\mu} - \partial A_{\nu} + g[A_{\mu}, A_{\mu}]$

Covariant derivative: $D_{\mu} = \partial_{\mu} + gA_{\mu}$

- At high energy, perturbative (asymptotic freedom)
- At low energy, non-perturbative (confinement)

Compute proton's mass in QCD

Two-point correlation function:

$$G(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 | T[\chi(x)\overline{\chi}(0) | 0 \rangle$$

Intepolating field

$$\chi(x) = \varepsilon^{abc}[u^{aT}(x)C\gamma_5 d^b(x)]u^c(x)$$

- On the hadronic level, saturated by intermediate states

$$G(\vec{p},t) = \sum_n \lambda_n^2 e^{-E_n(\vec{p})t}$$ with $$E_n(\vec{p}) = \sqrt{\vec{p}^2 + M_n^2}$$

- On the quark level, contracting out quark pairs

$$G(\vec{p},t) = -\varepsilon^{abc}\varepsilon^{a'b'c'}\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\left[S_u^{aa'}\gamma_5 C S_d^{bb'T} C\gamma_5 S_u^{cc'} + S_u^{aa'}\mathrm{Tr}\left(\gamma_5 C S_d^{bb'T} C\gamma_5 S_u^{cc'}\right)\right]$$

Where S_q^{ab} is the fully-interacting quark propagator.

Quark Propagator in QCD

$$M^{-1}(x,0) = \langle 0 | T[q(x)\bar{q}(0)] | 0 \rangle = \left\langle \frac{1}{D + m_q} \right\rangle$$

$$= \frac{\int DADqD\bar{q}[\bar{q}(x)q(0)]e^{-S_{QCD}}}{\int DADqD\bar{q}e^{-S_{QCD}}} = \frac{\int DAM^{-1}\det Me^{-S_G}}{\int DA\det Me^{-S_G}}$$

- Path Integrals
  - Evaluated numerically on a space-time lattice
  - Monte-Carlo sampling method
- Quenched Approximation (set $\det M = 1$)
  - Physically, suppress bubbles in the vacuum

Electric and Magnetic Polarizabilities

- In classical physics

$$\Delta m = -\frac{1}{2}\alpha E^2$$ or $$\Delta m = -\frac{1}{2}\beta B^2$$

  - Where α is electric polarizability, β is magnetic polarizability

- We determine the electric/magnetic polarizability by calculating the mass shift of hadrons

$$\Delta m(E) = m(E) - m(0)$$ or $$\Delta m(B) = m(B) - m(0)$$

Fitting the Data

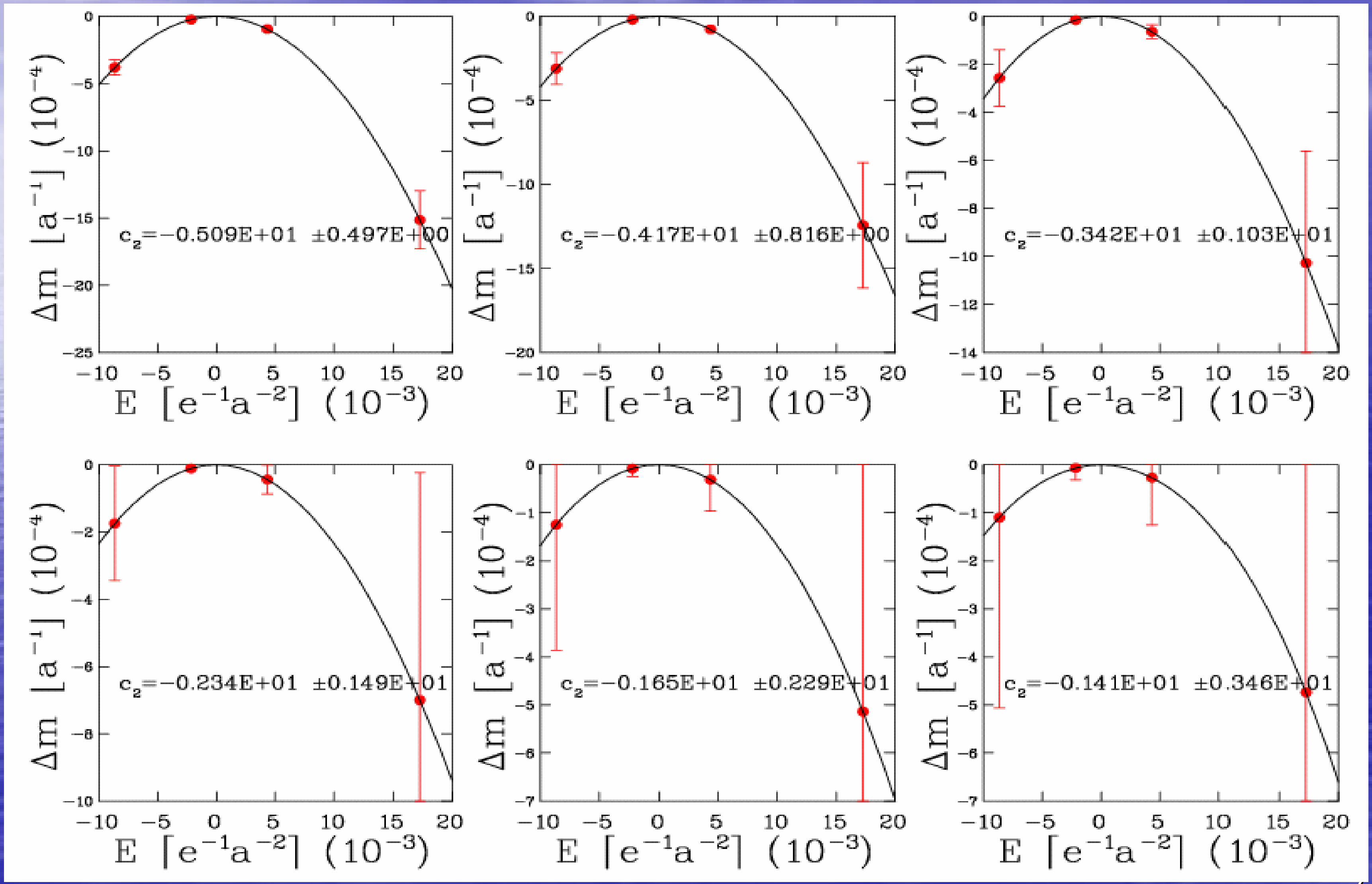
- We fit the data with a polynomial

$$\Delta m(E) = c_1 E + \frac{1}{2} c_2 E^2$$ and $$\Delta m(B) = c_1 B + \frac{1}{2} c_2 B^2$$

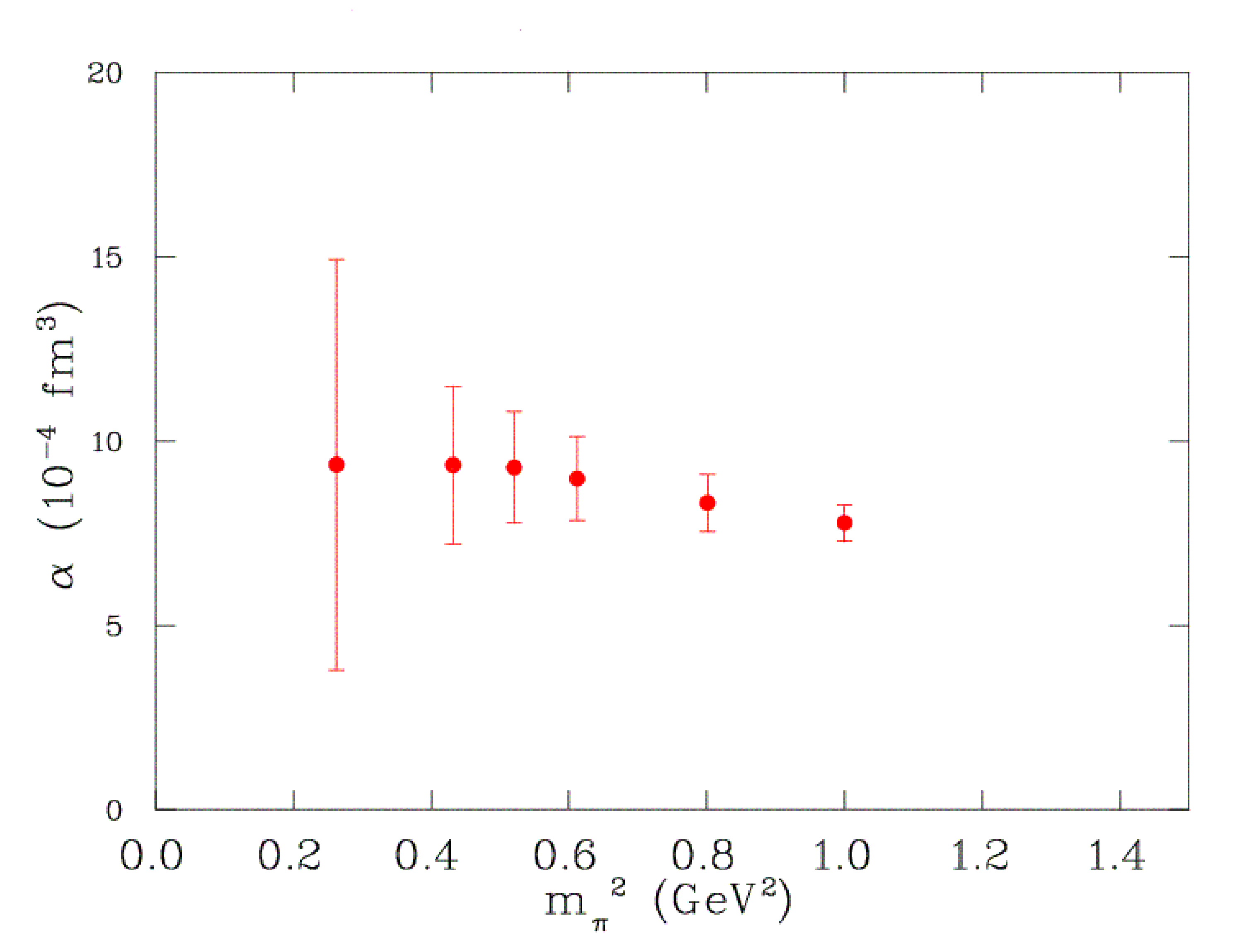
- We calculate mass shifts both in the field E (B) and its reverse –E (B), then average them.

$$\alpha = -c_2$$ $$\beta = -c_2$$

Mass Shift in Electric Field



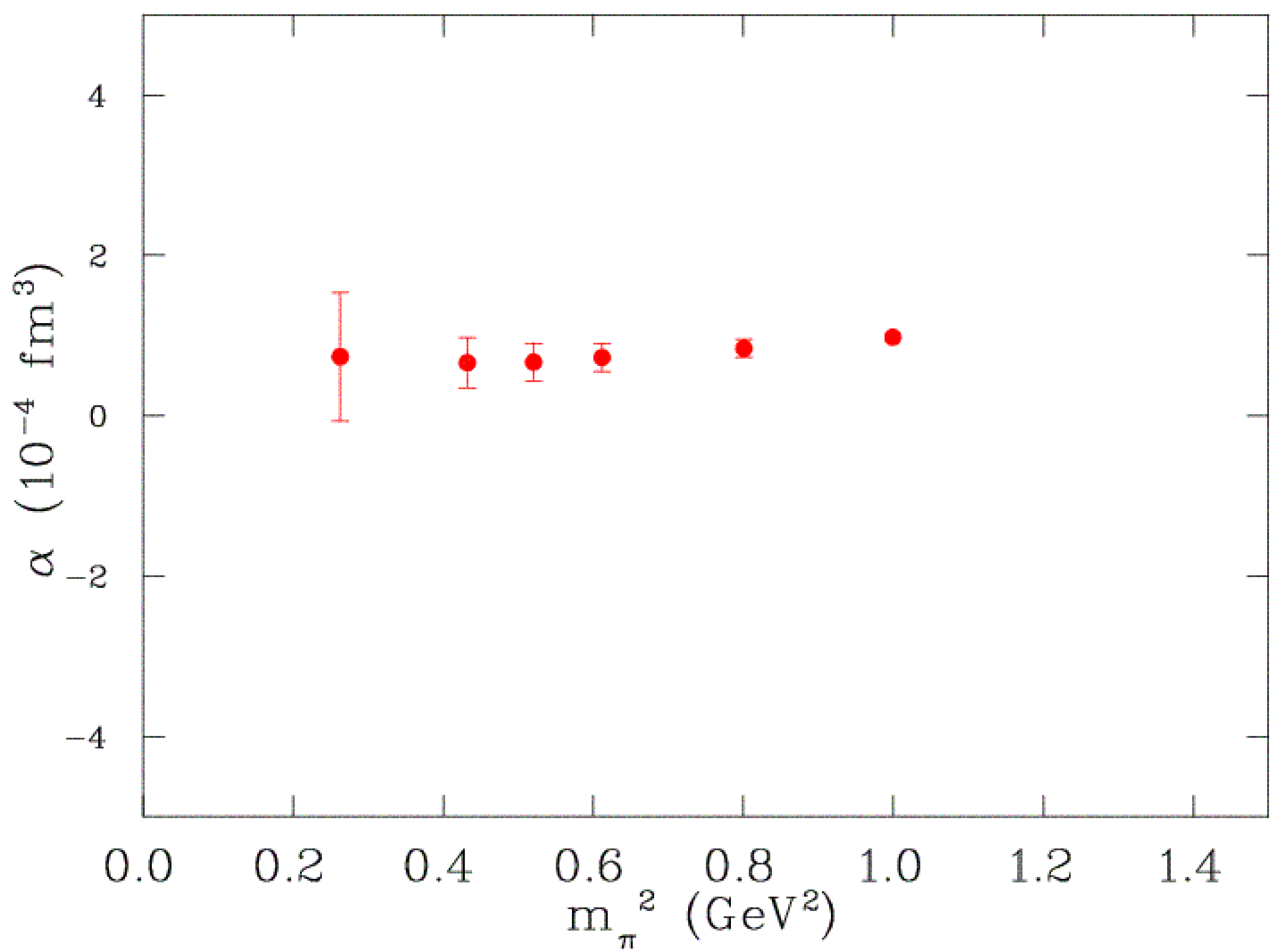
Electric Polarizability of Neutron



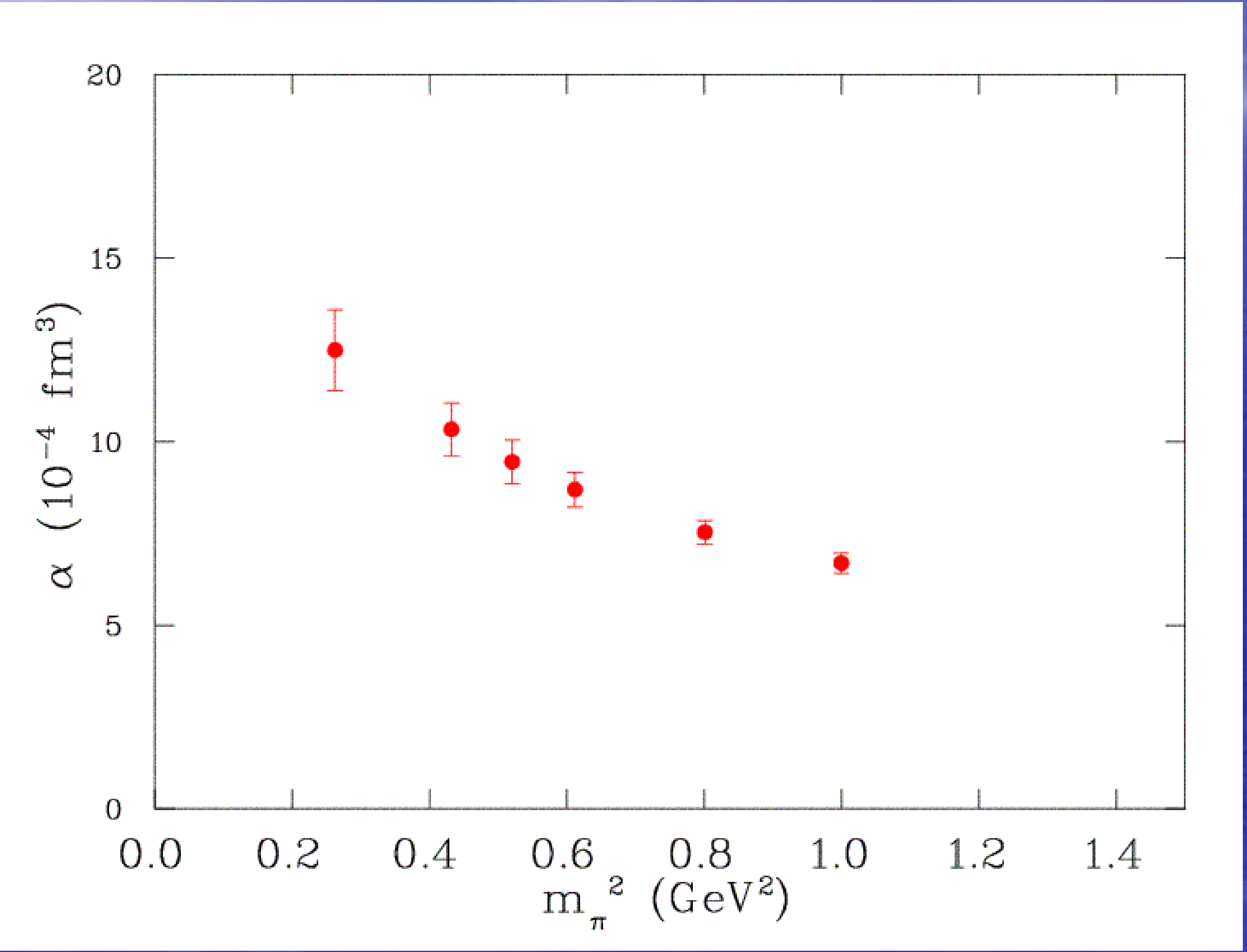
Exp. & Theoretical Value of α_n

$\alpha_n (\times 10^{-4}\ \mathrm{fm}^3)$	Method
$10.7^{+3.3}_{-10.7}$	Experimental(1990)
$12.6 \pm 1.5 \pm 2.0$	Experimental(1991)
$13.6^{+0.4}_{-6.0}$	Experimental(2000)
$9.8^{+1.9}_{-2.3}$	Particle Data Group (2002)
9 ± 3	Theoretical (2000)
12.7	Theoretical (2001)
$12.5 \pm 1.8^{+1.1}_{-0.6} \pm 1.1$	Theoretical (2002)
12.5+/-2.3	Experimental (2003)

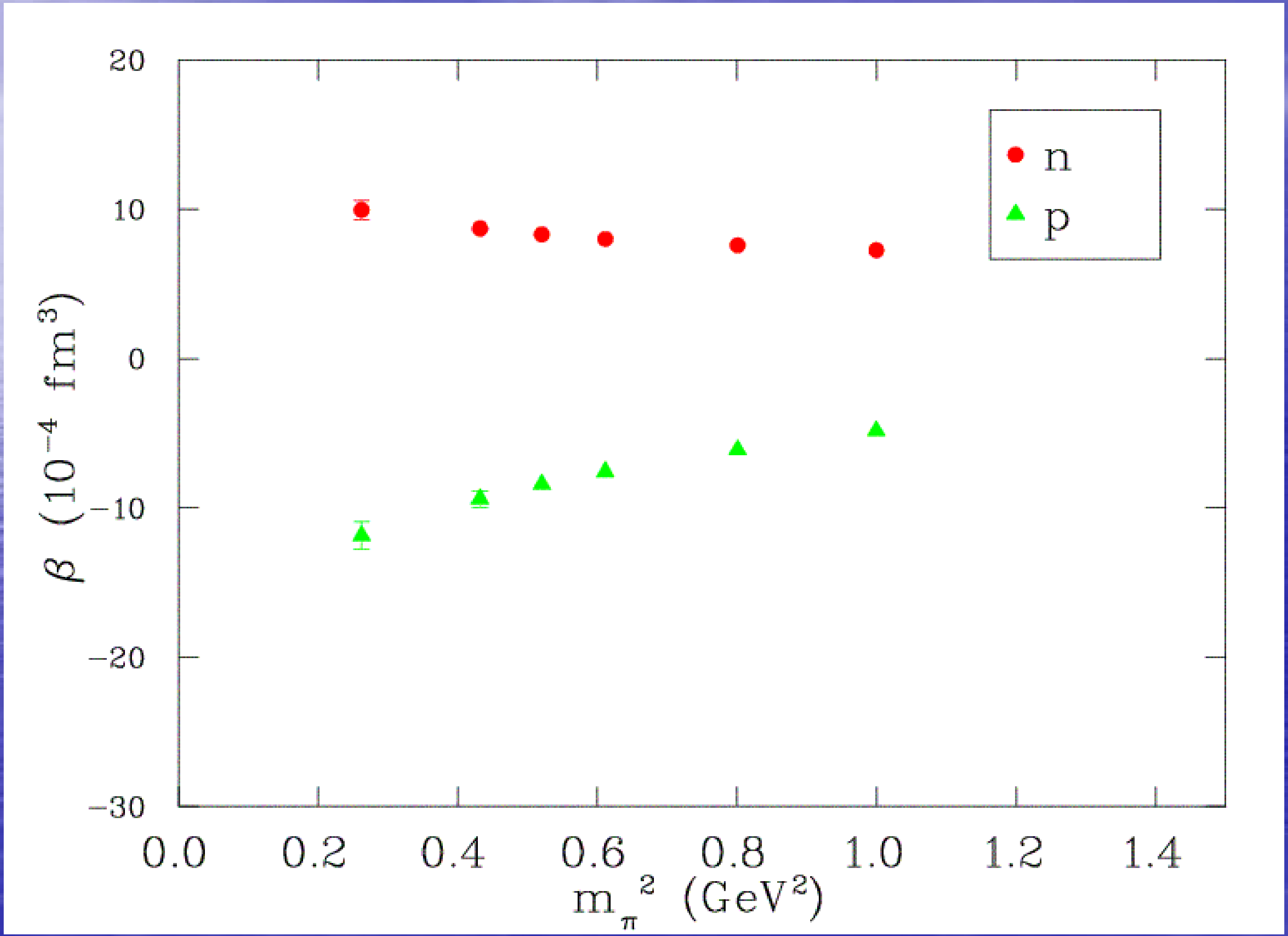
Electric Polarizability of Δ^0



Electric Polarizability of π^0



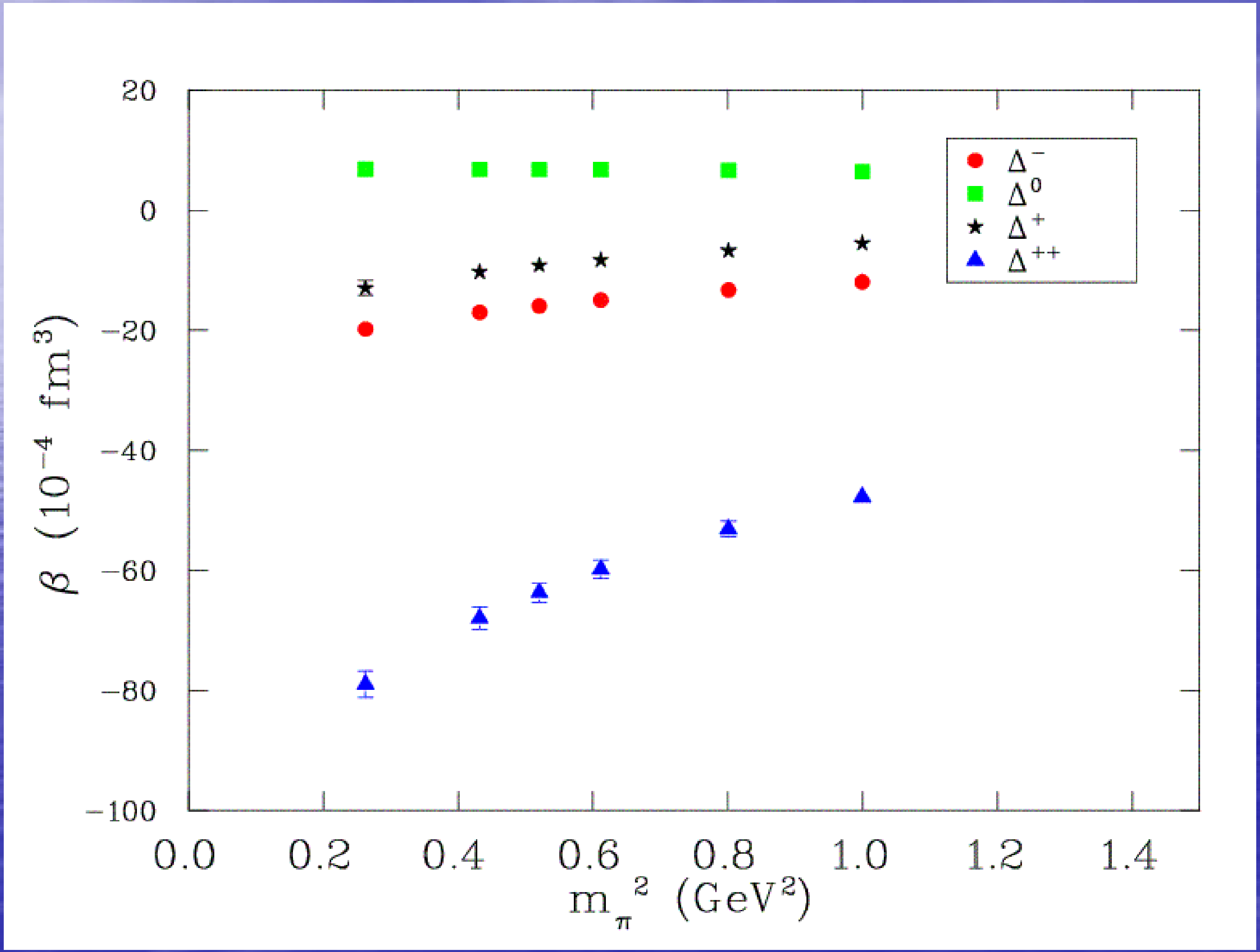
β of Neutron and Proton



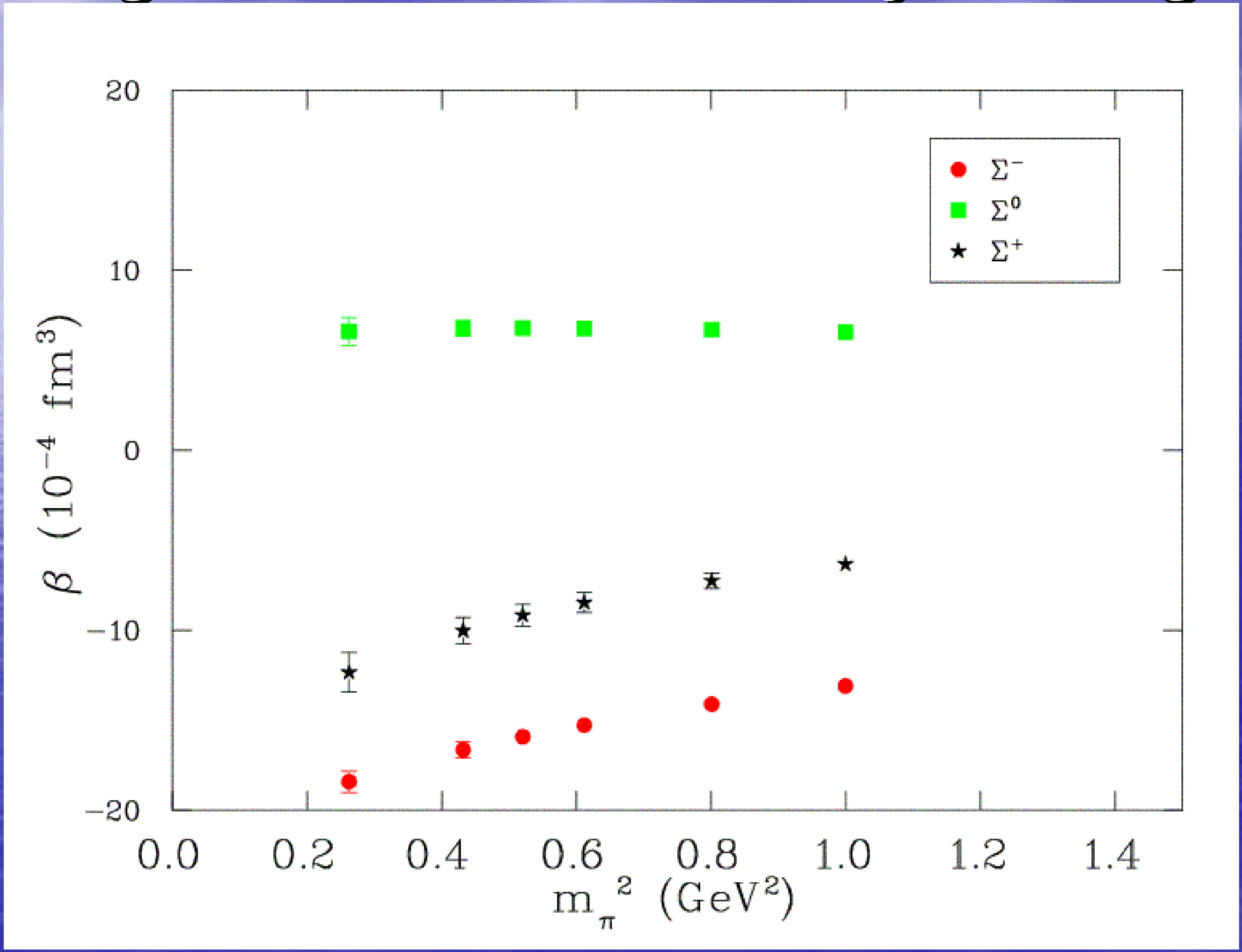
Exp. & Theoretical Value of β

Particle	$\beta(\times 10^{-4}\ \mathrm{fm}^3)$	Method
n	7.8	Theoretical (1993)
n	$6.7^{+1.3}_{-0.7}$	Theoretical (1994)
n	11 ± 3	Theoretical (2000)
n	$2.7 \mp 1.8^{+0.6}_{-1.1} \mp 1.1$	Theoretical (2002)
n	10.3 ∓ 2.0	Experimental (1999)
n	2.7-/+2.3	Experimental (2003)
p	$1.8 \pm 0.4 \pm 0.4$	Theoretical (2002)
p	$3.58^{+1.19+1.03}_{-1.25-1.07}$	Experimental (1992)
p	$4.4 \pm 0.4 \pm 1.1$	Experimental (1993)
p	$2.1 \mp 0.8 \mp 0.5$	Experimental (1995))

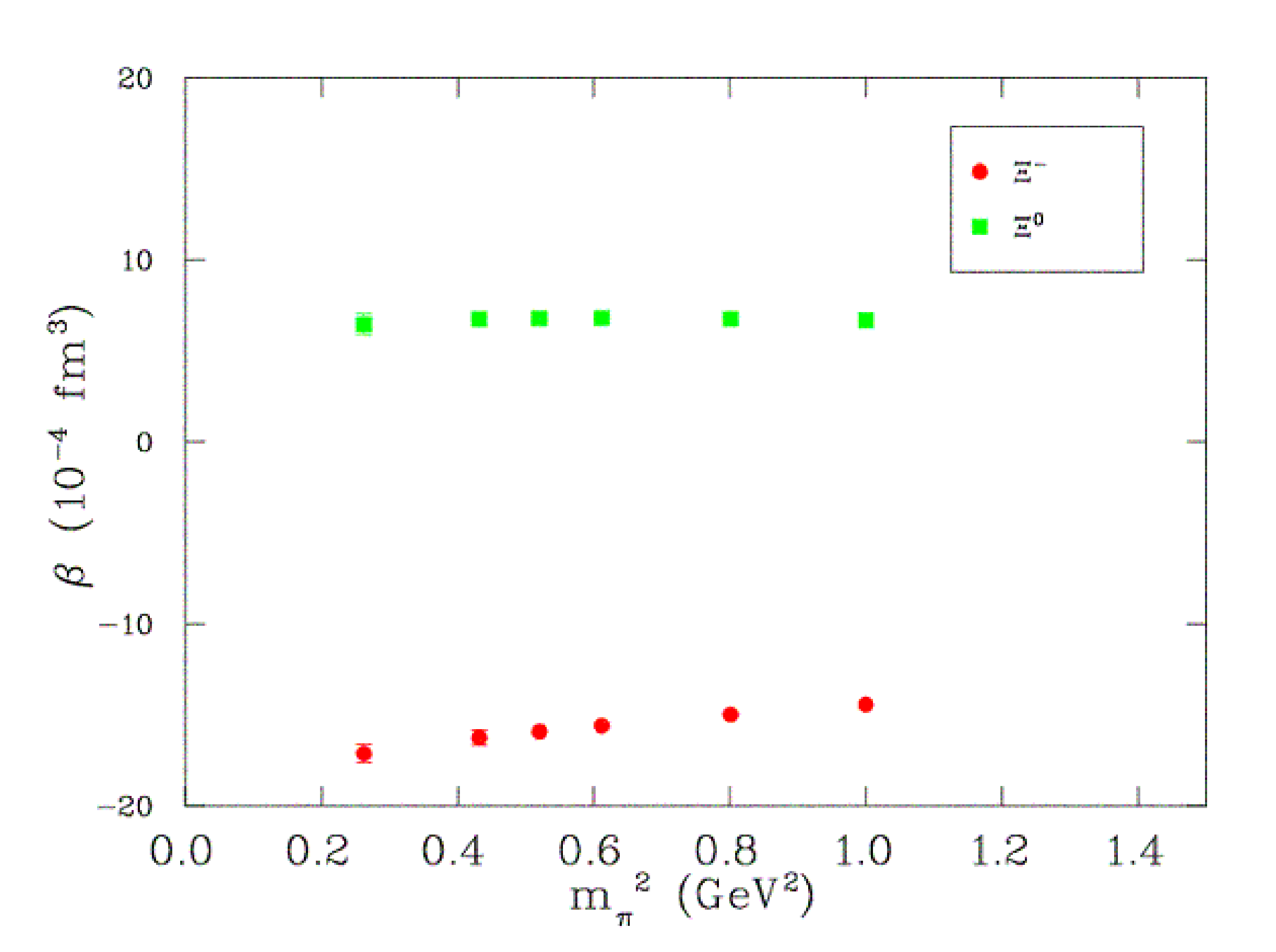
Magnetic Polarizability of Delta



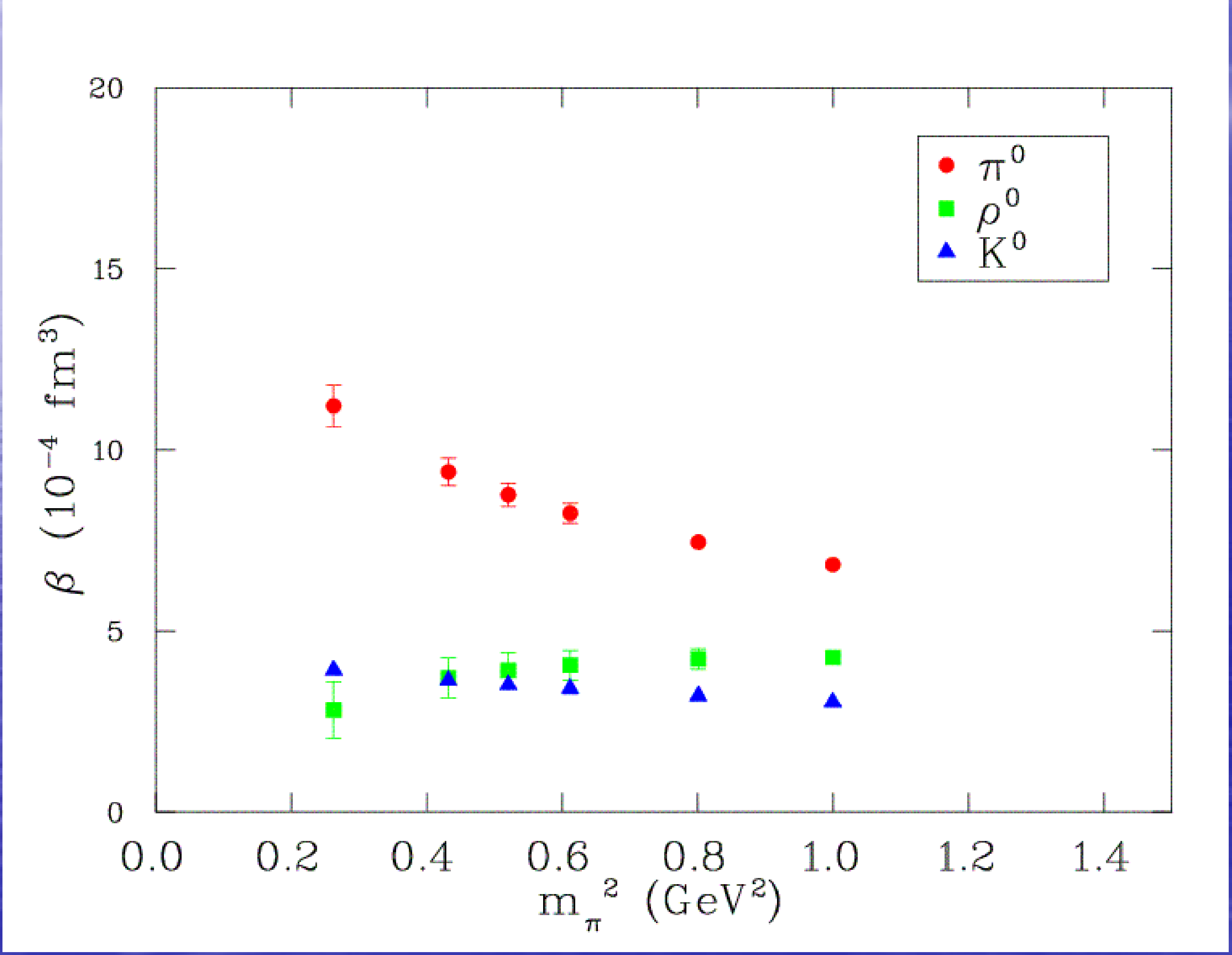
Magnetic Polarizability of Sigma



Magnetic Polarizability of Xi



β of Pion, Kaon and Rho



Summary

- We have computed the electric and magnetic polarizabilities of hadrons on the lattice.
- We are presenting preliminary results.
- Our electric polarizabilities calculations have a good agreement with other people’s results.
- Our magnetic results look greater than others.