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About the Error Estimation in Charge Asymmetry Measurements - A MonteCarlo Simulation

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Abstract

The error that affects the extraction of the charge asymmetry in the EG1 run is discussed as a function of the various beam parameters including the beam current and the resolution charge integrator used to digitize the Faraday Cup signal. Using a Monte Carlo simulation we proved that the process can be asimpthotically described by a pure statistical model that, however, overestimates the error when the beam current is too low or the charge integrator conversion factor is too small.

1 Introduction

The recent analysis of the EG1 data has pointed out the importance of the charge asymmetry between the two different beam helicities that could be a significant source of systematic error in the asymmetry measurement. This effect is strongly related to the adopted charge integrator that has been used to digitize the Faraday Cup signal. Due to the small conversion factor the measurement of the charge accumulated in a single bunch is not precise enough because the integer read-out of the charge integrator could differ by several % from the real value of the measured charge. Furthermore, by extending the measurement to several consecutive bunches, the statistical fluctuations

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increase and a non-negligible fraction of the measured charge is erroneusly assigned to the wrong helicity state of the electron beam. These two effects are better described later in this section. We will prove that the estimate of the error is not trivial because the read-out of two consecutive helicity states is strongly correlated and a pure statistical approach is working only asimpthotically. In this note we investigate the problem using a MonteCarlo simulation and compare the results with the expectation from a pure statistical approach. We will show how the uncertainties depend on the conversion factor of the charge integrator and that the MonteCarlo results asympthotically coincide with the results of the statistical approach.

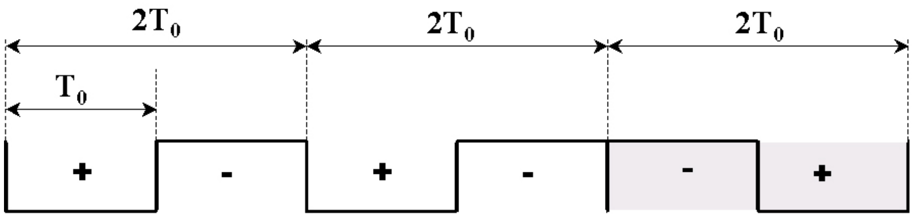

Fig. 1. Polarized beam time-structure. The grey area represents a bunch where the positive-negative order is reversed due to the random assignement.

The polarized beam time structure is reported in Fig.1. The electron pulses are divided into pairs of bunches with opposite electron polarization and the order in each pair is randomly chosen. The time duration of each bunch is constant and equal to $2T_0$ ($T_0 = 1$ s in the EG1A runs). The total charge q_+ in the positive ("+") part of the bunch is, in average, larger by a factor 1.006 that the charge in the negative ("-") part. The charge is measured by means of a charge integrator, with a charge conversion factor χ $[nC/count]$, whose integer output $k = q/\chi$ approaches the true value q. The precision on the measure of a single bunch is given by the difference $\delta q = q - k\chi$ that gives rise to a relative error of:

$$\frac{\delta q}{q} = 1 - \frac{k\chi}{q} \tag{1}$$

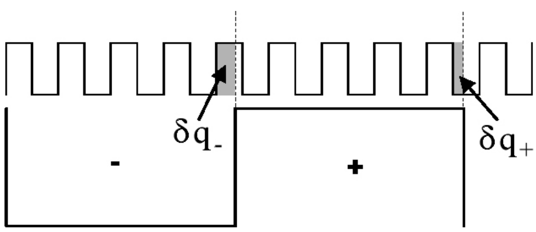

Fig. 2. Charge integration and error propagation. The grey area represents the charge that is counted in the wrong part of the bunch.

The error propagation is however more complicated than it appears from the previous equation. First of all, as shown in Fig.2, for a loss of charge due to the missing δq_+ of the "+" part of the bunch that is erroneusly added to the next bunch, there is correspondingly an increase due to the missing δq_- that is added from the negative part of the same bunch. When two consecutive bunches are analyzed together, two different possibilities could occur: a) the next bunch shows the same order in the electron polarization and therefore the δq_+ is erroneously added to the next q_-; b) the next bunch shows a reversed order in the electron polarization and therefore the δq_+ is added to the next q_+ and is not miscounted in the integral. The situation is summarized in Table 1 where the left column represents the possible configurations and the right column the contribution to the charge evaluation of the positive part of the second bunch ($\oplus$).

possible bunch configuration	error for the $\oplus$ part of the bunch
+ - $\oplus$ -	$\delta q_+ - \delta q_-$
- + $\oplus$ -	δq_+

Table 1
Counting leakage between bunches.

In summary for each bunch there is a charge leakage from the first part to the second part of the bunch, and every two bunches in average there is also a charge leakage to the next bunch. Obviously these considerations apply equally to the negative part of the bunch. As a result, this effect produces the erroneus assignement of a fraction of the integrated charge to the opposite helicity state.

2 The Statistical Approach

Before the discussion of the MonteCarlo simulation and results we will analyze the expectation from a statistical approach to the process. Assuming that the error in the evaluation of a single q_+ is independent for each bunch we can assign an average error obtained from the squared sum of the standard deviations of the two distributions $\delta_1 = \delta q_+$ and $\delta_2 = \delta q_+ - \delta q_-$.

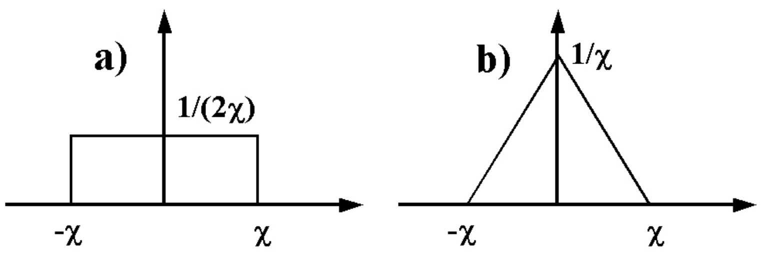

Fig. 3. a) distribution functions for δ_1 (assumed uniform; b) distribution function for δ_2 (the shape is a direct consequence of the assumption on δ_1).

The two distributions, reported in Fig.3, have zero mean value and the following standard deviations:

$$
\sigma_1^2 = \int_{-\chi}^{\chi} \frac{1}{2\chi} x^2 dx = \frac{\chi^2}{3} \tag{2}
$$

$$
\sigma_2^2 = \int_{-\chi}^{0} \left(1 + \frac{x}{\chi}\right) x^2 dx + \int_{0}^{\chi} \left(1 - \frac{x}{\chi}\right) x^2 dx = \frac{\chi^2}{6}
$$

After an elapsed time t corresponding to $n = \frac{t}{2T_0}$ bunches, the error affecting the sum Q_+ of all positive q_+ would become:

$$
\sigma_+^2 = \sum_{i=1}^{n/2} \left(\sigma_1^2 + \sigma_2^2\right) = \frac{n}{2} \left(\frac{\chi^2}{3} + \frac{\chi^2}{6}\right) = \frac{t}{4T_0} \frac{\chi^2}{2} \tag{3}
$$

For symmetry reasons this is the error also for Q_- giving rise to the final relative error:

$$
\frac{\sigma}{Q} = \frac{\sqrt{\sigma_+^2 + \sigma_-^2}}{Q_+ + Q_-} = \frac{\sqrt{2}\sigma_+}{tI} = \frac{1}{2\sqrt{T_0 t}} \frac{\chi}{I} \tag{4}
$$

where I is the beam current. Therefore the relative error should decrease as the inverse of the beam current, while the dependence from the elapsed time t shows a slower behaviour.

3 The MonteCarlo Results

We wrote a simple FORTRAN code to simulate the time evolution of the charge asymmetry measurement assuming a half-bunch length $T_0 = 1\ s$ and using a charge integrator with a conversion factor χ ranging between 0.01 and 0.5 $nC/count$. We studied the process in the beam current interval $0.1 < I < 20\ nA$, therefore with conditions similar to those used in EG1A. A charge asymmetry of $A = 3 \cdot 10^{-3}$ has been simulated. In Fig.4 the time evolution of a single simulation is reported for three different values of the beam current. The main feature is that the simulated asymmetry approaches the "true" values in shorter time at large beam current values. Particularly the results of the simulation with low I values show large fluctuations and in the case of $I = 0.1\ nA$ the asymmetry does not converge to the expected value.

More interesting is the study of the behaviour of the relative error. To have a statistically significant sample, we simulated 10000 times for each current value the measurement of the charge integration. For each simulated measurement the time evolution of the asymmetry has been followed from $t = 0\ s$ up to

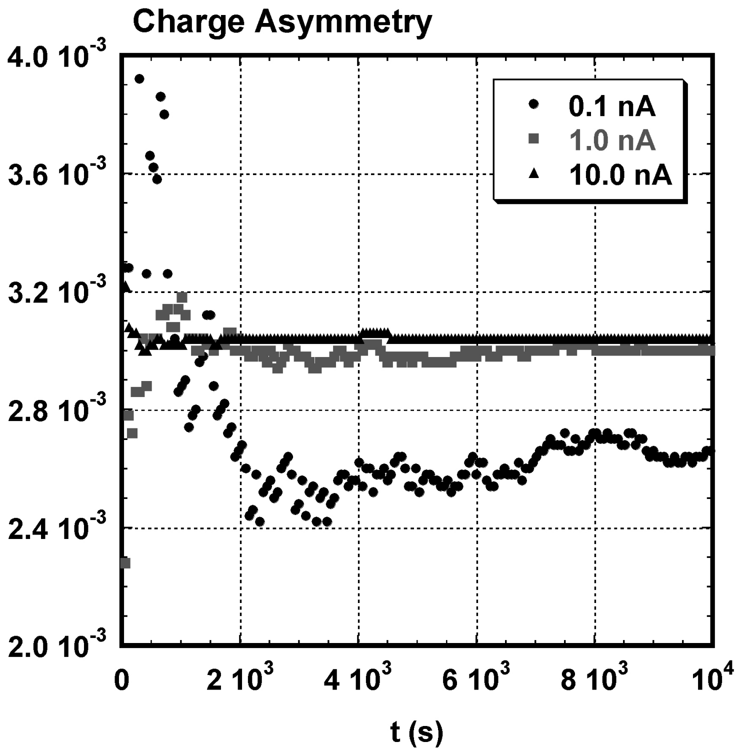

Fig. 4. Time evolution of the charge asymmetry plotted at three different values of the beam current.

$t = 10^5\ s$. At each $2T_0$ step a new bunch has been added with the polarization order randomly generated, while every 100 seconds the asymmetry average value and the relative standard deviation σ/Q has been evaluated. In Fig.5 the time evolution of σ/Q is reported: the dependence form $\frac{1}{\sqrt{t}}$ is well verified in a large interval of the elapsed time ($10^2 \leq t \leq 10^5\ s$) and the beam current ($0.1 \leq I \leq 20\ nA$) and is qualitatively in agreement with Eq.4. In the log-log plot of Fig.5 this behaviour corresponds to a linear dependence with a -2 power.

In Fig.6 the dependence from I is reported at different t values. Here the behaviour is slightly different from what expected from the purely statistical approach previously discussed, in fact at all t values, the plot shows a different behaviour for $I < 4\ nA$ where the dependence from I has been fitted by the expression

$$\frac{\sigma}{Q} = \frac{1}{40\sqrt{tI}} \tag{5}$$

while for $I > 4\ nA$ the dependence is in perfect agreement with what reported in Eq.4. In this particular configuration $I_c = 4\ nA$ is the critical current value above which the statistical approach begins to work. It is also interesting

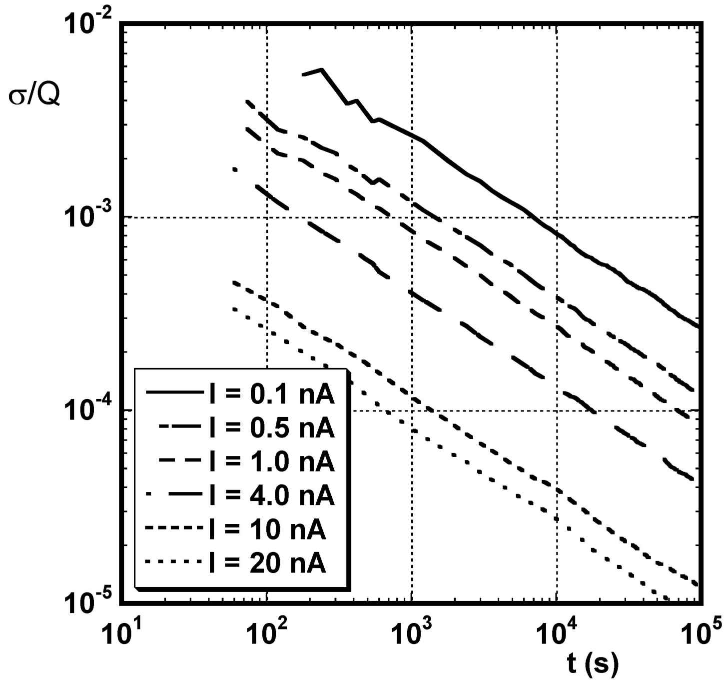

Fig. 5. Time evolution of the relative error of the charge asymmetry as a function of the beam current.

noting that for $I < 4\ nA$ the obtained error is smaller that expected from the statistical approach. This is certainly related to the way how the charge leakage is distributed among the two polarization states.

It is evident, however, that this is not the correct way to describe the phenomena. In fact the presence of two expressions with a different dependence from the beam current I would require the introduction of dimensional constants while the dependence on T_0 and χ has not yet been investigated. The correct way to approach this problem, therefore, requires the definition of the previous expression in terms of adimensional variables. It is not difficult to be convinced that the correct parameter must be the average counts in a half-bunch k that can be obtained by the following function of the three parameters:

$$k = \frac{q}{\chi} = \frac{IT_0}{\chi} \tag{6}$$

and that provides, in correspondence of I_c the value $k_c \approx 40$. To study the dependence from k a set of simulations at different values of χ and T_0 have been performed.

In Fig.7 the dependence on χ is reported. For sake of simplicity the evolution of the relative error has been studied at the value of the elapsed time $t = 10^4\ s$ corresponding to approximately one day of measurement. The relative error

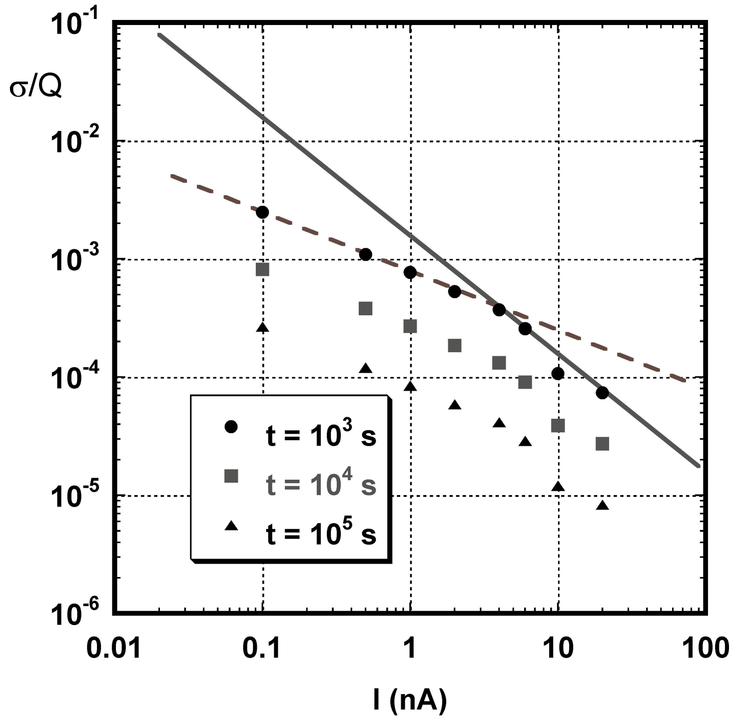

Fig. 6. Evolution of the relative error of the charge asymmetry as a function of the beam current at three different elapsed times. The continuous line is the result of the statistical approach and the dashed line is a fit to the relative error at low I values as described in the text.

shows the same behaviour in a large interval of χ values and for $k \geq 36$ it fairly agrees with the results of the statistical approach, while for $k < 36$ the dependence has been fitted. Finally the relative error can be described using the following expressions:

$$\frac{\sigma}{Q} = \frac{1}{12}\sqrt{\frac{T_0}{t}\frac{1}{\sqrt{k}}} \qquad k < 36 \tag{7}$$

$$\frac{\sigma}{Q} = \frac{1}{2}\sqrt{\frac{T_0}{t}\frac{1}{k}} \qquad k \geq 36$$

where the dimensionless ratio $\frac{T_0}{t}$, related to the number of bunches, has been introduced to correctly describe the time evolution of the relative error. The previous equations can be equivalently written as follows:

$$\frac{\sigma}{Q} = \frac{1}{12}\sqrt{\frac{\chi}{It}} = \frac{1}{12}\sqrt{\frac{\chi}{Q}} \qquad \frac{IT_0}{\chi} < 36 \tag{8}$$

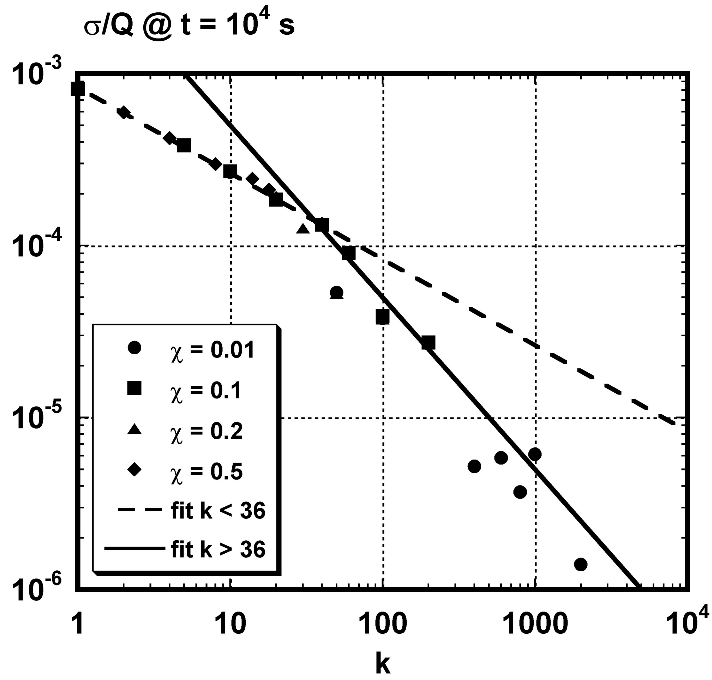

Fig. 7. Evolution of the relative error of the charge asymmetry as a function of k at fixed $t = 10^4$ s and $T_0 = 1$ s and different χ values. The dashed line is the fit for $k < 36$ and the continuous line is the fit for $k \geq 36$.

$$\frac{\sigma}{Q} = \frac{1}{2} \frac{1}{\sqrt{tT_0}} \frac{\chi}{I} \qquad\qquad \frac{IT_0}{\chi} \geq 36$$

The dependence from T_0 of Eq.7 has been also verified. The results of the Monte Carlo are reported in Fig.8 where the relative error, corrected by $1/T_0$ to take into account the explicit dependence of Eq.7 from T_0, clearly shows the expected dependence from k.

4 The EG1 run

We are now able to apply the obtained formulas to the EG1 runs. The running conditions have been summarized in Table 2. The conversion factor for the charge integrator used in the EG1 was $\chi = 0.1$ $nC/count$ and therefore the corresponding k value is generally less that 36. We also reported the estimated systematic error for the Hydrogen run we analyzed for the measurement of the asymmetry in the exclusive π^+ electroproduction. The relative error is less than 10^{-4} that corresponds, for a typical asymmetry measurement with CLAS, to a systematic error $\frac{\delta A}{A} < 1\%$.

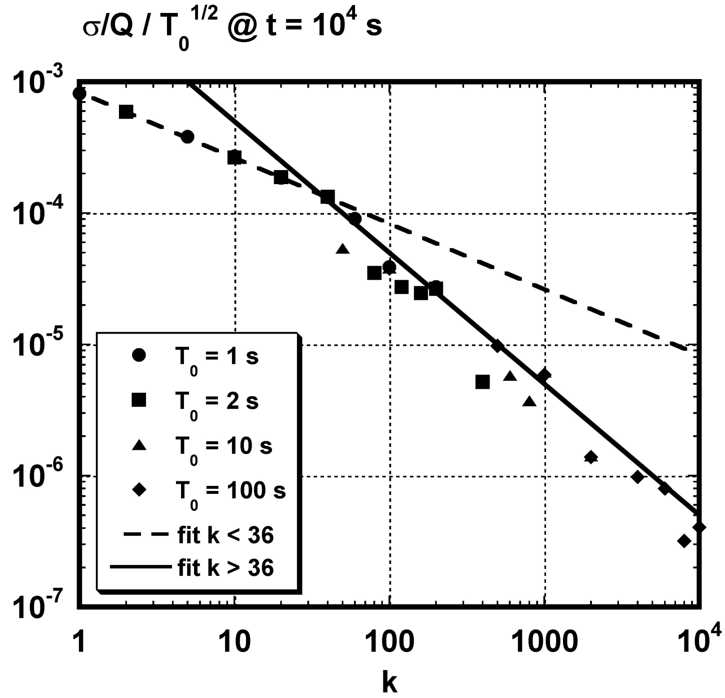

Fig. 8. Evolution of the relative error of the charge asymmetry as a function of k at fixed $t = 10^4$ s and $\chi = 0.1$ $nC/count$ and different T_0 values. The dashed line is the fit for $k < 36$ and the continuous line is the fit for $k \geq 36$.

5 Conclusions

We have evaluated the error that would affect the charge asymmetry measurement in electron-scattering experiments with longitudinally polarized electrons. Using a Monte Carlo simulation we have studied the dependence from the average number of counts k, the bunch half-length T_0, and the measurement time t. We have also shown that these parameters describe the error better than the beam current I and the charge integrator conversion factor χ could do. The dependence is summarized by Eqs.7 and 8 that coincide, for $k \geq 36$ with the expression that can be obtained using a pure statistical approach. The error is generally small and decreases rapidly with long measurement times. It is our opinion that it would not drastically affect the EG1 results.

Beam Energy (GeV)	Target	Torus Field (A)	Current (nA)	P_t	$\lambda/2$	Trigger Rate (Hz)	Events (x10^6)	k	Integrated Charge Q (μC)	σ/Q (x10^{-5})
2.562	NH_3	-1500	0.5	+	IN	1500	182.2	5		
2.562	NH_3	-1500	0.5	−	IN	1500	184.9	5		
2.562	NH_3	-1930	0.5	+	IN	1500	13.5	5		
2.565	NH_3	+1500	1.6	+	IN	1300	106.6	16		
2.565	NH_3	+1500	1.6	−	IN	1300	86.6	16		
2.565	NH_3	+2250	2.0	+	IN	1000	143.0	20	120.9	7.6
2.565	NH_3	+2250	2.0	−	IN	1000	101.4	20	192.6	6.3
4.278	NH_3	-2250	2.0	+	IN	2000	129.1	20		
4.278	NH_3	-2250	2.0	−	IN	2000	272.1	20		
4.239	NH_3	+1930	2.5	+	IN	1500	176.6	25		
4.278	NH_3	+1930	3.2	−	ININ	2000	88.1	32		
4.068	NH_3	+1930	3.2	+	ININ	2000	172.2	32		
4.068	NH_3	+1930	3.2	−	ININ	2000	187.7	32		
4.278	NH_3	+2250	4.0	+	IN	2000	35.8	40		
4.278	NH_3	+2250	4.0	+	ININ	2000	230.2	40	325.9	4.4
4.278	NH_3	+2250	4.0	−	ININ	2000	175.4	40	259.4	4.9
2.562	ND_3	-1500	0.5	+	IN	1500	153.9	5		
2.494	ND_3	-1500	0.5	−	IN	1500	83.2	5		
2.565	ND_3	+2250	2.2	+	IN	1500	338.8	22		
2.565	ND_3	+2250	2.2	−	IN	1500	23.5	22		

Table 2

Run summary for the EG1 run period. Each data set is characterized by the energy, toroidal field value, target type, beam current, target polarity, trigger rate and event number. In addition in the sixth column, the presence of the $\lambda/2$ plate used to reverse the laser polarization and therefore the electron helicity is reported.