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Computing Invariant Masses and Missing Masses

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Abstract

Introduction

We have examined resolution issues associated with the calculation of the *invariant mass* of two daughter particles, and the *missing mass* in a reaction where some part of the final state is observed. We find that there is a broad range of situations where the *invariant mass* is a much more stable quantity to calculate, and issues of the resolution being dependent on the incident beam energy in the case of *missing mass* calculations. We should also note that all of the resolution issues discussed in this report vanish if correct kinematic fitting [1] of the events is done. However, in the situation where either a kinematic fit is not possible, or it is not desirable to do this, the following is relevant.

We can understand the issue with the following simplified explanation. Consider the case where we have a particle of mass M that decays to two observed daughter particles of masses m_1 and m_2. Let us define the quantity $Q = M - m_1 - m_2$. In this scenario, the *invariant mass* and the error in the *invariant mass* of M can be expressed as follows:

$$M = m_1 + m_2 + Q \tag{1}$$

$$\Delta M = \Delta Q\,. \tag{2}$$

The quantity Q is a constant, independent of the actual reaction. If we make the assumption that $\frac{\Delta Q}{Q} \approx$ constant. then the error is

$$\Delta M \approx \text{constant} \frac{Q}{m_1 + m_2 + Q}. \tag{3}$$

In the case where Q is small compared to $m_1 + m_2$, the error in M can be driven to extremely small values, and be independent of the production of M.

Now consider the case where we have a particle of mass M that we observe through missing mass. In this case, we can simplify the evaluation of the mass to the following:

$$M = E_{tot} - E_{obs} \tag{4}$$

$$\Delta M = \Delta E_{obs}. \tag{5}$$

Where E_{tot} qualitatively represents the total energy and momentum in the initial state, while E_{obs} qualitatively represents the total observed energy and momentum used to compute the *missing mass*. Now consider the case where we are producing M with a broad range of initial E_{tot} (*e.g.* The CLAS photon beam). As E_{tot} gets larger, E_{obs} also has to get larger to maintain a constant difference. If we now assume that the error in E_{obs} is proportional to E_{obs}, then the error in M must grow as E_{tot} is increased.

While both of these explanations are over simplified, the conclusions are still valid. For particles with small Q values, it is better to compute an *invariant mass* rather than a *missing mass*. Whether the *invariant mass* becomes worse than the *missing mass* is dependent on the exact detector resolutions, the Q value of the reaction, and the difference $E_{tot} - M$. There is no guarantee that this is broadly true for all possible situations.

Given the fact that an *invariant mass* calculation is guaranteed to be more accurate for small Q decays, it would be useful to be able to convert the *missing mass* calculation to an *invariant mass* calculations. We propose a scheme for doing this in some specific situations.

Our conclusion is that this procedure would be useful if applied to the following analysis. We anticipate that there are other analysis that would benefit as well.

$$\begin{aligned}
\gamma p &\rightarrow K^+(\Lambda \rightarrow p\pi^-_{missing})_{missing} \;\; (Q = 30\, MeV,\, m = 1107\, MeV) \\
\gamma d &\rightarrow K^- p(\Theta^+ \rightarrow K^+ n_{missing})_{missing} \;\; (Q = 104\, MeV,\, m = 1540\, MeV) \\
\gamma p &\rightarrow K^- \pi^+(\Theta^+ \rightarrow K^+ n_{missing})_{missing} \;\; (Q = 104\, MeV,\, m = 1540\, MeV)
\end{aligned}$$

Toy Monte Carlo: $\gamma d \to pnK^+K^-$

We have looked at the reaction $\gamma d \to K^+K^-pn$ where the neutron is detected through a missing mass cut. In this reaction channel, there is clear evidence for the $\Lambda(1520) \to K^-p$. In addition, this channel contains the data for the first CLAS publication on the $\Theta^+ \to nK^+$. In this study, we have looked at the effects of detector resolution on the observed mass and width of both the $\Lambda(1520)$ and the Θ^+. We find that for the $\Lambda(1520)$, a measured width is very close to the known width over a broad range of incident photon energies, while for the Θ^+, the observed width is dependent on the incident photon energy, with observed widths rising rapidly as photon energy in increased.

This effect is a result of the way the mass of the two states are computed. The former being an invariant mass combination whose two daughter particles have a small Q value. The calculated mass is strongly dominated by the mass of the proton and kaon daughters.

$$m(\Lambda) = \sqrt{(E_p + E_K)^2 - \mid \vec{p}_p + \vec{p}_K \mid^2} \tag{6}$$

In the case of the Θ, the mass is computed as a missing mass in the reaction $\gamma d \to pK^-X_{miss}$. No such daughter mass constraint exists in this case, and the observed width of the Θ rises as the incident photon energy increases. Essentially, the missing mass formula throws away all knowledge of the neutron and kaon daughter particles.

$$m(\Theta) = \sqrt{(E_\gamma + m_d - E_p - E_K)^2 - \mid (\vec{p}_\gamma - \vec{p}_p - \vec{p}_K) \mid^2} \tag{7}$$

For the reaction of interest, an alternate procedure can used which restores the strength of the low Q value of the decay. In this procedure, we compute the momentum of the neutron through 3-momentum conservation, and then use the know neutron mass to compute its energy. We then combine this neutron 4-vector with the K^+ using an invariant mass formula.

$$\vec{p}_n = \vec{p}_\gamma - \vec{p}_p - \vec{p}_{K^-} \tag{8}$$

$$E_n = \sqrt{\mid \vec{p}_n \mid^2 + m_n^2} \tag{9}$$

$$m(\Theta) = \sqrt{(E_n + E_{K^+})^2 - \mid \vec{p}_n + \vec{p}_{K^+} \mid^2} \tag{10}$$

As an example of this procedure, we have built a toy Monte Carlo that generates $\gamma d \to \Theta^+(1550)pK^-$ according to 3-body phase space. The Θ is given a Breit-Wigner width of 10 MeV, and it is assumed that the momentum of all charged particles in the lab frame are measured with 2% errors. The incident photon beam energy is generated uniformly from 1.4 to 3.0 GeV. In the Figure 1, we plot the reconstructed Θ mass using both methods above and compare them with the true mass spectra as seen in **a**. It is clear that the method in equn. 10 produces a significantly better peak than that of equn. 7.

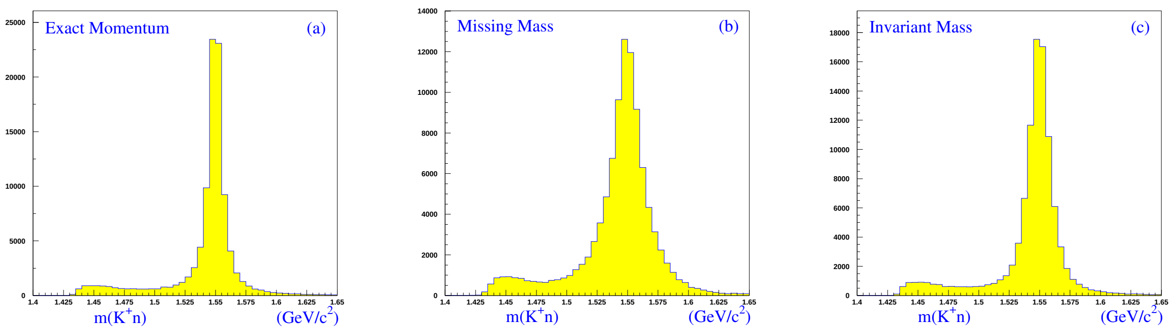

Figure 1: These figures are for all photon energies from 1.3 to 3.0 GeV. Figure **a** shows the invariant mass of the K^+n system computed using the exact values of the K^+ and n momenta and energies. **b** shows the missing mass as calculated using equn. 7. The measured width is substantially larger than the true width seen in **a**. Figure **c** is the invariant mass as calculated using equn. 10.

$E_\gamma\, GeV$	Natural	Missing Mass	Invariant Mass
1.5	7.3 MeV	MeV	10.1 MeV
1.7	7.3 MeV	10.1 MeV	10.1 MeV
1.9	7.3 MeV	13.1 MeV	10.4 MeV
2.1	7.3 MeV	16.2 MeV	10.8 MeV
2.3	7.3 MeV	19.4 MeV	11.0 MeV
2.5	7.3 MeV	22.2 MeV	11.1 MeV
2.7	7.3 MeV	24.7 MeV	11.2 MeV
2.9	7.3 MeV	27.2 MeV	11.5 MeV

Table 1: The measured width of the Θ as a function of the photon beam energy. The *Natural* width is built using the true values of momentum and energy. The *Missing Mass* value is built using equn. 7. The *Invariant Mass* is built using equn. 10.

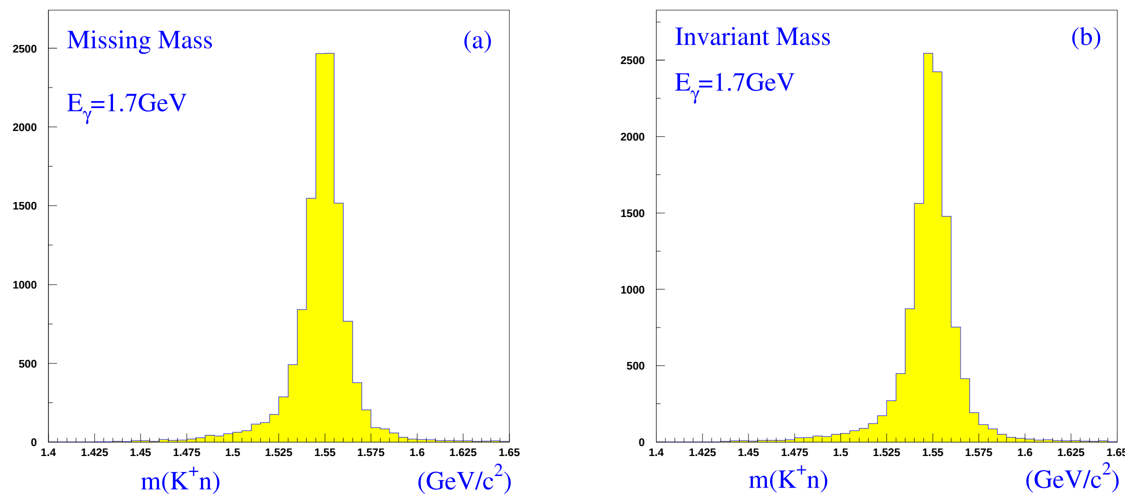

Figure 2: Events in which the incident photon energy is between 1.6 and 1.8 GeV. **a** shows the *missing mass* calculation, (equn. 7) for the Θ, while **b** shows the *invariant mass* calculation, (equn. 10) for the Θ.

GSIM, gpp and a1c: $\gamma d \rightarrow pnK^+K^-$

To continue with this study, we have generated 500000 Monte Carlo events of the form $\gamma d \rightarrow \Theta^+(1550)K^-p$. The photon energies are generated uniformly between 1.4 and 3.0 GeV, while the three particles are thrown according to 3-body phase space. The Θ is generated with a Breit-Wigner mass of 1.550 GeV/c^2 and a Breit-Wigner width of 10 MeV and is then allowed to decay isotropically in its rest frame into K^+n. These events are then located inside the g2a target and given to GSIM for tracking. The resulting events are then passed through GPP and finally A1C. Of the generated 500000 events, 10223 are reconstructed with a proton, K^+ and K^- in the final state. Of these, 8556 have a missing mass within $\pm 27 MeV/c^2$ of the neutron mass. If these, 8260 events satisfy a kinematic fit at the 2% confidence level cut. Of interest is the photon energy spectrum. Figure 4 shows this for both the generated events (left) and the events that pass all the cuts (right). There is a very clear depletion of events at the lowest photon energies that make it into the analysis.

In order to determine the effects of detector resolution, we have fit the resulting spectrum using a Voigtian function [2]. This function is the convolution of a Breit-

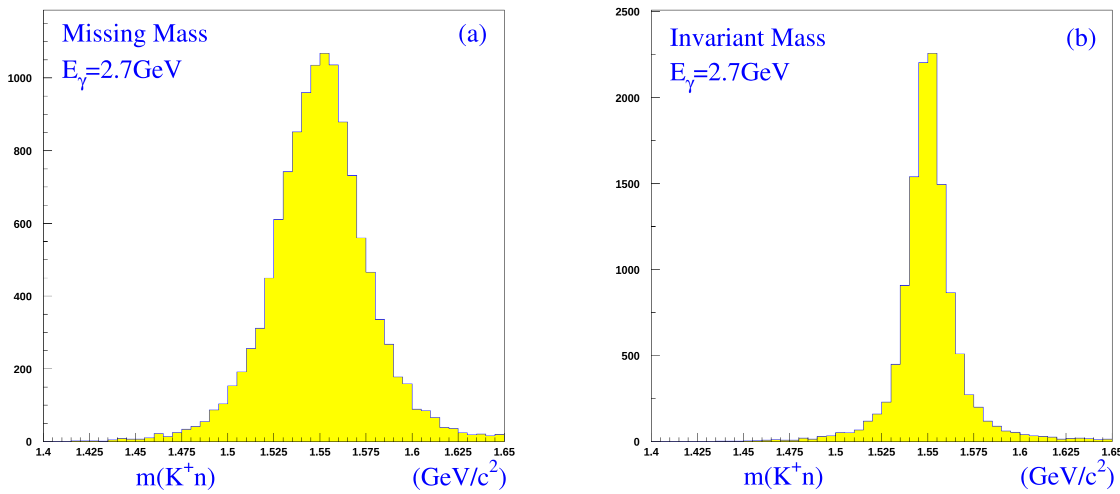

Figure 3: Events in which the incident photon energy is between 2.6 and 2.8 GeV. **a** shows the *missing mass* calculation, (equn. 7) for the Θ, while **b** shows the *invariant mass* calculation, (equn. 10) for the Θ.

Wigner of mass m_0 and width Γ and a Gaussian with a width of σ. Equn. 11 gives the explicit form of this function while equn. 12 provides an analytic way to evaluate this.

$$V(m) = C\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma}}\frac{\Gamma^2/4}{(m-m_0-x)^2+\Gamma^2/4}dx \tag{11}$$

$$= C\frac{\sqrt{2\pi}}{4}\frac{\Gamma}{\sigma}Re\left(w(v+ia)\right) \tag{12}$$

The function $w(z)$ is the complex error functions, and $v=(m-m_o)/(\sqrt{2}\sigma)$ and $a=\Gamma/(2\sqrt{2}\sigma)$. The fits to the distributions have been made using $\Gamma=10MeV$, and leaving the mass and sigma as free parameters. Table 2 shows the fit mass using each method as a function of photon energy, while Table 3 shows the corresponding resolution, σ. (Recall that this is on top of the natural Breit-Wigner width of 10 MeV.) First, we note that in this channel the mass computed using the *missing mass* technique is about 8 MeV too high, while that using the *invariant mass* technique is about 3 MeV too low and that from the kinematic fit is dead on right! Similarly, the resolution, σ, runs from about 5.5 MeV to 9.5 MeV using the *missing*

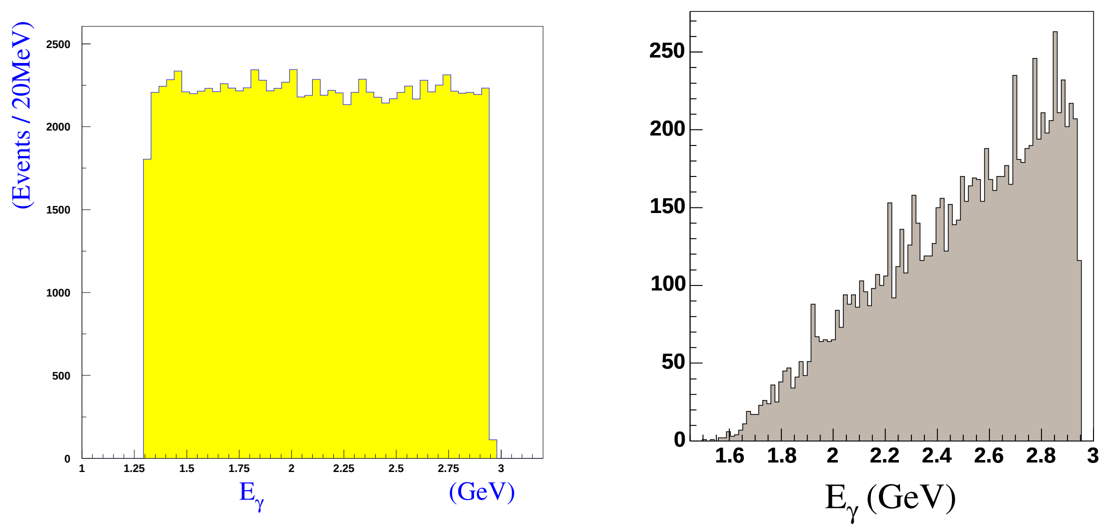

Figure 4: The photon energy spectrum of the generated events, (left figure) and of the events with a proton, K^+ and a K^-.

mass technique, while the *invariant mass* procedure produces a relatively constant $5\,MeV$ width, and the kinematic fit yields a σ of about $3\,MeV$. Clearly the *Invariant Mass* procedure yields a significantly better σ than the *missing mass* technique. We

$E_\gamma\,GeV$	Missing Mass	Invariant Mass	Kinematic Fit
1.6 to $2.9\,GeV$	$1558MeV$	$1547MeV$	$1550MeV$
2.00 to $2.20\,GeV$	$1557MeV$	$1546MeV$	$1550MeV$
2.25 to $2.45\,GeV$	$1558MeV$	$1547MeV$	$1550MeV$
2.50 to $2.70\,GeV$	$1559MeV$	$1548MeV$	$1550MeV$
2.75 to $2.95\,GeV$	$1558MeV$	$1548MeV$	$1550MeV$

Table 2: The fit Breit-Wigner mass to the Θ^+ as a function of the photon energy. The Θ was produced with a Breit-Wigner mass of $1550\,MeV$ and a Breit-Wigner width of $10\,MeV$.

can also see this improvement in the following Figures. Figure 5 shows the Θ mass for all photon energies. Figures 6,7,8 and 9 shows the three peaks for the various photon energy cuts in the previous tables.

$E_\gamma\, GeV$	σ(Missing Mass)	σ(Invariant Mass)	σ(Kinematic Fit)
1.6 to 2.9 GeV	7.9 MeV	5.2 MeV	3.7 MeV
2.00 to 2.20 GeV	5.6 MeV	4.8 MeV	3.1 MeV
2.25 to 2.45 GeV	7.0 MeV	4.7 MeV	3.3 MeV
2.50 to 2.70 GeV	8.0 MeV	4.6 MeV	3.2 MeV
2.75 to 2.95 GeV	9.4 MeV	5.0 MeV	4.1 MeV

Table 3: The fit detector resolution of the Θ^+ as a function of the photon energy. The Θ was produced with a Breit-Wigner mass of 1550 MeV and a Breit-Wigner width of 10 MeV.

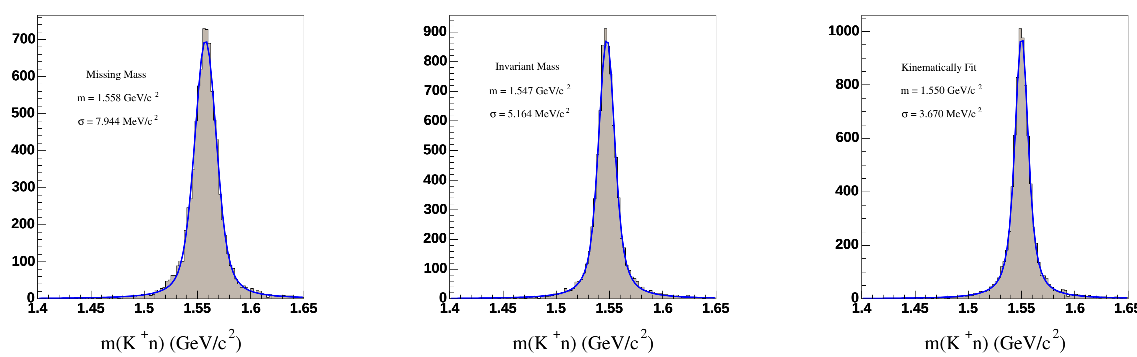

Figure 5: The Θ^+ mass for all events. The left hand figure is is computed using the *missing mass* formula, (equn. 7). The middle figure is computed using the modified *invariant mass* formula, (equn. 10) and the right hand figure is the result of kinematic fitting.

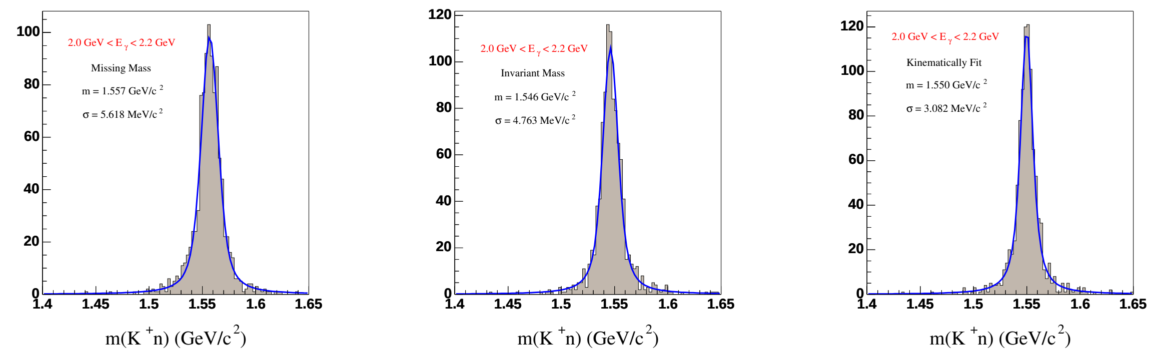

Figure 6: The Θ^+ mass for events with photon energy in the range: $2.00\,GeV < E_\gamma < 2.20\,GeV$. The left hand figure is is computed using the *missing mass* formula, (equn. 7). The middle figure is computed using the modified *invariant mass* formula, (equn. 10) and the right hand figure is the result of kinematic fitting.

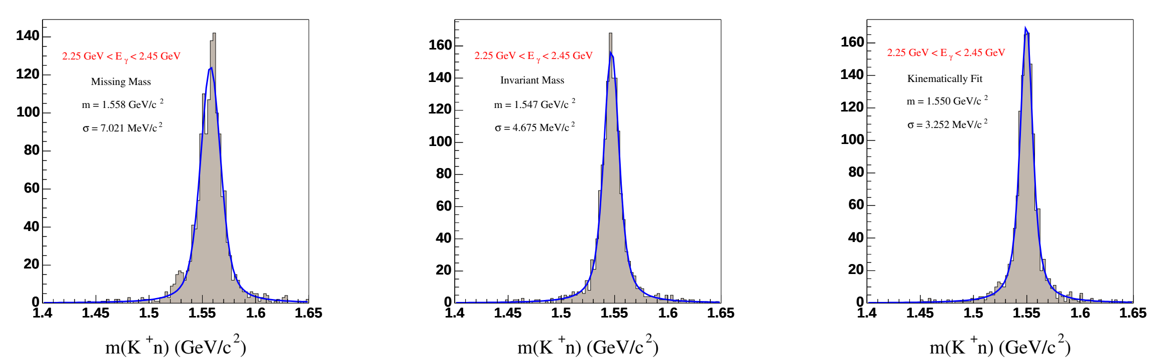

Figure 7: The Θ^+ mass for events with photon energy in the range: $2.25\,GeV < E_\gamma < 2.45\,GeV$. The left hand figure is is computed using the *missing mass* formula, (equn. 7). The middle figure is computed using the modified *invariant mass* formula, (equn. 10) and the right hand figure is the result of kinematic fitting.

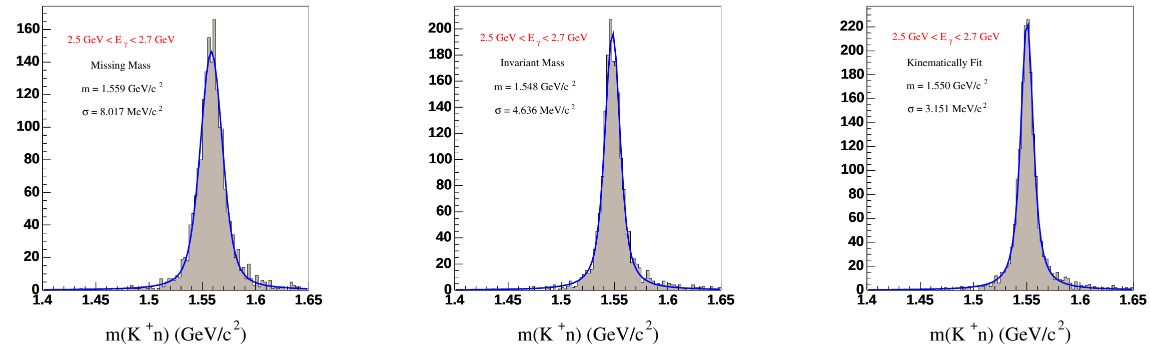

Figure 8: The Θ^+ mass for events with photon energy in the range: $2.50\,GeV < E_\gamma < 2.70\,GeV$. The left hand figure is is computed using the *missing mass* formula, (equn. 7). The middle figure is computed using the modified *invariant mass* formula, (equn. 10) and the right hand figure is the result of kinematic fitting.

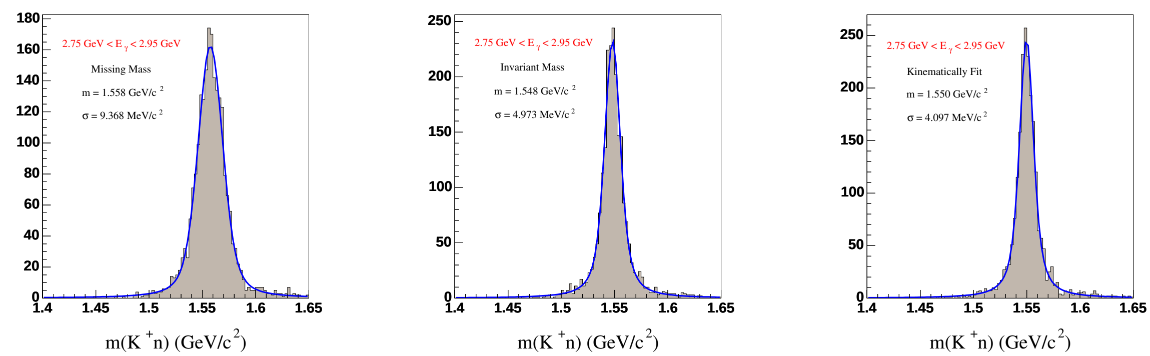

Figure 9: The Θ^+ mass for events with photon energy in the range: $2.75\,GeV < E_\gamma < 2.95\,GeV$. The left hand figure is is computed using the *missing mass* formula, (equn. 7). The middle figure is computed using the modified *invariant mass* formula, (equn. 10) and the right hand figure is the result of kinematic fitting.

g1c Data: $\gamma p \rightarrow K^+\Lambda$

Finally, we have collected data on the photoproduction of the $\Lambda(1107)$. These data come from the g1 run period, where the mass of the Λ has been computed using the *missing mass* technique. The Λ peak is then fit with a Gaussian function whose σ is plotted as a function of E_γ. These plots are shown in Figure 10, where a clear increase in the fit width of the Λ is seen as a function of the photon energy. If the analysis is repeated using the measured proton momentum, and evaluating the momentum of the π^- from missing 3-momentum and then imposing the π^- mass, the lower plot in Figure 10 is obtained. In the situation, *missing mass* procedure is better for $E_\gamma < 1.25\,GeV$, while the *invariant mass* procedure is better for all higher momentum. This is due to the fact that close to threshold, the proton is slow, and has a relatively large error on its reconstructed momentum. In the evaluation of the *missing mass*, we are not sensitive to the exact proton measurement. Presumably, the *invariant mass* would be better if the proton, and not the π^- were the missing particle.

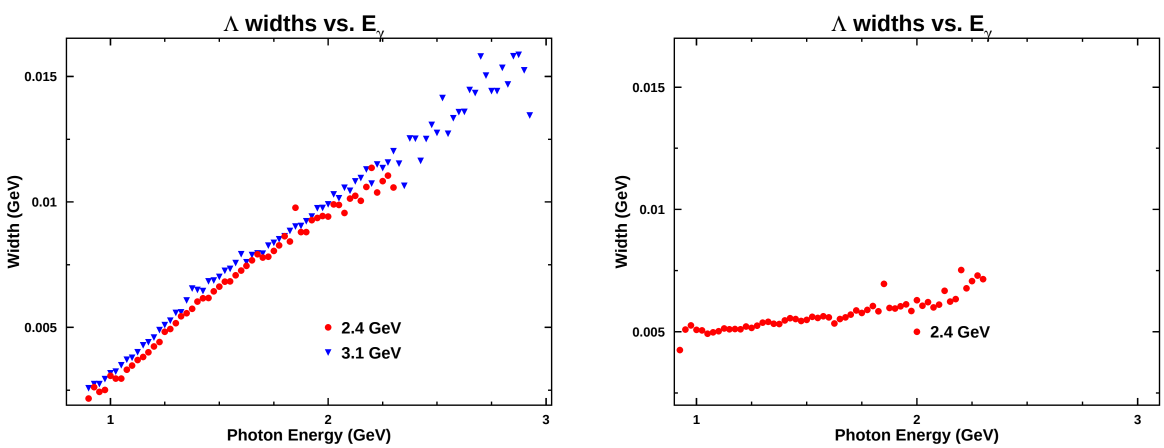

Figure 10: The width, (σ) of the $\Lambda(1107)$ as a function of the photon energy for data from the g1c run period. The upper plot is the *missing mass* width for the Λ in both the $2.4\,GeV$ and $3.1\,GeV$ data sets. The lower plots if the width of the Λ as computed using the *invariant mass* technique for the $2.4\,GeV$ data set.

Summary

We have examined the procedure for determining the mass of unstable particles via the *missing mass* technique. We have shown that in the case where the Q-value of the decay is small relative to the mass of the missing a particle, a significantly better measurement of both the mass and width can be made by using the modified *invariant mass* procedure. In all case, the best values are obtained using a kinematic fit to the data.

References

[1] Mike Williams and Curtis A. Meyer, **Kinematic Fitting in CLAS**, CLAS-note 2003-017, November, 2003.

[2] C. J. Batty *et al.*, Nucl. Instrum. Methods **137**, 179, (1976).