About the Resolution in Θ^+ Mass Reconstruction A MonteCarlo Simulation

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The width of the distribution of the Θ^+ mass reconstructed from the analysis of the reaction $\gamma p \to \pi^+\pi^- K^+(n)$ has been studied as a function of the various parameters including the beam energy and the momentum resolution of reconstructed particles. The two methods based on Missing Mass and Invariant Mass evaluation are discussed and compared. Using a Monte Carlo simulation we proved that the Missing Mass procedure works better at lower beam energy and the Invariant Mass procedure is more effective at higher energies.

I. INTRODUCTION

The recent analysis note by C.A. Meyer et *al.*[1] pointed out that the width of the distribution of the reconstructed Θ^+ mass in the reaction $\gamma d \to pnK^+K^-$ strongly depends on the procedure used to evaluate it. The authors showed that, by evaluating the invariant mass of the system nK^+ after having assigned the correct mass to all detected and intermediate particle, the width of the Θ^+ mass distribution is minimized. In this CLAS note we report the study performed with a toy Monte Carlo, of the photoproduction on the proton of the Θ^+ and the reconstruction on the mass when the three mesons π^+, π^- and K^+ are detected. We assumed that the Θ^+ is photoproduced in the reaction $\gamma p \to \Theta^+\bar{K}^0$ with the two particles decaying into K^+n and $\pi^+\pi^-$ respectively. The detected final state is $\pi^+\pi^-K^+$ with a missing neutron. From the information on the detected particle it is however possible to reconstruct the momentum of the missing neutron. Using the results of the Monte Carlo we compared the reconstructed Θ^+ mass obtained with two different procedures: as Missing Mass in the reaction $\gamma p \to \bar{K}^0X$ and as Invariant Mass of the system K^+n. We studied the dependence of the width of the Θ^+ reconstructed mass distribution from several parameters like the photon energy, the momentum of the detected particles and the emission angle of the $\bar{K}^0$ meson in the hadronic frame. The result is that the Θ^+ mass distribution obtained as the Invariant Mass of the system K^+n is generally narrower, but at low energy, close to the reaction threshold, the Missing Mass obtained from the reaction $\gamma p \to \bar{K}^0X$ is narrower. This difference is larger and in favour of the Missing Mass, when the $\bar{K}^0$ meson is produced at backward angles in the hadronic center of mass.

II. THE MONTECARLO DESCRIPTION

The number of involved variable is too large to attempt an analytical approach. However using the analytical approach it is possible to draw the general features of the problem. In the case of the Invariant Mass evaluation, the Θ^+ mass can be obtained from the 4-momenta of the final particles as:

$$M^2_{\Theta^+} = \left|n_\mu + K^+_\mu\right|^2, \tag{1}$$

from which one obtain in the non-relativistic limit:

$$M^2_{\Theta^+} = (M_n + M_{K^+})^2 + p_n^2\frac{M_{K^+}}{M_n} + p^2_{K^+}\frac{M_n}{M_{K^+}} + \frac{p_n^2 p^2_{K^+}}{M_n M_{K^+}} - 2p_n p_{K^+} cos\theta_{nK^+}, \tag{2}$$

where the neutron momenta is a function of the 3-momenta of all detected particles (π^+, π^- and K^+):

$$p_n^2 = \left|\vec{E}_\gamma + \vec{p}_{\pi^+} + \vec{p}_{\pi^-} + \vec{p}_{K^+}\right|^2. \tag{3}$$

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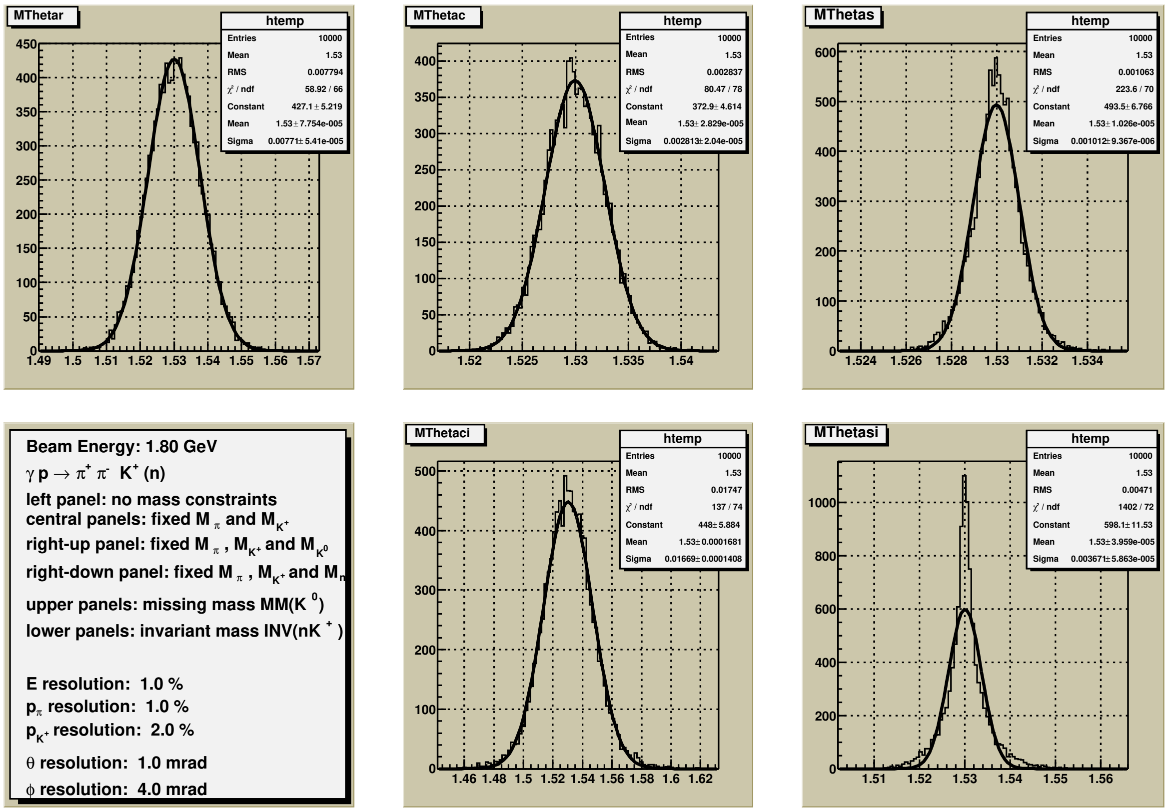

FIG. 1: Typical output of the toy Monte Carlo. Top-right panel corresponds to Θ^+ mass reconstructed as Missing Mass in the reaction $\gamma p \to \bar{K}^0 X$; bottom-right panel corresponds to Θ^+ mass reconstructed as Invariant Mass in the decay $\Theta^+ \to K^+ n$. From the distributions the RMS is extracted.

In the case of the Missing Mass evaluation, the Θ^+ mass can be obtained from the 4-momenta of the initial and final particles as:

$$M^2_{\Theta^+} = \left| E_{\gamma_\mu} + p_\mu - \bar{K}^0_\mu \right|^2, \tag{4}$$

from which one obtain in the non-relativistic limit:

$$M^2_{\Theta^+} = (M_p - M_{\bar{K}^0})^2 + 2(M_p - M_{\bar{K}^0})E_\gamma + 2p_{\bar{K}^0 z}E_\gamma - p^2_{\bar{K}^0}\frac{E_\gamma + M_p}{M_{\bar{K}^0}}, \tag{5}$$

where the $\bar{K}^0$ momenta is a function of the 3-momenta of the two detected pions.

We could therefore conclude that to evaluate the Missing Mass it is necessary to use only the two pions 3-momenta while in the Invariant Mass also the K^+ 3-momentum contribute. Another interesting feature is that in our kinematics

$$M_n + M_K = 0.940GeV + 0.494GeV \approx 0.94 M_\Theta, \tag{6}$$

and

$$M_n - M_K = 0.940GeV - 0.494GeV \approx 0.30 M_\Theta. \tag{7}$$

That is in Eq.2 the masses of the neutron and K^+ almost saturate the sum, while in Eq.5 the masses of the proton and $\bar{K}^0$ contribute only by 1/9 to the sum. This does not imply that the Invariant Mass would provide a narrower mass distribution because of the small value of the difference

$$p^2_n \frac{M_{K^+}}{M_n} + p^2_{K^+}\frac{M_n}{M_{K^+}} + \frac{p^2_n p^2_{K^+}}{M_n M_{K^+}} - 2p_n p_{K^+} cos\theta_{nK^+}, \tag{8}$$

in fact the fluctuations sum in quadrature, and do not cancel out. The most important feature is that in order to saturate the sum in Eq.5, the contributions of the particle 3-momenta

$$2p_{\bar{K}^0 z}E_\gamma - p^2_{\bar{K}^0}\frac{E_\gamma + M_p}{M_{\bar{K}^0}}, \tag{9}$$

and their uncertainties are linearly amplified by the photon energy E_γ:

It is clear that at large beam energy the resolution obtained with the invariant mass should be better because there are no explicit contribution from E_γ. On the contrary it is not evident what is the relative behaviour at threshold.

We wrote a simple C^{++} code based on ROOT to simulate the reaction $\gamma p \to \Theta^+\bar{K}^0$. The Θ^+ is generated isotropically in the hadronic mass frame, the three mesons (π^+, π^- and K^+) that originated from the decay are boosted to the Laboratory frame and their momentum are rotated accordingly to the direction of the primary particles. The energy is determined from the value of the 3-momentum and the particle mass. The beam energy is assumed fixed and the mass of the Θ^+ is set to 1.53 GeV. The module and the direction of the 3-momenta of the three detected mesons are modified with gaussian distributions with sigma values close to the CLAS resolution assuming a simplified momentum dependence, that is $\sigma_\theta = 1$ mrad for the θ angle, $\sigma_\phi = 4$ mrad for the ϕ angle and $\sigma_p = \alpha p$ for the momentum. The α_π and α_K parameters (for pion and Kaon respectively) range between 0.5 % and 2 %. We also run with the momentum dependence reported in the CLAS detector paper[2] without appreciable differences. The Θ^+ mass is evaluated in two ways: a) as Missing Mass of the reaction $\gamma p \to \bar{K}^0 X$ setting the $\bar{K}^0$ mass to the correct value; b) as Invariant Mass of the system $K^+ n$ setting the neutron mass to the correct value. From the typical output of the Monte Carlo as reported in Fig.1 it is possibile to draw the main features of the two reconstruction procedures:

1. the top-left panel shows the reconstructed Missing Mass when no constraints are applied to the mass of the reconstructed particles;

2. the top-central panel shows the reconstructed Missing Mass when the mass of the three mesons (π^+, π^- and K^+) are constrainted to the Particle Data value, in this case the width of the distribution is smaller than in the original one;

3. the top-left panel shows the reconstructed Missing Mass when also the mass of the intermediate $\bar{K}^0$ is constrainted to the Particle Data value, as a result the width of the distribution becomes smaller than the previous case;

4. the bottom-central panel shows the reconstructed Invariant Mass when the mass of the three mesons (π^+, π^- and K^+) are constrainted to the Particle Data value, in this case the width of the distribution is larger than the original;

5. the bottom-left panel shows the reconstructed Invariant Mass when also the mass of the intermediate neutron is constrainted to the Particle Data value, as a result the width of the distribution becomes smaller.

In all cases the average value of the reconstructed Θ^+ mass coincide with the generated one.

The σ of the Θ^2 mass distribution has been studied as a function of the beam energy, the momentum resolution and the $\bar{K}^0$ emission angle in the hadronic frame. Let us first present separately the results for the Missing Mass and the Invariant Mass procedures. We define σ_{MM} the width of the mass distribution when the Missing Mass procedure is applied and σ_{INV} the width of the mass distribution when the Invariant Mass procedure is applied.

A. The Missing Mass Procedure and the σ_{MM} Width

In Fig.2 the width σ_{MM} of the Θ^+ mass distribution is reported as a function of the beam energy for three different values (0.5 %, 1 % and 2 %) of the α_π 3-momentum resolution. The σ_{MM} value linearly increases with the beam energy and is larger for larger α_π values. In Fig.3 we reported the previous values divided by the corresponding α_π value. In this case all results align along the same line showing that the general beaviour of σ_{MM} when cuts have been applied to the $\bar{K}^0$ emission angle, can be written as:

$$\sigma_{MM} \approx 100[0.9 + 5.22(E_\gamma - E_{th})]\alpha_\pi \ (MeV), \tag{10}$$

being $E_{th} = \frac{(M_{\Theta^+} + M_{\bar{K}^0})^2 - M_p^2}{2M_p}$ the energy threshold of the Θ^+ photoproduction.

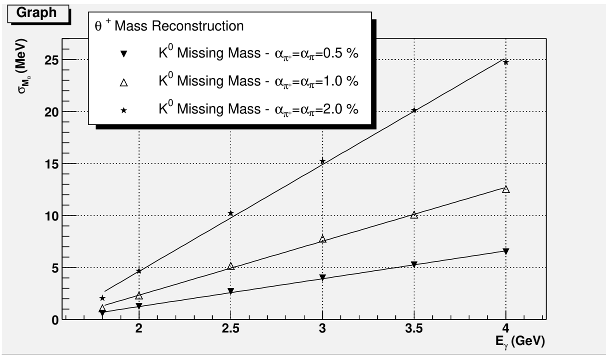

FIG. 2: σ_{MM} of the Θ^+ mass distribution when evaluated with the missing mass procedure in the reaction $\gamma p \rightarrow \bar{K}^0 X$. The value of σ_{MM} is reported as a function of the beam energy E_γ for three different values of the α_π momentum resolution. No cuts have been applied to the $\bar{K}^0$ emission angle.

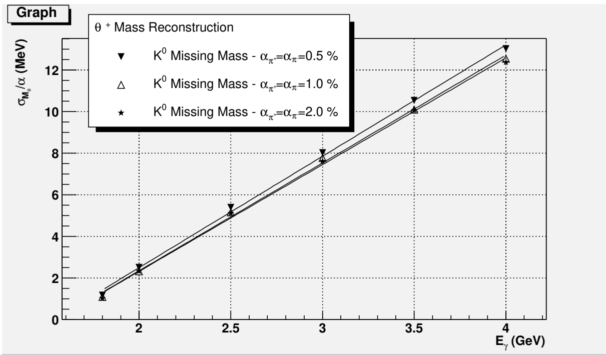

FIG. 3: σ_{MM} of the Θ^+ mass distribution when evaluated with the missing mass procedure in the reaction $\gamma p \rightarrow \bar{K}^0 X$. The value of σ_{MM} divided by the corresponding α_π is reported as a function of the beam energy E_γ for three different values of the α_π momentum resolution. No cuts have been applied to the $\bar{K}^0$ emission angle.

A similar behaviour has been obtained applying the kinematical cut $cos\theta_{\bar{K}^0} < -0.35$, based on the assumption that the Θ^+ is produced forward in the hadronic mass frame. The result is shown in Fig.4. In this case the slope is smaller and the expression that describes the behaviour is given approximately by:

$$\sigma_{MM}(cos\theta_{\bar{K}^0} < -0.35) \approx 440(E_\gamma - E_{th})]\alpha_\pi \ (MeV). \tag{11}$$

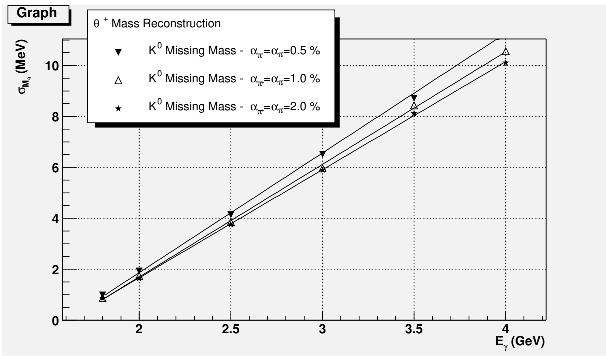

FIG. 4: σ_{MM} of the Θ^+ mass distribution when evaluated with the missing mass procedure in the reaction $\gamma p \rightarrow \bar{K}^0 X$. The value of σ_{MM} divided by the corresponding α_π is reported as a function of the beam energy E_γ for three different values of the α_π momentum resolution. The cut $cos\theta_{\bar{K}^0} < -0.35$ has been applied to the $\bar{K}^0$ emission angle.

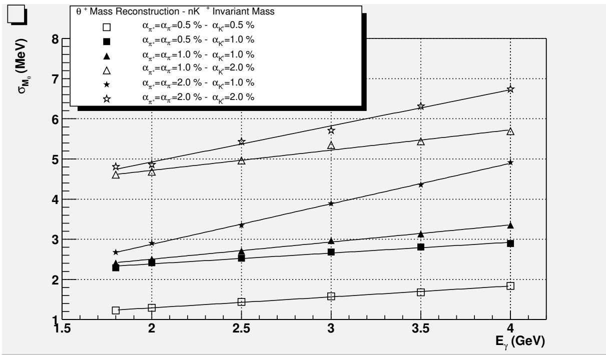

FIG. 5: σ_{INV} of the Θ^+ mass distribution when evaluated with the invariant mass procedure in the reaction $\Theta^+ \to nK^+$. The value of σ_{INV} is reported as a function of the beam energy E_γ for three different values of the α_π and three values of the α_K momentum resolutions. The results are independent from the cut to the $\bar{K}^0$ emission angle.

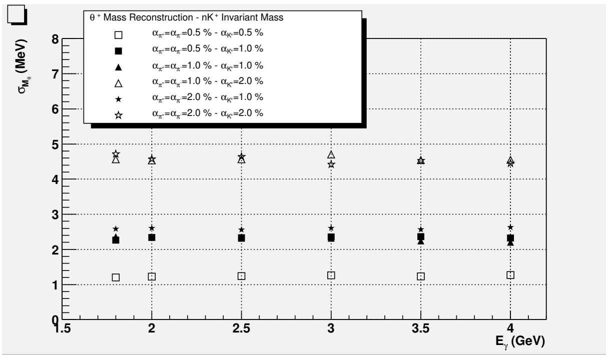

FIG. 6: σ_{INV} of the Θ^+ mass distribution when evaluated with the invariant mass procedure in the reaction $\Theta^+ \to nK^+$. The contribution of the pion momentum resolution is subtracted according to the expression $\sigma_{INV} - 0.5(E_\gamma - E_{th})\alpha_\pi$. The value of σ_{INV} is reported as a function of the beam energy E_γ for three different values of the α_π and two values of the α_K momentum resolutions. The results are independent from the cut to the $\bar{K}^0$ emission angle.

B. The Invariant Mass Procedure and the σ_{INV} Width

In Fig.5 the width σ_{INV} of the Θ^+ mass distribution is reported as a function of the beam energy for three different values (0.5 %, 1 % and 2 %) of α_π and three values (0.5 %, 1 % and 2 %) of α_K. The σ_{INV} values linearly increase with the beam energy, but the slope is not the same for all data sets. It is larger for larger α_π and α_K values and the dependence from α_K is less pronounced. To show the dependence from single contribution we first report in Fig.6 the previous values after subtracting the contribution of α_π evaluated as $\sigma_{INV} - 50(E_\gamma - E_{th})\alpha_\pi$. In this case all results shows the same flat behaviour. The slope is now only function of α_K as shown in Fig.7. Finally the general beaviour of σ_{INV} can be summarized as:

$$\sigma_{INV} \approx 230\alpha_K + 50(E_\gamma - E_{th})\alpha_\pi \ (MeV). \tag{12}$$

It is worth noting that we verified that σ_{INV} does not depend on the cut on $cos\theta_{\bar{K}^0}$.

C. Comparison

We can now compare the σ_{INV} and σ_{MM} previously obtained with different procedures and cuts. The three equations 10, 11 and 12 are plotted in Fig.8 for $\alpha_\pi = \alpha_K = 1\%$. It is evident that the Invariant Mass procedure is more effective at larger beam energy, but close to threshold ($E_\gamma < 2$ GeV) the Missing Mass procedure provides a smaller width. In particular, when the angular cut $cos\theta_{\bar{K}^0} < -0.35$ is applied, the Missing Mass procedure is more effective for beam energies up to $E_\gamma = 2.3$ GeV.

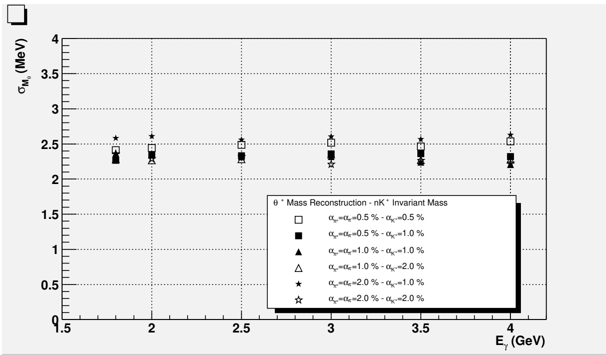

FIG. 7: σ_{INV} of the Θ^+ mass distribution when evaluated with the invariant mass procedure in the reaction $\Theta^+ \to nK^+$. The contribution of the Kaon momentum resolution is subtracted according to the expression $(\sigma_{INV} - 0.5(E_\gamma - E_{th})\alpha_\pi)/\alpha_K$. The value of σ_{INV} is reported as a function of the beam energy E_γ for three different values of the α_π and two values of the α_K momentum resolutions. The results are independent from the cut to the $\bar{K}^0$ emission angle.

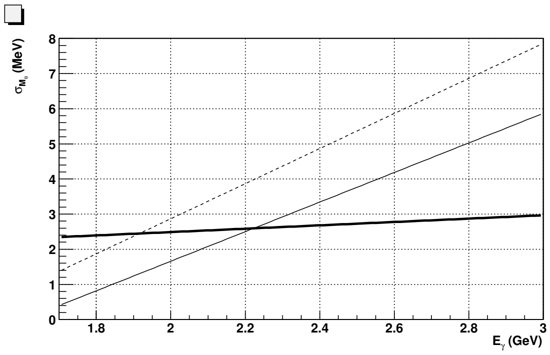

FIG. 8: Comparison between the resolutions obtained with different procedures: thick continuous line - Invariant Mass; dashed line - Missing Mass; thin continuous line - Missing Mass with $cos(\theta_{\bar{K}^0}) < -0.35$.

III. CONCLUSIONS

We studied with a toy Monte Carlo the effect of momentum resolution on the width of the Invariant Mass and Missing Mass distribution when the Θ^+ state is investigated in the reaction $\gamma p \to \pi^+\pi^- K^+(n)$. We showed that the width of the Θ^+ mass distribution is generally smaller when the Invariant Mass of the system nK^+ is evaluated, but at beam energies close to the threshold the Missing Mass of the reaction $\gamma p \to \bar{K}^0 X$ works better. The reason of this behaviour is that the K^+ 3-momentum uncertainty contributes only to the σ_{INV} and the pion 3-momentum uncertainty is amplified by the beam energy E_γ and the increment is larger when the σ_{MM} is evaluated.

[1] C.A. Meyer, M. Williams and R. Bradford, CLAS-NOTE 04-008.
[2] B. Meking and the CLAS Collaboration, *Nucl. Instrum. Methods* A **502**, (2003) 513.