The Choice of Analyzer and Isospin Considerations for gamp Keyfiles

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Abstract

We describe how to generate amplitudes with the program gamp using user-written keyfiles. We point out that in the keyfile the ordering of the final state particles is important in order not to introduce a phase difference between waves and can be understood using an isospin argument. A specific example will be given.

1 Introduction

The program gamp allows one to generate decay amplitudes for partial wave analyses (PWA) for both baryons and mesons in the final state. The program makes use of the isobar model, in which the breakup of a produced state X into the detected final state particles is assumed to proceed via successive two-body decays until the final state is reached. Fig. 1 gives an example for such a decay chain where a meson resonance X breaks up into three pions via the ρ isobar.
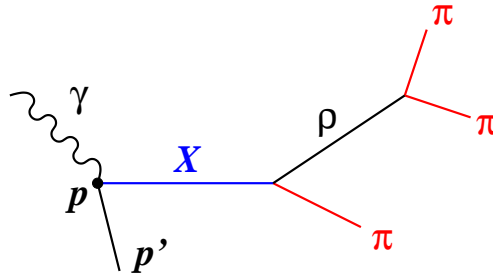

Fig. 1. Decay of a resonance X into three pions via a ρ isobar.

Before going into details, it is important to point out that a physics amplitude for a particular decay chain involves several pieces that as a whole, must be written in a self-consistent fashion. In particular, there is a piece that reflects the isospins of the mother, daughter and granddaughter particles. There is a piece that describes the decay angular distributions and the resonsnace pieces of intermediate particles. Finally, there are spinors that are needed for final state particles that have non-zero spin. Any of these can in principle be written down in the abscence of information on the others, but there are consistency rules that must be applied when putting the amplitudes together. The purpose of this note is to lay out those consistency rules to facilitate the writing of proper physics amplitudes.

2   The gamp keyfile

gamp takes as input a user-written keyfile, which allows rather general final states to be analyzed. An example of a keyfile for a $J^P = 1^-$ resonance X produced in a t-channel process is given below. The produced resonance X first decays into a ρ^0 and a π^0 in a relative P wave $(L = 1^1)$. The ρ^0 then decays into $\pi^+\pi^-$ with orbital angular momentum $l = 1$, thus populating the final state $\pi^+\pi^-\pi^0$.

```
mode=ascii;
channel=t;
J=2 P=-1 M=2 {
    rho(770)[1] {
            pi+[1]
            pi-[1]
            2
    }
    pi0[1]
    2
}
;
```

_____

[1] Note, that in the keyfile and gamp all angular momenta in the process are multiplied by a factor of 2. This allows to always deal with integers, even when baryons are involved.

3 Calculation of decay amplitudes

Decay amplitudes for a resonance are calculated in the rest frame of that resonance, where the axes are always aligned in a unique manner. For example the amplitude of the decay of X into $\rho\pi$ is determined in the so-called Gottfried-Jackson (GJ) frame, where $\hat{z}_{GJ}$ is defined by the direction of the beam momentum and $\hat{y}_{GJ}$ is chosen to be perpendicular to the production plane of the reaction [2]. The $\hat{x}_{GJ}$-axis is then given by defining a right handed coordinate system: $\hat{x}_{GJ} = \hat{y}_{GJ} \times \hat{z}_{GJ}$. The decay angles, θ_{GJ} and ϕ_{TY}, are distributed depending on the spin of the X and are determined from the ρ momentum in the GJ frame.

In a similar manner the amplitude for the decay of the ρ into $\pi^+\pi^-$ is calculated in the helicity frame, where $\hat{z}_h$ is in the direction of the ρ momentum before the boost into its rest frame and $\hat{y}_h$ is perpendicular to both, $\hat{z}_{GJ}$ and $\hat{z}_h$. Again $\hat{x}_h$ is defined by requiring a right handed coordinate system: $\hat{x}_h = \hat{y}_h \times \hat{z}_h$. In order to determine decay angles one has to choose one of the two pions from the ρ decay as the analyzer and θ_h and ϕ_h are then given by the angles of the momentum of the analyzer in the helicity frame. However, this causes an ambiguity since one can choose either one of π^+ or π^- as the analyzer. Since the the helicity frame is the rest frame of the ρ, the pions are decaying back-to-back and the decay angles are related by

$$\theta_h^+ = \pi - \theta_h^- \qquad \phi_h^+ = \pi + \phi_h^-. \tag{1}$$

In this case the `gamp`-generated amplitude will result in a relative minus sign if the order of the π^+ and π^- from the ρ decay is reversed in above example keyfile [3]. This is due to the symmetry of the D functions from the decay of the $\rho \to \pi\pi$ in the calculation of the amplitude for the example given above

$$D^1_{1\,0}{}^*(\phi_h^+, \theta_h^+, 0) = D^1_{1\,0}{}^*(\pi + \phi_h^-, \pi - \theta_h^-, 0) = -D^1_{1\,0}{}^*(\phi_h^-, \theta_h^-, 0)$$

$$D^1_{0\,0}{}^*(\phi_h^+, \theta_h^+, 0) = D^1_{0\,0}{}^*(\pi + \phi_h^-, \pi - \theta_h^-, 0) = -D^1_{0\,0}{}^*(\phi_h^-, \theta_h^-, 0) \tag{2}$$

$$D^1_{-1\,0}{}^*(\phi_h^+, \theta_h^+, 0) = D^1_{-1\,0}{}^*(\pi + \phi_h^-, \pi - \theta_h^-, 0) = -D^1_{-1\,0}{}^*(\phi_h^-, \theta_h^-, 0)$$

It is worth pointing out that the flip of the sign is due to the fact that the ρ meson has spin 1. For example. the decays of $a_{0,2}(980) \to \pi\eta$ are independent of the choice of analyzer because in this case the D functions are symmetric under the transformation given in Eq. 1. Interested readers may verify this by themselves, by using the D functions given in Appendix A. In the case

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[2] The production plane is defined by the direction of the momentum of X and the beam momentum in the center of mass system (*cms*).

[3] The order in the keyfile determines which of the two pions will be chosen as the analyzer.

of baryons the choice of analyzer may result in a factor i, because of the half-integer spin.

The relative minus signs appearing in Eq. 2 are disturbing because they may become important if one has several interfering amplitudes in the PWA fit and it is not immediately clear which phase will be picked for each wave by `gamp`. As an example we consider again the reaction

$$X \to \pi^+ \pi^- \pi^0 \tag{3}$$

The final state can be reached in three different ways with an intermediate ρ meson

$$
\begin{aligned}
X &\to \rho^0 \pi^0 \to (\pi^+ \pi^-) \pi^0 \\
X &\to \rho^- \pi^+ \to (\pi^- \pi^0) \pi^+ \\
X &\to \rho^+ \pi^- \to (\pi^+ \pi^0) \pi^-
\end{aligned}
\tag{4}
$$

It is not clear which phase each individual ρ decay will pick up, since in each of the three cases the analyzing π can be chosen arbitrarily.

The intensity in this case may then be written[4]

$$I = \left| V_0 \Psi_0 + V_- \Psi_- + V_+ \Psi_+ \right|^2 \tag{5}$$

where V_i are the production and Ψ_i the decay amplitudes, with $i = 0, -, +$, corresponding to the waves containing a ρ°, ρ^-, or a ρ^+, respectively. In principle it would be possible to absorb a minus sign from a decay amplitude Ψ_l into the production amplitude V_l of that wave and still get the same intensity as without the minus sign. However, this is only true if the production amplitudes were unrelated to each other. If one knows, for example from isospin splitting, how the production strengths should be related to each other, then a realtive minus sign between any two waves becomes problematic, since it will result in a wrong interference between these two waves.

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[4] For simplicity we assume that only these three decay modes are present and only one state X (for example $J^P = 1^-$) is produced, decaying into the three $\rho\pi$ states.

4 Isospin

A solution to the above problem is offered by considering the isospin of the problem. The decay of the ρ obeys the following isospin symmetry

$$| \rho^+ \rangle = \frac{1}{\sqrt{2}} \left[| \pi^+\pi^0 \rangle - | \pi^0\pi^+ \rangle \right]$$

$$| \rho^- \rangle = \frac{1}{\sqrt{2}} \left[| \pi^0\pi^- \rangle - | \pi^-\pi^0 \rangle \right] \tag{6}$$

$$| \rho^0 \rangle = \frac{1}{\sqrt{2}} \left[| \pi^+\pi^- \rangle - | \pi^-\pi^+ \rangle \right]$$

From this it is clear that there is a flip in the relative sign between the three decays if the ordering of the decay is not taken into account properly. For example generating two decays of the $J^P = 1^-$ wave in the following order

$$X \to \rho^+\pi^- \to (\pi^+\pi^0)\pi^-$$

$$X \to \rho^-\pi^+ \to (\pi^-\pi^0)\pi^+ \tag{7}$$

would have a relative minus sign as is evident from the first two lines from Eq. 6, whereas

$$X \to \rho^+\pi^- \to (\pi^+\pi^0)\pi^-$$

$$X \to \rho^-\pi^+ \to (\pi^0\pi^-)\pi^+ \tag{8}$$

would not exhibit this behavior.

Therefore it is crucial that, when writing keyfiles, one pays very close attention to the ordering of the different π in the reaction. Keeping the order of the pions in the keyfile the same as in the order for the isospin will ensure that no relative minus sign between the decay modes is introduced. It is important that this order is maintained throughout all of the amplitude generation. The overall sign is not important, therefore one can choose to follow either the plus or the minus signs in Eq. 6.

Below we give as an example the keyfiles to calculate the three waves from the reaction in Eqs. 3 and 4 for a $J^P = 1^-$ $\rho\pi$ P wave.

5 Example

$$X(J^P = 1^-) \rightarrow \rho^+ \pi^- \rightarrow (\pi^+ \pi^0) \pi^-$$

```
mode=ascii;
channel=t;
J=2 P=-1 M=2 {
    rho(770)[1] {
            pi+[1]
            pi0[1]
            2
    }
    pi-[1]
    2
};
```

$$X(J^P = 1^-) \rightarrow \rho^- \pi^+ \rightarrow (\pi^- \pi^0) \pi^+$$

```
mode=ascii;
channel=t;
J=2 P=-1 M=2 {
    rho(770)[1] {
            pi0[1]
            pi-[1]
            2
    }
    pi+[1]
    2
};
```

$$X(J^P = 1^-) \rightarrow \rho^0 \pi^0 \rightarrow (\pi^+ \pi^-) \pi^0$$

```
mode=ascii;
channel=t;
J=2 P=-1 M=2 {
    rho(770)[1] {
            pi+[1]
            pi-[1]
            2
    }
    pi0[1]
    2
};
```

Appendix

A D functions

A.1 Spin 0 particles (e.g. a_0)

$$D^0_{0\ 0}(\alpha, \beta, \gamma) = const. \tag{A.1}$$

A.2 Spin 1 particles (e.g. ρ)

$$D^1_{1\ 0}(\alpha, \beta, \gamma) = e^{-i\alpha} \left[-\frac{1}{\sqrt{2}} \sin\beta \right]$$

$$D^1_{0\ 0}(\alpha, \beta, \gamma) = \cos\beta \tag{A.2}$$

$$D^1_{-1\ 0}(\alpha, \beta, \gamma) = e^{i\alpha} \left[\frac{1}{\sqrt{2}} \sin\beta \right]$$

A.3 Spin 1 particles (e.g. a_2)

$$D^2_{1\ 0}(\alpha, \beta, \gamma) = e^{-i\alpha} \left[-\sqrt{\frac{3}{2}} \sin\beta \cos\beta \right]$$

$$D^2_{0\ 0}(\alpha, \beta, \gamma) = \frac{3}{2} \cos^2\beta - \frac{1}{2} \tag{A.3}$$

$$D^2_{-1\ 0}(\alpha, \beta, \gamma) = e^{i\alpha} \left[-\sqrt{\frac{3}{2}} \sin\beta \cos\beta \right]$$