CLAS-NOTE 90-015

Geometric Definition of Distorted Hexagonal Cells for CLAS Wire Chambers

by

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A. Purpose

My objective is to create cells which closely resemble hexagons, are proportionally identical from one layer to the next, and fit perfectly with the layers above and below them.

B. Introduction

There are three regions in the CLAS which will contain wire chambers. The third region, outside of the CLAS coil, will have "hexagonal" cells. For the third region the CEBAF Physics Staff previously decided that the hexagons would lie on arcs in the midplane of the wire chamber, that there would be 192 cells per layer, and that groups of wires would lie on radial lines from the center of the arc, not the target center of the CLAS. Please note that the cell pattern on the endplates of region III are distorted hexagons which are arrayed in an elliptical pattern.

C. Assumptions

Some assumptions must be made about the geometry. I assume within one cell the distance from the center to each corner will be identical. I also assume the sides of the hexagon will have the same length as the center to corner distance. Therefore 8 out of 12 of the distances will be exactly the same.

To determine that distance I also approximate the arc length from sense wire to sense wire is the same as the chord length. See Figure 1.

$$\text{chord length} = \left(\frac{\theta_2 - \theta_1}{191}\right)(R_0)\left(\frac{2\pi}{360^0}\right)$$

$$\text{center to corner distance} = l = \frac{1}{2}\ \text{cord length}\ \frac{\sin 90^\circ}{\sin 60^\circ}$$

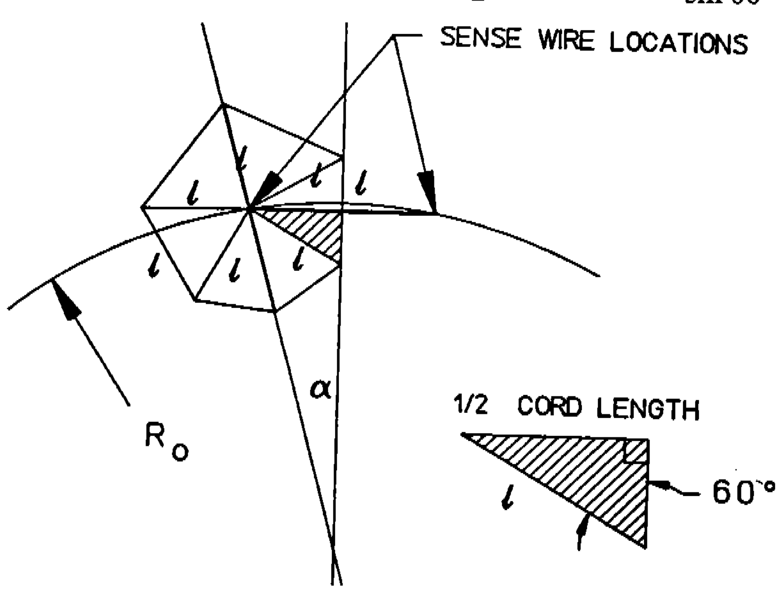

Figure 1. Center to corner distance (l) for a single cell.

D. Relationship From One Layer to the next - A Derivation

The relationship from one layer to the next is dependent on alpha. Alpha is dependant on the forward and backward limits, and the radius to the center of each cell from a distant point (which is the center of the superlayer arcs.) Alpha is defined as $(\theta_2 - \theta_1)/(193.5)\cdot(2)$. See Figure 2.

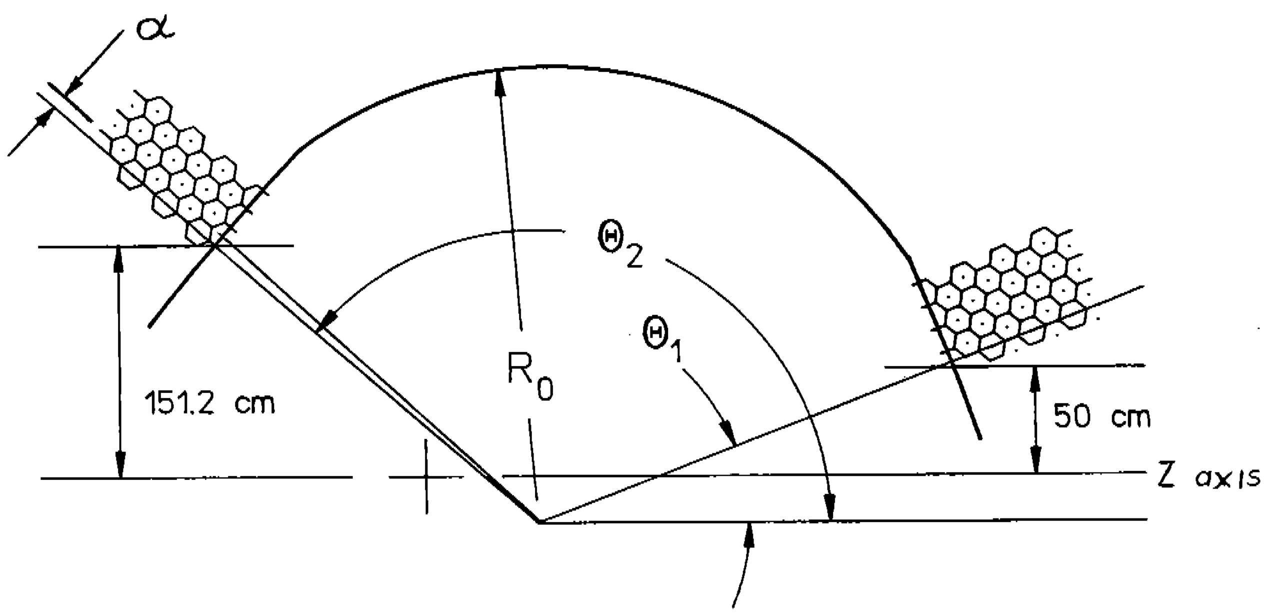

Figure 2. Definition of alpha.

Given the three assumptions and the definition of alpha, a mathematical relationship from one cell layer to the next can be defined. See Figure 3.

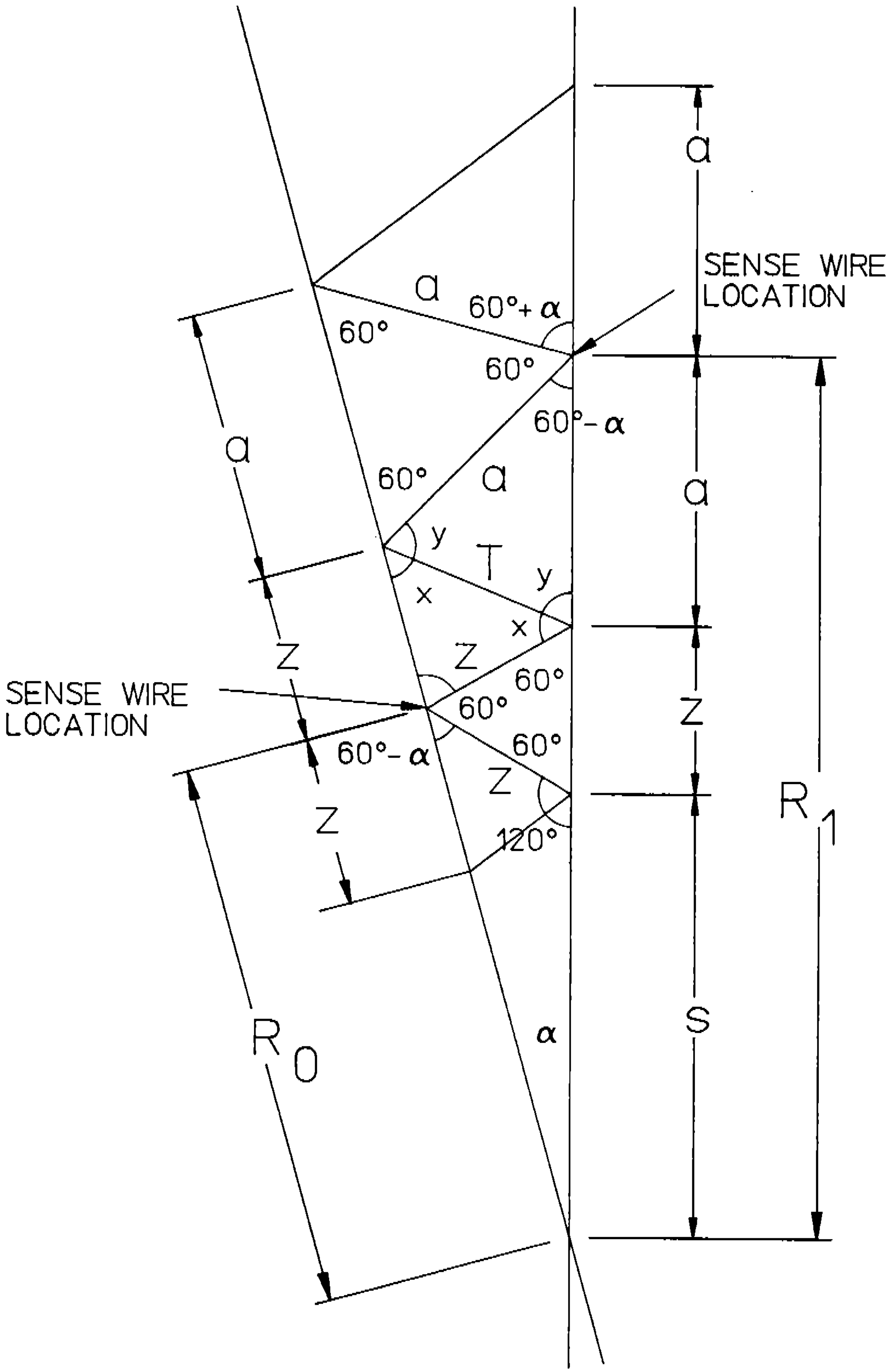

Figure 3. Sense wire to sense wire geometric relationship.

$$(1) \qquad R_1 = s + z + a$$

$$(2) \qquad \frac{R_0}{\sin 120^\circ} = \frac{s}{\sin(60^\circ - \alpha)} = \frac{z}{\sin \alpha}$$

$$\frac{T}{\sin(60^\circ + \alpha)} = \frac{z}{\sin x}$$

$$x = \frac{180^\circ - (60^\circ + \alpha)}{2} = \frac{120^\circ - \alpha}{2}$$

$$\frac{T}{\sin(60^\circ - \alpha)} = \frac{a}{\sin y}$$

$$y = \frac{180 - (60 - \alpha)}{2} = \frac{120 + \alpha}{2}$$

$$T = \frac{z}{\sin(\frac{120^\circ - \alpha}{2})} \sin(60^\circ + \alpha) = \frac{a}{\sin(\frac{120^\circ + \alpha}{2})} \sin(60^\circ - \alpha)$$

$$(3) \qquad a = z \left(\frac{\sin(60^\circ + \alpha) \sin\left(\frac{120^\circ + \alpha}{2}\right)}{\sin\left(\frac{120^\circ - \alpha}{2}\right) \sin(60^\circ - \alpha)} \right)$$

To get the relationship between one sense wire radius and the next I combine equations 1, 2, and 3.

$$R_1 = z \frac{\sin(60^\circ - \alpha)}{\sin \alpha} + z + z \left(\frac{\sin(60^\circ + \alpha) \sin\left(\frac{120^\circ + \alpha}{2}\right)}{\sin(\frac{120^\circ - \alpha}{2}) \sin(60^\circ - \alpha)} \right)$$

$$z = R_0 \left(\frac{\sin \alpha}{\sin 120^\circ} \right)$$

$$(4) \qquad R_1 = R_0 \left(\frac{\sin \alpha}{\sin 120^\circ} \right) \left(1 + \frac{\sin(60^\circ - \alpha)}{\sin \alpha} + \frac{\sin(60^\circ + \alpha) \sin\left(\frac{120^\circ + \alpha}{2}\right)}{\sin\left(\frac{120^\circ - \alpha}{2}\right) \sin(60^\circ - \alpha)} \right)$$

Let

$$K = \left(\frac{\sin \alpha}{\sin 120^\circ} \right) \left(1 + \frac{\sin(60^\circ - \alpha)}{\sin \alpha} + \frac{\sin(60^\circ + \alpha) \sin\left(\frac{120^\circ + \alpha}{2}\right)}{\sin\left(\frac{120^\circ - \alpha}{2}\right) \sin(60^\circ - \alpha)} \right)$$

then the relationship is

$$R_1 = R_0 K^n \tag{5}$$

E. Summary

It is possible to create distorted hexagonal cells which are proportionally identical from one layer to the next, and fit perfectly with the layers above and below them.

The relationship is

$$R_n = R_o k^n$$

where

$$K = \left(\frac{\sin\alpha}{\sin 120^\circ}\right)\left(1 + \frac{\sin(60^\circ - \alpha)}{\sin\alpha} + \frac{\sin(60^\circ + \alpha)\sin((\frac{120^\circ + \alpha}{2})}{\sin(\frac{120^\circ - \alpha}{2})\sin(60^\circ - \alpha)}\right)$$

and

$$\alpha = \frac{(\theta_2 - \theta_1)}{2(\text{number of cells})}$$

F. Current Specifications

Date	Oct 10, 1990
Target Center	(0,0)
Bogdan Center 3	(-70 cm in Y, 48 cm in Z)
Number of active cells in a layer	192
Number of inactive cells in a layer	1 each end
Radial offset from Z axis, forward	50 cm
Radial offset from Z axis, backward	151.2 cm
Radial limit	331.3 cm
R_o (First layer of cells)	Unknown - dependent on inner wall of chamber
Theta 1	unknown - dependent on R_o
Theta 2	unknown - dependent on R_o
α	unknown - dependent on Theta 1 and Theta 2
k	unknown - dependent on α

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