TAGGER ELECTRON BEAM DUMP ACCEPTANCE

Daniel I. Sober
The Catholic University of America
Washington, DC 20064

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1. Introduction

The profile of the electron beam at the tagger electron beam dump depends on the radiation and scattering of the electrons in the radiator foil and the subsequent tranport of electrons through the tagger magnet. As a guide for the final design of the dump, we calculate here the fraction of the incident electron flux which falls outside a given dump aperture.

2. Beam Optics

In the notation of the TRANSPORT program, the important transport matrix elements from the radiator to the dump are R_{12}, R_{34} and R_{16}, defined by

$$x_{dump} = R_{12} \; x'_{rad} + R_{16} \; \delta p/p_0$$

$$y_{dump} = R_{34} \; y'_{rad}$$

The coordinates x_{dump} and y_{dump} are perpendicular to the reference ray in the radial (dispersive) and transverse directions respectively; x'_{rad} and y'_{rad} are the respective angular projections following the radiator.

Table 1 gives the nominal values of these coefficients from the radiator to floor level and from the radiator to the dump entrance (defined arbitarily as 3 meters past the floor level measured along the beam, or approximately halfway down the dump tunnel.) Values are given for the normal radiator position (50 cm from the effective field boundary of the entry edge) and for the maximum radiator distance envisioned for future out-of-plane polarized photon tagging.

At the full-energy orbit, because of proximity to the edge of the magnet, the field gradient index $n \equiv -r_0/B_0 \; dB/dr$ is approximately 0.30 for electron energies up to at least 4 GeV.   At higher

excitations, the gradient becomes larger as the magnet saturates, but the scattering becomes small so that the present considerations become unimportant. The 10-m radiator position would be necessary only in polarization work at high incident energies, where, again, scattering is small. Thus, for all interesting cases, the approximation

$$R_{12} \approx R_{34} \approx 1.6$$

can be used for calculating beam size at floor level.

Table 1

Beam transport coefficients from radiator to floor and radiator to dump (3 meters diagonally below floor) for several different sets of conditions. $n = 0.30$ is a good approximation to the field index for energies up to 4 GeV.

Radiator position	Field index n	To floor level			To dump		
		R_{12} (cm/ mr)	R_{34} (cm/ mr)	R_{16} (cm/ %)	R_{12} (cm/ mr)	R_{34} (cm/ mr)	R_{16} (cm/ %)
0.5 m (normal)	0	1.40	1.57	6.10	1.65	1.87	7.60
	0.30	1.45	1.52	6.17	1.71	1.81	7.69
10 m (maximum)	0	1.86	2.53	6.10	1.99	2.83	7.60
	0.30	2.05	2.32	6.17	2.23	2.57	7.69

3. Multiple and single scattering

The principal contribution to the angular divergence of the full-energy beam after the radiator is scattering of electrons in the radiator. At large angles (greater than ≈ 3.5 times the RMS multiple scattering angle) single Mott scattering dominates over multiple scattering.

The relative importance of multiple and single scattering is independent of energy and of radiator thickness, and depends only on the Z and A of the radiator. The probabilities of scattering by an angle θ are

$$dP_{mult}(\theta)/d\theta = (2/\pi)^{\frac{1}{2}} 1/\theta_0 \exp(-\theta^2/2\theta_0^2) \tag{1}$$

$$dP_{Mott}(\theta)/d\theta \quad = 2\pi \ \sin\theta \ d\sigma/d\Omega \ N_0\rho x/A$$

$$\approx 32\pi \ N_0\rho x/A \ (Z\alpha/2E_0)^2 \ \theta^{-3} \tag{2}$$

where the last expression uses the small-angle approximation and also assumes that the elastic form factor is 1, both of which are valid at the small angles at which the integral of (2) converges. In the above expressions, Θ_0 is the RMS multiple scattering angle:

$$\Theta_0 = (21 \text{ MeV})/E_0 \ (x/X_0)^{1/2} \ , \tag{3}$$

E_0 is the electron energy and x/X_0 is the radiator thickness in radiation lengths.

The fraction of electrons outside a cone of given half-angle θ_{max} turns out to have a simple description if θ is measured in units of the RMS multiple scattering angle Θ_0. Substituting $\theta = (\theta/\Theta_0)\Theta_0$ and using (3), it can be seen that both (1) and (2) are of the form

$$dP/d\theta = x^{1/2} \ E_0 \ f(\theta/\Theta_0) \ . \tag{4}$$

When integrated over a range of angles, the results for the two processes are

$$P_{mult}(\theta > \theta_{min}) = f_1(\theta/\Theta_0) \tag{5}$$

$$P_{Mott}(\theta > \theta_{min}) = (Z^2 X_0/A) \ f_2(\theta/\Theta_0) \tag{6}$$

where the functions f_1 and f_2 are independent of E_0, x, and material.

The function f_1 of Equation (5) can be calculated by numerical integration of Equation (1). The explicit form of Equation (6) is

$$P_{Mott}(\theta > \theta_{min}) = 4\pi N_0\rho(\hbar c\alpha)^2/(21 \text{ MeV})^2 \ (Z^2 X_0/A) \ (\Theta_0/\theta_{min})^2 \ . \tag{7}$$

Figure 1 shows these integrated probabilities for two materials, platinum and carbon. Note that these curves are universal for a given material: the dependences on energy and radiator thickness are hidden inside Θ_0. It is seen that for θ_{max} greater than about $3.5 \ \Theta_0$, the single scattering contribution dominates for all materials.

4. Fraction of beam scattered outside a dump aperture

The fraction of the full-energy electron beam which falls outside any given dump aperture can be estimated by the following steps:

(i) Divide the aperture radius (in a plane normal to the beam) by the larger of the transport coefficients R_{12}, R_{34} to estimate the half-angle θ_{max} of the accepted angular cone at the radiator.

This is an overestimate of the excluded beam if the two coefficients are appreciably different. As seen in Table 1, 1.60 is a good conservative estimate for both R_{12} and R_{34} to the plane of the floor.

(ii) Using the radiator thickness and the incident electron energy, calculate the RMS multiple scattering angle Θ_0 from Equation (3).

(iii) Using θ_{max}/Θ_0 and Equations (5) and (6), find the probability $P(\theta > \theta_{max})$, which is identical to the fraction of the beam which falls outside the dump radius.

Figure 2 shows this fraction as a function of diameter at floor level, for an 800 MeV electron beam incident on carbon and platinum radiators of thickness 10^{-2}, 10^{-3} and 10^{-4} radiation lengths. A rough rule of thumb is that the excluded fraction is $\approx x/X_0$ for a dump diameter of ≈ 20 cm. Note that for the 10^{-2} X_0 radiator at diameters less than ≈ 25 cm, the multiple scattering distribution is playing an appreciable role; everywhere else the calculation is dominated by single scattering.

Remembering that $\Theta_0 \propto 1/E_0$, it is easy to scale from Figure 2 to other energies:

$$P_{outside}(E_2 , D_2) = P_{outside}(E_1, (E_2/E_1)D_1). \tag{8}$$

For example, the fraction outside a 20 cm aperture at $E_0 = 1600$ MeV is equal to the fraction outside a 40 cm aperture at 800 MeV,

Although Figure 2 is calculated for an aperture at floor level, it can also be used to estimate the fraction of beam striking the dump aperture at 3 meters (diagonally) below floor level by using the transport coefficients in Table 1. Since R_{12} and R_{34} to the dump are approximately 19% larger than the corresponding coefficients to the floor, the fraction of beam outside a given dump diameter D_{dump} is given by scaling from Equation (8):

$$P'_{outside}(E, D_{dump}) = P_{outside}(E, D_{floor}=D_{dump}/1.19). \tag{9}$$

5. Radiative tail and limits on beam acceptance

Although Figure 2 implies that increasing the aperture may allow the fraction of beam lost outside the dump to be reduced to an arbitrarily small value, there is a natural limit to how far this can be pushed. In passing through the radiator, the electron beam loses energy by producing bremsstrahlung photons. The distribution of radiated energy per incident electron is given approximately by

$$dE/dk \approx x/X_0. \tag{10}$$

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and the probability per electron of producing a photon of energy k
is given by

$$dP/dk \approx x/X_0 \; 1/k \; .$$

(11)

The "full-energy" electron beam emerging from the radiator thus has
an energy distribution which results in a 1-sided "radiative tail"
in space after passing through the tagger magnet. The fraction of
the beam which falls outside a given dump radius r_{dump} is then given
by

$$P_{radiative}(r > r_{dump}) \approx x/X_0 \int_{kmin}^{kmax} dk/k$$

$$= x/X_0 \; \log(k_{max}/k_{min})$$

(12)

with $k_{min} = (r_{dump}/R_{16}) \cdot E_0$, where R_{16} is the dispersion coefficient of
the magnetic transport system. k_{max} is not well defined, but must
be somwhere between k_{min} and 0.2 E_0, the value at which the tagger
focal plane ends. The dispersion coefficient R_{16} from the radiator
to the floor is approximately 6.1 cm/percent.

Table 2 gives the results for a radiator thickness of 10^{-4} X_0 for
k_{max}/E_0 = 0.1 and 0.2.

Table 2

Fraction of beam outside an aperture at floor
level due to radiative tail from 10^{-4} radiation
length target, using $k_{max} \equiv 0.1 \; E_0$ and 0.2 E_0.

Diameter at floor (cm)	k/E0	$P_{outside}$ at floor per 10^{-4} X_0	
		$k_{max} = 0.1 \; E_0$	$k_{max} = 0.2 \; E_0$
10	0.0082	2.5×10^{-4}	3.2×10^{-4}
20	0.0164	1.8×10^{-4}	2.5×10^{-4}
30	0.0246	1.4×10^{-4}	2.1×10^{-4}
40	0.0328	1.1×10^{-4}	1.8×10^{-4}

The fraction of the beam outside the dump due to radiation is
proportional to the radiator thickness, and decreases much more
slowly with increasing aperture diameter than does the fractional
loss due to scattering (Figure 2).

Comparing with Figure 2, we see that

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(a)   radiation produces more beam outside the aperture than does scattering when the aperture diameter at floor level is greater than $\approx$ 15 cm,

(b)   the fraction of beam falling outside the aperture cannot be made smaller than $\approx x/X_0$, and

(c)   there is very little to be gained in making the aperture diameter greater than $\approx$ 20 cm at floor level or $\approx$ 25 cm at 3 m diagonally below the floor.

6. Choice of radiator thickness

In general, the thinnest possible radiator (10^{-4} X_0 or less) is always preferable in order to keep the multiple scattering contribution to the photon angular distribution small compared to the characteristic bremsstrahlung angular distribution and thus minimize collimation losses.

If the dominant dump-related background is due to the fraction of electrons which arrive outside the dump aperture, then in most cases the background per photon is insensitive to the choice of radiator thickness. A thicker radiator produces proportionately more backgound per incident electron, but requires proportionately fewer incident electrons per produced photon. The exception to this is when the radiator exceeds $\approx 10^{-2}$ X_0 (perhaps to produce an intense untagged bremsstrahlung beam). For such thick radiators, the multiple-scattering angular distribution contributes significantly (the upper pair of curves in Figure 2). In this case, the background per produced photon will increase with radiator thickness, and it would be preferable to increase the beam current and hold the radiator thickness to a few times 10^{-3} X_0.

7. Conclusions

The fraction of the tagger full-energy electron beam falling outside a dump aperture has been calculated for a variety of possible conditions. In general, this fraction is limited by radiative effects to be of the order of x/X_0 (the radiator thickness in radiation lengths), and thus is minimized by using the thinnest possible radiator, a choice which is also desirable from the point of view of collimation efficiency. The apertures which limit the full energy beam should have diameters of at least $\approx$ 20 cm at floor level and $\approx$ 25 cm at the true dump entry ($\approx$ 3 m beyond floor level). Increasing the apertures further will not produce much improvement. Background per produced photon should not depend strongly on radiator thickness as long as the latter is less than $\approx 10^{-2}$ X_0.
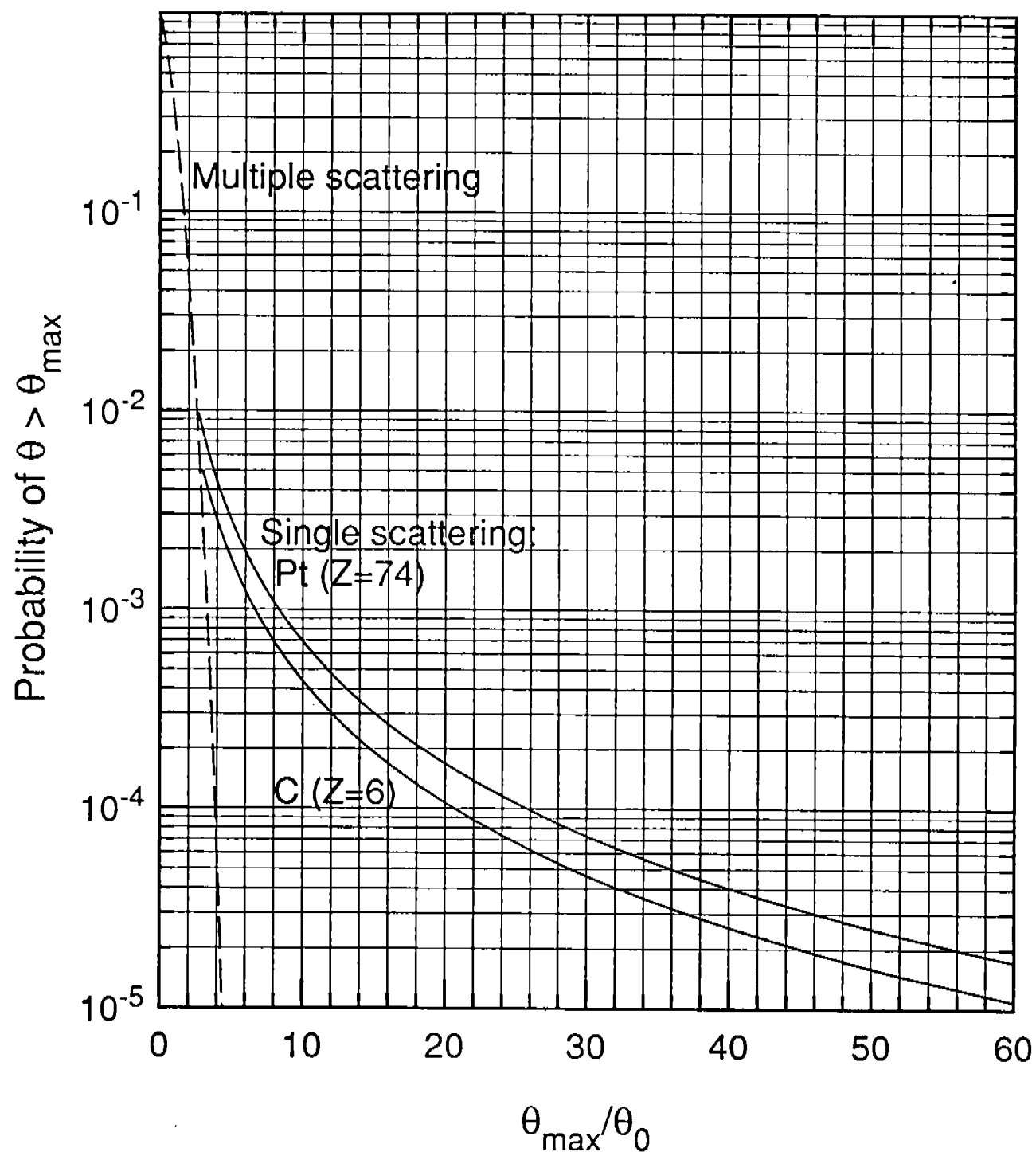
Figure 1 Probability of electron scattering by an angle $\theta > \theta_{max}$ as a function of θ_{max}/Θ_0, where Θ_0 is the RMS multiple scattering angle defined by Equation (3). <u>Solid curves</u>: single (Mott) scattering contribution for carbon and platinum foils. <u>Dashed curve</u>: multiple scattering contribution.

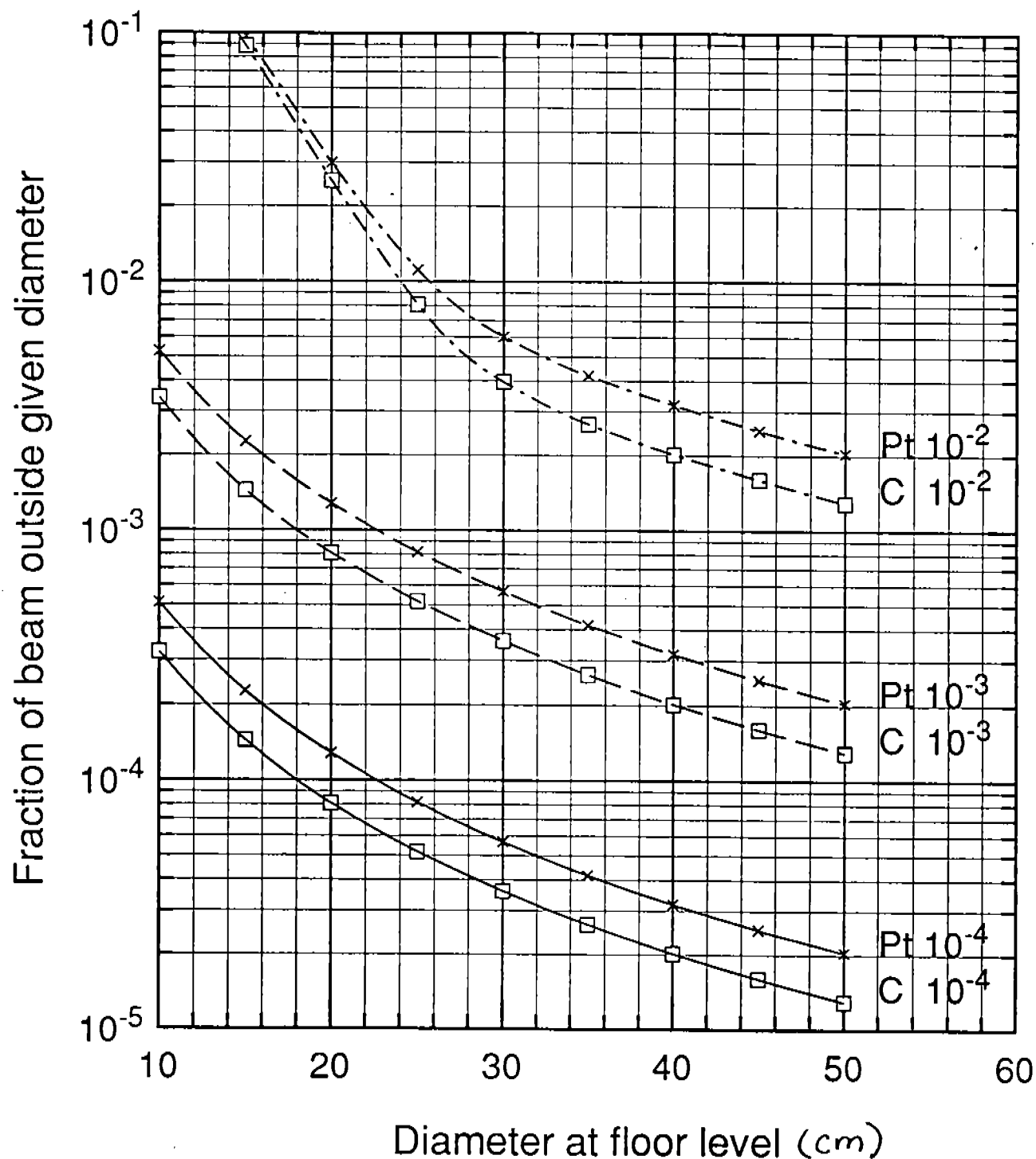**Tagger Beam Dump at E_0 = 800 MeV**

Figure 2 Fraction of electron beam scattered outside a circular aperture of given diameter at floor level. (The aperture lies in a plane normal to the beam). Each curve is labeled with the radiator material (carbon or platinum) and thickness (in radiation lengths).