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Monte-Carlo Event Generator of Inclusive Electron - Nucleon Scattering.

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Abstract

A phenomenological model and the corresponding algorithm of a computer program together comprise a Monte-Carlo event generator for inclusive electron - nucleon scattering processes at four-momentum transfers squared from approximately 0.1 GeV^2/c^2 to 10 GeV^2/c^2. This generator program may be used to estimate cross sections and acceptances in the planning and in the data analysis stages of high energy electro-nuclear experiments. The reaction cross section is calculated as a sum of concurrent processes including elastic scattering on the nucleon, excitations of nucleon resonances, and an incoherent part which is modeled as scattering on valence and sea quarks. A comparison of the generated inclusive electron cross sections with the experimental data is given.

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I. INTRODUCTION

The nature of highly excited nuclear matter is a subject of many experiments proposed for the new experimental facilities at CEBAF and other medium-energy electron accelerators. Important topics being addressed include nuclear structure at high energy and momentum transfers, the measurements of high energy components of the nuclear wave function, production and interactions of meson- and nucleon- resonances in the nuclear medium, correlations in nuclei, and properties of the multihadron final states in electronuclear interactions [1]. Most of the experiments require large solid angle detectors, such as the CLAS detector at CEBAF, which are able to measure exclusively many-body final states. Such complex detectors have to be provided with sophisticated simulation and event reconstruction software, and realistic input generators of electro-nuclear interaction events in order to make correct calculations of detector and trigger efficiencies and acceptances. Correct estimations of the reaction cross sections, and therefore of the required beam time for an experiment, are also of great importance.

The computer program GENIE ("Generator of Electro-Nuclear Interaction Events") has been developed to be used as such a tool for the experimentalist. The model principles which have been implemented in the program may also be of interest as an attempt to qualitatively understand the complicated physics of electro-nuclear reactions at a phenomenological level. The part of the GENIE algorithm which is discussed in the present paper models the processes of inclusive electron - nucleon scattering.

II. BRIEF REVIEW OF THE MODEL

Electron inclusive scattering on the nucleon has been measured in many experiments which show, in general, three major contributions to the cross section:

— elastic scattering;

— nucleon resonance excitations;

— non-resonant, incoherent scattering which may include meson exchange currents, final state interactions, and deep inelastic scattering (DIS) on quarks.

The relative contributions of the processes depend on Q^2 such that all may be neglected except DIS because of the smallness of their form factors as Q^2 is increased. At high Q^2 the parton model is qualitatively successful. In the range of initial energies and Q^2 under consideration (CEBAF and CLAS parameters, $E_o = 2 - 8\, GeV$, $Q^2 = 0.1 - 6\, GeV^2/c^2$) all the processes have comparable cross sections, so an attempt has been made to model them all, to calculate them, and to take their sum.

Within the framework of the phenomenological model proposed here we consider a nucleon to be composed of quarks and gluons, and try to model all incoherent processes as electron scattering on the quark-partons. The high Q^2 limit of the model should correspond to the standard parton model, if the correct momentum distribution of quarks and gluons in the nucleon could be estimated. The structure functions, measured at high Q^2, indicate that quarks carry about 50% of the momentum of a nucleon, so it is assumed that there are three light valence quarks in a nucleon, and the average number of additional gluons and the quark-antiquark pairs in the quark sea is taken as a parameter. The number of additional partons in the nucleon is assumed to take the form of Poisson distribution with the mean value n_{add}, and one more parameter is used to determine the proportion between the gluons and the sea quarks. The parton momentum distribution is modeled as the phase space distribution for the given number of light partons, with the sum of their four-momenta equal to the four-momentum of the nucleon. The limit of very light (massless) partons has been used to model the phase space momentum distribution using a fast algorithm developed by M.V. Kossov [2]. Small four-momentum non-conservation, negligible in comparison with the portion of nucleon four-momentum carried by the gluons, is introduced when the quark masses are set to be non-zero, with the values taken from reference [3].

The plane wave impulse approximation (PWIA) cross section of eN scattering is then

$$\sigma_{eN} = \sigma_{eN}^{elastic} + \sigma_{eN}^{resonance} + \sum_i \sigma_{eq_i}^{elastic}$$

for a given momentum configuration of the valence and sea quarks q_i. The integration over the quark momenta is carried out using the Monte-Carlo method: q_i are generated in accordance with the quark momentum distribution, then $\sum_i \sigma_{eq_i}^{elastic}$ is calculated, σ_{eN} is thus obtained for this generated configuration, and then averaged over the large number of the quark and momentum configurations.

However, certain restrictions should be applied. Due to baryon number conservation we are not allowed to create, as a result of an electron scattering on a free nucleon, a system which is lighter than a nucleon, or even a system which is lighter than the nucleon plus pion mass because there is no allowed eigenstates for the γ-nucleon system with the center-of-mass energy $W < m_N + m_\pi$. Furthermore, the suppression of the incoherent processes of electron scattering on quarks at the poles in the amplitudes of the resonance excitation is modeled as a probability to forbid an incoherent final state with center-of-mass energy W less than a resonant mass which is chosen at random in accordance with the relative resonance excitation cross sections for different resonances.

In order to get rid of the double counting in the cross-section, we introduce the Q^2 - dependence of the DIS part in a way that is similar to the use of the nucleon form factors in electron elastic scattering. Qualitatively, the nucleon form factor reflects the Q^2 - dependence of the relative contribution of scattering on its composite parts, as compared with scattering from a "pointlike nucleon", i.e. a nucleon without internal structure (for example, see Ref. [4], pages 188–198). Similarly we may hypothesize an effective form factor for the DIS, which reflects the growing competition of the coherent processes at low Q^2 with the DIS. The Q^2 dependence of the effective form factor will be discussed below.

The restrictions given above, and the Q^2 - dependent suppression of the DIS cross section may be estimated in the Monte-Carlo procedure in which, for the given quark momenta configuration, we may select one quark, generate the electron scattering on the quark, look at the resulting configuration, calculate the generated Q^2 and W, and apply the restrictions. Then if the scattering event is forbidden, zero is added in the averaging cross section sum.

III. GENERATORS OF ELEMENTARY SCATTERINGS

The generators of the elementary scatterings involve the calculation of electron scattering events in accordance with the cross section for the elementary partial processes. Expressed in terms of relativistic invariants (s,t,u), the cross section for unpolarized electron scattering on a pointlike Dirac (spin 1/2) particle of mass $m \gg m_e$ and charge ζ, at $s - m^2 \gg m_e^2$ is (cf. Ref. [4], Eq. 9.58):

$$\varphi_q(s,t) \equiv \frac{d\sigma_{eq}^{elastic}}{dt}(s,t) = \frac{4\pi\alpha^2\zeta^2}{(s-m^2)^2}\left[\frac{1}{2} + \frac{s}{t} + \frac{(s-m^2)^2}{t^2}\right]. \tag{1}$$

The example of the function $\varphi_q(t)$ for given $s = 8\,GeV^2$, and a quark mass $m = 10\,MeV$ is shown in Fig. 1(a). The range of the variable t for an arbitrary $1 + 2 \rightarrow 3 + 4$ reaction [3] is determined by $t_{lowlimit} = t_+$ and $t_{uplimit} = t_-$, where

$$t_{\mp} = \left[\frac{m_1^2 - m_3^2 - m_2^2 + m_4^2}{2\sqrt{s}}\right]^2 - \left\{\left[\left(\frac{s + m_1^2 - m_2^2}{2\sqrt{s}}\right)^2 - m_1^2\right]^{1/2} \mp \left[\left(\frac{s + m_3^2 - m_4^2}{2\sqrt{s}}\right)^2 - m_3^2\right]^{1/2}\right\}^2. \tag{2}$$

For elastic scattering $t_{uplimit} = 0$. The function $\varphi_q(t)$ is diverging at $t \rightarrow 0$. But if our consideration is limited by some $t_{max}(\equiv -Q^2_{min}) < 0$, then all terms in $\varphi_q(t)$ are finite, and an analytic form for the function may be obtained:

$$\Phi_q(s,t) = \int_{t_{lowlimit}}^{t} \varphi_q(s,t')dt'. \tag{3}$$

An analytic form for the function $\Phi_q(s,t)$ is essential for calculating the cross section of the electron scattering with Q^2 greater than a given Q^2_{min}, for a given generated momentum configuration, reasonably fast. It is equally important for the event generation procedure. If we want to generate t (or Q^2) distributions in accordance with the function $\varphi_q(t)$, then knowing the analytic form of $\Phi_q(t)$ helps to make it fast. The method is illustrated in Fig. 1(b). If y_i is a uniformly distributed random number from 0 to $\Phi_q(t_{max})$, then the values $t_i = \Phi_q^{-1}(y_i)$ are distributed as $\varphi_q(t)$. If we take the ordinate interval dy, then the number of events dN in this interval is proportional to the total number of events N_0 and the interval itself: $dN = dy\, N_0/\Phi_q(t_{max})$. The corresponding interval dt for these events is equal to $dy/\Phi'_q(t)$, which means that $dN/dt = (N_0/\Phi_q(t_{max}))\varphi_q(t)$. Although the direct analytical calculation of the inverse function $\Phi_q^{-1}(y)$ is impossible, the analytic forms for $\Phi_q(t)$, and $\varphi_q(t)$ give a possibility of effectively calculating it numerically using Newton's method of solving equations. The generated value of t (Q^2) may then be applied to determine the scattering angle of the electron in the center-of-mass frame. Assuming azimuthal symmetry, the ϕ angle is generated from the uniform $(0-2\pi)$ distribution.

The considerations leading to the generator algorithms for elastic eN scattering are essentially the same and lead to cross section (cf. Ref. [5], Eq. 19-25):

$$\varphi_N(s,t) \equiv \frac{d\sigma_{eN}^{elastic}}{dt}(s,t) = \frac{4\pi\alpha^2}{(s-m^2)^2}\left\{\frac{1}{2}(G_M^N)^2 + \left[\frac{s}{t} + \frac{(s-m^2)^2}{t^2}\right]\left[\frac{4m^2(G_E^N)^2 - t(G_M^N)^2}{4m^2-t}\right]\right\}, \tag{4}$$

where

$$G_{M,E}^N(t) = \frac{G_{M,E}^N(0)}{(1-t/b)^2}$$

are the nucleon form factors in the dipole approximation, for which $b = 0.71\, GeV^2$.

The case of electroexcitation of nucleon resonances is, in general, much more complicated. To be able to model these processes in the framework of the generator algorithms, the simplified form of the resonance excitation cross section formulas has been used, which takes into account only the kinematical differences between the elastic electron scattering and resonance production. Assuming that the amplitudes and the cross sections for the process

$$e + N \rightarrow e' + (nucleon\ resonance)$$

expressed in terms of s, t, u invariants take the same form as for elastic e-nucleon scattering, the following expression (see Appendix for some derivation details) for the cross section of resonance excitation results:

$$\varphi_R(s,t) \equiv \frac{d\sigma_{eN}^{resonance}}{dt}(s,t) = \frac{4\pi\alpha^2}{(s-m_i^2)^2}\left\{\frac{1}{2}(G_M^R)^2 + \left[\frac{2s-(m_i-m_f)^2}{2t}\right.\right.$$

$$+ \frac{s^2 - s(m_i^2 + m_f^2) + m_i^2 m_f^2}{t^2}\Bigg] \left[\frac{4m_i^2 (G_E^R)^2 - t(G_M^R)^2}{4m_i^2 - t}\right]\Bigg\}, \tag{5}$$

where the initial nucleon mass m_i and the mass of the resonance final state m_f correspond to the masses m_2 and m_4 in the standard notation for the $1 + 2 \rightarrow 3 + 4$ reaction, and the values of effective transition form factors differ from those of the nucleon. For example, to provide the transverse character of resonance electroproduction, the value G_E^R is set to zero. The range of t is determined by t_+ and t_- defined in Eq. 2.

To generate a resonance state and to calculate cross section it is necessary to generate m_f, the mass of the resonance. The standard Breit-Wigner form for the m_f distribution for electroexcitation of nucleon resonances [6] was used:

$$\sigma_{BW}(m) \propto \frac{dN}{dm} \propto \frac{1}{k_\gamma^2} \frac{m_R^2 \Gamma_\pi \Gamma_\gamma}{(m_R^2 - m^2)^2 + m_R^2 \Gamma_\pi^2}, \tag{6}$$

where

$$\Gamma_\pi = \Gamma_R \left[\frac{k_\pi}{(k_\pi)_0}\right]^{2l+1} \left[\frac{(k_\pi)_0^2 + X^2}{k_\pi^2 + X^2}\right]^l,$$

and

$$\Gamma_\gamma = \Gamma_R \left[\frac{k_\gamma}{(k_\gamma)_0}\right]^{2j} \left[\frac{(k_\gamma)_0^2 + X^2}{k_\gamma^2 + X^2}\right]^j.$$

Here Γ_R is the resonance width, m_R is the resonance mass, $k_{\gamma,\pi}$ is the value of the center of mass momentum of a photon or pion in the $\gamma - nucleon$, or $\pi - nucleon$ system with effective mass m, and index ($_0$) indicates the corresponding value calculated at $m = m_R$. The values of parameters l, j, and X are taken from Ref. [6]. The cross section in the Eq. 5 is strongly dependent on the resonance mass m_f, so the correction to $\sigma_{BW}(m_f)$, equal to the averaged $\varphi_R(m_f)$ is applied before the m_f generation.

The nucleon resonances $\Delta(1232)$, $N^*(1440)$, $N^*(1520)$ and $N^*(1680)$ are currently implemented in the generator. To take into account the large number of resonances at higher $W \approx 1.7 - 2.3\, GeV$ the effective resonance R^* with mass 2.0 GeV and width 0.4 GeV has been also introduced.

IV. THE ALGORITHM

The computer program has as input: E_0, and Q^2_{min}. At the first call the program performs initializations and cross section estimates. Then, and at the second and succeeding calls, it starts the loop with the following steps:

1. It chooses the process in accordance with the current accumulated partial cross sections, divided by the mean restriction rates for each partial process in order to keep the output ratios between the processes in a correspondence with the partial cross sections.

2. It generates a random configuration, i.e., the number, and the four-momenta of the quark-partons q_i, or resonance mass m_f.

3. It calculates $\sigma_{partial}(Q^2_{min})$. In the case of a DIS process, it calculates the sum of $\sigma_i(Q^2_{min})$ for each quark-parton.

4. It generates Q^2 and the angle ϕ. In the case of a DIS process, it chooses the i^{th} quark with the probability proportional to the $\sigma_i(Q^2_{min})$ cross sections calculated in the previous step, and then generates the scattering on this quark.

5. Restrictions are applied and the cross section is set to zero if the event is forbidden.

6. Cross sections are accumulated, and restriction rates are adjusted.

7. If the cross section is zero, it then returns to step 1 and makes another try. Otherwise the generation is successful and the generator returns the parameters of the scattered electron $(E', \mathbf{k}')$, and the estimate of the cross section $\sigma_{eN}(Q^2_{min}) = \sum < \sigma_{partial} >$, where $<>$ means averaging over all previously generated scatterings for the given partial process.

V. COMPARISON WITH EXPERIMENT

This algorithm has been tested in the generation of $p(e,e')$ events at $E_0 = 6$ and $12\,GeV$ and at different values of Q^2_{min} from 0.1 to 6 GeV^2/c^2. We accumulated the two-dimensional distributions $d\sigma/(dW\,dQ^2)$ and extracted values of

$$\Sigma(Q^2, W) = \frac{1}{\Gamma_t}\frac{d\sigma}{d\Omega\,dE'} = \sigma_t(Q^2, W) + \epsilon\sigma_l(Q^2, W), \tag{7}$$

where, following F. W. Brasse et al. [7], Γ_t is the flux of transverse polarized virtual photons, ϵ is the degree of polarization of the virtual photons, and σ_t and σ_l are the absorption cross section for transverse and longitudinal polarized virtual photons, respectively.

Figure 2 shows the comparison of the generated cross sections with the parameterization of $\Sigma(Q^2, W)$ given in [7] and shown as a function of W for the same values of Q^2 as in the original paper by F. W. Brasse. A reasonable agreement is achieved if we use as a parameter the additional number of partons in a proton, $n_{add} = 4.5$, and the proportion between the sea quarks and gluons $n_{sea-q}/n_{gluons} = 2/3$. The parameters of the resonance form factors taken in the dipole approximation

$$G_M^R(Q^2) = \frac{G_M^R(0)}{(1 + Q^2/b_R)^2},$$

are given in Table 1.

The effective DIS Q^2 - dependent suppression was assumed to be described by a factor $D_S = 1 - \sum_j [w_j/(1 + Q^2/b_j)^4]$, where j stands for elastic, and resonance excitation partial cross sections, b_j are the parameters of corresponding dipole form factor forms, and $w_j = \sigma^j / \sum_j \sigma^j$ are the relative probabilities of the partial processes.

A formal fitting procedure was not carried out because of the qualitative and phenomenological character of the basic assumptions of the model. Using more sophisticated modeling of the partial subprocesses in elastic scattering, resonance excitation processes, and DIS, including separate considerations for different polarization in the initial and final states, should provide better agreement with experiment. However the generator in its present form gives qualitative agreement with the experiment and may be used to model unpolarized ep scattering in the range of $Q^2 \approx 0.1 - 6\,GeV^2/c^2$ and $W < 2\,GeV$, with an accuracy of 10 to 30%.

From Fig. 2 it may be seen that the resonance excitation cross section at low $W < 2GeV$ is comparable and larger than the incoherent, or DIS part of the cross section, which reflects the quark momentum distribution in the nucleon. So the low-W data are difficult to use in the procedure of tuning the parameters of the quark distribution. To adjust the parameters we tried to model the cross sections at high $W > 2\,GeV$. The standard formalism of the nucleon structure function $F_2(x, Q^2)$ has been used, where $x = Q^2/2M\nu$, M is the nucleon mass, and ν is the energy transfer in the reaction. Following Ref. [8–10] and assuming that the ratio of longitudinal to transversal cross sections is zero,

$$\frac{d\sigma_{eN}}{dx\,dQ^2}(x, Q^2) = F_2(x, Q^2)\xi(x, Q^2), \tag{8}$$

where

$$\xi(x, Q^2) = \frac{4\pi\alpha^2}{x\,Q^4}\left(1 - \frac{\nu}{E_0} + \frac{\nu^2}{2E_0^2} + \frac{Q^2}{4E_0^2}\right). \tag{9}$$

Thus accumulating the distributions

$$\frac{d\sigma_{eN}}{dx\,dQ^2}(x, Q^2)$$

with the weight $1/\xi(x, Q^2)$ for the generated events, the generated value of the modeled $F_2(x, Q^2)$ structure function is obtained. The example is shown in Fig. 3 for the ${}^1H(e, e')$ reaction at 200 GeV and $Q^2_{min} = 3.7\,GeV^2/c^2$. The bins of this histogram at $Q^2 = 3.8 -$ $-4.3\,GeV^2/c^2$ were used to extract the x - dependence of the $F_2(x, Q^2)$ structure function at $Q^2 = 4\,GeV^2/c^2$ and compare it with the experimental data. The comparison with the data extracted from the approximations given in Ref. [10] is shown in Fig. 4 for three values of Q^2 : 1, 4, and 10 GeV^2/c^2. The agreement is good enough to reproduce main qualitative features of the cross section dependence on Q^2 and x. Using extra parameters, for example, modeling different momentum distributions of u and d quarks in a nucleon, a better agreement may be achieved.

VI. PROGRAM IMPLEMENTATION

The computer program GENIE implements the algorithm presented above at the initial stage of modeling the electron interaction with an atomic nucleus. The generalization of the algorithm for the case of electron scattering on a quasi-free nucleon in the nucleus is possible and will be discussed elsewhere [11]. Electron - nucleon scattering events may be generated by GENIE if the target is specified in the input parameters as a nucleon.

The program is written using FORTRAN77 and standard CERN library subroutines [12] are used. The program consists of the two major modules: PROGRAM GENIE_MAIN, and SUBROUTINE GENIE. The PROGRAM part performs user interface functions, providing the event generator with the input parameters, booking and storing histograms, and writing data summary tapes (DST) or disk files with the generated events. The PROGRAM module could be modified to adjust user's needs in the new histograms and/or different output event formats. The SUBROUTINE GENIE is the generator itself using input parameters, such as initial energy E_0, Q^2_{min}, and the number of protons and neutrons in the target nucleus to estimate the cross section, generate the momentum of the scattered electron, and if requested, to generate the momenta of the secondary hadrons - products of the fragmentation of the excited nucleus. The last option models the processes of multihadron fragmentation of medium- and heavy nuclei [11] and is not applicable for the nucleon targets yet.

The GENIE modules, together with the examples of programs, command files, and analysis routines, are available upon request. The generator is implemented for running under VAXVMS, and OpenVMS Operating Systems on VAX and AlphaAXP computers. A test run has been performed at the HP-ULTRIX machines at CEBAF.

VII. CONCLUSION

In conclusion, the part of the algorithm of the computer program GENIE is presented which models the processes of electron - nucleon scattering. The phenomenological parametrization of the processes of elastic scattering, nucleon resonance excitation, and deep inelastic scattering qualitatively reproduces the main features of the electron - nucleon cross sections. The program may be used to estimate cross sections and acceptances in the planning and in the data analysis stages of high energy electro-nuclear experiments.

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APPENDIX

In this Appendix we consider the derivation of the Eq. 5. Differential cross section for the process $e(k)N(p_1) \to e'(k')N^*(p_2)$ may be written in the form

$$d\sigma = \frac{1}{(2s_1+1)(2s_2+1)}\frac{1}{4p_1k}|A|^2 d\Phi \ ,$$
$$d\Phi = (2\pi)^4\delta^4(p_2+k'-p_1-k)\frac{d^3k'}{(2\pi)^3 2E'}\frac{d^3p_2}{(2\pi)^3 2\omega}, \qquad \text{(A1)}$$
$$|A|^2 = \frac{e^2}{q^4}L_{\mu\nu}H^{\mu\nu}$$

where we have neglected masses of leptons, and m_i and m_f are masses of the N and N^* respectively.

The differential cross section $d\sigma/dt$ can be obtained by integration of the double differential cross section

$$\frac{d^2\sigma}{dt\ d\nu} = -\frac{\pi}{EE'}\frac{d^2\sigma}{d\Omega dE'}, \qquad \text{(A2)}$$

where the distribution over Ω and E' turns out to be

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{16m_i^2t^2}\frac{E'}{E}|A|^2\delta(\nu+\nu_0), \qquad \text{(A3)}$$

where $t=q^2$, $\nu = E - E'$ in the laboratory frame and $\nu_0 = (t+m_i^2-m_f^2)/2m_i$.

Following the standard procedure we get for leptonic and hadronic tensors:

$$L_{\mu\nu} = Tr\{\not{k}'\gamma_\mu\not{k}\gamma_\nu\}$$
$$H^{\mu\nu} = Tr\{(\not{p}_1 - m_i)\Gamma^\mu(\not{p}_2 - m_f)\Gamma^\nu\} \qquad \text{(A4)}$$

As usual the leptonic tensor reads

$$L_{\mu\nu} = k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu}(k'\cdot k) \qquad \text{(A5)}$$

Since nucleon is not a point-like particle it is nesessary to introduce form-factors (in the case of $N \to N^*$ - transitions among spin-1/2 particles there are only three of them):

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + \kappa\frac{F_2(q^2)}{2m_i} i\sigma^{\mu\alpha}q_\alpha + F_3(q^2)q^\mu, \tag{A6}$$

with κ being the anomalous magnetic moment. Conservation of the current $J^\mu = \bar{u}\Gamma^\mu u$, i.e. $q_\mu J^\mu = 0$ leads to the following identity

$$F_3(q^2) = F_1(q^2)\frac{m_i - m_f}{q^2}, \tag{A7}$$

which expresses F_3 in terms of F_1. Evaluation of the hadronic part yields :

$$\begin{aligned} H^{\mu\nu} = {} & 4|F_1|^2\left[p_1^\mu p_2^\nu - g^{\mu\nu}(p_1\cdot p_2 - m_i m_f)\right] - 4|F_2|^2\left\{\left[p_1^\mu p_2^\nu q^2 - (p_1\cdot q)p_2^\mu q^\nu\right.\right. \\ & \left. - (p_2\cdot q)p_1^\mu q^\nu\right] + [\mu\leftrightarrow\nu] + 2g^{\mu\nu}(p_1 q)(p_2 q) - (p_1\cdot p_2)g^{\mu\nu}q^2 \\ & \left. + (p_1\cdot p_2)q^\mu q^\nu + m_i m_f q^\mu q^\nu - m_i m_f g^{\mu\nu}q^2\right\} + 4\bar{F}_2^* F_1\left\{m_f\left[p_1^\mu q^\nu - (p_1\cdot q)g^{\mu\nu}\right]\right. \\ & \left. - m_i\left[p_2^\mu q^\nu - (p_2\cdot q)g^{\mu\nu}\right]\right\} - 4F_1^*\bar{F}_2\left\{m_i\left[p_2^\nu q^\mu - g^{\mu\nu}(p_2\cdot q)\right]\right. \\ & \left. - m_f\left[p_1^\nu q^\mu - (p_1\cdot q)g^{\mu\nu}\right]\right\} + \ldots \end{aligned} \tag{A8}$$

where elipses represent contributions from F_3- dependent parts of the hadronic current vanishing upon contraction with the leptonic tensor, and $\bar{F}_2 = \kappa F_2/2m_i$. Convolution of this tensor with (A5) yields

$$\begin{aligned} \frac{1}{16}L_{\mu\nu}H^{\mu\nu} = {} & 2|F_1|^2\left[(kp_1)(k'p_2) + (k'p_1)(kp_2) - (kk')m_i m_f\right] \\ & - |\bar{F}_2|^2\left[2(p_1k')(kp_2)q^2 + 2(kp_1)(k'p_2)q^2 - 2(p_1q)(k'q)(kp_2)\right. \\ & - 2(p_1q)(k'p_2)(kq) - 2(p_2q)(k'p_1)(kq) - 2(p_2q)(kp_1)(k'q) \\ & + 2(k'q)(kq)(p_1p_2) + 2m_i m_f(k'q)(kq) - q^2(kk')(p_1p_2) \\ & \left. + m_i m_f(kk')q^2\right] + \bar{F}_2^* F_1\left\{m_f\left[(k'p_1)(kq) + (kp_1)(k'q) + (kk')(p_1q)\right]\right. \\ & \left. - m_i\left[(k'p_2)(kq) + (kp_2)(k'q) + (kk')(p_2q)\right]\right\} \\ & - F_1^*\bar{F}_2\left\{m_i\left[(k'q)(kp_2) + (kq)(k'p_2) + (kk')(p_2q)\right]\right. \\ & \left. - m_f\left[(k'q)(kp_1) + (kq)(k'p_1) + (p_1q)(kk')\right]\right\} + \ldots \end{aligned} \tag{A9}$$

In the laboratory frame the following identities hold:

$$kk' = -\frac{q^2}{2}, \quad kq = \frac{q^2}{2}, \quad k'q = -\frac{q^2}{2}$$
$$kp_1 = Em_i, \quad kp_2 = Em_i + \frac{q^2}{2} \tag{A10}$$
$$k'p_1 = E'm_i, \quad k'p_2 = E'm_i - \frac{q^2}{2}$$

Then modulus squared amplitude takes the form:

$$\begin{aligned} |A|^2 = 16 \cdot 2m_i^2 \Big\{ & \left[2EE' - \frac{t}{2m_i}(E - E') + \frac{t}{2m_i} m_f \right] |F_1|^2 \\ & + t \left[\frac{t}{2m_i} + \frac{\nu}{2m_i}(m_i - m_f) \right] (F_1^* \bar{F}_2 + \bar{F}_2^* F_1) \\ & - t \left[2EE' + \frac{t}{2m_i}\nu + \nu^2 - \frac{t}{2m_i} m_f \right] |\bar{F}_2|^2 \Big\} \end{aligned} \tag{A11}$$

Substituting (A11) and (A3) into (A2), integrating over ν (presence of the δ-function makes this integration trivial) and making use of the formulas

$$\begin{aligned} E &= \frac{s - m_i^2}{2m_i}, \quad E' = \frac{m_i^2 - u}{2m_i} \\ EE' &= \frac{(s - m_i^2)(m_i^2 - u)}{4m_i^2} = \frac{(s - m_i^2)(s + t - m_f^2)}{4m_i^2} \end{aligned} \tag{A12}$$

gives the expression for the cross-section in terms of F - form factors:

$$\begin{aligned} \frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{(s - m_i^2)^2} \Big\{ & \left[1 + \frac{2(s - m_i^2)(m_i^2 - u)}{t^2} + \frac{(m_i + m_f)^2 - 2m_f^2}{t} \right] |F_1|^2 \\ & + \left[\frac{\kappa}{2m_i}(m_i + m_f) \left(1 - \frac{[m_i - m_f]^2}{t} \right) \right] (F_1^* F_2 + F_1 F_2^*) \\ & - \left[-1 - \frac{(m_i + m_f)^2 - 2m_f^2}{t} + \frac{2(s - m_i^2)(m_i^2 - u)}{t^2} \right. \\ & \left. + \frac{(t + m_i^2 - m_f^2)^2}{t^2} \right] t \left| \frac{\kappa}{2m_i} F_2 \right|^2 \Big\} \end{aligned} \tag{A13}$$

which in the case of the point-like particles ($F_1 = 1, F_2 = 0$) takes the form

$$\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{(s - m_i^2)^2} \left[1 + \frac{2(s - m_i^2)(m_i^2 - u)}{t^2} + \frac{(m_i + m_f)^2 - 2m_f^2}{t} \right] \tag{A14}$$

This form corresponds to Eq. 5 with $G_M^R = G_E^R = 1$ and can be used to describe the shift of the chiral quark on-shell, i.e. transition $q^*(m_q = 0) \to q(m_q)$. Eq. A14 reduces to the well-known result from Ref. [4] as $m_i = m_f = 0$:

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{s^2}\frac{1}{2}\frac{(s^2+u^2)}{t^2} \tag{A15}$$

It is natural to use Eq. A14 to describe the electron scattering on the off-shell quasifree nucleon in the nucleus. m_i in this case is taken to be the bound nucleon mass which is smaller than the mass of a free nucleon m_f by a few percent. In the first approximation the free nucleon form factors can be used and we get the form of Eq. 5 for the cross section of quasielastic electron scattering. We believe this simplified form could also be used phenomenologically to parametrize *nucleon* $\rightarrow$ *nucleon resonance* transition cross section, though generally much more complicated forms with larger number of different form factors are valid.

The example of $N \rightarrow N^*$ - transition among spin-1/2 particles is given by the Eq. A13. Sometimes it is more convenient to express Eq. A13 in terms of electric and "magnetic" form-factors. Let's define

$$\begin{aligned} G_E &= F_1 + \frac{\kappa t}{4m_i^2}F_2 \\ G_M &= F_1 + \frac{\kappa}{2m_i}(m_i+m_f)\left(1-\frac{[m_i-m_f]^2}{t}\right)F_2 \\ \mu(t) &= \frac{m_i+m_f}{2m_i}\left(1-\frac{[m_i-m_f]^2}{t}\right) \end{aligned} \tag{A16}$$

As it can be easily seen, $\mu(t) \rightarrow 1$ as $m_f \rightarrow m_i$. In terms of electric and "magnetic" formfactors (A13) reads

$$\begin{aligned} \frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{t^2(s-m_i^2)^2} \Bigg\{ & \frac{t^2}{2}G_M^2 + \left[\left(\frac{m_i^2+m_f^2}{2}-s\right)^2 - \frac{(m_i^2-m_f^2)^2}{4} + st\right] \\ & \cdot \left[\frac{\left(\mu^2(t)-\frac{t}{4m_i^2}\right)G_E^2 - \left(1-\frac{t}{4m_i^2}\right)\frac{t}{4m_i^2}G_M^2 - \frac{1-\mu(t)}{2m_i^2}tG_EG_M}{\left(\mu(t)-\frac{t}{4m_i^2}\right)^2}\right] \\ & + \frac{t^2}{8m_i^2}\left[4m_i(m_f-m_i\mu(t)) - \frac{(m_i^2-m_f^2)^2}{t}\right]\frac{(G_M-G_E)^2}{\left(\mu(t)-\frac{t}{4m_i^2}\right)^2}\Bigg\} \end{aligned} \tag{A17}$$

which reduces to the result (Eq. 4 in the main text) from Ref. [5] as $m_f \rightarrow m_i$.

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FIGURES

FIG. 1. An example for the cross section t - dependence. See details in the text.

FIG. 2. The total cross section of $\gamma_v - proton$ interaction shown as a function of W in nine intervals of Q^2 from 0.1 to 6 GeV^2/c^2 compared to the data of F. W. Brasse et al. [7].

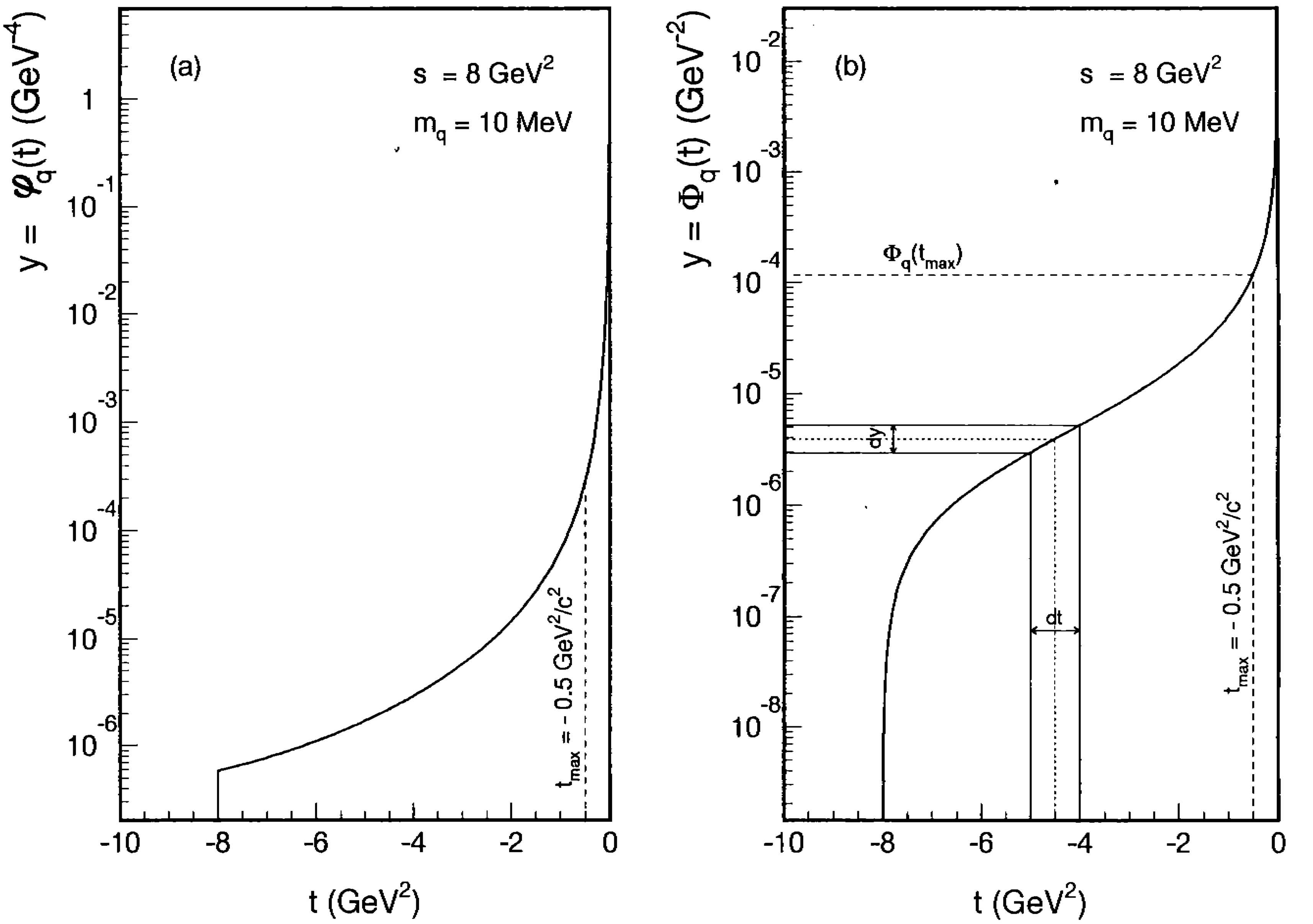
FIG. 3. The proton $F_2(x, Q^2)$ structure function generated by the GENIE Monte - Carlo event generator. The linear size of a rectangle at each bin of the histogram is proportional to the value of $F_2(x, Q^2)$ averaged over the bin. The result is obtained from the generation of events of electron - proton interaction at initial energy 200 GeV with $Q^2_{min} = 3.7\, GeV^2/c^2$ and $W > 2\, GeV$. The solid and dashed curves show maximum and minimum limits on x.

FIG. 4. The proton structure function at $Q^2 = 1, 4$, and $10\, GeV^2/c^2$ generated by the GENIE Monte - Carlo generator (lines) as compared with the experimental values (squares).

TABLES

TABLE I. Parameter values of the nuclear resonance form factors in the dipole approximation used in the generator.

Resonance	$\Delta(1232)$	$N^*(1440)$	$N^*(1520)$	$N^*(1680)$	$R^*(2000)$
$G_M^R(0)$	3.14	0.62	1.15	0.97	1.08
b_R (GeV^2)	0.57	1.80	1.20	1.40	2.50



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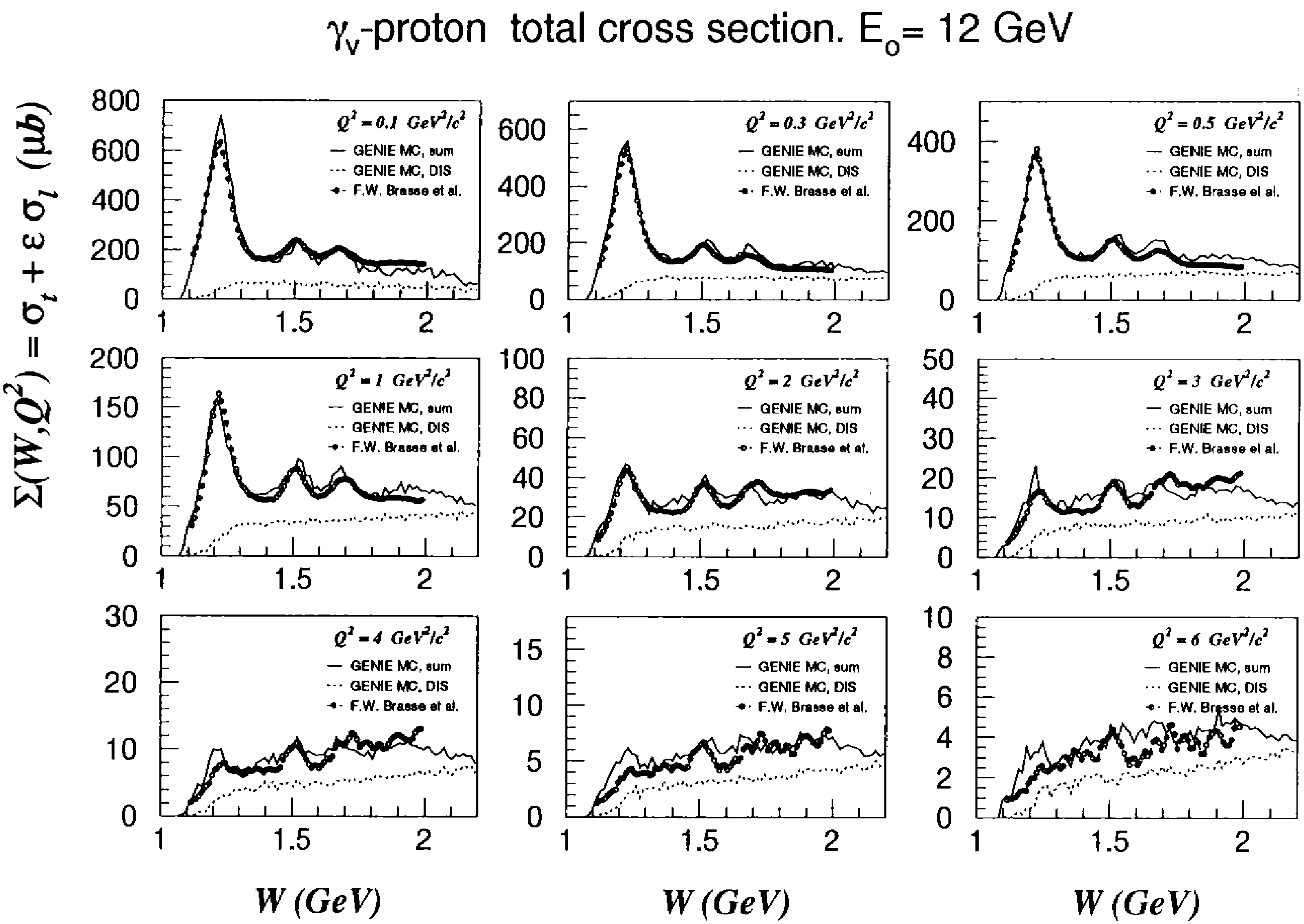

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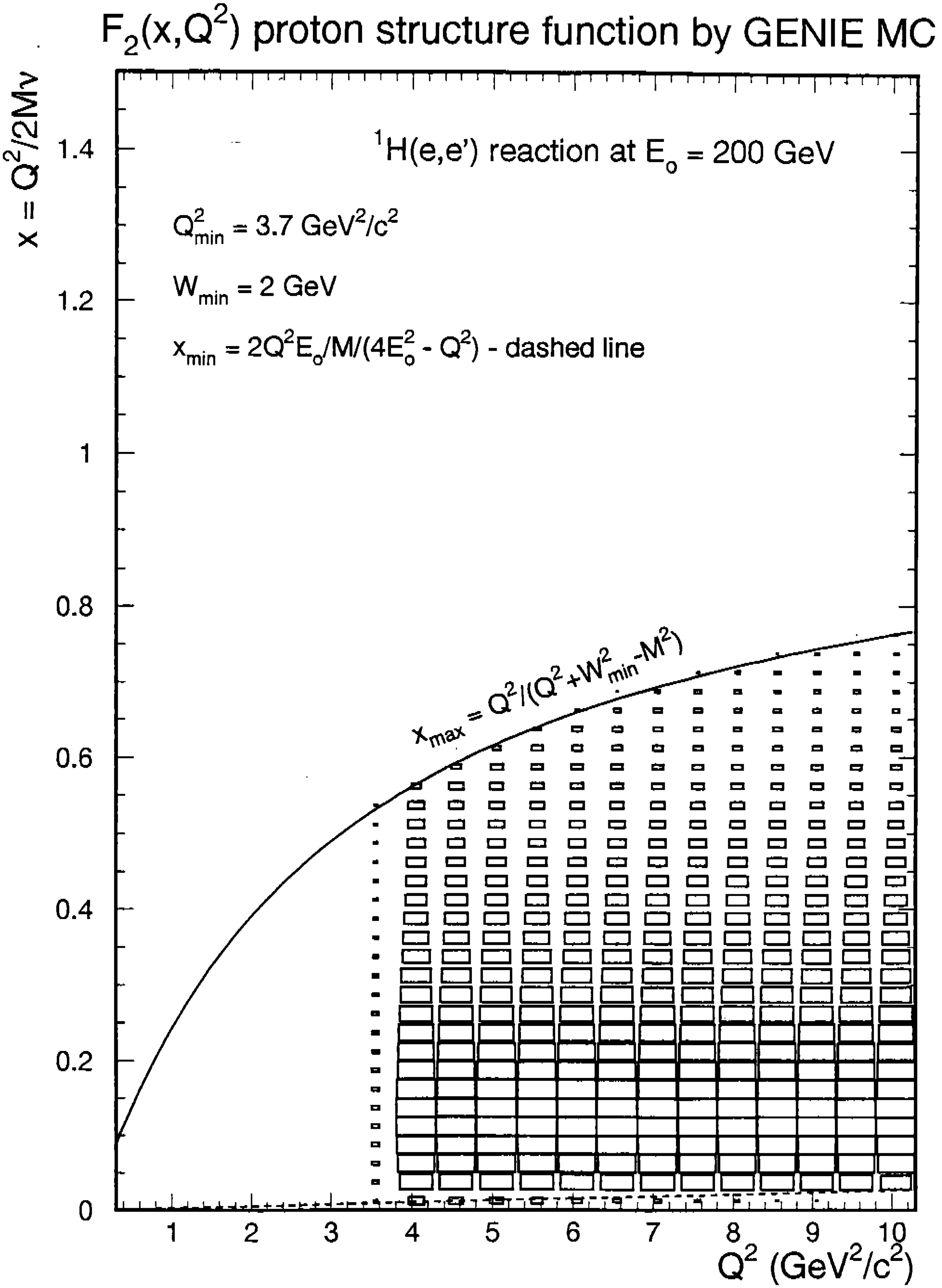

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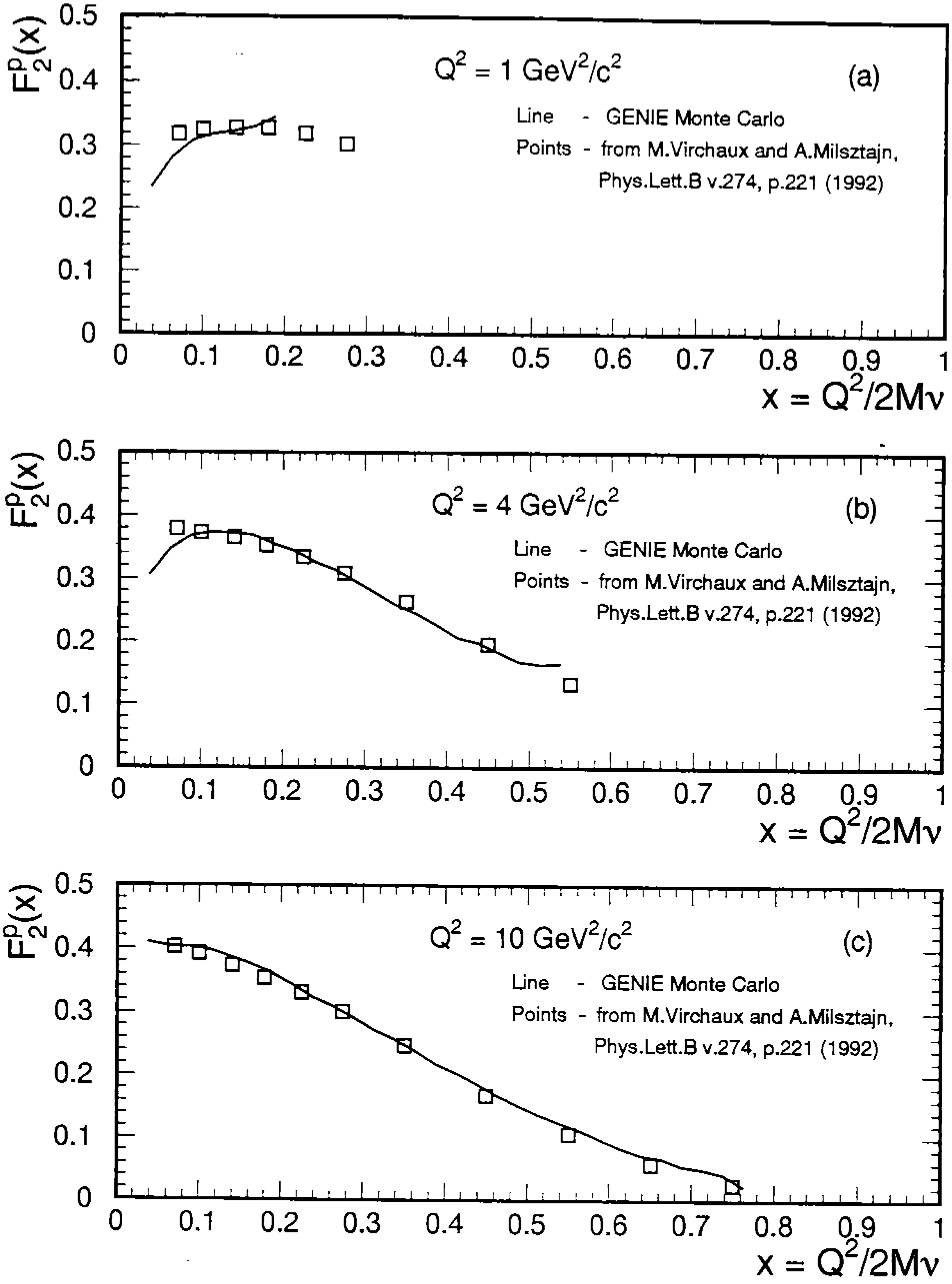

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