Effects of Discrete Ionization in the CLAS Drift Chambers

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Introduction

When a charged particle traverses a drift chamber it leaves behind a trail of ion pairs. The electrons drift to the anode wire and produce a charge avalanche. The density of ionization pairs is sparse; for example, in a 50:50 argon ethane mixture a minimum ionizing track produces on average one primary ionization event pair every 350 μm. The ions are not spaced evenly but follow a Poisson distribution. This "clumpiness" of ionization has several consequences:

- the drift time of tracks close to the sense wire is shifted to larger times,
- the drift time of tracks close to the field wires is also shifted to larger times,
- the spatial resolution for hits near the sense or field wires deteriorates, and
- the efficiency of such tracks in producing hits is reduced.

Using formulas for the ionization density along a track we calculate the functional dependences of these effects. Armed with these functions, we describe methods to correct the data for these effects (on the average) and to determine the expected additional resolution. We closely follow two sources:**Principles of Operation of Multiwire Proportional and Drift Chambers**, F. Sauli, (1977) and **Optimierung der Ortsauflosung der Zylindrischen Driftkammer des Detektor ARGUS**, G. Harder, (1984). Sauli's work is known as the "bible" for drift chamber operation, while the second is a thesis by a student at ARGUS which contains many useful studies of their drift chamber.

1 Ionization along a Track

The density of ionization along a track segment of length dy is proportional to the segment length, $k \cdot dy$, where $k = 28cm^{-1}$ is characteristic of argon-ethane (50:50), currently being used in the "nose-cone" prototype chamber. Note that this "k" is the number of **primary** ionization events per track length. In argon-ethane, an ionization event consists of 2.5 ion pairs on average. The distribution is sharply peaked at one ion, falling as $1/n^2$ where n is the number of ion pairs in one ionization event. Thus a minimum ionizing particle traversing one cm of argon-ethane will leave behind 28 clusters of ions and a total of 70 ion pairs. To simplify the calculations, we will assume that the wire "fires" when at least "j" ion clusters reach the sense wire, where j is a small number (3 or 4). This threshold depends upon the overall gas and electronics gain as well as the discriminator threshold. See the appendix for further discussion.

2 Spatial Distribution of Ion Clusters along a Track

We now define $P_1(y) \cdot dy$ to be the probability that the first ion pair along a track occur within the segment $(y, y+dy)$ or within $(-y, -y-dy)$, relative to the point of closest approach of the track to the wire. See Figure 1. To derive the form of P_1, consider the function $\bar{P}(y)$ which is the probability that there is **no** ion within ± y of the origin. The differential equation which defines $\bar{P}(y)$ is the following:

$$\bar{P}(y+dy) = \bar{P}(y) \cdot (1 - k \cdot 2 \cdot dy)$$

In other words, the probability of no ion being within y+dy of the origin is the probability of no ion being within y of the origin times the probability that no ion is produced within (y,y+dy) or within (-y,-y-dy). So,

$$\bar{P}(y+dy) = \bar{P}(y) - 2k\bar{P}(y)dy$$

$$\frac{d\bar{P}}{dy} = -2k\bar{P}.$$

The solution is

$$\bar{P}(y) = e^{-2ky}.$$

Now, $P_1(y)dy$ is the probability that the first ion along a track occurs in the interval (y,y+dy) or (-y,-y-dy).

$$P_1(y)dy = \bar{P}(y) \cdot 2kdy$$

So,

$$P_1(y) = 2ke^{-2ky}.$$

The probability that **two** ions occur within a segment from -y to y is the integral over the product $P_1(y_1) \cdot P_1(y - y_1)$ where y_1 is intermediate between 0 and y. So,

$$P_2(y) = \int_0^y [2ke^{-2ky_1}][2ke^{-2k(y-y_1)}]dy_1$$

and

$$P_2(y) = y(2k)^2 e^{-2ky}$$

and following Sauli,

$$P_j(y) = \frac{y^{j-1}}{(j-1)!}(2k)^j e^{-2ky}$$

is the probability that j clusters occurs within the segment -y to y.

The average value of y for which j clusters are contained within -y to y is thus

$$< y > = \int_0^{infty} y \cdot P_j(y)dy.$$

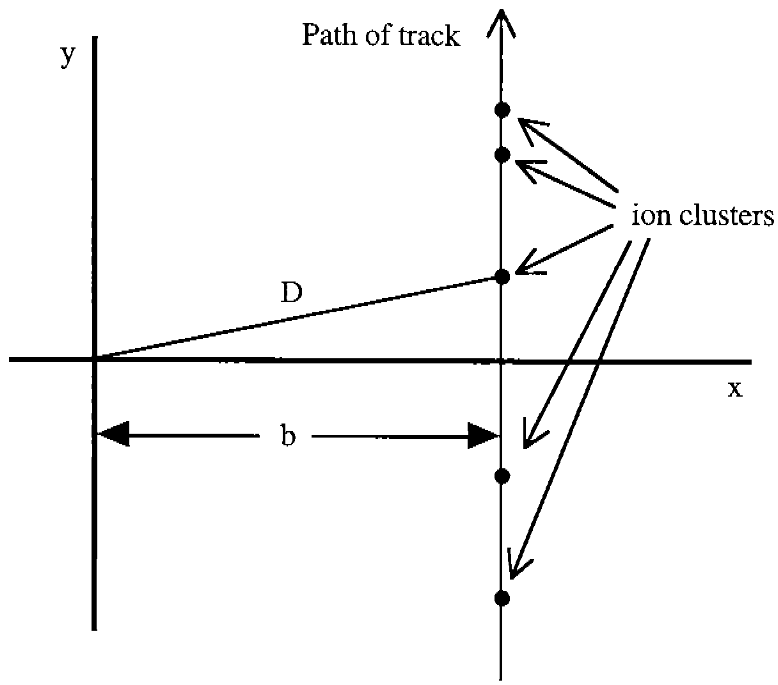

Figure 1: Finite deposition of ions along a track.

The solution to this integral is

$$< y >= \frac{j}{2k} = j \cdot 179 \mu m$$

for $k = 28 cm^{-1}$.

3 Effect upon Observed Time (and Distance) of Hits

Discrete ionization shifts the apparent time of a hit to larger values. Referring again to Figure 1, suppose a track is at a distance, b, from a wire. Also suppose that there is no hit at this distance of closest approach but that the first hit occurs at distance y along the track. Then the observed distance of the track from the wire, D, is the quadratic sum of b and y,

$$D = \sqrt{b^2 + y^2}.$$

So, knowing the ideal track impact parameter, b, and average ionization displacement $< y >$ we can calculate the observed distance, D. We must invert this procedure; that is, given the **observed** distance, D, what is the average correction, δD, averaged over all contributing values of impact parameter. It's not as easy as saying that

$$b_{est} = \sqrt{D^2 - < y >^2}$$

As an example, what do you do when D is smaller than $< y >$?

We begin to answer this question by calculating three quantities of interest:

1. the shape of the leading edge of the observed time distribution for all tracks
2. the shape of the time distribution for tracks close to a wire
3. the average shift in the observed time as a function of the observed time.

The first two distributions will be compared with our data to estimate the threshold parameter, j. The third distribution is our best estimate of the **average** time shift as a function of the **observed** time, and will be subtracted from the observed time to yield the corrected time.

3.1 Leading Edge of the Observed Time Distribution

The shift of the distance of closest approach will reveal itself in changes to the observed time distribution of hits from a wire. The larger the shift (i.e. for larger j) the less steep is the leading edge of the distribution, and the more it is shifted to positive times.

If the hit threshold requires j primary ionization pairs then the probability distribution for the distance y of the hit position is given by $P_j(y)$. What is the resulting probability distribution for the observed (apparent) distance of closest approach, D?

Refer to figure 2. Here we have sketched the solution. Assume that tracks are along the y direction (the solution is valid for any angle). Suppose that the distribution of incident tracks is uniform in x. How many tracks are observed to be at distance, D, from the wire? For instance, if a track were at x=0 it could still appear to be at distance, D, if its 'jth' hit were at distance D from the wire. Here we are assuming that a 'hit' occurs only after the arrival of j electrons.

The number of events observed at distance, D, is thus given by the following integral:

$$N(D)dD = \int_0^{\pi} (D \cdot d\phi \cdot dD) P_j(y) P_{track}(x)$$

where $P_{track}(x)$ is the probability distribution for tracks occuring at some distance x from the wire and is assumed to be constant, and $P_j(y)$ is defined in section 2.

We have done the integral numerically and present the solution in figure 3 for different assumptions on the hit threshold (j = 1, 2, 3 or 4). As the threshold, j, increases there are three effects: first, the slope of the leading edge decreases, second, the straight-line extrapolation point is shifted to larger times, and third, the overshoot increases.

We can use the leading edge slope as a measure of the effective threshold, j, and use this measure of j to estimate the shift in T_0 from the straight line extrapolation point. Referring again to figure 3, we see that the data distribution (the solid histogram) slightly favors the j=3 solution. We point out that this data were taken with the "nose-cone" prototype at 2700 V, with 75' of signal cable, with electronic amplification of 22 $mV/\mu A$, and with a discriminator threshold of -10mV. In future work, we will

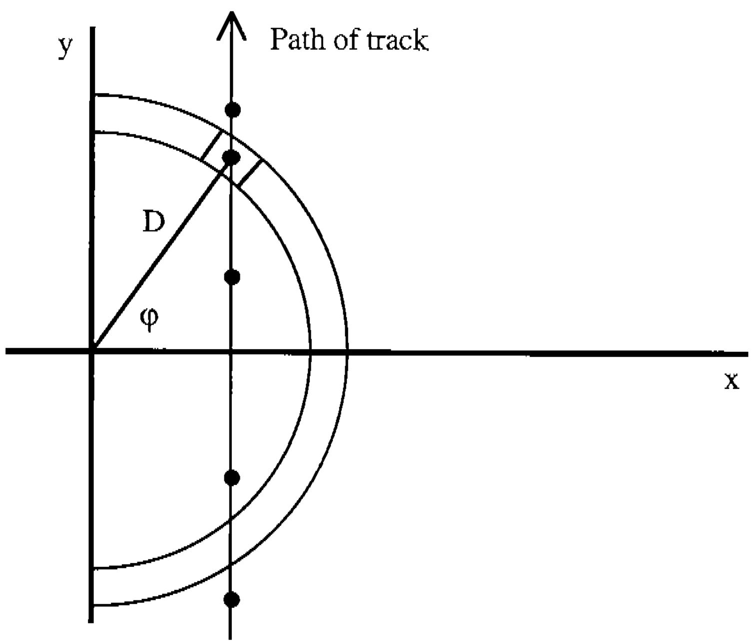

Figure 2: One example of an event at apparent distance D; the probability of an event observed at D is the integral over all such cases.

change the gas amplification and check whether the threshold parameter, j, changes accordingly.

3.2 Observed Time Distribution for Zero-Impact Parameter Tracks

The expected distribution in times for hits from tracks very close to the wire is quite sensitive to the threshold parameter, j. The expected function form is simply

$$P_j(y) = \frac{y^{j-1}}{(j-1)!}(2k)^j e^{-2ky}$$

with y transformed to time by the saturated drift velocity, $V_0 = 56\mu m/ns$. In Figure 4 we plot $P_j(y)$ for j=1,2,3,4 over a histogram of raw times collected for the case when the fitted track was within $200\mu m$ of the wire position, once again for 2700 V data. The j=3 curve again seems to best fit the data. Also, the mean time of 9.5 ns is consistent with $< time >=< y > /V_0 = 3 * 179\mu m/56\mu m/ns = 9.6ns$.

3.3 Calculated Time Shift as Function of Observed Time

Refer again to figure 2 where the ideal impact parameter, b, is shifted to the obsreved distance, D, by adding the clusterization distance y in quadrature. We don't know y

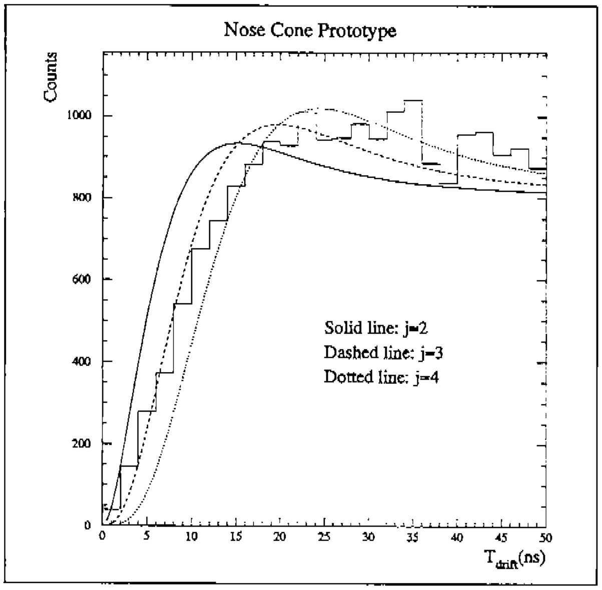

Figure 3: Expected time distribution for different thresholds. Data from a 2700 V run is shown as the histogram.

on an event-by-event basis but our best estimate of its average value is $< y >= j \cdot 2k$. We also know its expected probability distribution, $P_j(y)$. Therefore, we can calculate the average value for the **shift in distance,** δD with this formula

$$< \delta D >= \int_0^{\pi} ((b - D) \cdot d\phi) P_j(y)$$

as a function of observed distance, D.

In Figure 5 we plot the results of this calculation. The four dotted curves are the values of the **correction** δD plotted versus the observed distance, D. We then correct the observed distance by simply subtracting this value, δD.

4 Effect of Discrete Ionization upon Resolution Near the Wire

We have now corrected the observed time for the average shift. After this correction, what is the expected standard deviation on the observed distance of closest approach; i.e. what is the resolution? A solution to this problem is given in Harder's thesis. He obtains the RMS deviation of the hit position from 0 by using the formal definition for the standard deviation, σ,

$$\sigma^2(y) = < y >^2 - < y^2 >$$

where

$$< y >= \int y \cdot P_j(y) dy = \frac{j}{2k}$$

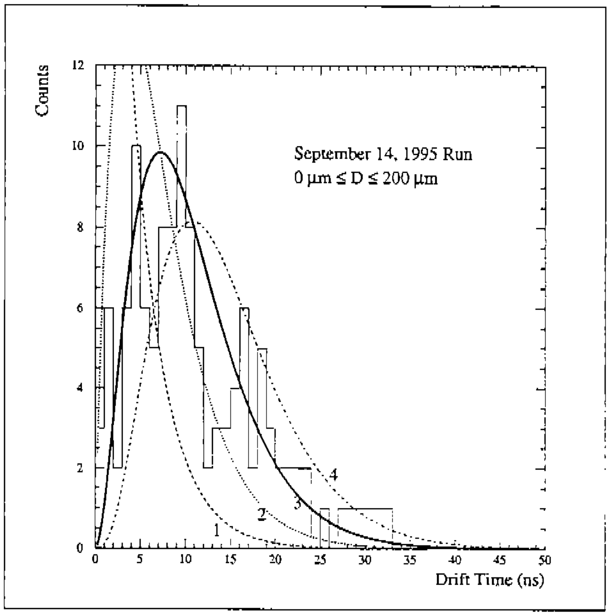

Figure 4: Expected time distribution of different threshold values, j. The data (hatched histogram) are the drift times for wires for which the track was within $200\mu m$.

and

$$< y^2 >= \int y^2 \cdot P_j(y)dy = \frac{j(j+1)}{4k^2}$$

so

$$\sigma^2(y) = \frac{j}{4k^2}$$

He then derives the standard deviation on the observed distance D using this formula

$$\sigma^2(D) = \sigma^2(y) \cdot (\frac{\partial D}{\partial y})^2 = \frac{j^3}{4k^2 \cdot (j^2 + 4k^2b^2)}$$

, where b is the actual distance of closest approach.

In Figure 6, we plot the expected resolution versus distance of closest approach for j=1,2,3,4 as calculated from the formula above. Note, this is the expected resolution **after** correcting for the average time shift.

5 Effect for Tracks Far from the Wire

Clusterization effects can worsen resolution and shift observed times to larger values for tracks near the field wires, that is, near the outer edges of the cell. In fact, the effects are more severe here than near the sense wire. Near the sense wire the effects are purely one of distance; for example, a track which is $50\mu m$ from a sense wire deposits its closest ion(s) at a distance of $150\mu m$ from the sense wire. This effect is greatest at 0 distance, resulting in a maximum time shift of the order of 5 - 10 ns.

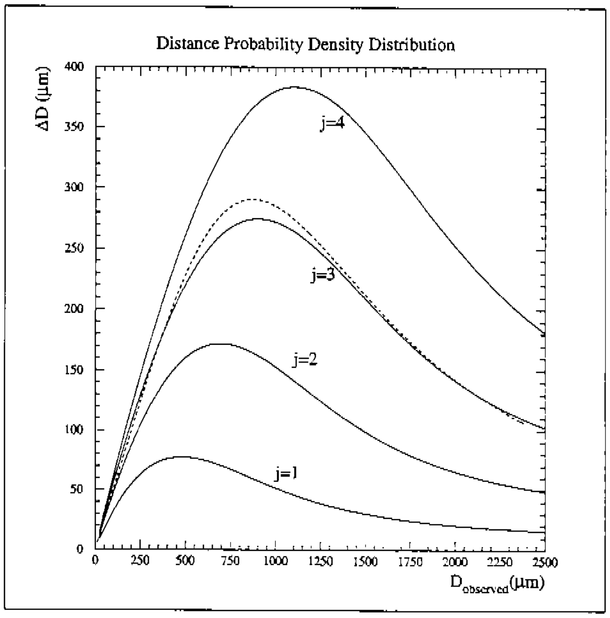

Figure 5: The calculated shift in distance, δD as a function of observed distance, D for j=1,2,3,4. Also shown is the functional form $.5(j/k)^3 D/((j/k)^3 + D^3)$ which approximates the j=3 curve.

However, near the field wires the electric field lines diverge. This means that ions which are deposited near one another can take widely divergent paths (with widely different transit times) from their point of deposition to the sense wire. For this reason, we do **not** correct for the average time shift for such tracks. We feel a more prudent approach is to not use these hits or to weight them appropriately less.

6 Effect on Cell Efficiency

Tracks near the sense wire have on average the largest track length and total deposited charge, but have small pulse heights because the collected ions are spread out in time. Ions which are spaced $350\mu m$ apart on average are spaced 6 ns apart in time. If the characteristic pulse formation time was about 6 ns (3 ns rise-time and 3 ns fall-time) then such a track would appear as a long (500 ns) train of 6 ns pulses with pulse height corresponding to 2 primary ionizations. In fact, the charge deposition is randomly spaced and the characteristic time for pulse formation is closer to 10 - 20 ns; however, at some voltage we expect a fall in hit efficiency for tracks very near the wire.

The situation is similar but worse for tracks near the cell edge, e.g. for "corner clippers". Here the track length is short and the arriving ions are well spread out in time. We definitely expect a drop in efficiency near the cell edge where a decrease in track length can not be compensated by a proportional drop in discriminator threshold.

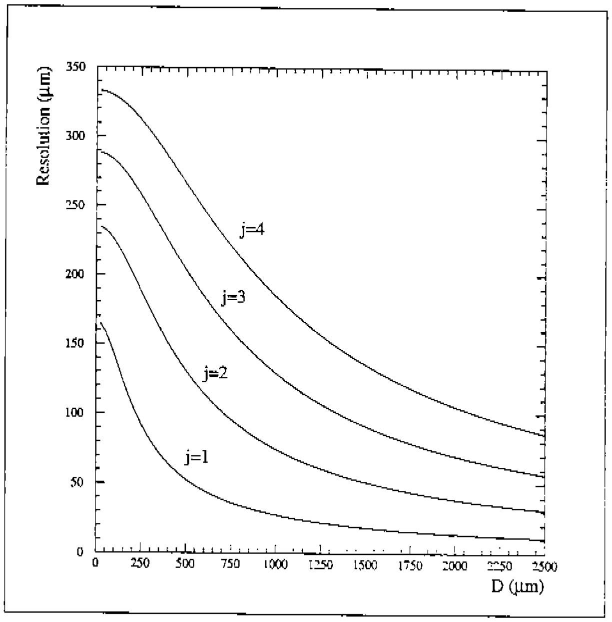

Figure 6: Expected resolution versus distance due to clusterization; for different thresholds.

7 Appendix: Variation of j with Voltage

The parameter j is the number of primary ionization events which are required to "fire" the discriminator on a particular wire. Since most ionization clusters ($\approx 70\%$) consist of a single ion pair, j is the number of electrons required to initiate an avalanche which will produce a voltage signal above threshold. Thus, j depends on the gas gain. For example, j will change from a value of 4 to a value of 2 if the gas gain doubles. For our chambers this amounts to a voltage change of about 100 volts.