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Evaluating Polarization Data

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Abstract A method is presented, which ensures that different polarization observables describing one reaction channel are consistent with each other. Using the connection of the observables to the same underlying reaction amplitudes, a constrained estimate of the observables is carried out using a Markov Chain Monte Carlo method. Initial results indicate that the new estimates are guaranteed to be physical, and will remove the need for artificial fudge factors when these processed data are used in subsequent fits.

1 Introduction

Different observables used in fits to determine the properties of the light baryon spectrum are often measured independently. If there exist inconsistencies between different data sets, this will render unsafe the findings obtained by fitting theoretical models to the inconsistent data.

There are instances of measurements of the same quantity that are clearly inconsistent when plotted on one graph, and a phenomenologist must decide whether to use the different data sets, ignore one or the other, or combine them in a more or less ad hoc manner. There is the more subtle problem of fitting to different observables, whose true values are actually correlated because they are different combinations of the same amplitudes.

Even within a single experiment, the method of extracting the observables from measured distributions often does not guarantee that the reported estimate is in the physical region. An example of this is the estimation of observables that are asymmetries. The true values are, by definition, bounded in the region $[-1, +1]$. However, one often sees experimental results where error bars and even the mean of the point estimates themselves are outside this region. This is a mathematical impossibility! There is zero probability that the true values exist outside the region, so anything to the contrary indicates that the method use to derive the estimator has been used too simplistically.

This contribution indicates a possible solution to the problems highlighted above by describing a method that takes the raw measured data, and finds the combination of different observables that represents a best estimate, subject to known constraints. The idea is similar in philosophy to the procedure of kinematic fitting. This is applied to the example of pseudoscalar meson photoproduction, where recent progress in experimental measurements has resulted in a large data set of different observables for several reaction channels.

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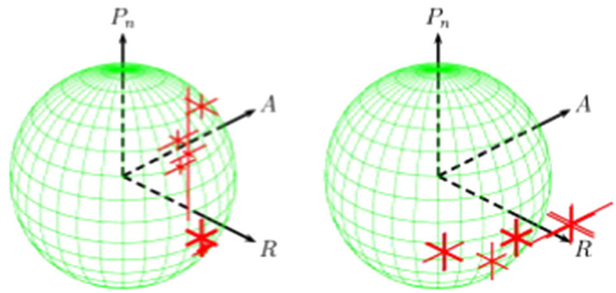


Fig. 1 (Taken from [1]) $\pi^- p$ (left) and $\pi^+ p$ (right) polarization observables

2 Connected Observables

One way to illustrate that observables are connected is firstly to look at elastic $\pi - N$ scattering. In this case, there are two amplitudes, which hence lead to four observables, one of which is the differential cross section:

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2$$
$$A = |f|^2 - |g|^2$$
$$R = -2\,\mathrm{Re}\left(fg^\star\right)$$
$$P = 2\,\mathrm{Im}\left(fg^\star\right)$$

where P is the polarization of the recoiling nucleon, and A and R are the spin rotation parameters. This algebra results in a constraint:

$$A^2 + R^2 + P^2 = 1.$$

This is the equation of a unit sphere in the space of the observables, and is therefore the surface on which the true values of the combination of A, R and P must exist.

Figure 1 shows plots of combination of observables, from which it was concluded that the combinations were *consistent with* the physical values that must lie on the sphere. One could (and should) turn this around and use the fact that since the true values lie on the spherical surface, the estimates of the measured values should be adjusted using the spherical surface as a constraint.

3 Constrained Parameter Estimation

One example of constrained parameter estimation is routinely used in nuclear and particle physics experiments: kinematic fitting. By way of a simple illustration, take a simulated measurement of a single particle in a detector (see Fig. 2).

In this case, consider a measured energy $E \pm \delta E$ and momentum $p \pm \delta p$, the combination of which is used to identify particle type through the calculation of *measured* mass:

$$m_{meas}^2 = E^2 - p^2$$

Taking values of measured energy (415 ± 10 MeV) and measured momentum (400 ± 5 MeV/c), the probability density functions (PDFs) can be depicted on a plot of mass and momentum (Fig. 2a). The combination will give a joint PDF whose maximum gives an estimated of the measured mass m_{meas}.

With the given numbers the measured mass is 110 ± 42 MeV/c^2. No particle exists with this mass, but it is close enough to the mass of the pion to allow the identification of this particle as a pion. This is illustrated in Fig. 2b, where the red horizontal line is the mass value of the maximum of the joint PDF and the black line is the accepted pion mass, which of course is known to very high accuracy.

Having identified the particle as a pion, we can use the mass constraint to give adjusted estimates of the energy (421 ± 10 MeV) and momentum (397 ± 5 MeV/c) that are consistent with the raw measurements *and* the known pion mass.

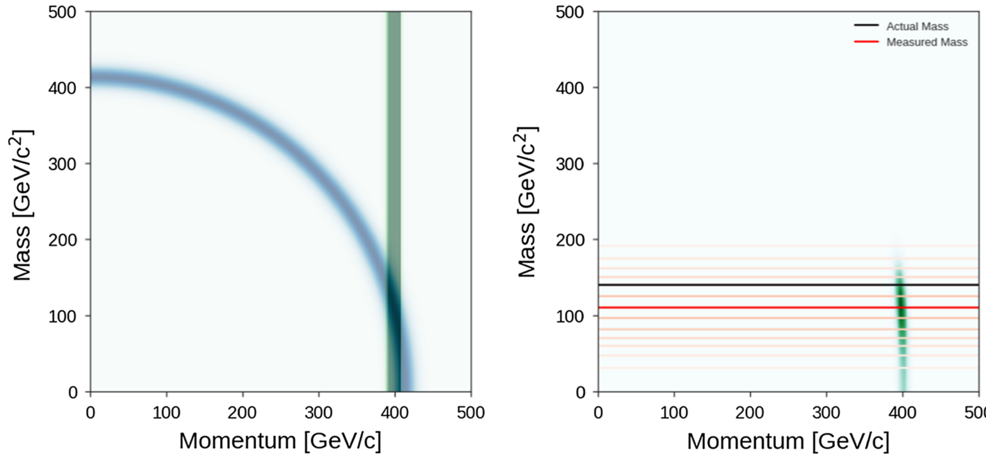


Fig. 2 Kinematic fitting for one particle uses the known particle mass to determine better estimates of the separately measured energy and momentum. The constraint is $E^2 - p^2 - m^2 = 0$. **a** Energy and momentum PDFs. **b** Particle mass is used as a constraint

4 Pseudoscalar Meson Photoproduction

Many of the most recent results pertinent to the study of the light baryon spectrum are from pseudoscalar meson photoproduction measurements, where one or more of the beam, the target or the recoiling baryon can have its polarization determined.

In this case there are four amplitudes. There are a number of equivalent representations, but transversity amplitudes b_j $(j = 1, 2, 3, 4)$ are a convenient description. Here the quantization axis is perpendicular to the reaction plane, and with linear photon polarizations J_x and J_y we have

$$\begin{aligned} b_1 &= {}_y\langle +|J_y|+\rangle_y, \\ b_2 &= {}_y\langle -|J_y|-\rangle_y, \\ b_3 &= {}_y\langle +|J_x|-\rangle_y, \\ b_4 &= {}_y\langle -|J_x|+\rangle_y. \end{aligned}$$

Assuming that the differential cross sections are measured to high accuracy, all other observables are different bilinear combinations of these amplitudes. It is further convenient to define normalized transversity amplitudes (NTA) a_j $(j = 1, 2, 3, 4)$

$$a_j \equiv \frac{b_j}{\sqrt{|b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2}},$$

noting that the a_j are functions of W (hadronic mass) and $\theta_{\text{c.m.}}$ (scattering angle).

The condition relating the normalized transversity amplitudes:

$$|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1 \tag{1}$$

defines a unit sphere in $\mathbb{R}^8$, i.e. a constraint in *amplitude space*. The key idea in this method is to recognise that a PDF in *amplitude space* can be mapped to a PDF in *observable space*, and vice versa.

5 Example Process

Taking an example measurement where pseudoscalar meson photoproduction is carried out with a linearly polarized photon beam, and the polarization of the recoil is determined, Table 1 gives example values and uncertainties for the observables Σ, P, T, O_x and O_z. The separate PDFs for these parameters are shown in Fig. 3. Note that the prior PDF for all observables in this reaction is quadratic, and this is illustrated in the lighter shaded distributions.

Table 1 Example measurements of polarization observables

Observable	Value	Uncertainty
Σ	0.222	0.037
R	− 0.419	0.041
T	− 0.979	0.095
O_x	− 0.962	0.099
O_z	− 0.040	0.099

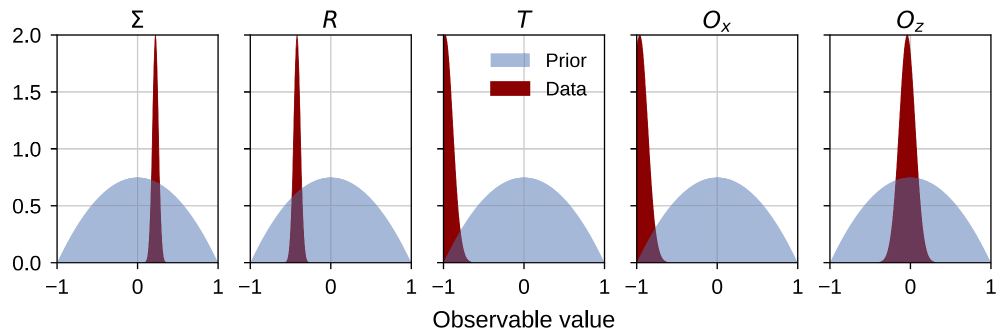


Fig. 3 Examples of the PDFs arising from the measurement of five observables. Prior PDFs for each observable are shown in light shading. The measured values are given in Table 1

We now proceed to impose the constraint (1) and adjust the estimates according to the following prescription. An ensemble of sample amplitudes $\{a_i\}; \quad i = 1, 2, 3, 4$ are generated at random. Observables $o_j = f_j(a_i); \quad j = 1, \ldots 16$ are calculated for each of the amplitude combinations and compared with the measured observables and their uncertainties. A probability is evaluated for each observable, based on the ratio of a gaussian PDF from the raw data and the quadratic prior PDF. A Markov Chain Monte Carlo (MCMC) is then used to wander about amplitude space to generated an amplitude PDF, which depend on the measured observables. This PDF is then projected back into observable space to give the constrained parameter estimates for the observables.

The procedure is illustrated in Figs. 4 and 5. The prior PDFs of the observables show the quadratic shape in Fig. 4, together with intriguing, but predictable, shapes for joint PDFs between pairs of observables. Figure 5 shows the resulting PDFs after the MCMC likelihood evaluation. The red lines indicate the values that were initially measured.

For the observables Σ, P, T, O_x and O_z, it can be seen that evaluated PDFs will lead to slightly adjusted values and uncertainties, although not significantly outside the original uncertainties. The method returns PDFs for all observables, regardless of whether there are initial measured values. In this case we also present the PDFs for the unmeasured observables C_x and C_z. It is clearly seen that the unmeasured observables are highly correlated with the measured observables in this case.

6 Application to Kaon Photoproduction

Recent measurements of $\gamma + p \to K^+\Lambda$ from the CLAS collaboration [3] produced data on the observables Σ, T, O_x and O_z, in addition to previous work that produced recoil polarization P data [2]. In order to demonstrate that the method for constrained parameter estimation could be applied to real data, the procedure described above was applied to the data set reported in [3].

Figure 6 shows the results by depicting angular distributions of the target asymmetry T at several values of hadronic mass W from 1.7 to 2.2 GeV. The exact energy and angle values are not relevant for the present purposes; the plots are simply to show that the method works in practice.

What is evident from the plots is that data points that were originally outside the physical region $[-1, +1]$ have been brought back inside, whilst there is little adjustment required for the majority of the points.

Perhaps a more convincing demonstration is to look at the combinations of the observables Σ, P, T. Figure 7 shows the raw (a) and adjusted (b) data plotted in three dimensions. The tetrahedron within the cube delimits

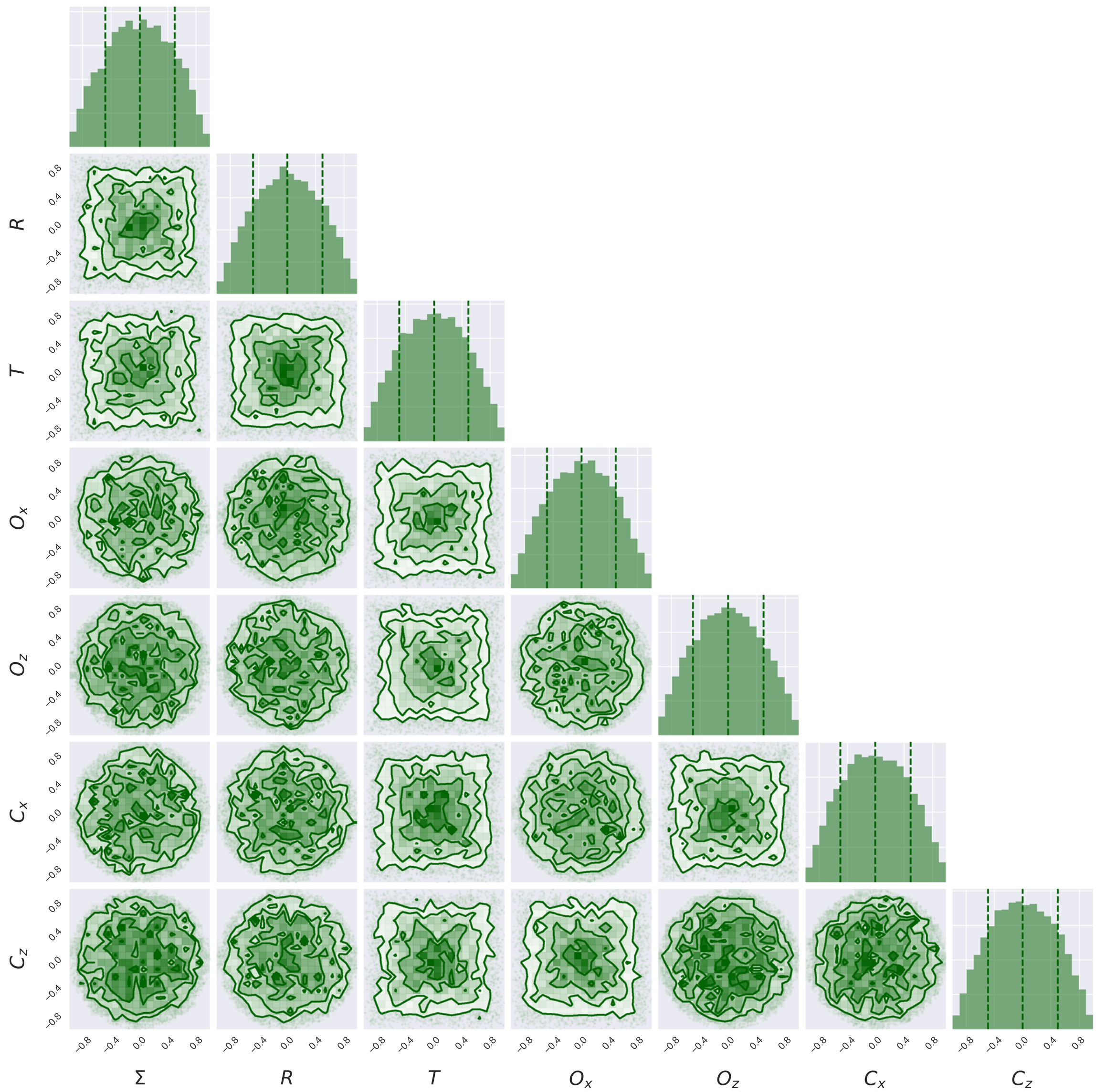


Fig. 4 Prior PDFs

the physical region for the combined values, and it is quite evident that several raw points lie outside the region, whereas the adjusted data have all been forced to take on physical values.

7 Conclusion and Outlook

A method designed to evaluate polarization data from measurements of pseudoscalar meson photoproduction experiments has been described. By using the constraint implied by the connection of the observables to the amplitudes, adjustments can be made to the raw observable estimates that are reported in experiments, in a manner similar to kinematic fitting.

Future work will focus on studies to show that the method is statistically robust, and also how to incorporate information from more than one experiment in one combined likelihood calculation. The net result will be a scheme that allows the processing of raw data to give data sets that are internally self-consistent, thus removing the need need for arbitrary fudge factors when fitting theoretical models.

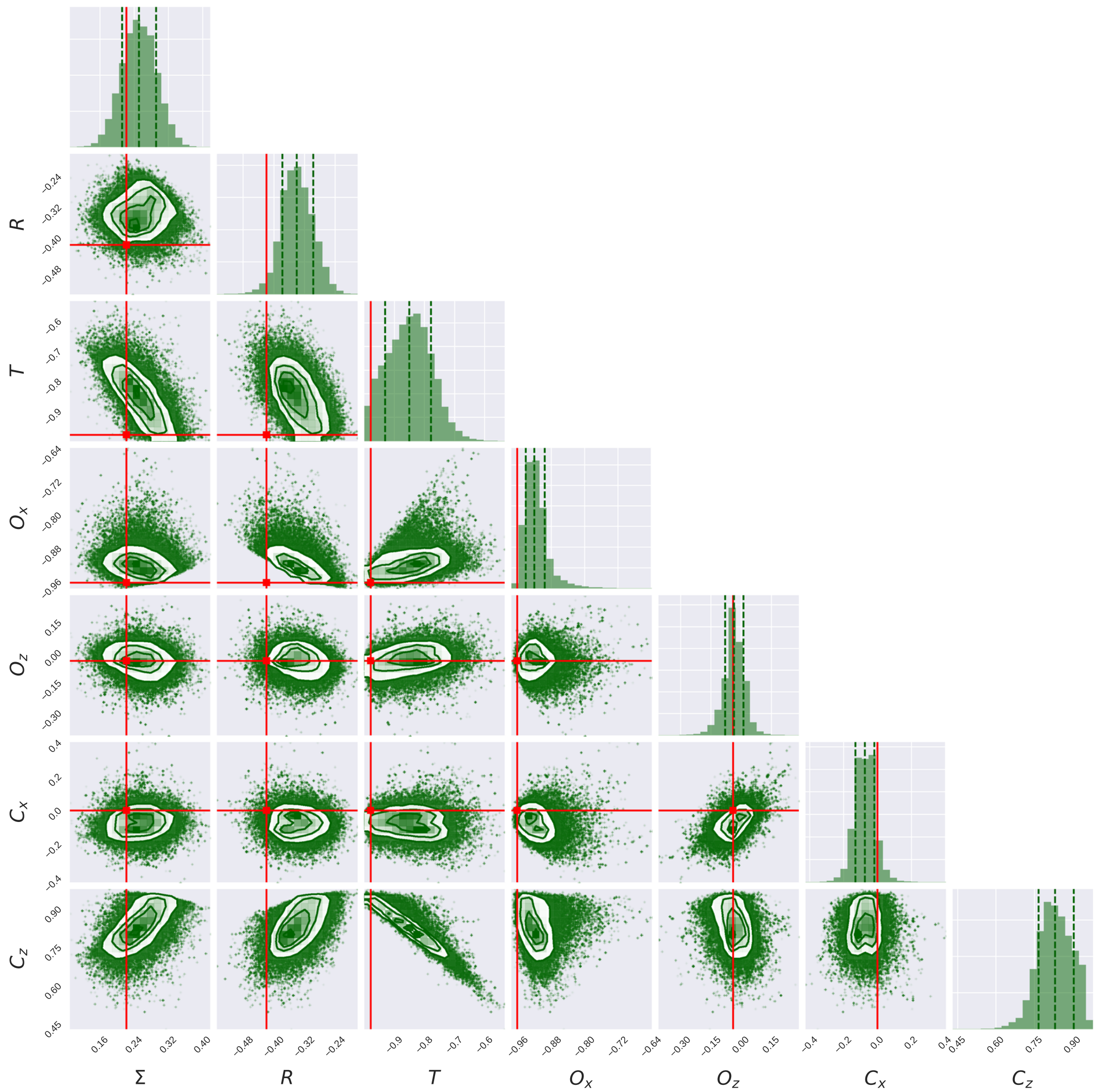


Fig. 5 Posterior PDFs

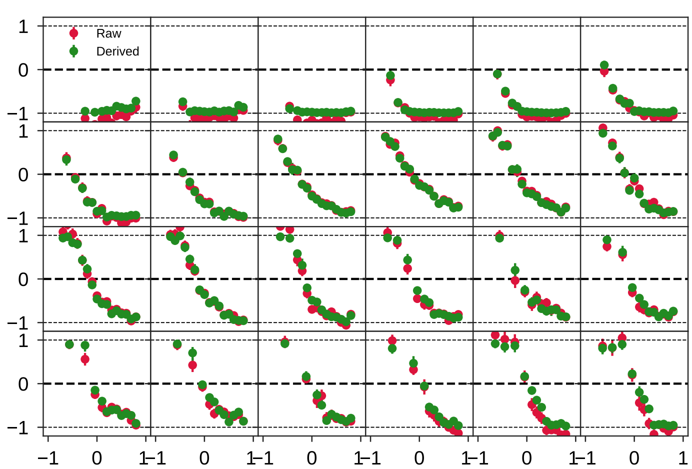
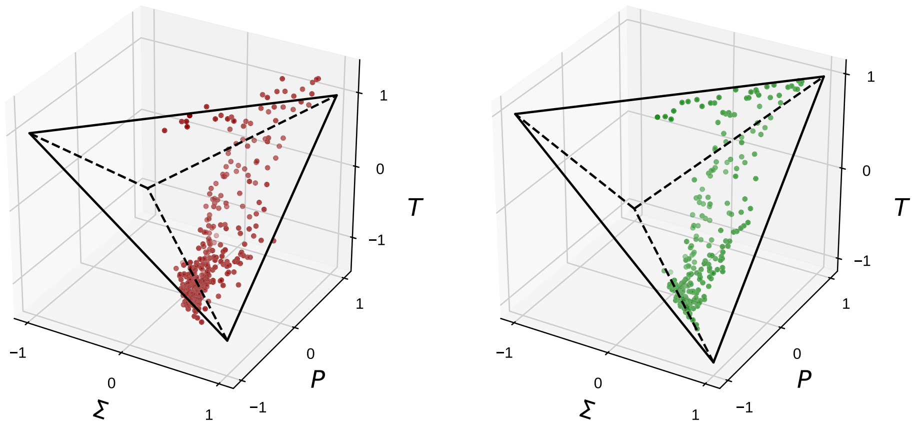


Fig. 6 Angular distributions of the target asymmetry T in kaon photoproduction



(a) **(b)**

Fig. 7 The raw and processed data compared to the physical region (the tetrahedron). **a** Raw data for Σ, P and T. **b** Processed data

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