Moments and Lattice QCD

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Outline

- What can Hall C @ 12 GeV do?

  $\longrightarrow$ Structure functions over large kinematic range

  $\longrightarrow$ L-T separation for $F_{1,2}(x, Q^2)$ up to $Q^2 = 12$ GeV^2

  $\longrightarrow$ Polarization asymmetries $A_{1,2}$ up to $Q^2 = 10$ GeV^2

  $\longrightarrow$ $Moments$ of $F_{1,2}$ and $g_{1,2}$ structure functions

- What can lattice QCD actually calculate?

  $\longrightarrow$ Lattice moments

  $\longrightarrow$ Chiral extrapolation (π cloud)

  $\longrightarrow$ Plans of LHPC (Lattice Hadron Physics Collaboration)

INELASTIC SCATTERING

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[2\sin^2\frac{\theta}{2}\, W_1 + \cos^2\frac{\theta}{2}\, W_2\right]$$

$$\left.\begin{aligned} Q^2 &= 4EE'\sin^2\frac{\theta}{2} \\ \nu &= E - E' \end{aligned}\right\} \quad x \equiv \frac{Q^2}{2M\nu}$$

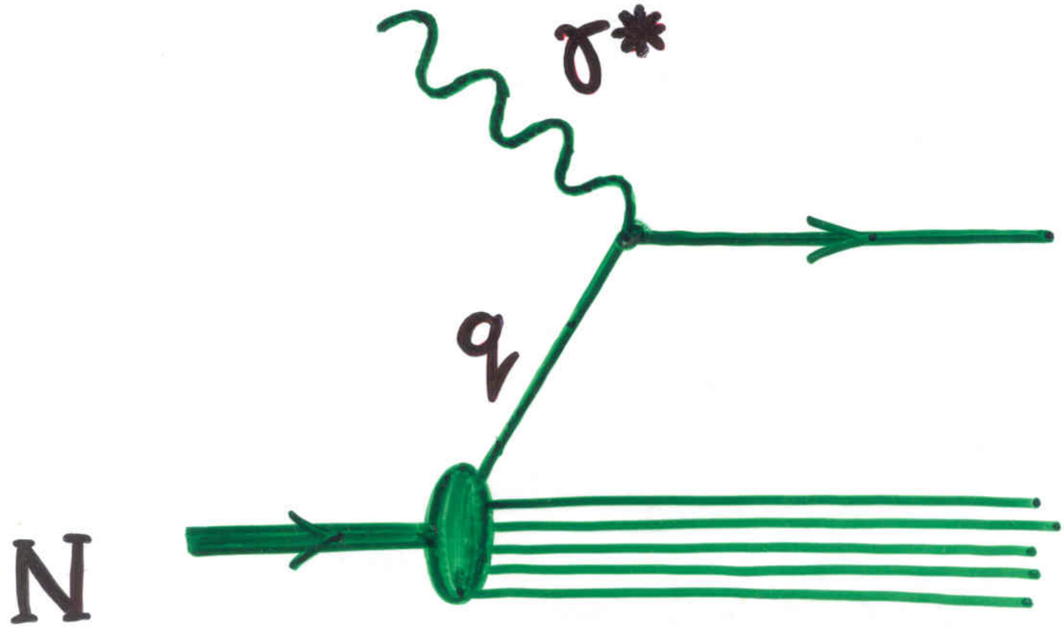
BJORKEN LIMIT $\quad Q^2, \nu \to \infty$

x FIXED

$$\boxed{\begin{aligned} M W_1(x, Q^2) &\longrightarrow F_1(x) \\ \nu W_2(x, Q^2) &\longrightarrow F_2(x) \end{aligned}}$$

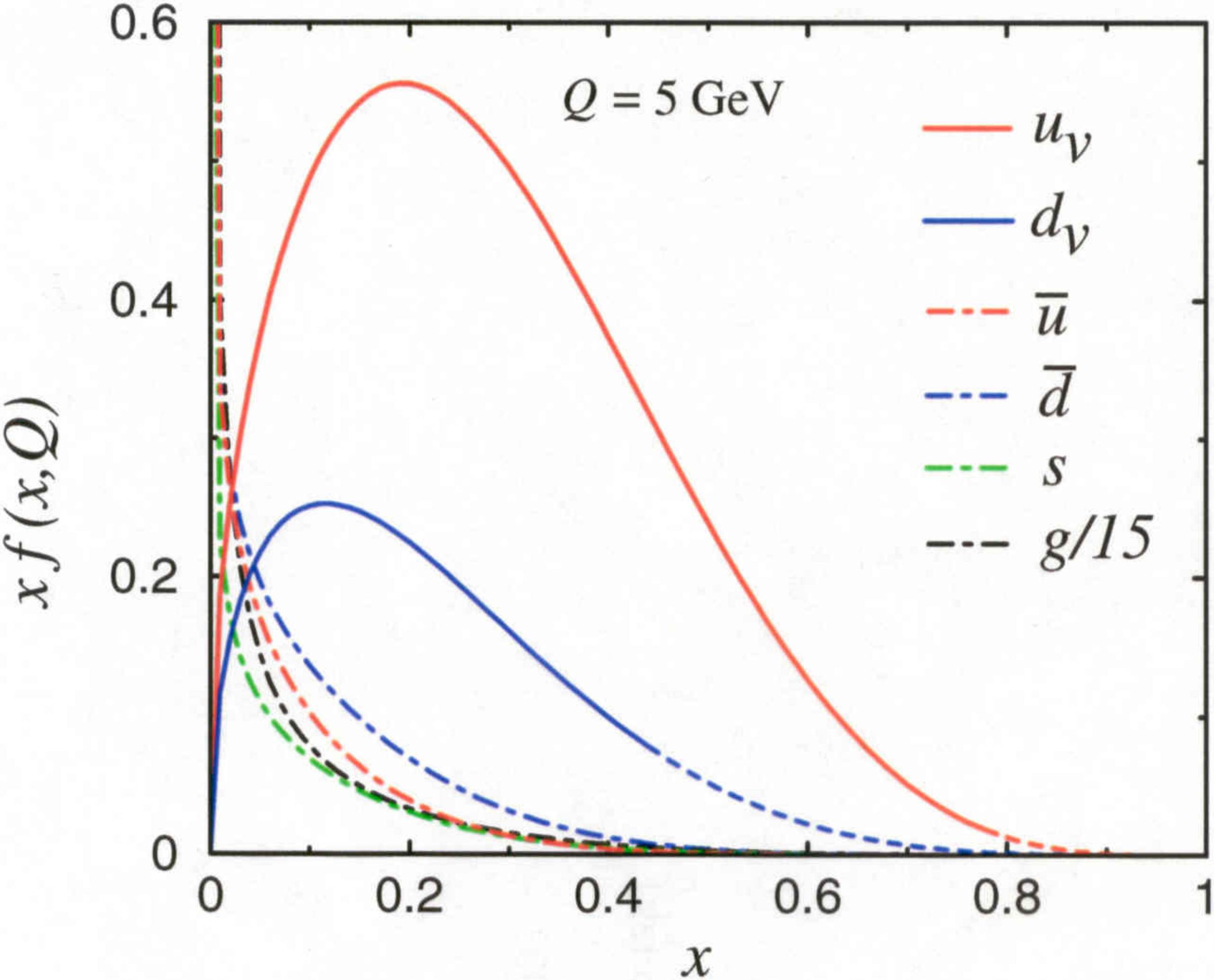
ALSO $\quad F_2(x) = 2x F_1(x)$

STRUCTURE FUNCTIONS IN PARTON MODEL



$$F_2(x) = x \sum_q e_q^2 \, q(x)$$

$$= \frac{4}{9} x u(x) + \frac{1}{9} x d(x) + \cdots$$



Parton distributions in the proton

POLARIZED SCATTERING

$$\frac{d^2\sigma^{\uparrow\uparrow-\uparrow\downarrow}}{d\Omega\, dE'} = \frac{4\alpha^2 E'}{M\nu Q^2 E}\left[(E+E'\cos\theta)\, g_1 - 2Mx\, g_2\right]$$

$$\frac{d^2\sigma^{\uparrow\rightarrow-\uparrow\leftarrow}}{d\Omega\, dE'} = \frac{4\alpha^2 E'}{M\nu Q^2 E}\sin\theta\left[g_1 + \frac{2E}{\nu}\, g_2\right]$$

QUARK-PARTON MODEL

$$g_1(x) = \frac{1}{2}\sum_q e_q^2\, \Delta q(x)$$

$$q^\uparrow - q^\downarrow + \bar{q}^\uparrow - \bar{q}^\downarrow$$

$$g_2(x) = g_2^{WW}(x) + g_2^{\text{twist-3}}(x)$$

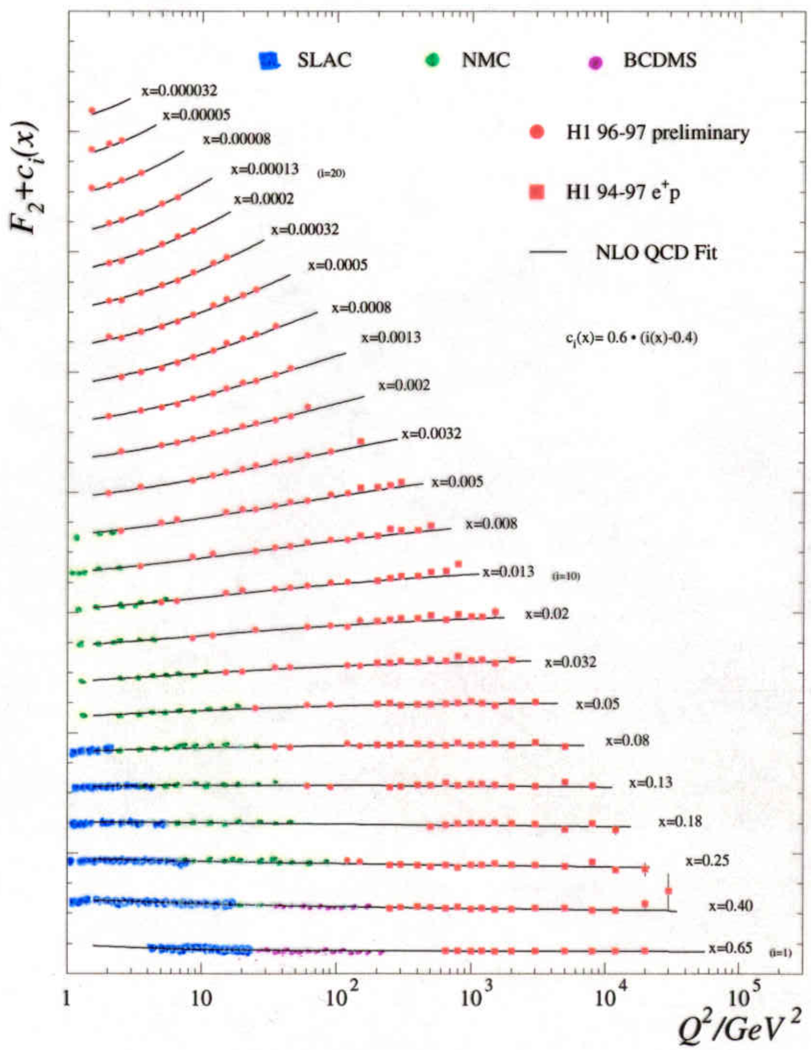
$$\int_x^1 \frac{dy}{y}\, g_1(y) - g_1(x)$$

Wandzura-Wilczek

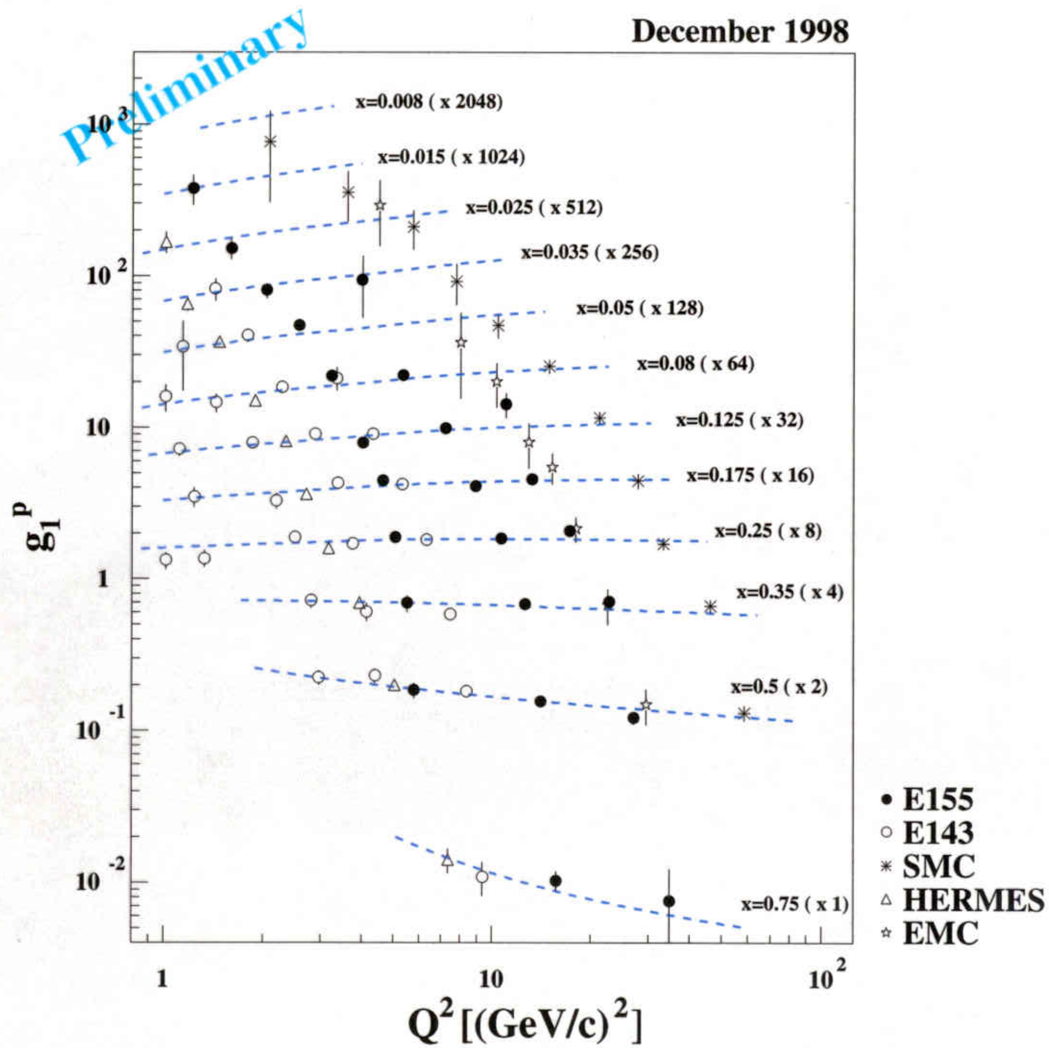
$$\langle \psi^\dagger \vec{B}\, \psi \rangle$$

quark-gluon correlations (color $\vec{B}$ & $\vec{E}$ polarizabilities)

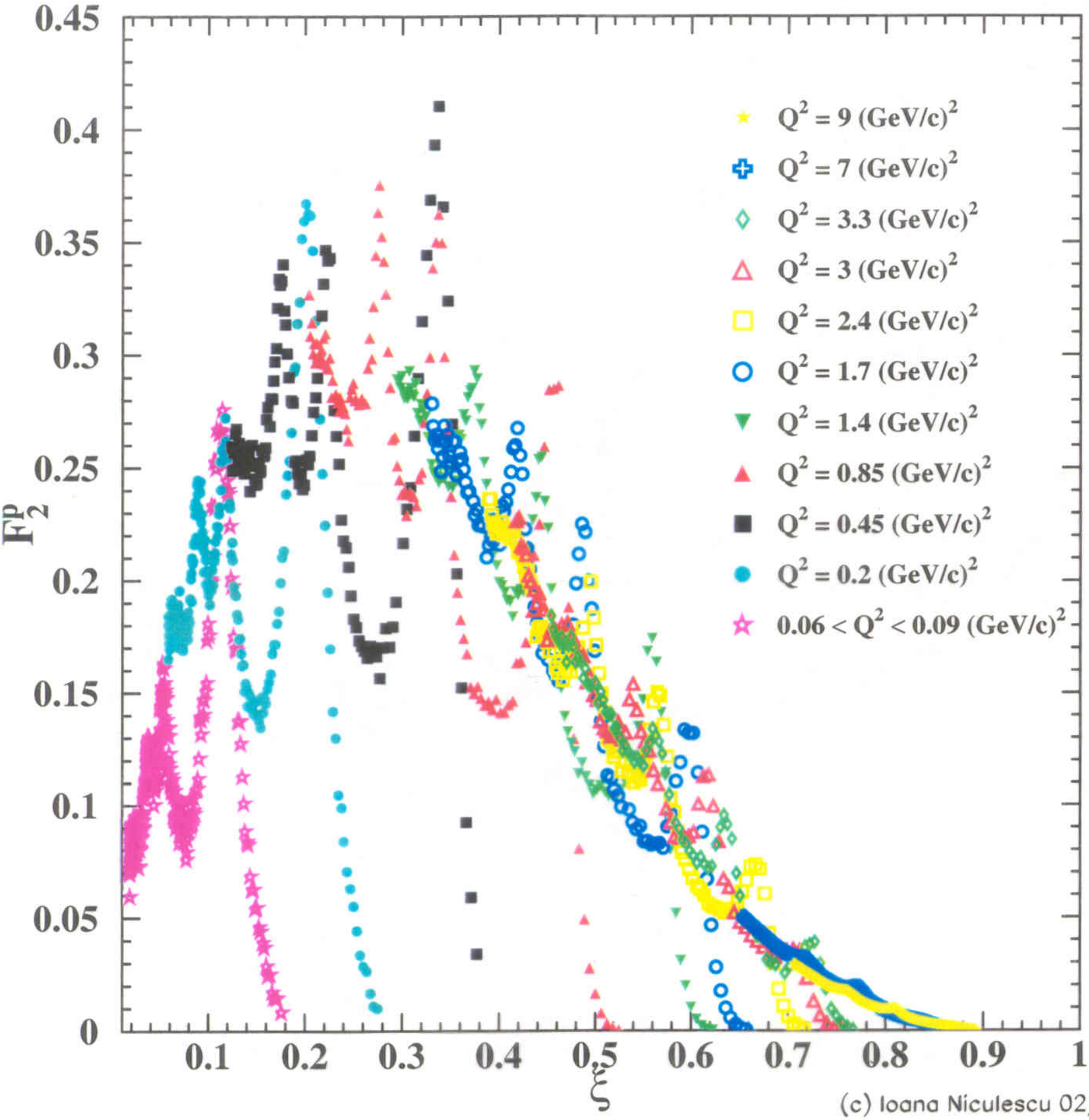
World data on F_1^p



World data on g_1^p



NICULESCU ET AL. PRL 85 (2000) 1182



(c) Ioana Niculescu 02/02/00

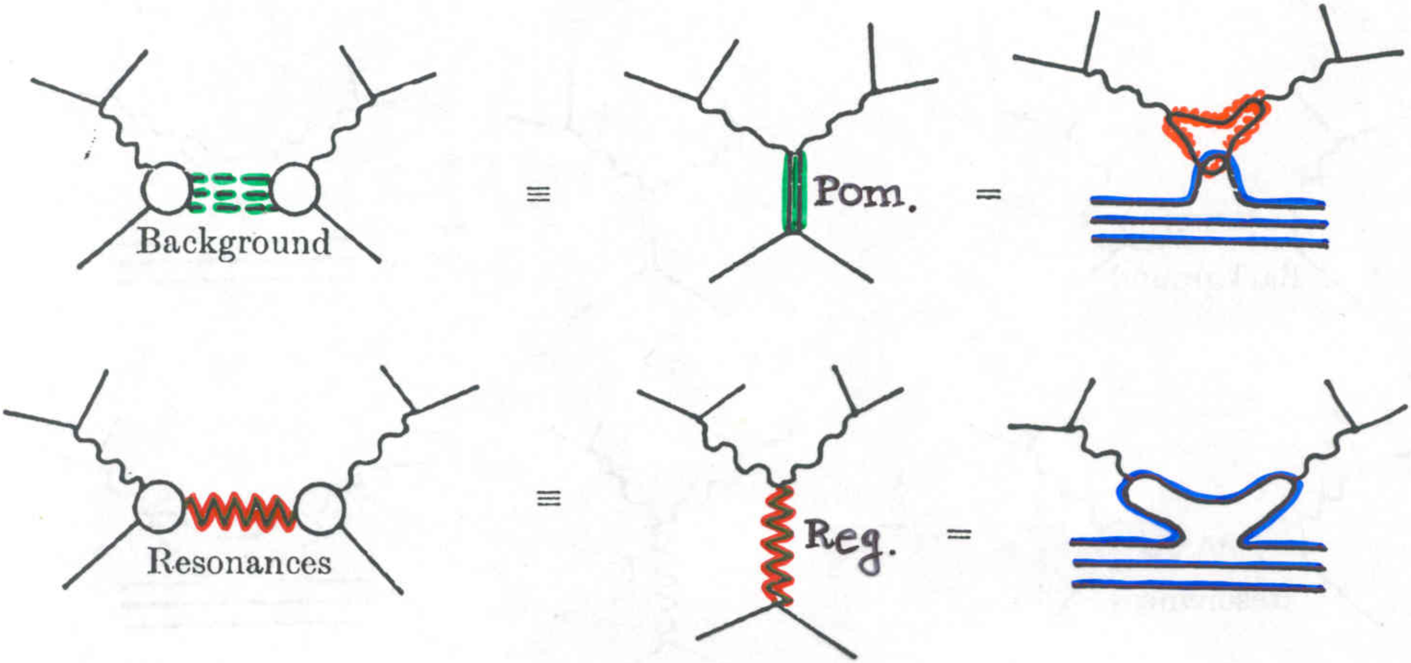
GAUGE INVARIANCE

$\sigma^{\gamma p}$ FINITE AT $Q^2 = 0$

$$\sigma^{\gamma^* p} \propto \frac{F_2^p}{Q^2}$$

$$\Rightarrow \quad F_2^p(Q^2 \to 0) \to 0$$

DUALITY DIAGRAMS FOR ep → eX



COLLINS 1977

For $\gamma^* N$ scattering at large s,

$$F_2 \sim x^{1-\alpha(0)}$$

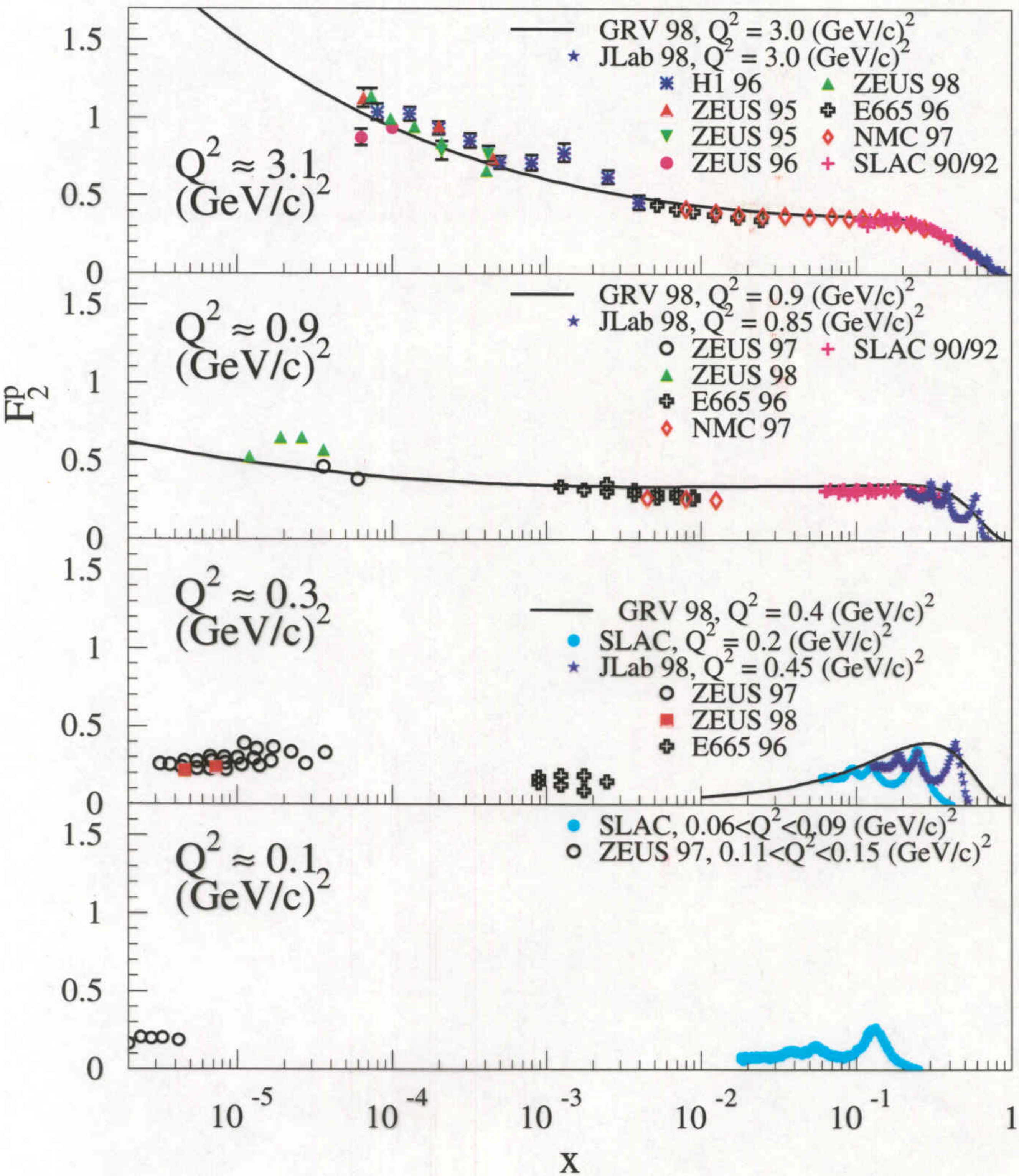
(since $x \sim 1/s$)

$\longrightarrow$ Valence (non-diffractive, Reggeon)

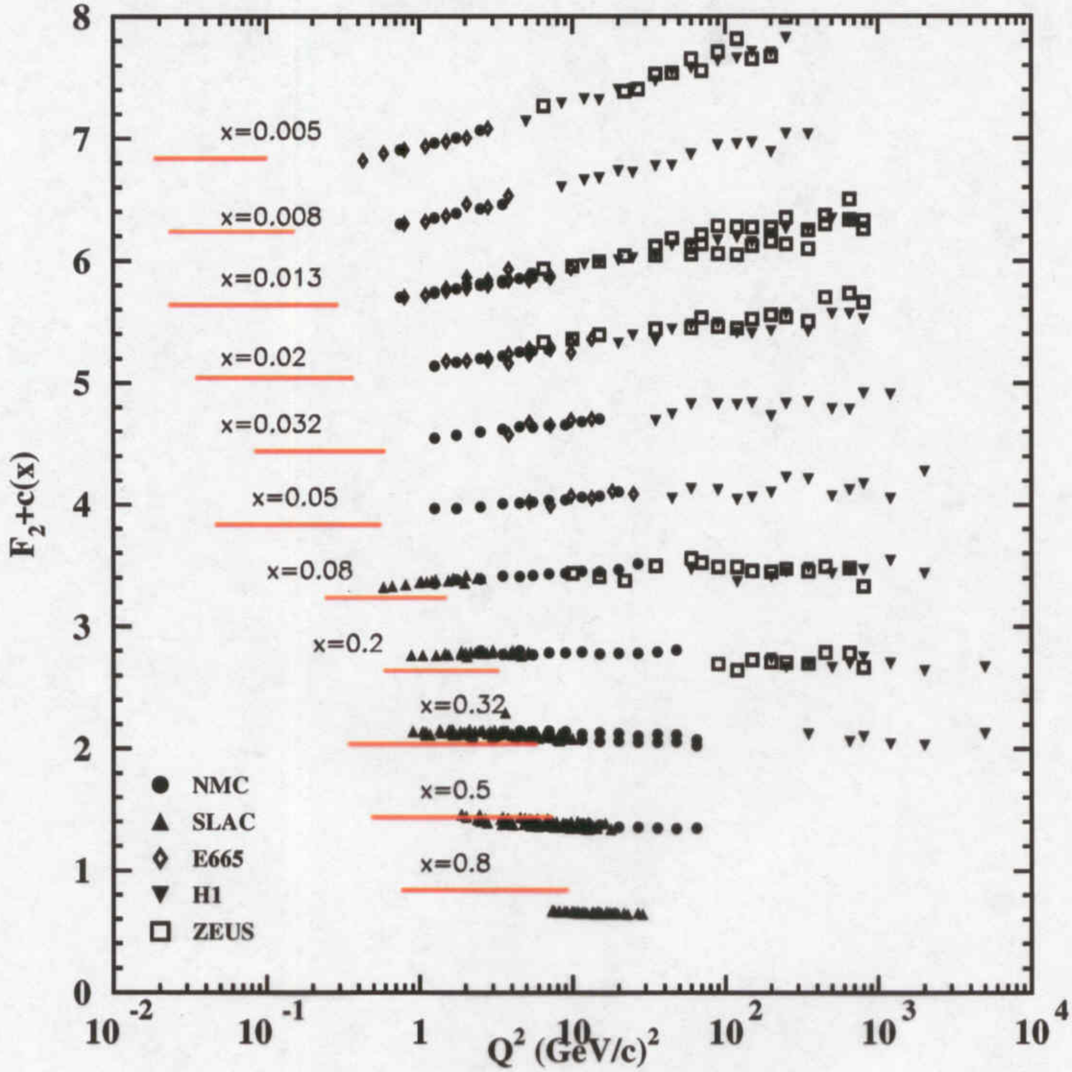
$$F_2^{\text{val}} \sim x^{1/2}$$

$\longrightarrow$ Sea (diffractive, Pomeron)

$$F_2^{\text{sea}} \sim x^0$$



GRV 98, Q² = 3.0 (GeV/c)²
JLab 98, Q² = 3.0 (GeV/c)²
H1 96
ZEUS 98
ZEUS 95
E665 96
ZEUS 95
NMC 97
ZEUS 96
SLAC 90/92
Q² ≈ 3.1 (GeV/c)²
GRV 98, Q² = 0.9 (GeV/c)²
JLab 98, Q² = 0.85 (GeV/c)²
ZEUS 97
SLAC 90/92
ZEUS 98
E665 96
NMC 97
Q² ≈ 0.9 (GeV/c)²
Q² ≈ 0.3 (GeV/c)²
GRV 98, Q² = 0.4 (GeV/c)²
SLAC, Q² = 0.2 (GeV/c)²
JLab 98, Q² = 0.45 (GeV/c)²
ZEUS 97
ZEUS 98
E665 96
Q² ≈ 0.1 (GeV/c)²
SLAC, 0.06<Q²<0.09 (GeV/c)²
ZEUS 97, 0.11<Q²<0.15 (GeV/c)²
F₂ᵖ
x



Kinematic regions covered by global data on F_2^p

TRANSITION REGION $Q^2 < 1$ GeV^2

FOR $0.005 \leq x \leq 0.2$ UNEXPLORED

Longitudinal/Transverse Separation

Measured cross section

$$\frac{d\sigma}{d\Omega dE'} = \Gamma\left(\sigma_T(x,Q^2) + \epsilon\, \sigma_L(x,Q^2)\right)$$

$\longrightarrow$ virtual photon flux Γ

$\longrightarrow$ virtual photon polarization

$$\epsilon = \left(1 + 2(1+\nu^2/Q^2)\tan^2(\theta/2)\right)^{-1}$$

$\sigma_T(\sigma_L)$ cross section for photoabsorption of transverse (longitudinal) photon

$$\sigma_T = \frac{4\pi^2\alpha}{2x(W^2-M^2)}\, 2xF_T$$

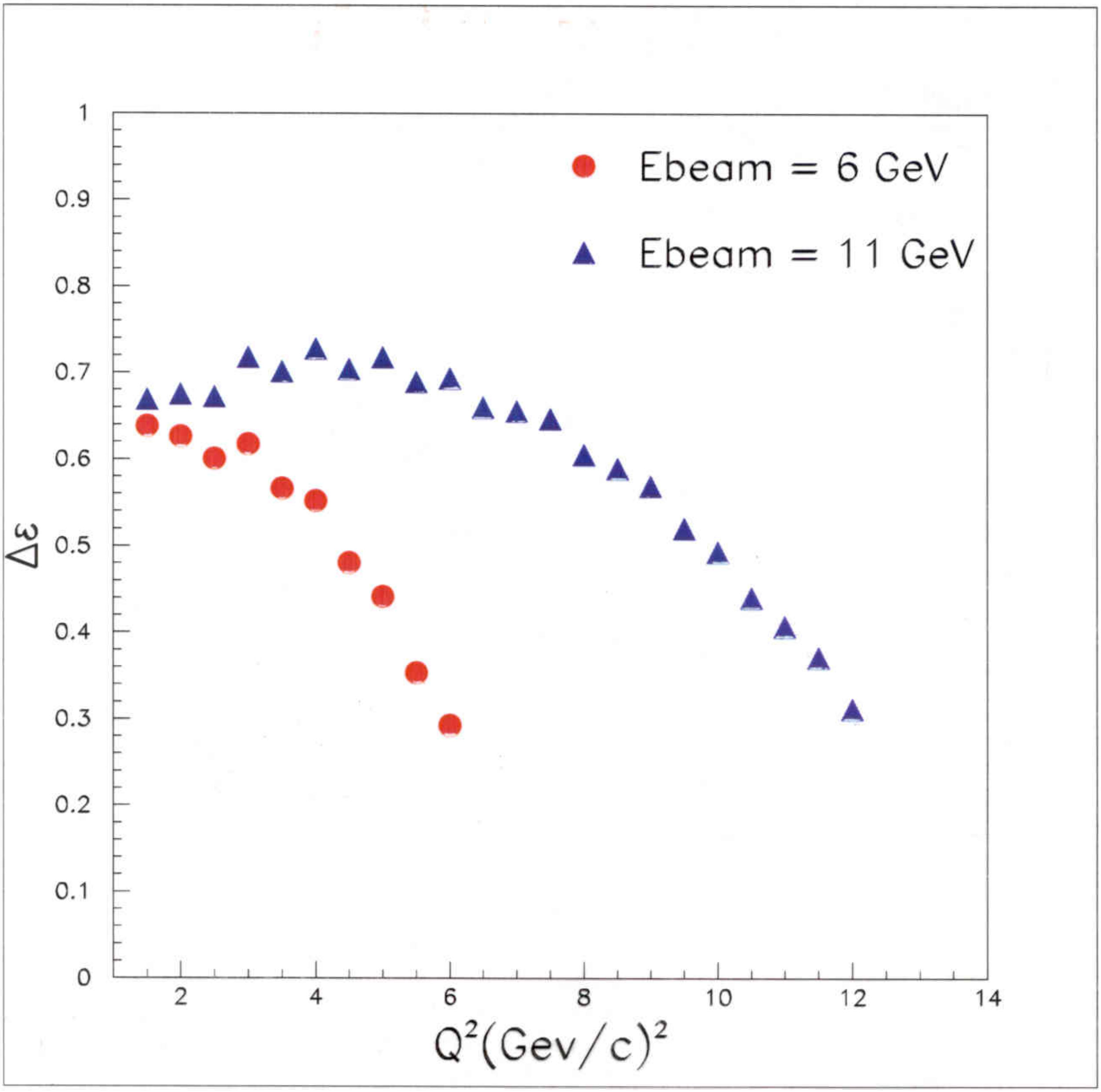
$$\sigma_L = \frac{4\pi^2\alpha}{2x(W^2-M^2)}\, F_L$$

where $F_T = F_1$, $F_L = (1+Q^2/\nu^2)F_2 - 2xF_1$

Rosenbluth separation

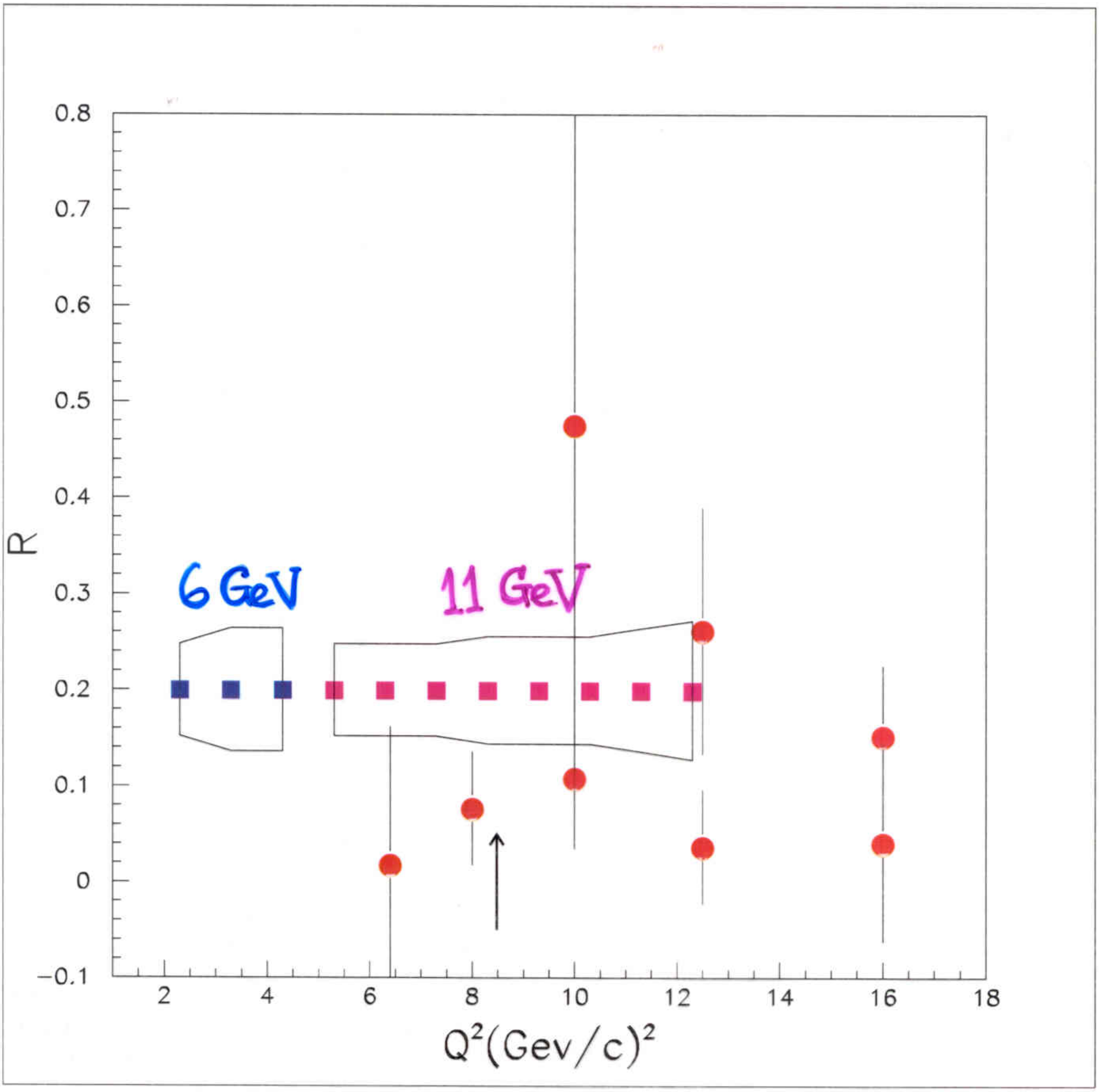
$\longrightarrow$ vary ϵ for fixed x and Q^2

$\longrightarrow$ fit intercept σ_T, slope σ_L



Range of $\Delta\epsilon$ accessible at $x = 0.8$
in Hall C at 12 GeV

$R = \sigma_L / \sigma_T$



World data on R for $x > 0.75$. Arrow indicates $W = 2$ GeV boundary for $x = 0.8$.

Spin Dependence of Transition

- Transition in spin dependent structure functions even more fascinating
  $\longrightarrow$ structure functions need not be positive!
  $\longrightarrow$ dramatic transition from (negative) GDH integral at $Q^2 = 0$ to (positive) Bjorken integral at $Q^2 \geq 2$ GeV^2

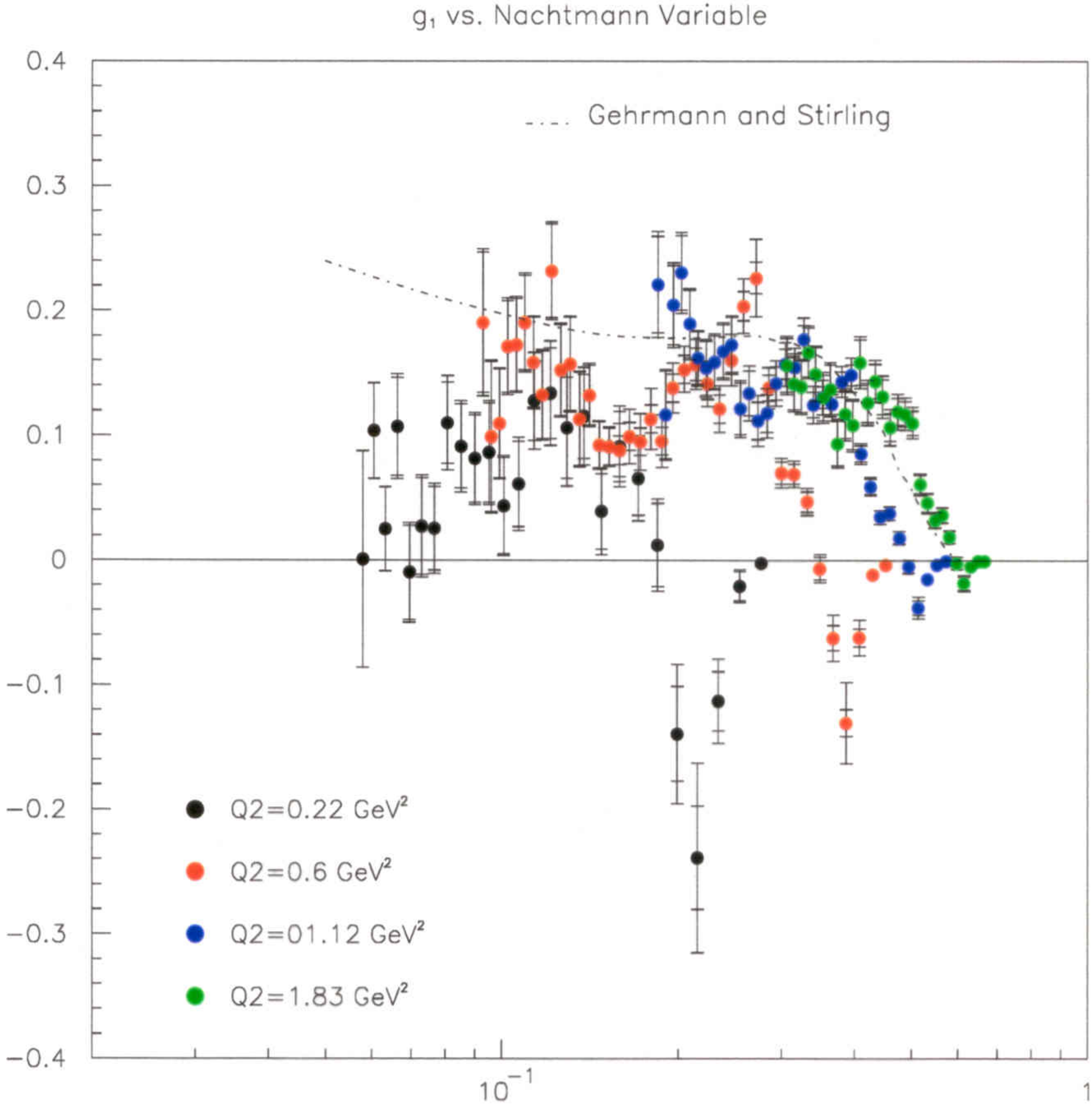
- Are higher twist effects more important for g_1 than F_2?

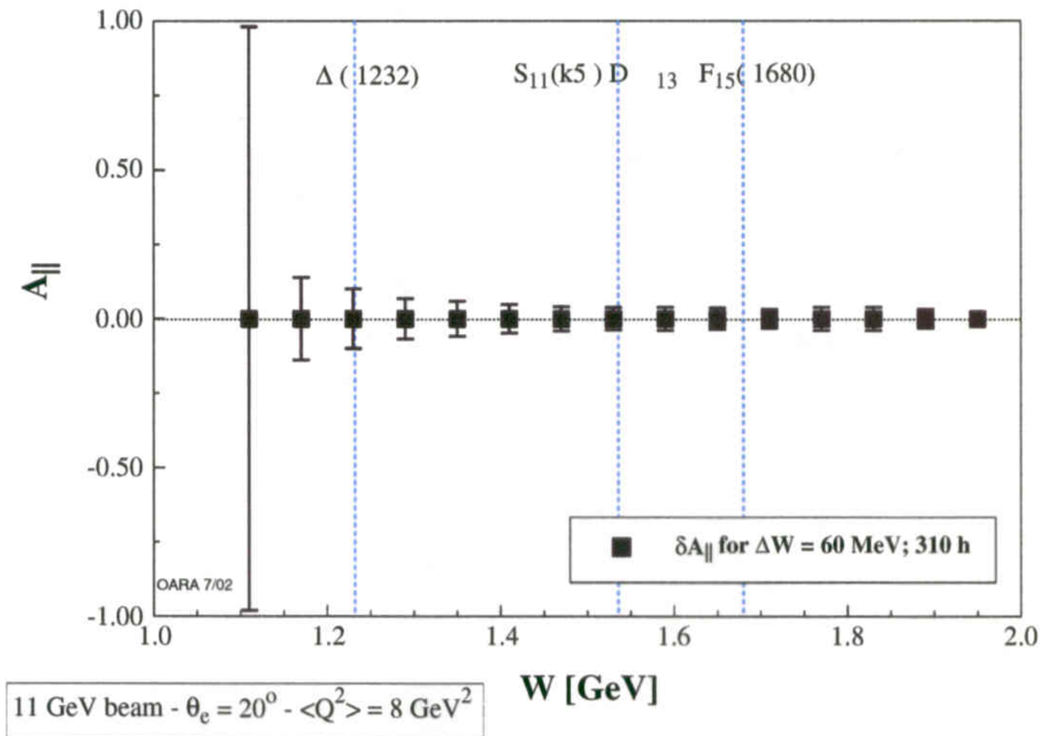
- How does g_1 at low Q^2 approach the scaling curve?

- Transition for (twist-3) g_2 structure function?

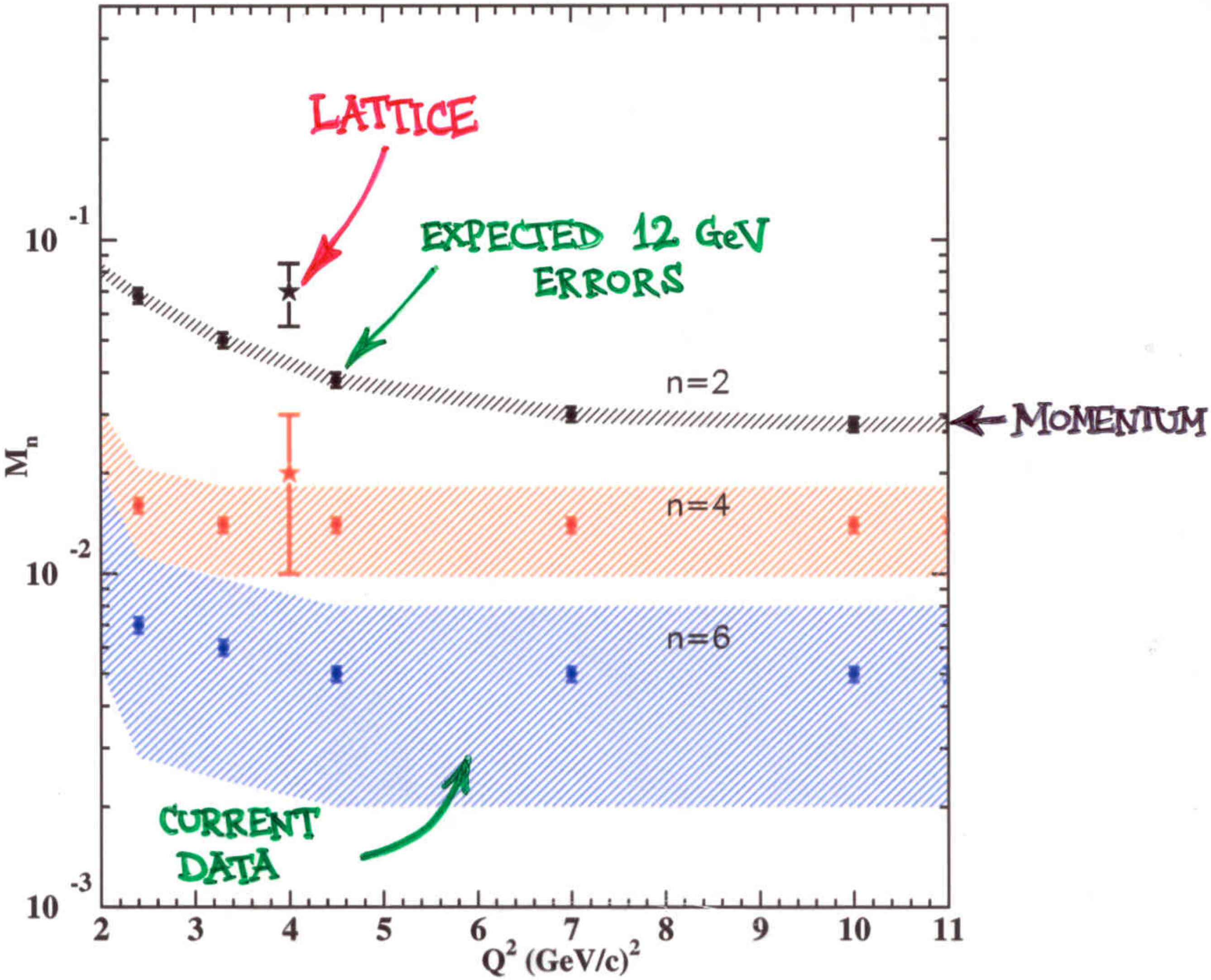
- Complete absence of data in resonance region at high x

Testing duality in g_1





Expected uncertainties for $A_\parallel$ at $Q^2 = 8\ \mathrm{GeV}^2$.



Lowest moments of F_2^p, compared with lattice calculations.

Structure Functions in Lattice QCD

Cannot calculate x-distribution, only *moments*

$$\langle x^n \rangle = \int_0^1 dx\, x^n \left(q(x) + (-1)^{n+1}\, \bar{q}(x) \right)$$

where $\langle x^n \rangle$ are matrix elements of twist-2 operators (twist = dimension − spin):

$$\langle x^n \rangle\, p_{\mu_1} \cdots p_{\mu_{n+1}} = \langle N |\, \mathcal{O}_{\{\mu_1 \cdots \mu_{n+1}\}} | N \rangle$$

where

$$\mathcal{O}_{\{\mu_1 \cdots \mu_{n+1}\}} = \overline{\psi}\, \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_{n+1}\}}\, \psi$$

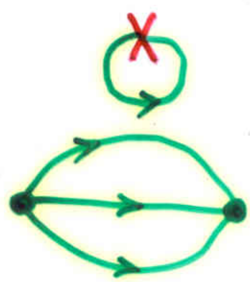
On the lattice, operators transform according to hypercubic $H(4)$ subgroup of $O(4)$

$\longrightarrow$ because of operator mixing, only $n \leq 3$ moments can be calculated

Lattice Calculation of SF Moments

Matrix elements of twist-2 operators have contributions from connected and disconnected diagrams:

CONNECTED



DISCONNECTED

Disconnected diagrams are very noisy
$\longrightarrow$ at present only connected diagrams can be reliably calculated on the lattice

However, disconnected contributions cancel in (non-singlet) $u - d$ difference!

DETMOLD ET AL. PRD66, 054501

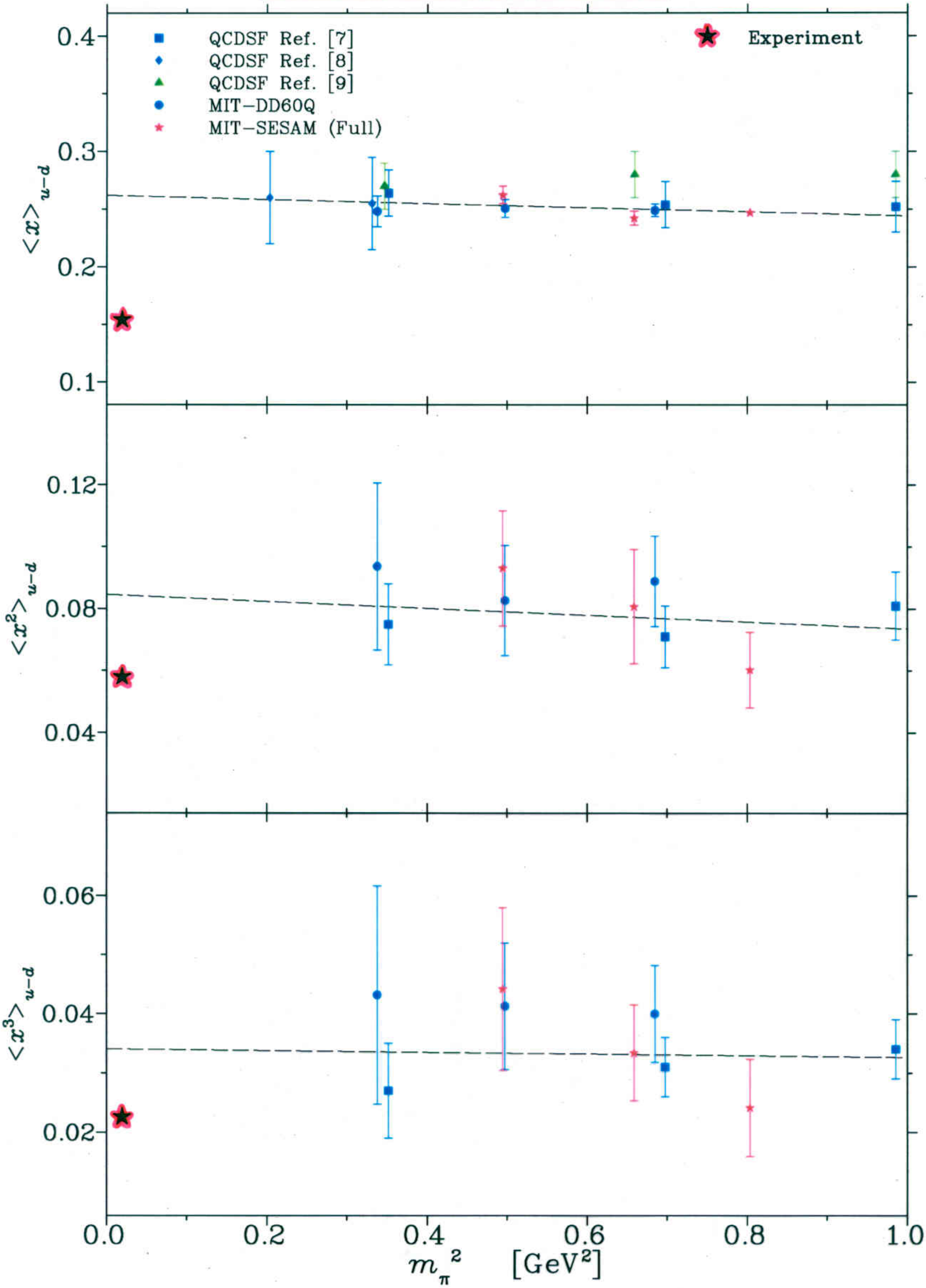
Reference	Q/U	Quark Action	Lattice	a [fm]	m_π [GeV]	Moments	Symbol
QCDSF [30]	Q	Wilson	$16^3 \times 32$	0.1	0.6 – 1.0	All	▲
QCDSF [31]	Q	Wilson	$24^3 \times 32$	0.1	0.35 – 0.6	All	■
QCDSF [32]	Q	NPIC	$16^3 \times 32$	0.1	0.6 – 1.0	All	×
QCDSF [33]	Q	NPIC	$16^3 \times 32$	0.1	0.65 – 1.2	$\langle 1\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$	♦
	Q	NPIC	$24^3 \times 48$	0.075	0.7 – 1.2	$\langle 1\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$	●
	Q	NPIC	$32^3 \times 48$	0.05	0.6 – 1.25	$\langle 1\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$	✳
MIT [20]	Q	Wilson	$16^3 \times 32$	0.1	0.58 – 0.82	All	★
MIT-SESAM [20]	U	Wilson	$16^3 \times 32$	0.1	0.63 – 1.0	All	◇
MIT-SCRI [20]	U	Wilson	$16^3 \times 32$	0.1	0.48 – 0.67	All	□
KEK [27, 28]	Q	Wilson	$16^3 \times 20$	0.14	0.52 – 0.97	$\langle 1\rangle_{\Delta q}$, $\langle 1\rangle_{\delta q}$	▼

TABLE I: Simulation parameters for lattice calculations of the moments of PDFs included in our analysis. Q/U corresponds to quenched/unquenched simulations, and NPIC denotes the nonperturbatively improved clover quark action. "All" moments correspond to $\langle x^i\rangle_q$ for $i = 1, 2, 3$, $\langle x^i\rangle_{\Delta q}$ for $i = 0, 1, 2$, and $\langle x^i\rangle_{\delta q}$ for $i = 0, 1$. The symbols shown in the final column correspond to those plotted in Figs. 8, 9 and 10.

VOLUME LARGE ENOUGH TO INCLUDE π CLOUD

$L \gtrsim 4/m_\pi$

Linear Extrapolation



Chiral Symmetry and QCD

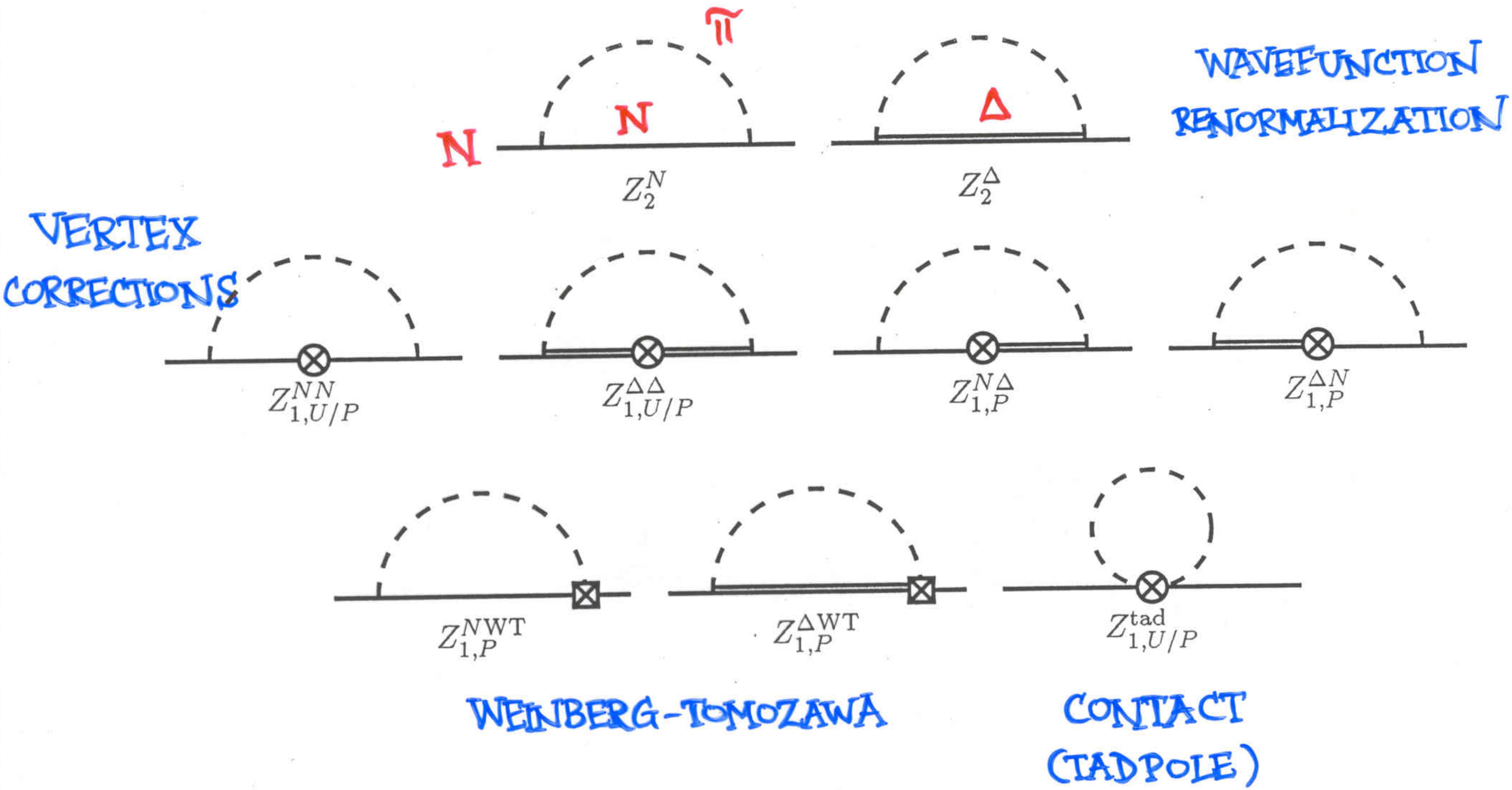
- In limit $m_q \to 0$, QCD has exact *chiral* $\mathrm{SU(N}_f)_L \times \mathrm{SU(N}_f)_R$ symmetry

- Spontaneous breaking of chiral symmetry
  $\longrightarrow$ appearance of pseudoscalar
  Goldstone bosons (π, K)

- Pions play important role at small $m_\pi^2 \sim m_q$
  $\longrightarrow$ expand observables in powers of m_π
  (e.g. χPT)

- Pion loops in χPT generate non-polynomial terms which are *non-analytic* in m_q
  (odd powers of m_π or log m_π)

- Coefficients of non-analytic terms are model-independent
  $\longrightarrow$ given in terms of g_A and f_π



WAVEFUNCTION RENORMALIZATION
N
N
π
Δ
Z_2^N
Z_2^Δ
VERTEX CORRECTIONS
$Z_{1,U/P}^{NN}$
$Z_{1,U/P}^{\Delta\Delta}$
$Z_{1,P}^{N\Delta}$
$Z_{1,P}^{\Delta N}$
$Z_{1,P}^{N\mathrm{WT}}$
$Z_{1,P}^{\Delta\mathrm{WT}}$
$Z_{1,U/P}^{\mathrm{tad}}$
WEINBERG-TOMOZAWA
CONTACT (TADPOLE)

Chiral Extrapolation of SF Moments

Extrapolation formula for n-th moment of $u-d$:

$$\langle x^n \rangle_{u-d} = a_n + b_n \frac{m_\pi^2}{m_\pi^2 + m_b^2} + c_n \, m_\pi^2 \log\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)$$

Detmold, WM, Negele, Renner, Thomas,
Phys.Rev.Lett. **87** (2001) 172001

Thomas, WM, Steffens
Phys.Rev.Lett. **85** (2000) 2892

Coefficient of non-analytic term:

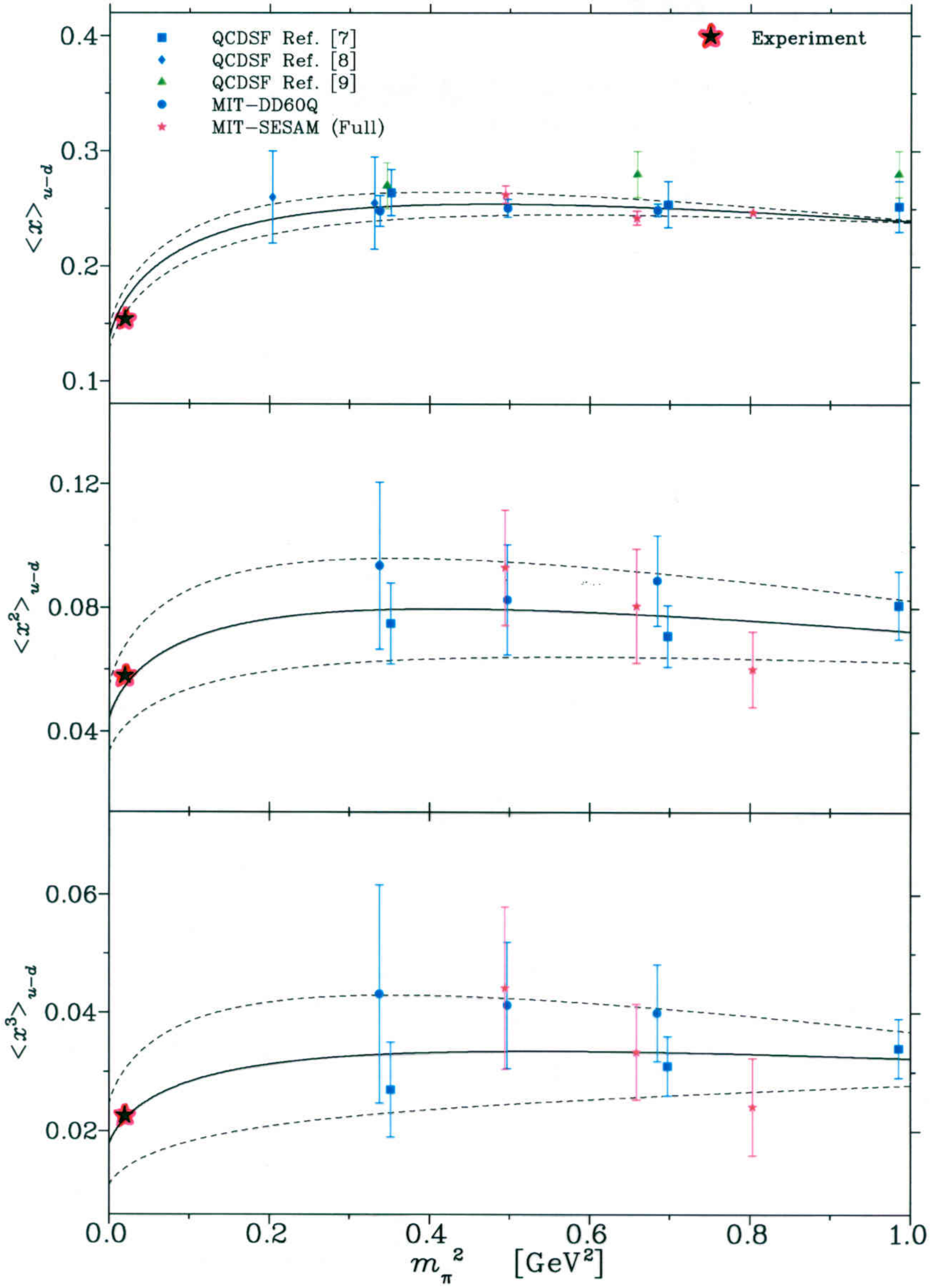
$$c_n = -a_n \frac{3g_A^2 + 1}{4\pi^2 f_\pi^2}$$

calculated in chiral perturbation theory

Arndt, Savage, `nucl-th/0105045`
Chen, Ji, `hep-ph/0105197`

DETMOLD, WM, THOMAS PRD 66, 054501

Chiral Extrapolation ($\mu = 550$ MeV)



x dependence and Regge Trajectories

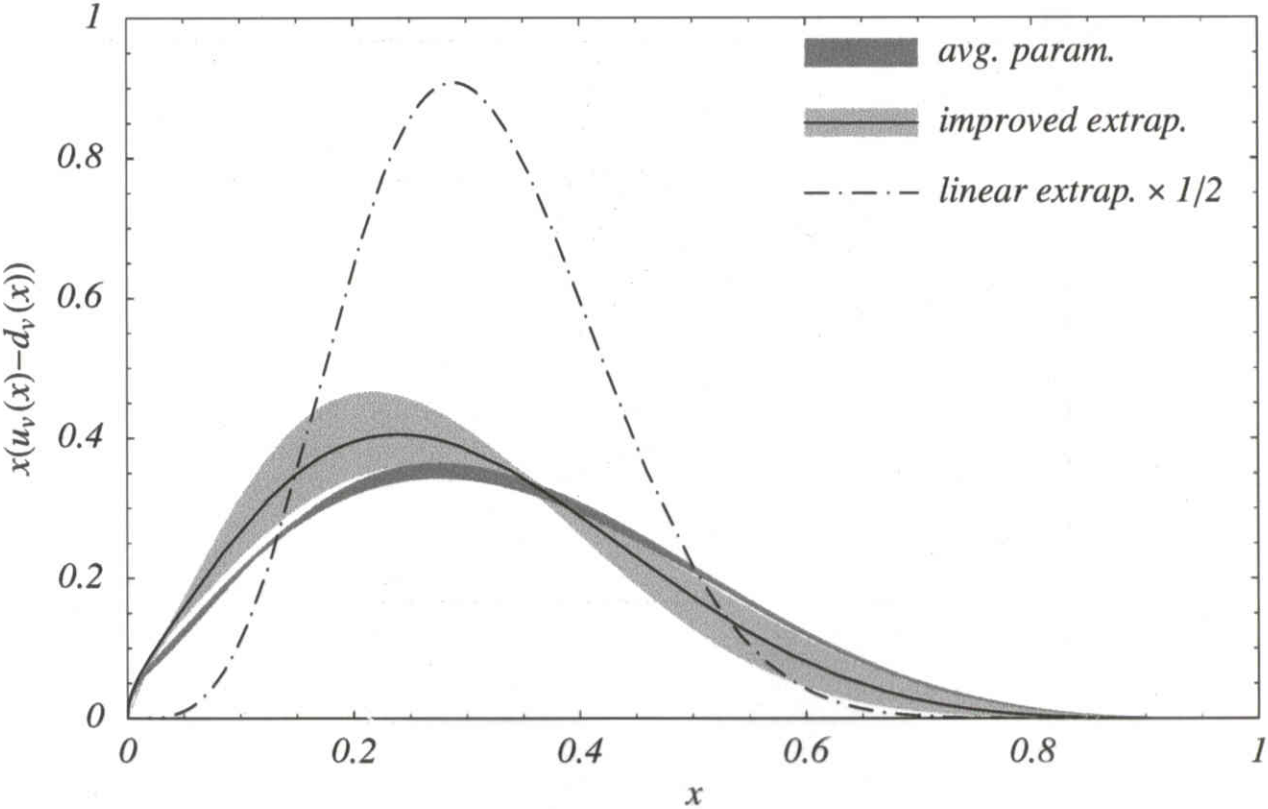
- What can lattice moments tell us about x dependence of parton distributions?

- Typical parameterization of (non-singlet) distributions (CTEQ, MRS, GRV)

$$x\ q(x) = a\ x^b\ (1-x)^c\ (1+\epsilon\sqrt{x}+\gamma x)$$

- Fit parameters b and c to lattice data using extrapolation formula
  $\longrightarrow$ quark distribution as function of m_q!

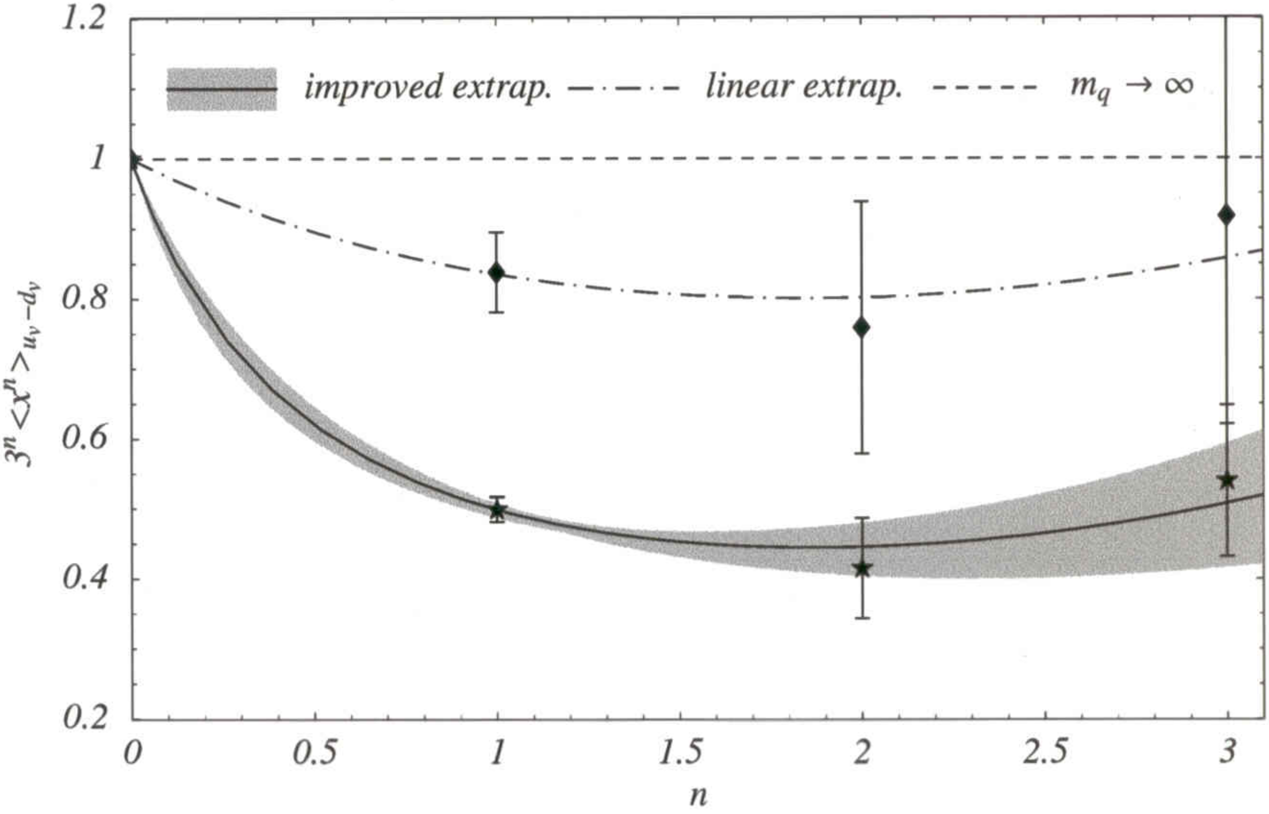
- Exponent b related to intercept of ρ Regge trajectory
  $\longrightarrow$ relate m_q dependence of masses of 1^{--}, 2^{++}, 3^{--}, $\cdots$ mesons to $x \to 0$ behavior of structure function moments!



Extracted $x(u_v - d_v)$ distribution

at the physical quark mass

Detmold, WM, Thomas
Eur.Phys.J. C (2001)



Moments of $u_v - d_v$ distribution

extracted from lattice data (scaled by 3^n)

Detmold, WM, Thomas
Eur.Phys.J. C (2001)

Spin Depdendent Distributions

Moments of spin dependent distributions:

$$\langle \Delta x^n \rangle \;=\; \int_0^1 dx x^n \left(\Delta q(x) + (-1)^n \; \Delta\bar{q}(x)\right)$$

helicity

$$\langle \delta x^n \rangle \;=\; \int_0^1 dx x^n \left(\delta q(x) + (-1)^{n+1} \; \delta\bar{q}(x)\right)$$

transversity

obtained from matrix elements of twist-2 operators:

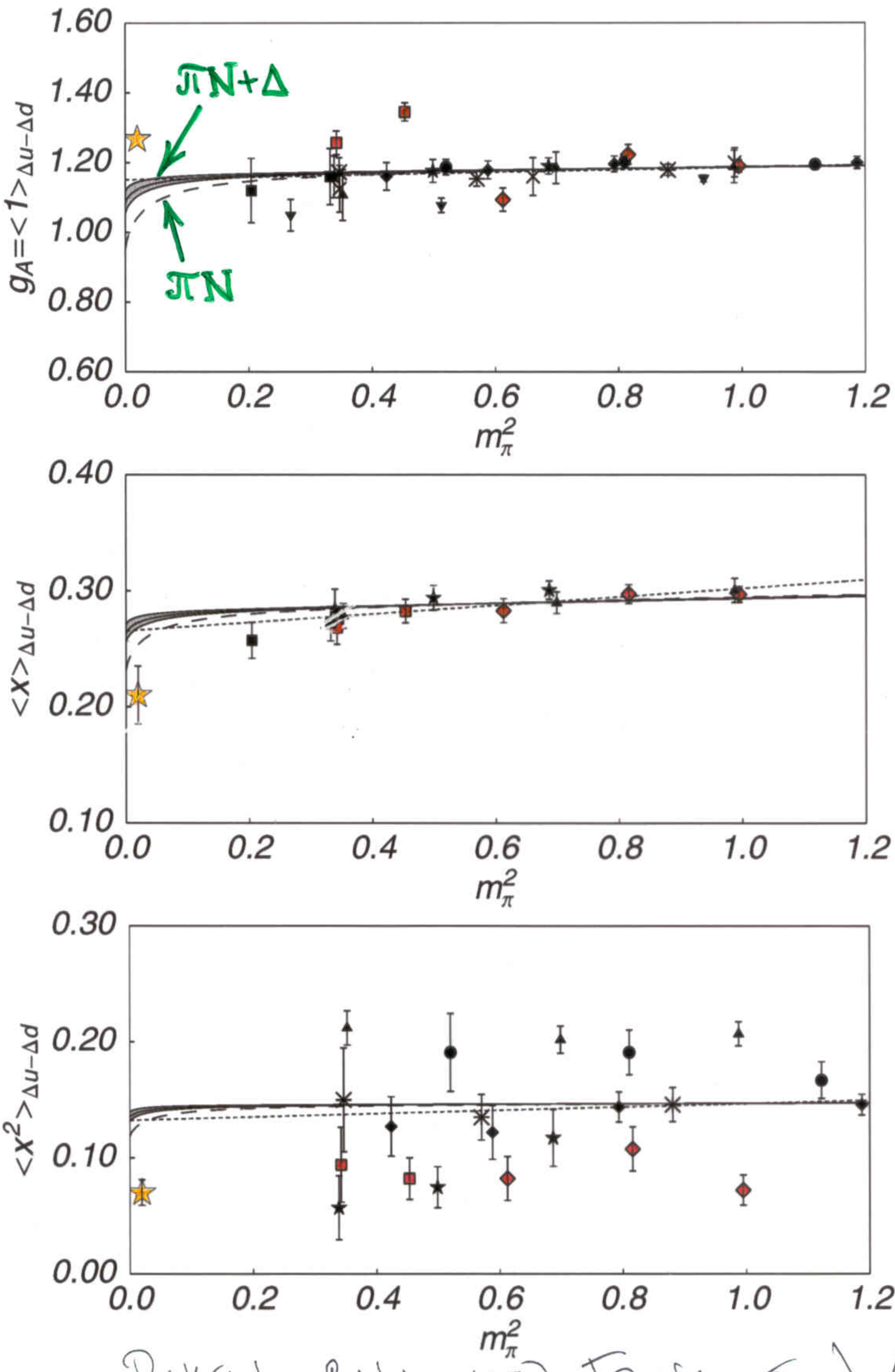
$$\langle N|\bar{\psi}\; \gamma_{\{\mu_1}\gamma_5 \; D_{\mu_2} \cdots D_{\mu_{n+1}\}} \;\psi|N\rangle \;\longrightarrow\; \langle \Delta x^n \rangle$$

$$\langle N|\bar{\psi}\; \sigma_{[\alpha\{\mu_1]}\gamma_5 \; D_{\mu_2} \cdots D_{\mu_{n+1}\}} \;\psi|N\rangle \;\longrightarrow\; \langle \delta x^n \rangle$$

Leading nonanalytic chiral coefficients recently calculated for both $\langle \Delta x^n \rangle$ and $\langle \delta x^n \rangle$

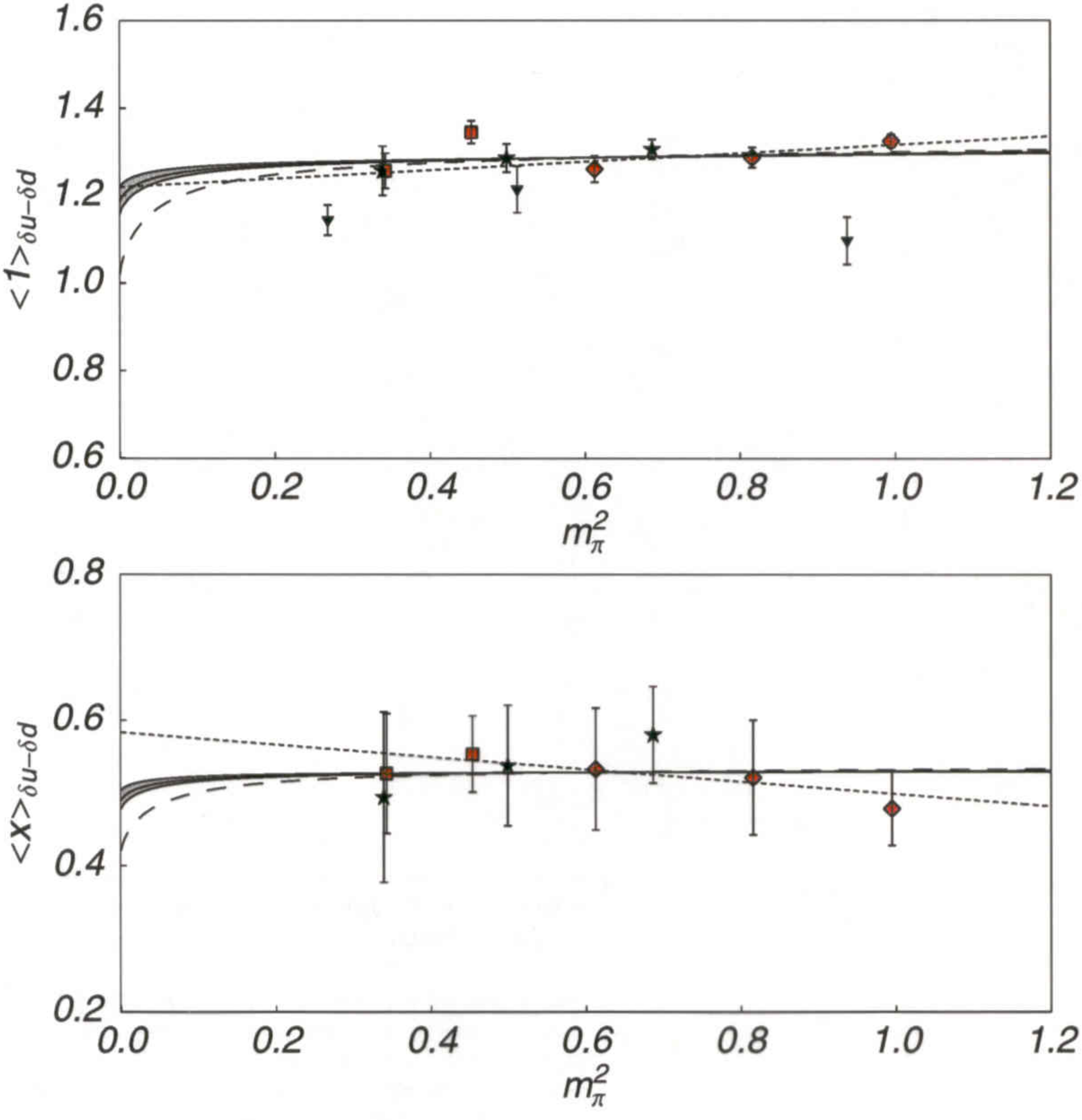
(Chen & Ji, hep-ph/0105197, hep-ph/0105296)

DETMOLD, WM, THOMAS hep-lat/0206001



RIKEN-BNL $\Longrightarrow$ FINITE V

$\longrightarrow$ JURY STILL OUT ON g_A^{lat}



1.6
1.4
1.2
1.0
0.8
0.6
$<1>_{\delta u-\delta d}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
m_π^2
0.8
0.6
0.4
0.2
$<x>_{\delta u-\delta d}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
m_π^2

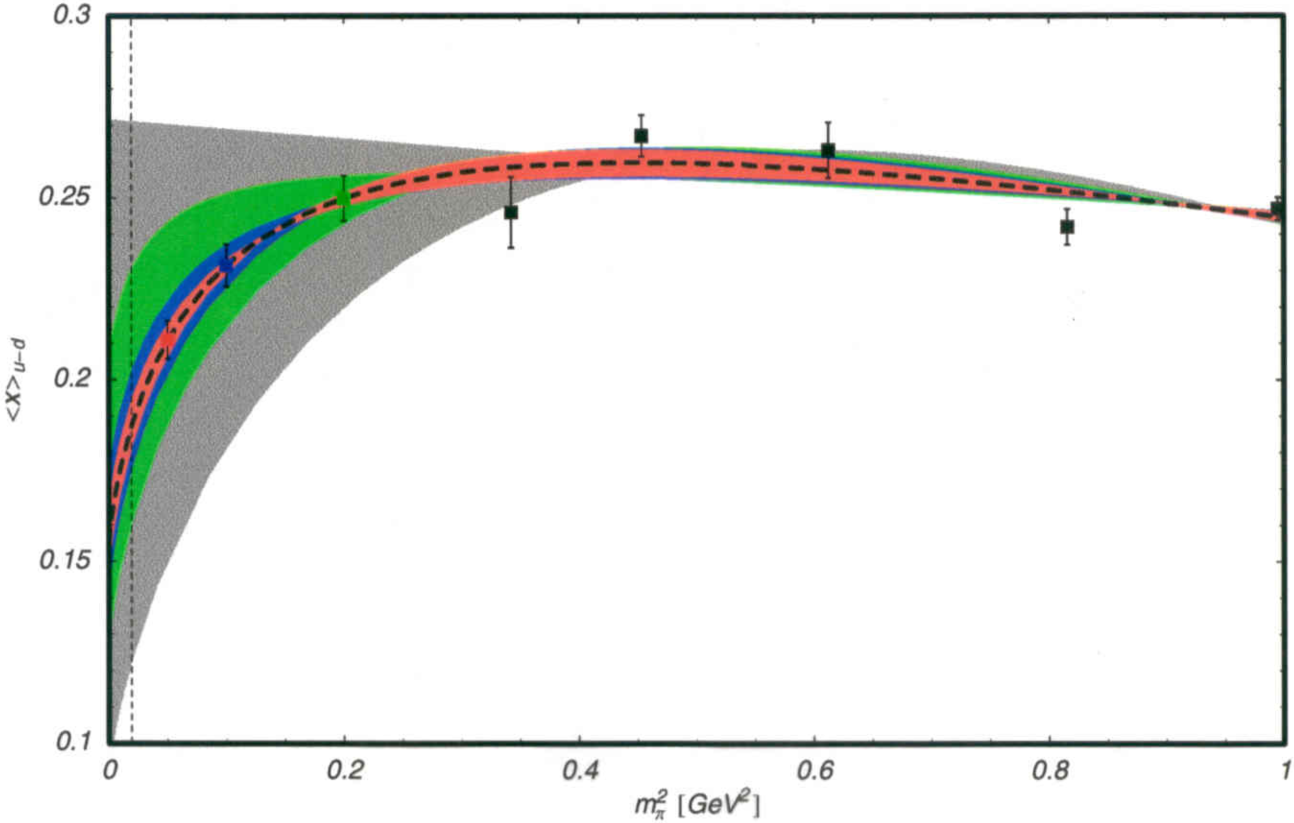
The Promise of Lattice

- Lattice Hadron Physics Collaboration has ambitious long-term plan to calculate structure function moments from first principles
  $\longrightarrow$ currently 128 nodes, 0.1 Tflops
  $\longrightarrow$ another 256 nodes in FY03, 0.5 Tflops
  $\longrightarrow$ expect 1 Tflops by FY04
  $\longrightarrow$ 8 Tflops by FY06
  (c.f. largest cluster today 5 Tflops,
  Earth simulator $\sim$ 30 Tflops)

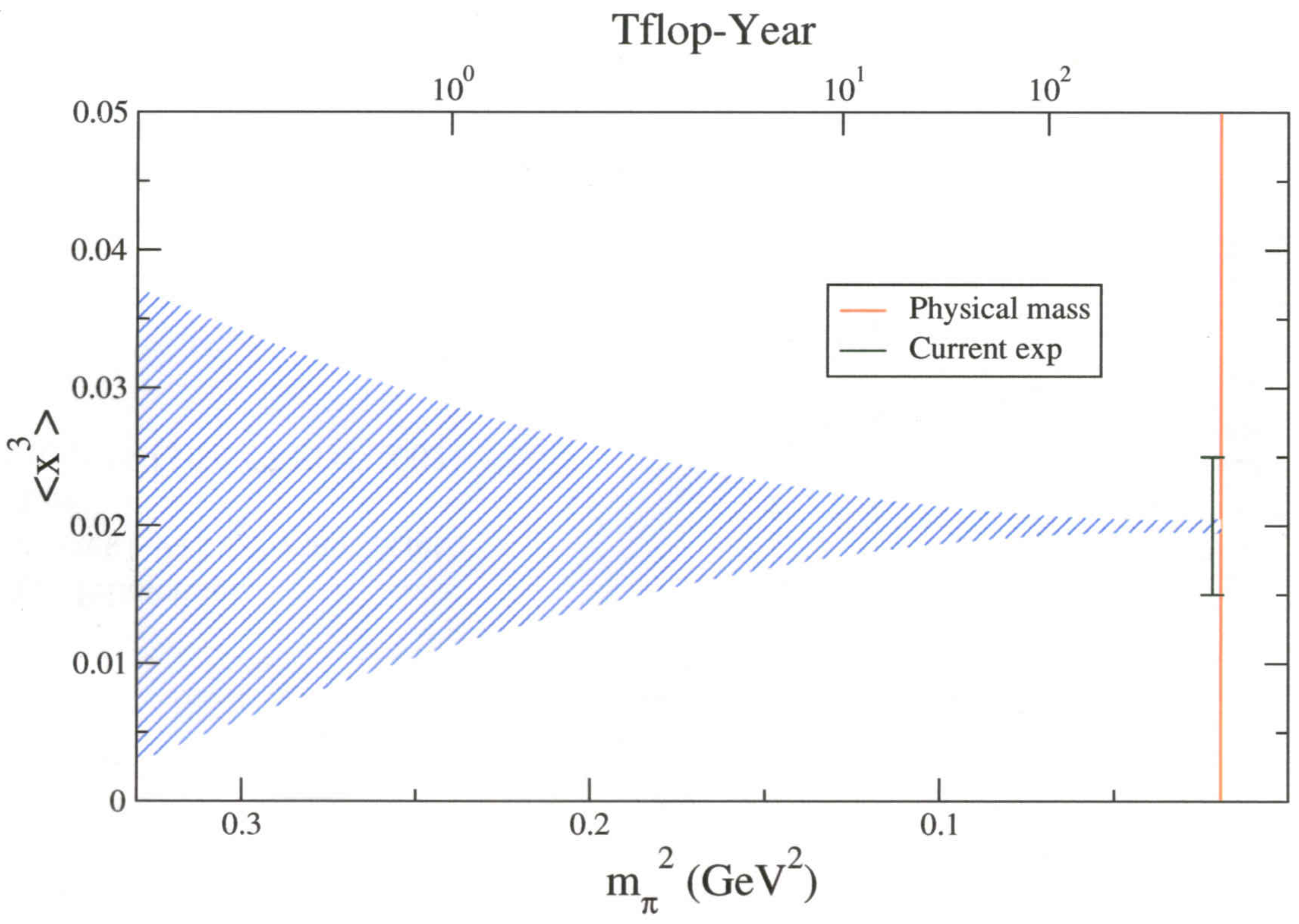
- Simulations usually in two stages
  $\longrightarrow$ computation of quark propagators (expensive)
  $\longrightarrow$ tying propagators together into matrix elements (easy)

- Push closer to physical quark masses, $m_q \to m_q^{phys}$ $(m_q \propto m_\pi^2)$
  $\longrightarrow$ simulations with $m_\pi \approx 300$ MeV currently running

- Larger volumes to include pion cloud



Momentum carried by u-d quarks in proton,
with projected lattice errors on chiral extrapolation.


Tflop-Year
10^0
10^1
10^2
0.05
0.04
0.03
0.02
0.01
0
<x^3>
Physical mass
Current exp
0.3
0.2
0.1
m_π^2 (GeV^2)

- Scale Q^2 determined by (inverse) lattice spacing, $Q^2 \sim 1/a^2$
  $\longrightarrow$ currently $a \approx 0.1$ fm $\Leftrightarrow$ $Q^2 \approx 4$ GeV^2
  $\longrightarrow$ extend to $a \approx 0.05$ fm to access $Q^2 \approx 16$ GeV^2
  $BUT \sim 2^4$ times more expensive
  $\longrightarrow$ larger spacings, e.g. $a = 0.2$ fm $\Leftrightarrow$ $Q^2 \approx 1$ GeV^2, but must match onto perturbative renormalization (e.g. $\overline{\mathrm{MS}}$)

- Singlet quantities (e.g. $F_2^p + F_2^n$) require computationally difficult *disconnected insertions*
  $\longrightarrow$ ~ 20 harder for currently accessible m_q
  $\longrightarrow$ worse for smaller m_q

- Similar expectations for spin-dependent moments
  $\longrightarrow$ possibly larger finite V effects for g_A

- Unquenching — inclusion of quark loops
  $\longrightarrow$ 1000 times more expensive than quenched for quark propagators
  $\longrightarrow$ more important at small m_q where quark loops not suppressed
  $\longrightarrow$ expect to match today's accuracy of quenched simulations by ~ 2006

Summary

- Reconstructing complete quark structure of nucleon requires measurement of structure functions over wide range of kinematics

- Hall C @ 12 GeV, with HMS-SHMS
  $\longrightarrow$ (high $\mathcal{L}$, small θ, high $\vec{p}$)
  will provide unprecedented access to unpolarized ($F_{2,L}$) and polarized ($g_{1,2}$) structure functions and their *moments*

- Measurements will confront QCD directly through moments calculated in lattice QCD
  $\longrightarrow$ complement long-term plans by LHPC