The Charged Pion Form Factor at 12 GeV

The electromagnetic form factors of hadrons are a convenient meeting place between hadronic structure theory and electron scattering experiments. With the trivial Mott cross section effects removed from the data, the theorist needs to know nothing about the experiment other than the form factor at a particular 4-momentum transfer, Q^2. For vanishing energy transfers, the electromagnetic form factors are the Fourier transform of the Lab-frame charge and magnetization distributions. This is a good approximation at low Q^2, but at high Q^2 this simple intrepretation only holds for a special frame of reference called the Breit frame. Despite this conceptual complication, the electromagnetic form factors are important quantities which can unambiguously be calculated and compared to experiment.

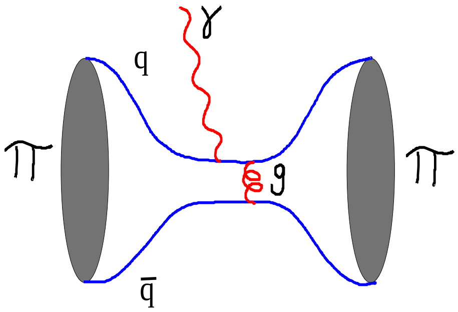

Figure 1: Cartoon showing a high momentum virtual photon coupling to pion which has fluctuated into a rare pure $q\bar{q}$ configuration.

The structure of the pion is of key interest because it is the lowest mass hadron which can be built from the light up and down quarks. It is, in some sense, the "hydrogen atom" of light quark QCD. Whereas a spin 1/2 hadron like the proton has two electromagnetic form factors, the spin 0 charged pion has only a charge form factor, denoted F_π. The asymptotic behavior of F_π is rigorously calculable[1] in perturbative QCD (pQCD), with the value

$$F_\pi = \frac{8\pi\alpha_s f_\pi^2}{Q^2}$$

where f_π is the $\pi^\pm$ axial-vector decay constant. By contrast, the asymptotic behavior of the nucleon form factors is not predicted in such an unequivocal manner. (The β decay of protons has not yet been observed!) Furthermore, the larger number of valence quarks in the nucleon (3 versus 2) means that the asymptotic regime won't be reached until $Q^2 \geq 100$ (GeV/c)^2. Due to the $1/Q^4$ expected asymptotic dependence of the nucleon form factors, this means the resulting nucleon cross sections will be relatively small. For these reasons, the pion may be the only accessible "pQCD Laboratory" which nature has provided.

The theoretical prediction for F_π at experimentally accessible Q^2, below about 10 $(GeV/c)^2$, is less certain

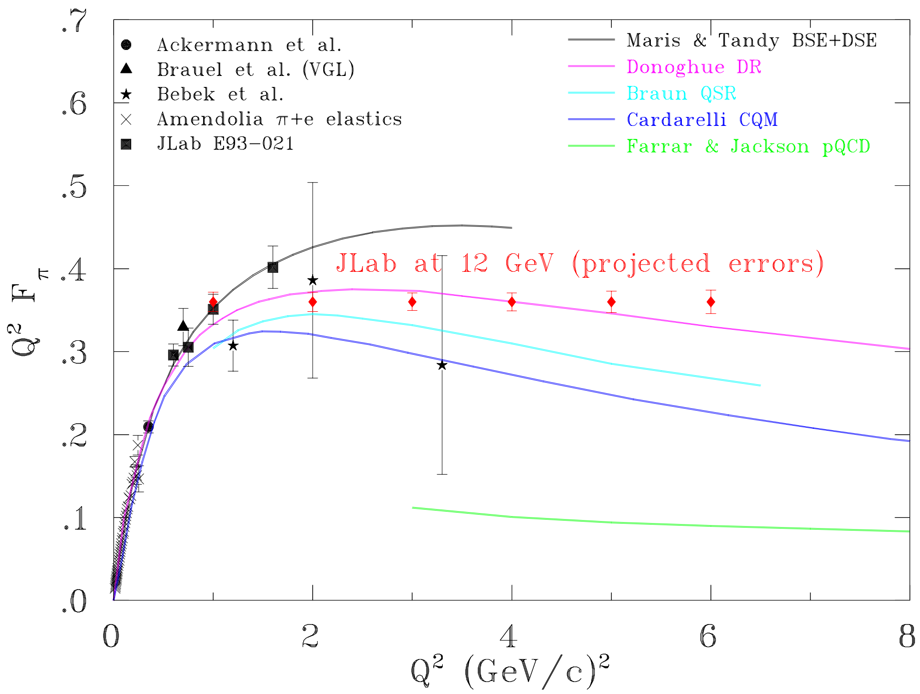

Figure 2: World data set and theoretical predictions for F_π at intermediate Q^2. Projected errors for a measurement following the JLab upgrade, assuming the HMS+SHMS and 100 days of beam.

since the complex physics of confinement cannot be ignored. Figure 2 compares the world's experimental data [2, 3, 4, 5, 6] and several calculations in this intermediate Q^2 regime. For the highest Q^2 data, there is a weak suggestion that the condition $Q^2F_\pi \simeq$ constant is being approached. However, since the data are several times larger than the pQCD prediction, higher order effects are still obviously dominant at Q^2 = 2-3. Only a few of the calculations can be discussed here, and not all of them are predictions. Braun, Khodjamirian, and Maul [7] performed light-cone sum rule calculation up to twist 6, and found that the non-perturbative hard contributions of higher twist strongly cancel the soft components, even at relatively modest Q^2. The models of references [8, 9, 10] obtain good agreement with the experimental data over a broad region of Q^2 by incorporating a confining potential, which dominates at low Q^2, and a QCD-based interaction, which dominates at high Q^2. Finally, Maris and Tandy [11] use the Bethe-Salpeter plus Schwinger-Dyson equations to determine the pion form factor. In this case, the model's parameters are adjusted to reproduce m_π, f_π, and $< \bar{q}q >$ and then the **predicted** F_π has good agreement with the existing data. Reliable experimental data are clearly needed to delineate the role of hard versus soft contributions at intermediate Q^2, and so aid the further development of these models.

Existing data for F_π above $Q^2 = 1.6$ $(GeV/c)^2$ are of poor quality. Hall C will soon extend these

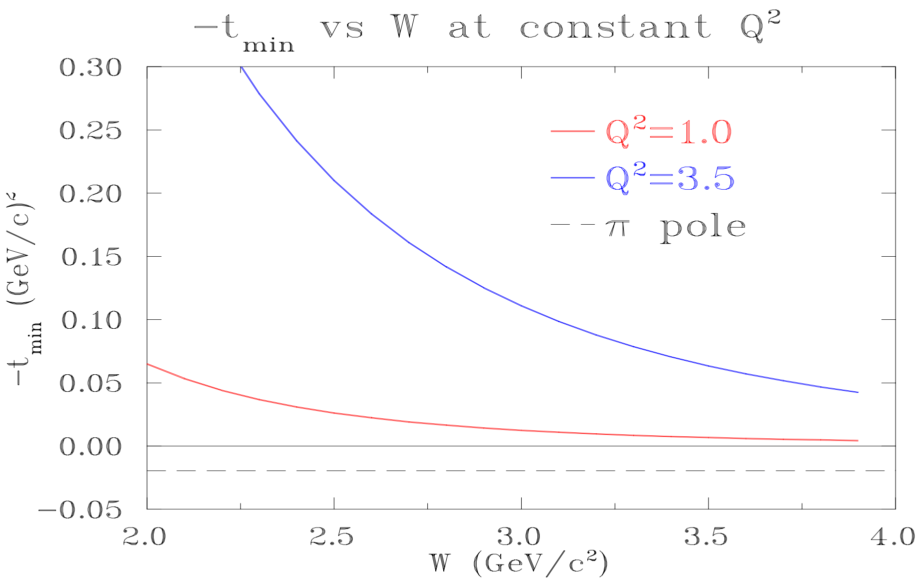

Figure 3: The pion pole in the physically inaccessible region at $t = m_\pi^2 = 0.02(GeV/c)^2$ is denoted by the dashed line. For fixed Q^2, the minimum distance to the pion pole decreases with increasing invariant mass W.

measurements to $Q^2 = 2.5$ (GeV/c)^2, but further advances must then await the 12 GeV upgrade and the construction of a new high momentum, small angle spectrometer like the SHMS. The higher energy of the upgraded JLab electron beam is essential for not only obtaining higher momentum transfers, but to allow one to approach the pion pole closely and so maximize its contribution to the cross section (Figure 3). Because free pion targets are impractical, the experimental target used is the free proton, with the kinematics chosen to maximize sensitivity to scattering from the proton's virtual pion field. Although the mass of the virtual pion may be a factor of several times different than the on-shell pion mass, the good agreement for F_π between low Q^2 pion electroproduction and elastic $\pi + e$ elastic scattering experiments is clear evidence that these off-shell effects are small.

The unpolarized cross section for $e + p \to e + \pi^+ + n$ contains contributions from 4 response functions which can be separated by their different kinematic dependences. The longitudinal response is essential for F_π measurements because at small $-t$ it is proportional to F_π^2 and has a marked dependence on Mandelstam t due to pion exchange:

$$\sigma_L \propto \frac{-2tQ^2}{(t - m_\pi^2)^2} F_\pi^2.$$

Operationally, one must obtain high statistics $N(e, e'\pi^\pm)N$ data in parallel kinematics at fixed Q^2 for two beam energies. Because the longitudinal photon polarization is different at the two beam energies, the longitudinal response can be extracted.

Once the separated $d\sigma_L/dt$ data are in hand, a modern, gauge-invariant model of pion electroproduction [14] is used to extract F_π. The suitability of the Regge model will improve at higher beam energies since all measurements can be made well above the nucleon resonance region. Because most of the Regge model's few free parameters have already been fixed by photoproduction data, the value of $F_\pi(Q^2)$ is obtained by an essentially one-parameter fit of our $d\sigma_L/dt$ data to the model.

In this experiment, energetic pions will be emitted at rather small angles with the beam. The proposed SHMS spectrometer is **essential** for this measurement because of

- its excellent 5.5^o forward angle capability (Figure 4)
- its ability to identify and determine the momentum of very energetic pions (Figure 5
- its rigid and reproducible connection to the pivot (to control systematic errors in the extraction of the Longitudinal response), and
- the high luminosity capability of Hall C.

Figure 2 (above) shows the expected error bars that could be obtained with the SHMS+HMS combination and a 10.9 GeV beam after 100 days of running time. A 4 cm LH_2 target and 50 μA[1] of beam were assumed, for a luminosity of about $6x10^{37}cm^{-2}sec^{-1}$. It is easily seen that the CEBAF upgrade would allow a very significant advance in the understanding of the pion form factor.

[1]The 1 MWatt limit on total beam power corresponds to exactly 100 μA of beam available at 10 GeV, so 50 μA is a reasonable maximum current assuming simultaneous scheduling of Halls A and C.

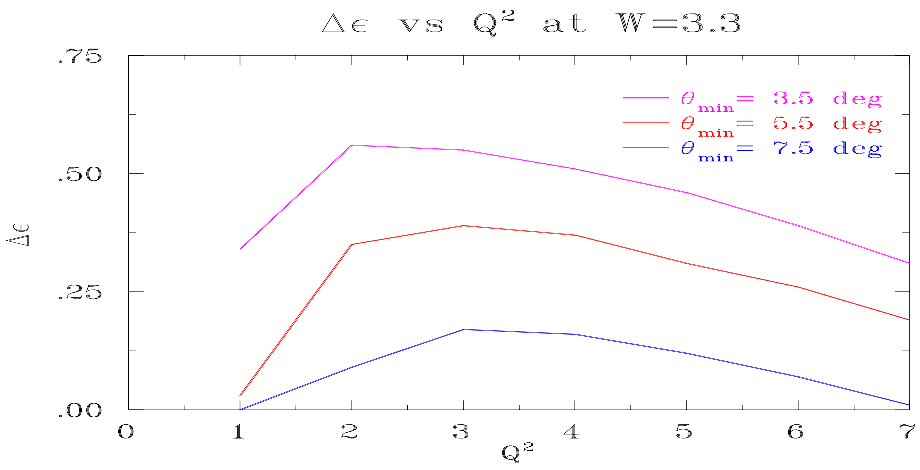

Figure 4: The minimum pion angle has a large effect on the accessible range in the longitudinal photon polarization, ϵ. While the minimum central SHMS scattering angle of 5.5^o yields a satisfactory $\Delta\epsilon$, a central angle only a few degrees larger would be problematical.

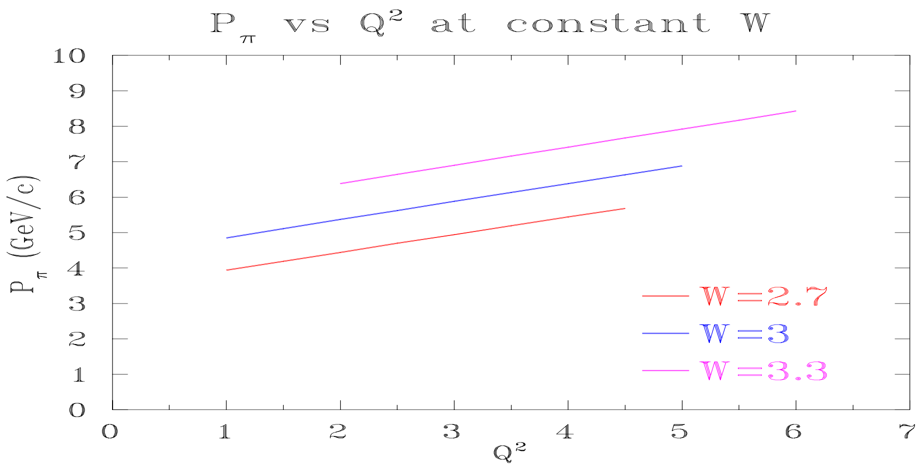

Figure 5: A spectrometer with a maximum momentum of approximately 9 GeV/c is required for the F_π program. The maximum momentum of the SHMS, 11 GeV/c, would allow F_{pi} measurements to be extended beyond $Q^2 = 6$ if improvements in SRF technology increase the Hall C beam energy beyond 10.9 GeV.

References

[1] G.R. Farrar, D.R. Jackson, Phys. Rev. Lett. **43** (1979) 246.

[2] L.J. Bebek et al., Phys. Rev. D **9** (1974) 1229.
L.J. Bebek et al., Phys. Rev. D **13** (1976) 25.
L.J. Bebek et al., Phys. Rev. D **17** (1978) 1693.

[3] P. Brauel et al., Z. Phys. C **3** (1979) 101.

[4] H. Ackerman et al., Nucl. Phys. **137B** (1978) 294.

[5] S.R. Amendolia et al., Phys. Lett. **146B** (1984) 116.
S.R. Amendolia et al., Nucl. Phys. **277B** (1986) 168.

[6] J. Volmer et al., Phys. Rev. Lett. **86** 1713 (2001).

[7] V.M. Braun, A. Khodjamirian, M. Maul, Phys. Rev. D **61** (2000) 073004.

[8] O.C. Jacob, L.S. Kisslinger, Phys. Lett. **243B** (1990) 323.

[9] P.C. Tiemeijer, J.A. Tjon, Phys. Lett. **277B** (1992) 38.

[10] H. Ito, W.W. Buck, F. Gross, Phys. Lett. **287B** (1992) 23.

[11] P. Maris, P.C. Tandy, Phys. Rev. C bf 62 055204.

[12] J.F Donoghue, E.S. Na, Phys. Rev. D **56** (1997) 7073.

[13] F. Cardarelli et al., Phys. Lett. **332B** (1994) 1.
F. Cardarelli et al., Phys. Lett. **357B** (1995) 267.

[14] M. Vanderhaeghen, M. Guidal, J.-M. Laget, Phys. Rev. C **57** (1998) 1454.