BARYON CHIRAL DYNAMICS

Baryons 2002 @ JLab

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Overview

- Chiral dynamics … with nucleons
- Higher, faster, stronger, …
  - Formulation of the effective Theory
  - Full one loop results: $O(q^4)$ (Two loop result for m_N)
  - Isospin breaking, electromagnetism.
  - Pionic hydrogen
  - Two nucleon sector
  - Connection to lattice QCD: Quenched BCHPT
  - Photons: RCS, VCS, OMC, RMC,… $\rightarrow$Merkel's talk
  - Higher energies: Resonances, dispersion relations
- … but still puzzled: $g_{\pi N}$, σ-term, SU(3)

"QCD Lite" … (QCD for N=2 massless flavors)

- One "parameter" $g_s \leftrightarrow \Lambda_{QCD}$
  - Parameter free predictions for dimensionless quantities
- High degree of symmetry: $SU_V(2)\times SU_A(2)$

$SU_V(2)$: No distinction between flavors

Isospin

$SU_A(2)$: Left- $\leftrightarrow$ right-handed quarks

Chiral symmetry

$SU_V(2)$ x $SU_A(2)$

$Q_V|0\rangle = 0$

Isospin multiplets

$Q_A|0\rangle \neq 0$

SSB

3 Goldstone bosons

$\rightarrow$ Pions

Chiral Expansion

- Goldstone bosons decouple as $E_\pi \to 0$
  - Low energy singularities of the Geen's functions from the propagation of π's.
  - Account for those, expand amplitudes in external momenta.
- $\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm Lite} + \sum_i m_i \, \bar{q}_i \, q_i$
  - But m_u,m_d (and m_s ?) happen to be light
  - Treat mass term as a perturbation
- Simultaneous expansion in q_μ, m_u,m_d

CHiral Perturbation Theory

- Chiral symmetry leads to relations between different amplitudes
- PCAC, Current Algebra
  - Work out structure of amplitudes and symmetry relations by hand. *Tedious!*
- CHPT: Use effective Lagrangian ☹$_{eff}(\pi)$
  - most general ☹$_{eff}$ compatible with symmetries
  - QCD dynamics encoded in coupling constants.
  - Order ☹$_{eff}$ by the number of derivatives on the π-field and by powers of m_q

Goals

- Low energy QCD in terms of a small number of parameters
  - Connect different processes
  - Experiment vs. lattice simulations
- Extrapolation to QCD Lite
  - Predictions of the symmetry.
  - Mechanism of symmetry breaking?
  - E.g. nucleon mass in chiral limit?

Baryon CHPT

- Include nucleon
  - ☹$_{eff}$= ☹ $_{\pi}$ + ☹$_{N}$
  - Lowest order

$$\mathcal{L}_N^{(1)} = -\frac{g_A}{2F_\pi}\bar{\psi}\,\gamma^\mu\,\gamma_5(\partial_\mu\pi)\,\psi + \frac{1}{8F_\pi^2}\bar{\psi}\,\gamma^\mu\,i[\pi,(\partial_\mu\pi)]\,\psi + \ldots$$

- $m_N \gg M_\pi$
  - $P_\mu \sim (m_N,0,0,0)$ is $O(q^0)$
  - Interactions with soft pions:Nucleon remains nearly static, surrounded by cloud of π's
- ☹$_{N}$contains odd powers of derivatives

Low Energy Constants

- Pion sector
  - Only even powers
- Nucleon sector
  - Odd and even powers

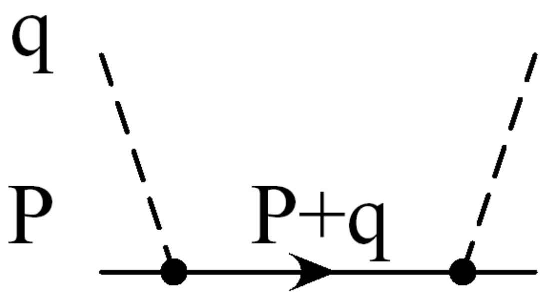
$$\mathcal{L}_{\pi\pi} = \underset{(2)}{\mathcal{L}^{(2)}_{\pi\pi}} + \underset{(7)}{\mathcal{L}^{(4)}_{\pi\pi}} + \underset{(53)}{\mathcal{L}^{(6)}_{\pi\pi}}$$

$$\mathcal{L}_{\pi N} = \underbrace{\underset{(1)}{\mathcal{L}^{(1)}_{\pi N}} + \underset{(7)}{\mathcal{L}^{(2)}_{\pi N}}}_{\text{Tree-level}} + \underbrace{\underset{(23)}{\mathcal{L}^{(3)}_{\pi N}} + \boxed{\underset{(118)}{\mathcal{L}^{(4)}_{\pi N}}}}_{\text{1-loop}} \underbrace{+ \ldots}_{\text{2-loop}}$$

Fettes, Meissner, Mojzis & Steininger

Expansion of the *N*–Kinematics…

- Chiral expansion of the *N*-propagator



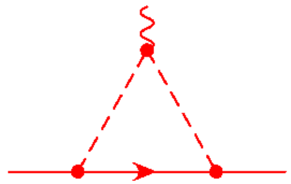
$$\rightarrow \frac{1}{(P+q)^2 - m_N^2}$$

$$= \frac{1}{2Pq} + O(q^0) \qquad \text{for } P^2 = m_N^2$$

- Can be implemented into ☹$_{\text{eff}}$: HBCHPT

… is delicate …

- Choice of kinematical variables
- Threshold singularities:
  - Breaks down at the threshold



$\rightarrow \infty$ for $t \rightarrow 4M_\pi^2$

  - Slow convergence at threshold

… and can be avoided.

- Relativistic formulation
  - Dimensional regularization
    - Loop graphs are of the same order as tree level.
    - Problems to organize the perturbation series
  - Infrared regularization
    - Variant of dim. reg.
    - Well organized perturbation series
    - Avoids expansion of *N*-kinematics

Ellis & Tang; Becher & Leutwyler; Goity, Lehmann, Prezeau, Saez

Infrared Regularization

- Split dim. reg. loop graphs $L = R + I$
  - R : Large loop momentum $k \sim m_N$
    - Violates chiral counting
    - Trivial chiral expansion
  - I : Small loop momentum $k \sim M_\pi$
    - Contains all low energy singularities
- Absorb R into definition of ☹$_{eff}$ and set $L = I$
  - Powercounting
  - Controlled expansion of kinematics
  - Manifest Lorentz invariance

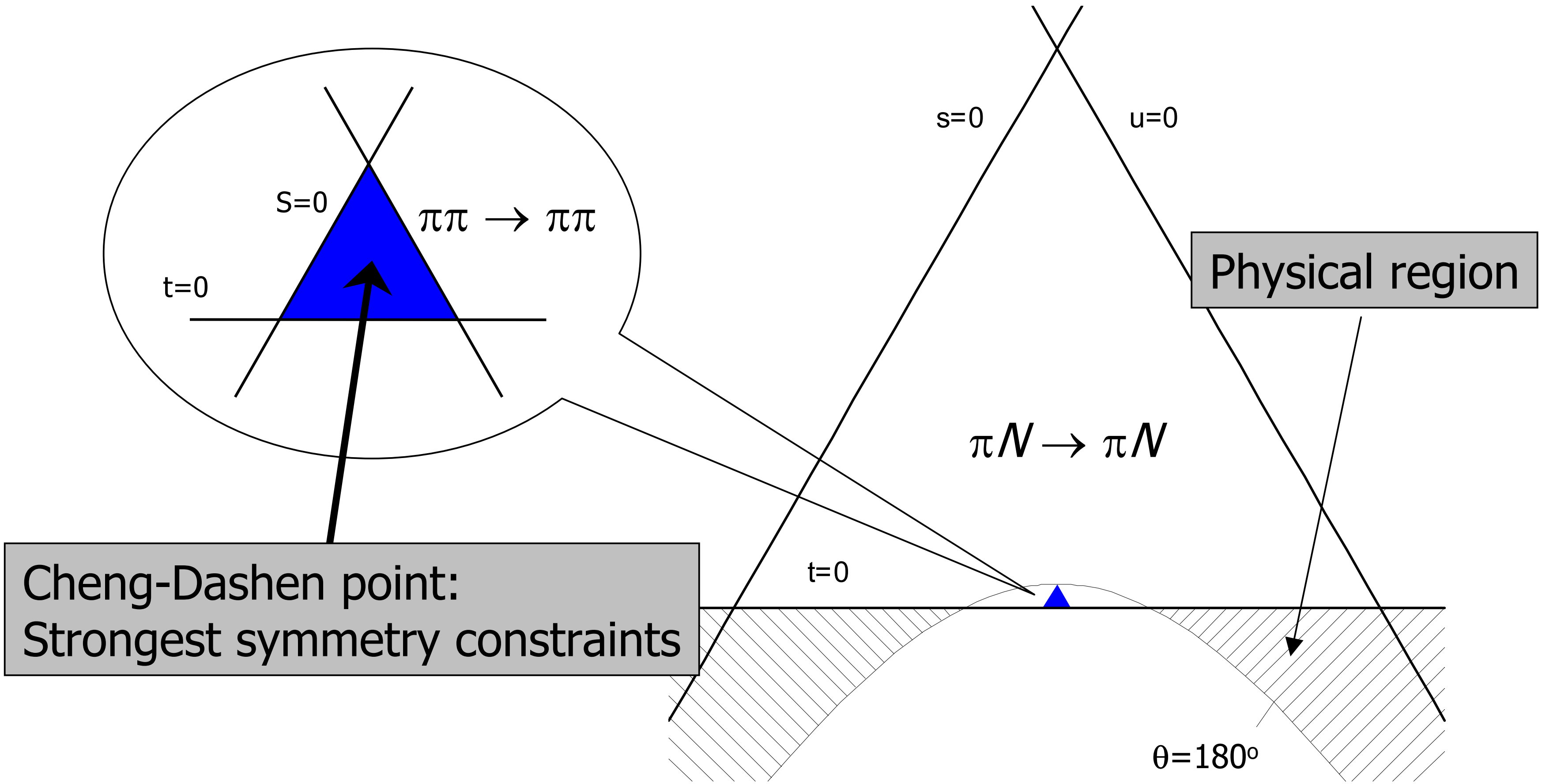
$\pi N \to \pi N$ Scattering Amplitude

- $O(q^4)$ result in isospin limit
  - HBCHPT Meissner and Fettes
  - Infrared regularization Becher and Leutwyler
- Isospin breaking (strong and e.m.)
  - $O(q^3)$ Meissner and Fettes
- Inclusion of Δ-resonance
  - $O(\varepsilon^3)$ Ellis and Tang; Meissner and Fettes

$\pi\pi \rightarrow \pi\pi$ versus $\pi N \rightarrow \pi N$

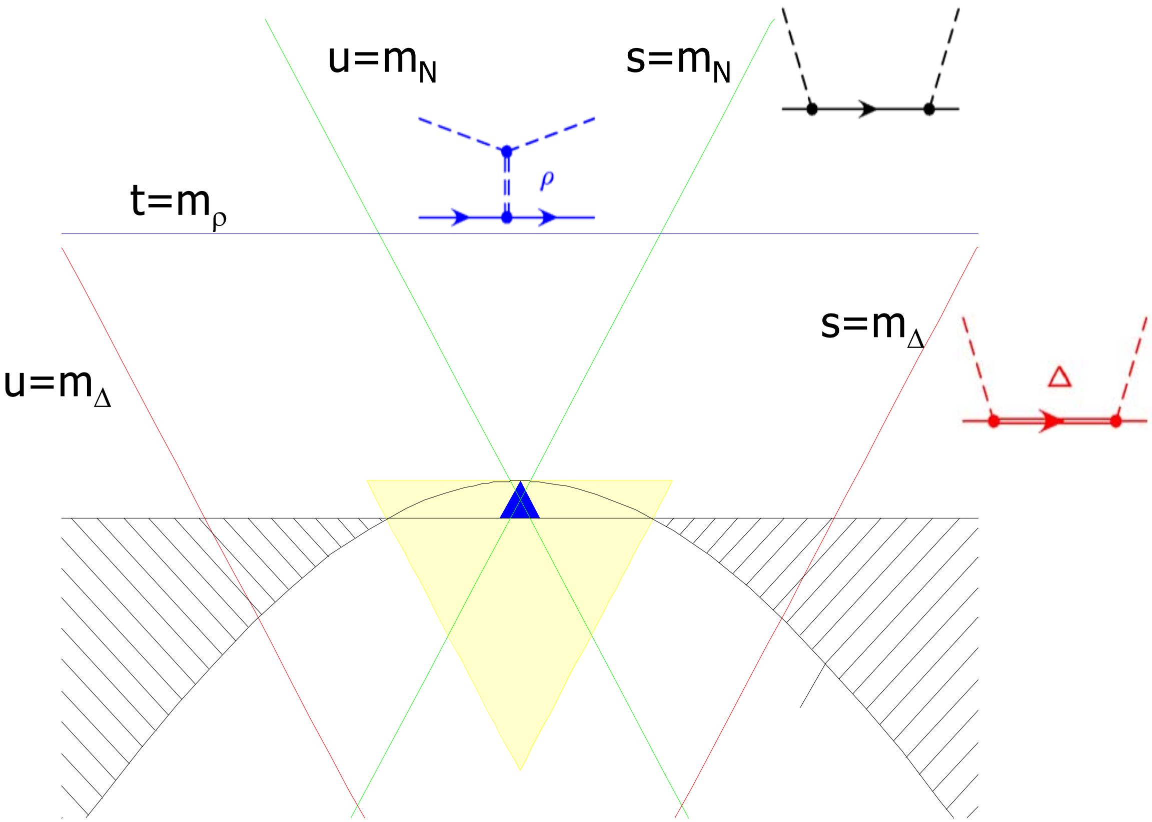
- *More LEC's* for πN-Scattering
  - 4 parameters for $\pi\pi \rightarrow \pi\pi$ to $O(q^4)$
  - 14 for $\pi N \rightarrow \pi N$ to $O(q^4)$
- But also *more data* !
- πN experimental region is at "higher" energies.

Mandelstam Triangle



Low Energy Region

- Low energy expansion breaks down, once resonances are produced!



Scattering amplitude at O(q^4)

- Simple parameterization

$$A(\nu,t) = B + P(\nu,t) + F(s) + F(u) + G(t), \quad \text{where} \quad \nu = (s-u)/4m_N$$

  - B : Nucleon pole terms
  - P : Polynomial $\leftarrow$ LEC's
  - F : s-channel cut, linear in t .
  - G : t-channel cut, linear in ν.
  - Four (2 spin $\times$ 2 isospin) amplitudes in terms of 9 functions of a *single* variable.
- Constraints from Chiral Symmetry
  - Goldberger-Treiman relation
  - Value of the amplitude at the CD-point

Goldberger Treiman Relation

- Relation between g_A and $g_{\pi N}$

$$g_{\pi N} = \frac{g_A m_N}{F_\pi}\{1 + \Delta_{GT}\}$$

- Δ_{GT} *vanishes* for m_q =0. Note $M_\pi^2 \propto m_q$

$$\Delta_{GT} = c\, M_\pi^2 + O(M_\pi^4)$$

- No chiral logarithm, no M_π^3 term!
- Experimentally: Δ_{GT} =2-4%

GWU → 2 KH80 → 4

Isospin Odd Amplitude

Cheng Dashen point:

$$C = 2\,F_\pi^2\,\frac{\bar{D}^-(\nu,t)}{\nu}\bigg|_{\nu=0,\ t=2M_\pi^2}$$

- C =1 for m_q =0

$$C \;=\; 1 - \frac{1+5\,g_A^2}{24\,\pi^2 F^2}\,M^2\,\ln\frac{M}{\mu} + k_1\,M^2 + k_2\,M^3 + O(M^4)$$

  - Contains chiral log: Expect O(10%) symmetry breaking
  - "Experimentally" $C \sim$ 1.06-1.13
  - Contraint on subthreshold coefficients

Isospin Even Amplitude

- Scalar form factor $\leftrightarrow$ Amplitude at CD
  - Amplitude at CD: $\Sigma = F_\pi^2 \bar{D}^+(0, 2M_\pi^2)$
  - And scalar form factor

$$\langle N' | \, m_u \, \bar{u}u + m_d \, \bar{d}d \, | N \rangle = \sigma(t) \, \bar{u}' u \, .$$

- Relation

$$\Sigma = \sigma(2M_\pi^2) + \Delta_{CD}$$

$$\Delta_{CD} = k_{CD} M^4 + O(M^5)$$

  - No low E singularities in $\Delta_{CD} \approx$ 1-2MeV.

σ -Term

- Quark mass dependence of the nucleon mass

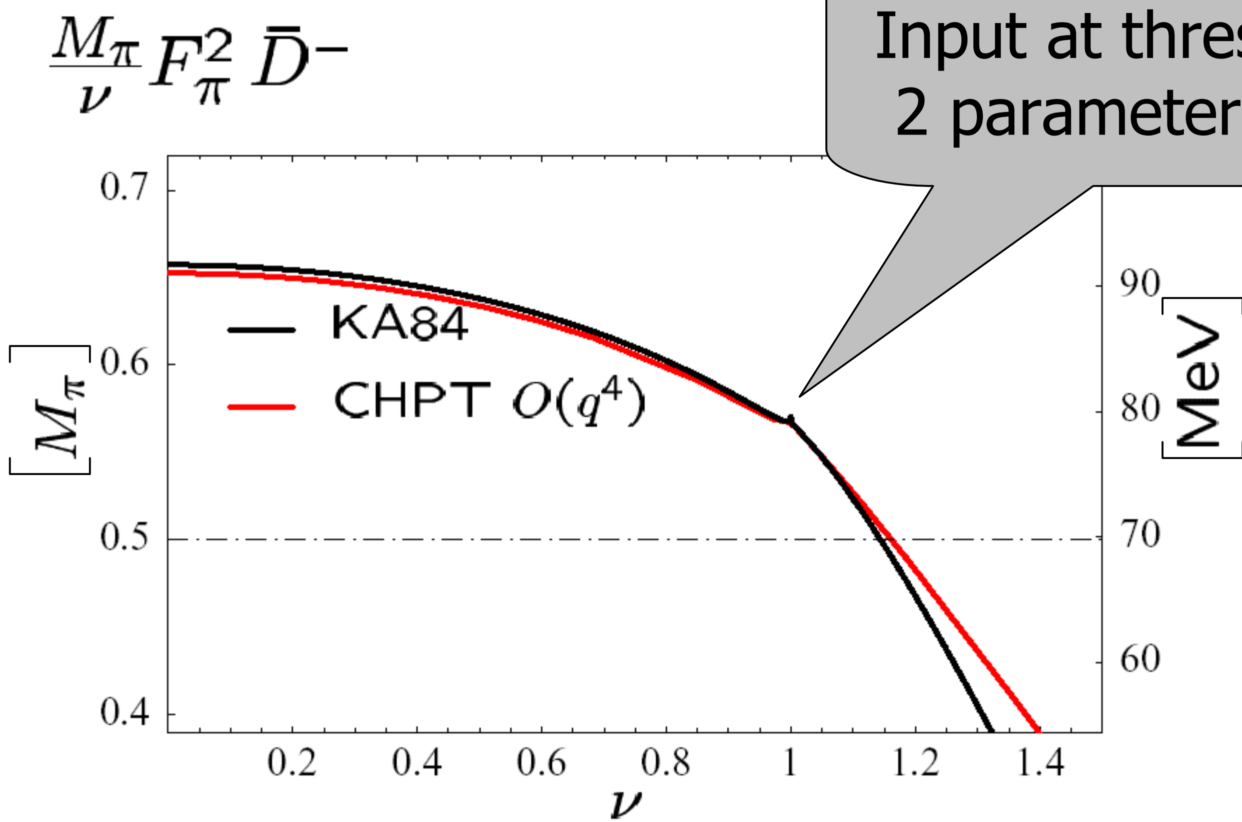
$$\sigma \equiv \sigma(0) = m_u \frac{\partial m_N}{\partial m_u} + m_d \frac{\partial m_N}{\partial m_d}$$

- Extrapolation t= $2M_\pi^2 \rightarrow$ t=0.
  - CHPT confirms dispersive result

$$\sigma(2M_\pi^2) - \sigma = 15.2 \pm 0.4$$

- Value of $\Sigma \rightarrow \sigma$

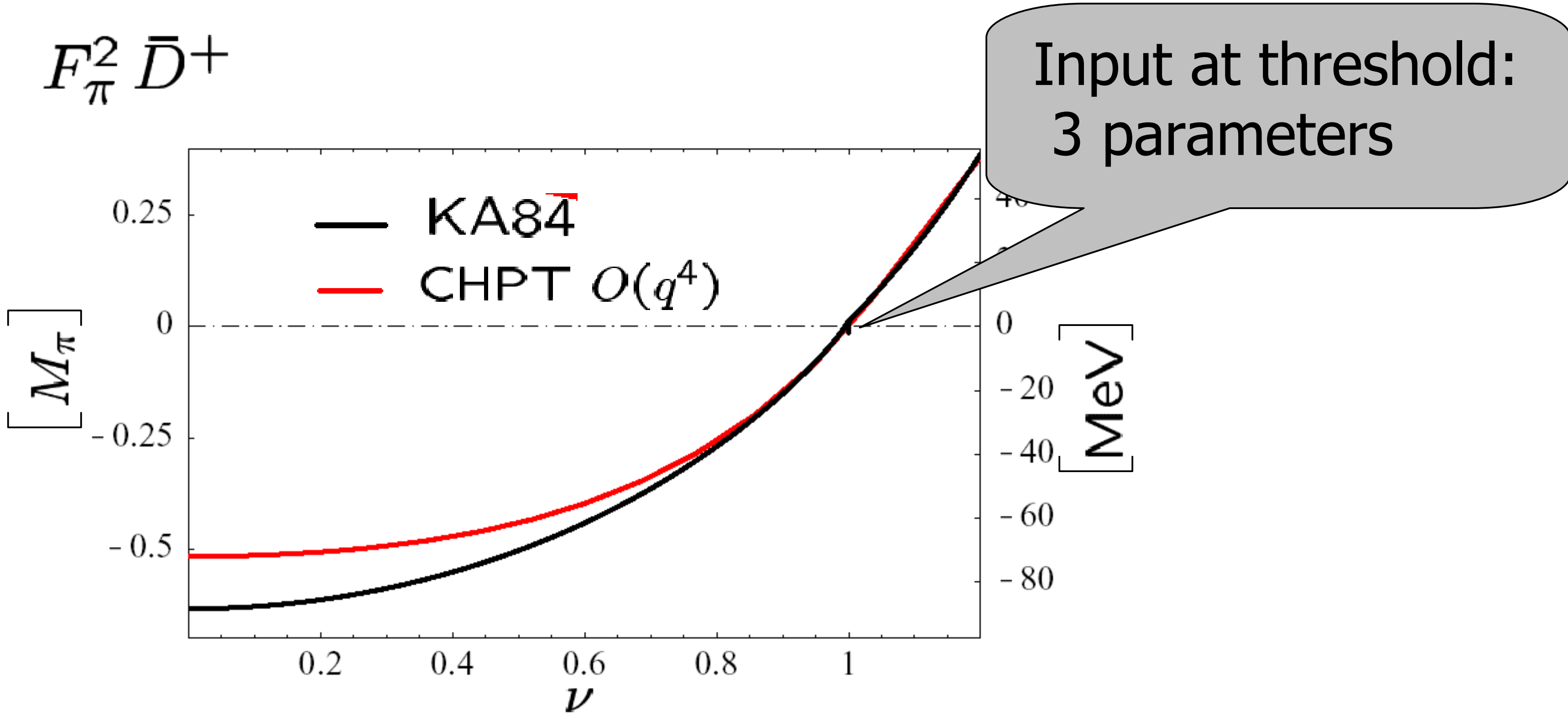
Amplitude for t=0



Input at threshold:
2 parameters

Starts to deviate soon above threshold…

Isospin Even Amplitude



Chiral representation is not precise enough to extrapolate experimental results to the CD region

Higher Energies

- Two issues
  - Unitarity is fulfilled only perturbatively in CHPT.
    - One loop imaginary parts are “tree level” squared and not very accurate.
  - Resonances
    - Δ-Resonance close to threshold and $g_{\Delta N\pi}$ is large.

Δ-Resonance

- Can be taken into account systematically

  Jenkins & Manohar; Hemmert, Holstein & Kambor

  - Incorporate Δ into ☹$_{\text{eff}}$.
  - Count both q_π and $\Delta = m_\Delta - m_N$ as $O(\varepsilon)$.
- Scattering amplitude known to $O(\varepsilon^3)$.

  Ellis & Tang, Fettes & Meissner

  - Pole term is dominant contribution.
  - Accuracy similar to $O(q^4)$ CHPT result
  - Unitarity remains an issue.
  - Nonrelativistic expansion of the Δ-propagator…

Implement Unitarity

- Unitarization
  - Unitarize model or CHPT result

E.g. Oller and Meissner

  - Good description of experimental data
  - Various unitarization prescriptions
  - Model dependent, distort structure of amplitude
- Dispersion relations
  - Use experimental imaginary parts, generate real part with dispersion relatios
  - Integral equations

Dispersive approach

- Complicated! Function of *two* variables.
- Implement low energy structure found in CHPT (functions of *single* variable)
  - Leads to set of integral equations similar to the Roy-equations in $\pi\pi$-scattering
  - Input:
    - Exp. data above elastic region
    - Four subtraction constants

Becher, Leutwyler; Mojzis; Stahov

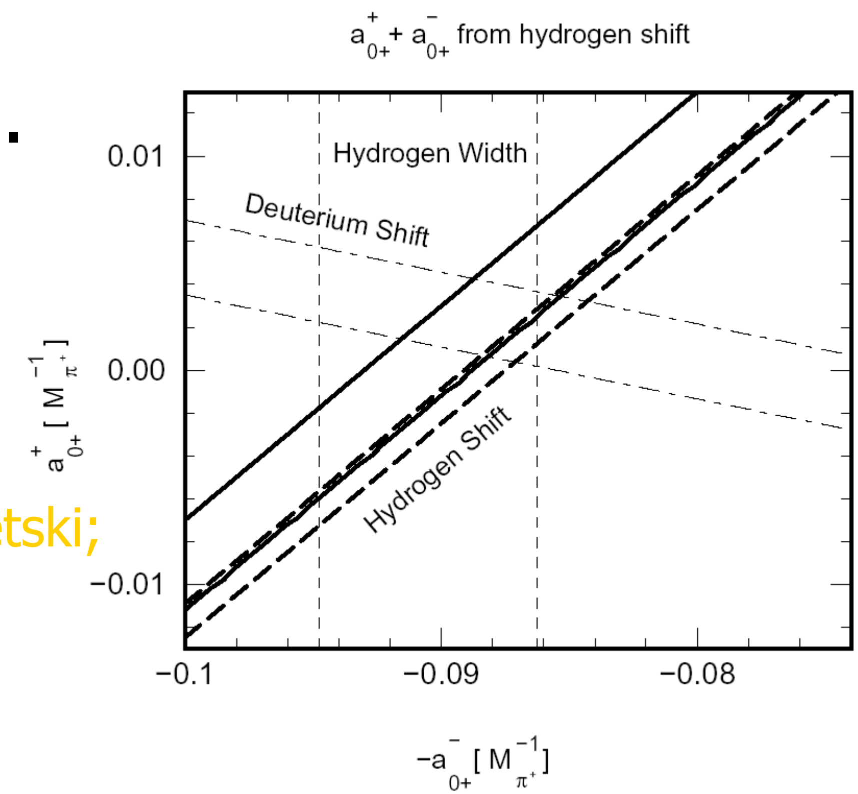
Isospin violation

- Two sources
  - Strong: Quark mass difference m_u-m_d
  - Electromagnetic
- Need to treat both on equal footing
  - Count e^2 as O(q)
  - $\pi N \rightarrow \pi N$ to $O(q^3)$ Meissner and Fettes
    - Isospin breaking small, mostly static
    - Dynamical effects only in S-wave O(1%).
    - Models incomplete

π^-p Bound State

- Scattering lengths from π^-p energy shift and decay width.
  - Theory developed.
  - Results for $\pi^-\pi^+$.
  - Level shift for π^-p.

Gasser, Lyubovitskij & Rusetski; Eiras, Soto



Conclusion

- Have
  - Reached high precision
  - Studied wide range of processes
  - Good understanding of the role of chiral symmetry in the baryon sector.
- Chiral symmetry breaking is a *small* effect
  - Minimize model dependence
  - Push dispersive methods to connect with experimental results.
  - Resolve discrepancies in basic parameters