Heavy quark physics on the lattice

BARYONS2002

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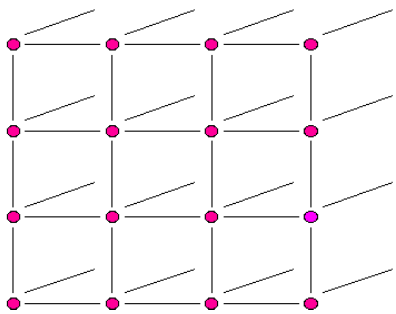
Lattice QCD =

Euclidean space-time lattice



+ QCD Lagrangian (discretised)

$$\begin{aligned}\mathcal{L}_{QCD} &= \mathcal{L}_g + \mathcal{L}_q \\ &= \frac{1}{2g^2} Tr F_{\mu\nu}^2 + \overline{\psi}(\gamma \cdot D + m)\psi\end{aligned}$$

Parameters are those of QCD:
Bare gauge coupling $\beta = 6/g^2$
Quark masses, $m_i a$.
Lattice spacing (a) is implicit u.v. cutoff
Determine a and fix m_i from $1 + n_f$ hadron masses.

Difficulties

Systematic errors:

- Discretisation errors - physical results are dependent on a. Reduce the dependence and/or extrapolate to $a = 0$.

- Matching errors - must renormalise lattice matrix elements to obtain continuum results. Requires pert. or nonpert. matching calculation.

- Quenching errors - v. expensive to include light dynamical (sea) quarks. Error from using quenched approx. = 10-20% (?).

A short history of lattice QCD

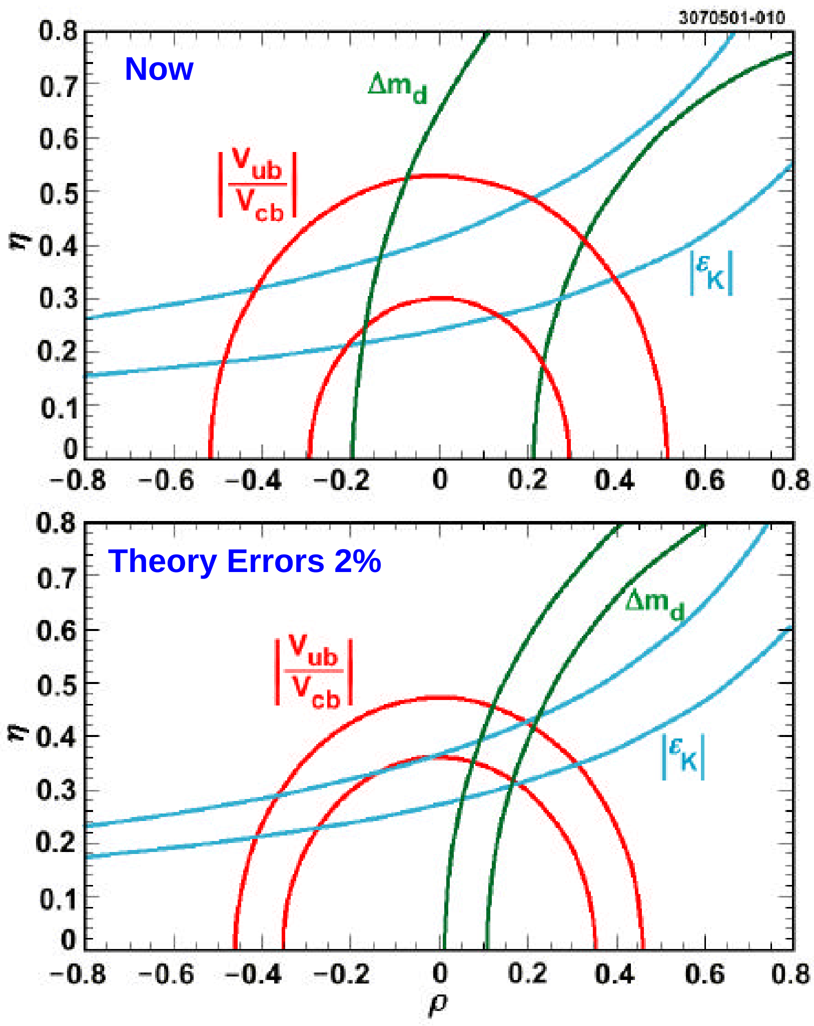
- Invented 1974
  slow progress ....

- Renaissance in 1990s
  - Much improved understanding of systematic errors
  - 10-20% errors possible on spectrum, form factors etc

- Second lattice revolution - early 2000s
  - Teraflop computing power will enable simulations with 'real' dynamical quarks
  - carry improvement of systematic errors further
  - 2-3% errors will be possible

Why do you care?

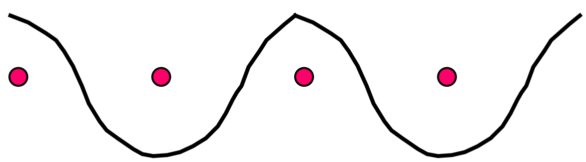
Improved theoretical precision will add huge value to experiment, e.g. from B factories.



(CLEO-c - new expts will check lattice errors.)

Heavy Quark Physics

Heavy (b and c) quarks ($m >> \Lambda_{QCD}$) present special challenges to lattice QCD because $m_Q a \geq 1$. $m_b a \sim 2-3$, $m_c a \sim 0.5-1$.
$\vec{p} \approx m_Q$ very distorted and naive use of relativistic quark formulations (Wilson, clover) give large errors ($\propto m_Q a, (m_Q a)^2$).



BUT, b and c are non-relativistic in their bound states.
m_Q and $\vec{p} \approx m_Q$ are irrelevant dynamical scales.
Can treat b and c quarks accurately on the lattice with non-relativistic techniques.

Several ways to proceed:

1. Static Quarks : $m_Q = \infty$ limit. Spinless and flavourless. Quark prop = string of gluon fields in time dirn. Useful limit for understanding HQET.

2. NRQCD : Non-relativistic effective theory.

$$\mathcal{L}_Q = \overline{\psi}(D_t - \frac{\vec{D}^2}{2m_Q a} - c_B \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q a} + \ldots)\psi$$

ψ a 2-component spinor.

$m_Q a$ fixed by requiring one heavy hadron mass correct. $E_h(p) = E_h(0) + p^2/2m_h$.

c_i fixed by pert. or nonpert. matching to QCD.

$m_Q \to \infty$ is static.

Cannot take a to 0 but improve until a-dependence small enough.

3. Heavy Wilson quarks (FNAL method). :

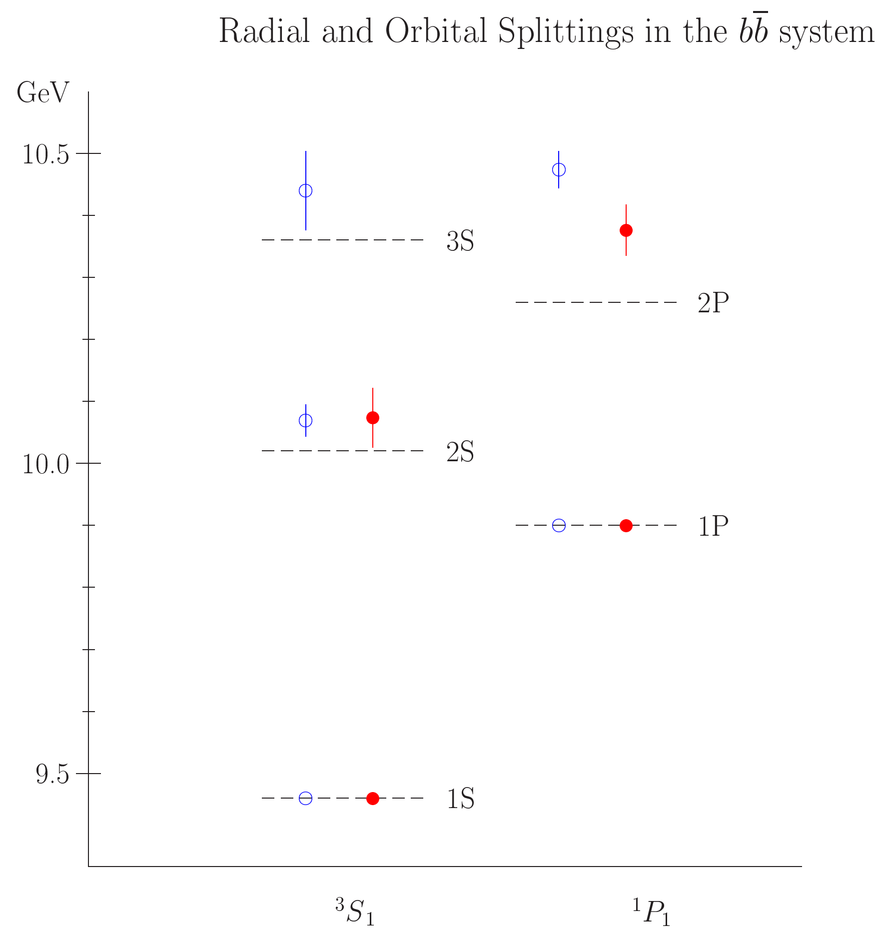
$$\mathcal{L}_Q = \overline{\psi}(\not{D} + m_Q a - \frac{iac_{sw}}{4}\sigma_{\mu\nu}F^{\mu\nu})\psi$$

but interpret non-relativistically to fix $m_Q a$. Match to QCD with $m_Q a$-dependent coefficients.
Small $m_Q a$ limit is light quarks.
Large $m_Q a$ limit is NRQCD.

4.Wilson/clover quarks : Same action as above but work only at small ma (OK for m_c ?). Extrapolate to large $m \Rightarrow$ large errors *and* expensive. Anisotropic lattices may be better. (fine $a_t \Rightarrow$ small ma_t even with large a_s). See X. Liao parallel talk.

Results on the spectrum

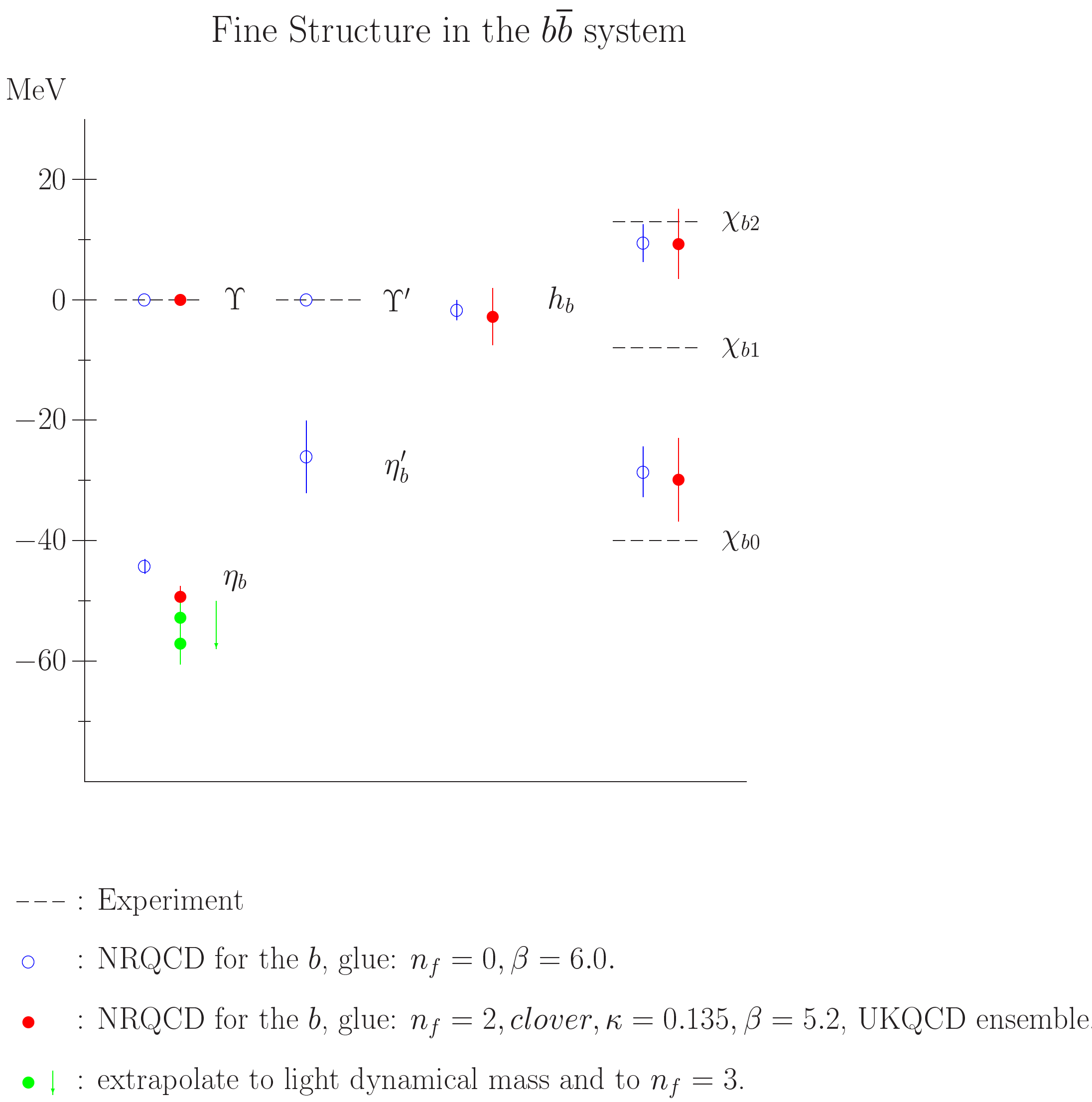
1. $b\bar{b}$ (Υ) spectrum (UKQCD collaboration - Marcantonio et al)



--- : Experiment

○ : NRQCD for the b, glue: $n_f = 0, \beta = 6.0$.

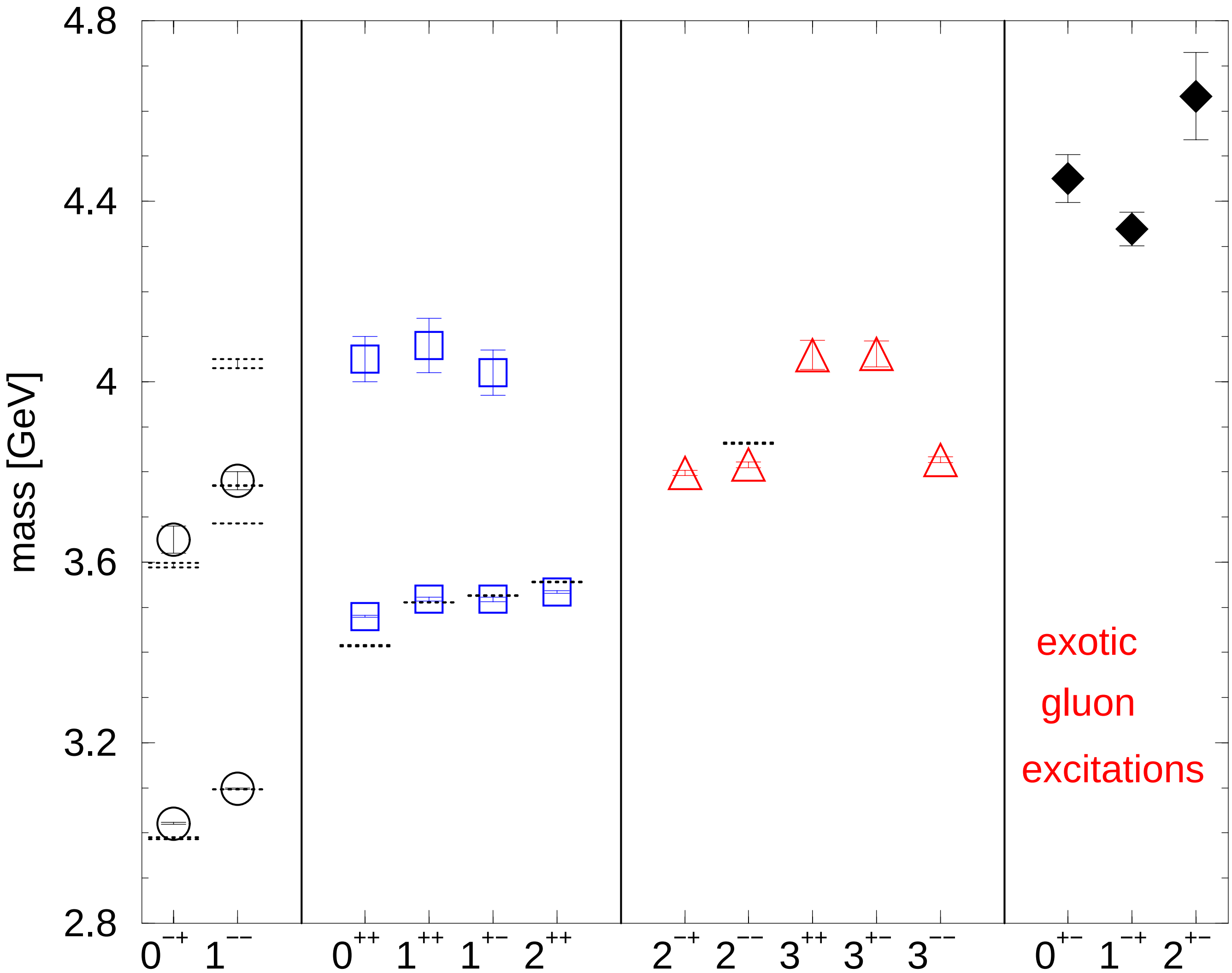
● : NRQCD for the b, glue: $n_f = 2, clover, \kappa = 0.135, \beta = 5.2$, UKQCD ensemble.



Hyperfine splitting sees dynamical quarks. Predict $m(\Upsilon) - m(\eta_b) = 60 \pm 15$ MeV. Aim for 1 – 2 % error for CLEO-c.

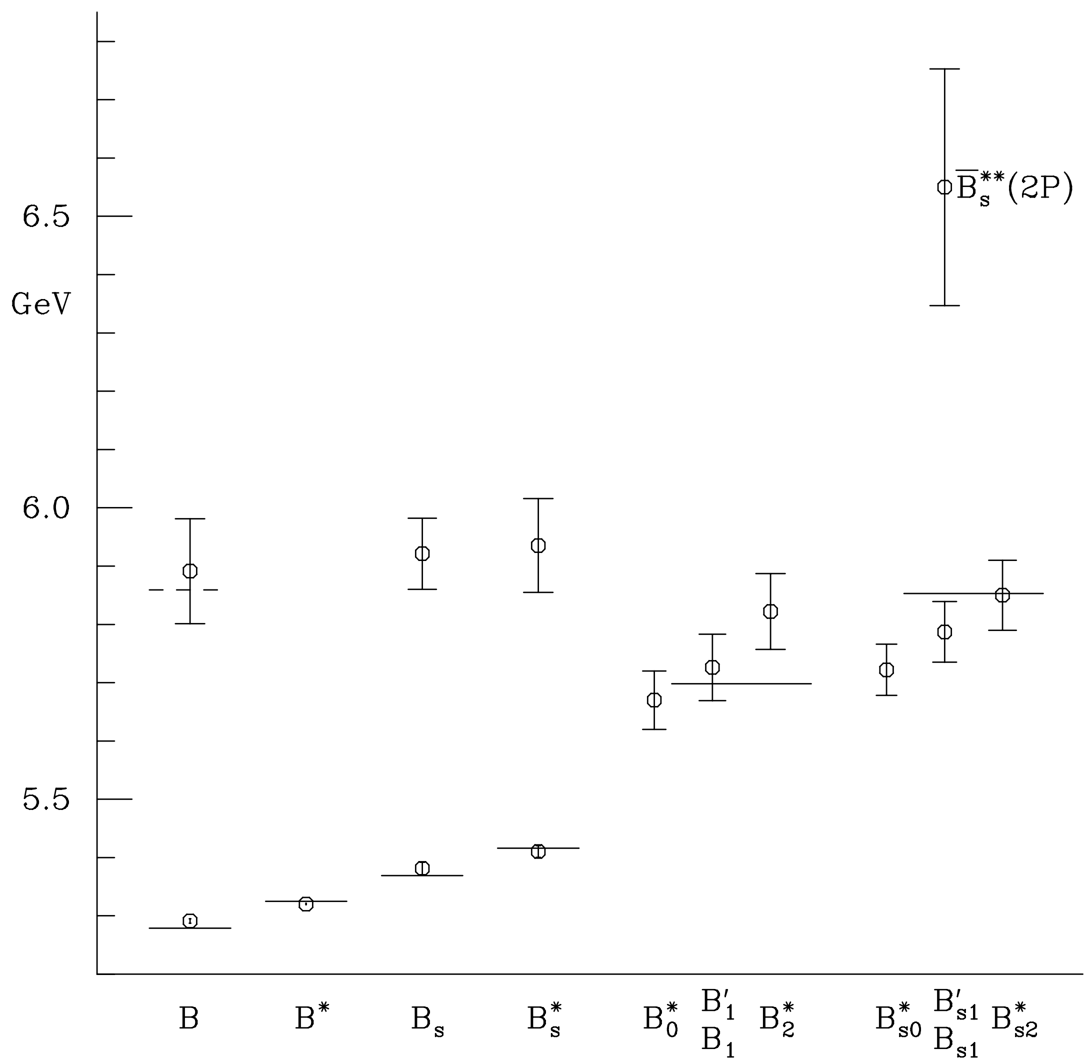
2. $c\bar{c}$ (ψ) spectrum

Columbia results (Chen et al) on anisotropic lattices in the QA, including $c\bar{c}g$ hybrids.



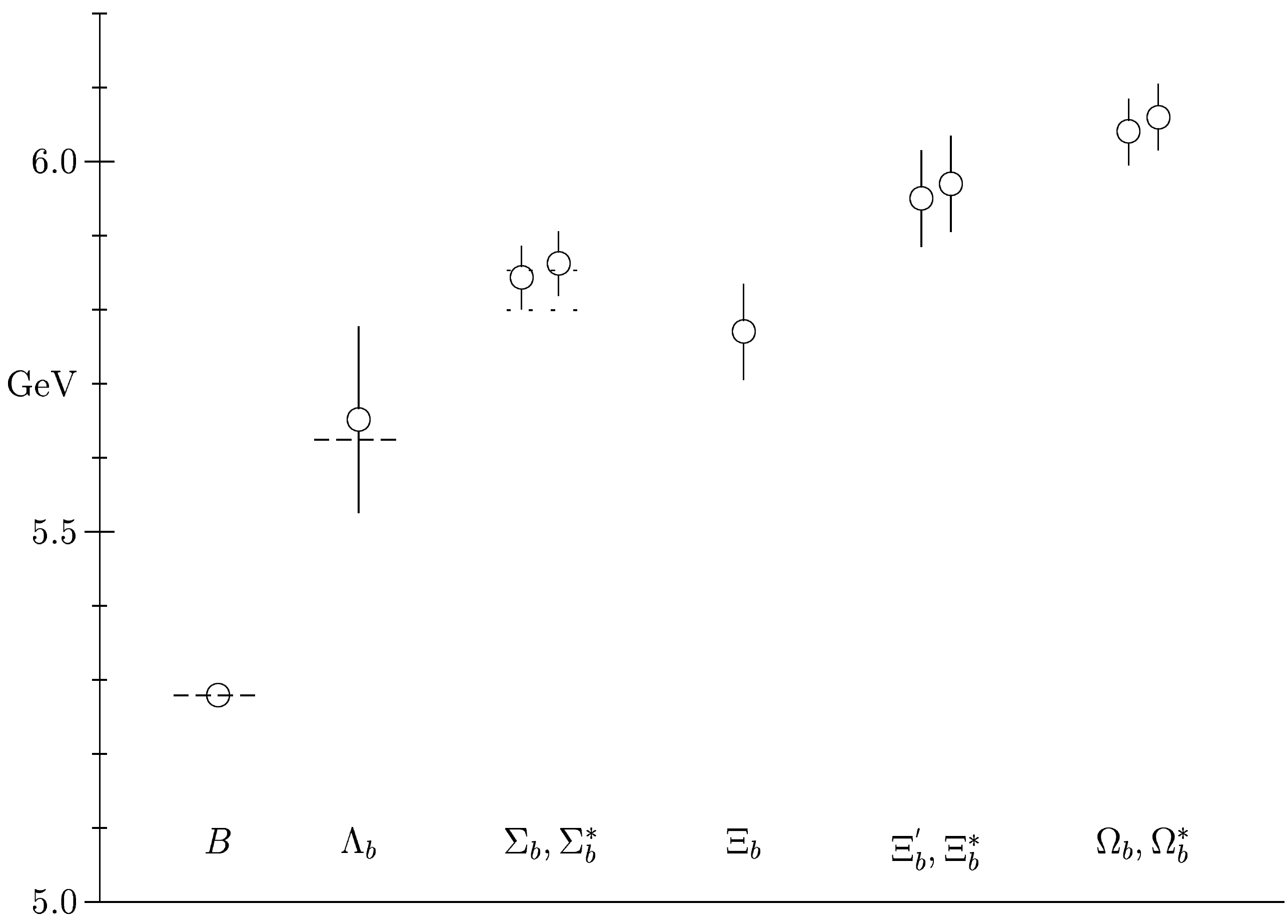
3. b-light (B) spectrum

NRQCD results in the QA (Hein et al).



4. b-light-light baryon spectrum

NRQCD results in the QA (Ali Khan et al) for udb, usb, ssb states.



Quark masses

m_b fixed s.t. B or Υ mass correct. Best determination is from B in static limit in QA.

$$\overline{m}_b(\overline{m}_b) = Z_{cont}(m_B - E_B(\vec{p} = 0) + E_0)$$

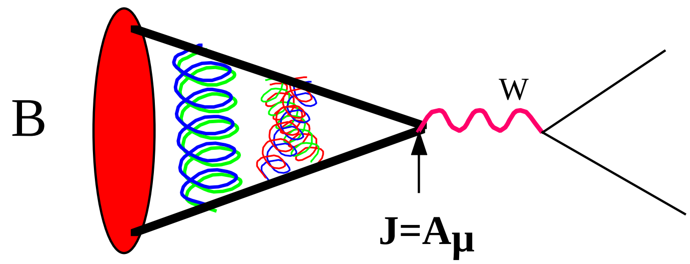
Z_{cont} and E_0 known in pert. th. through α_s^3.
'World average' is 4.30(10) GeV. (Ryan, LAT01)
$m_b \approx 50$ MeV smaller unquenched (?)
New non-pert methods in progress (Sommer et al).

m_c from ψ and α_s^2 lattice mass renorm for non-rel. case (LAT01: Juge et al)
$m_c(M_c) = 1.28(4)$ GeV, QA
m_c from D_s and non-pert renorm. for rel. case (Becirevic et al)
$m_c(m_c) = 1.26(13)$ GeV, QA.

Determining B matrix elements

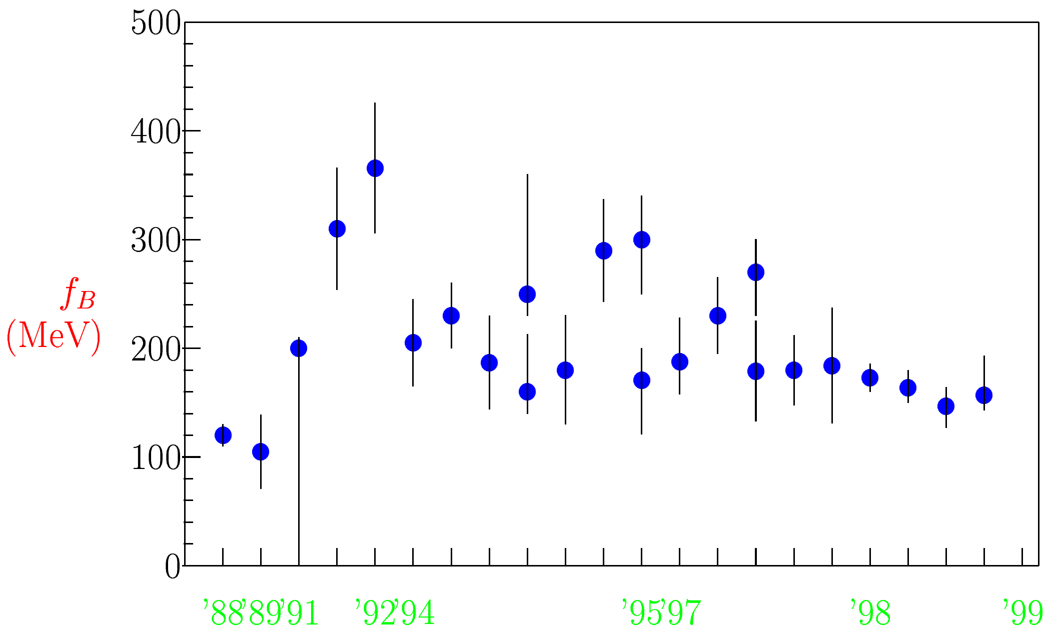
1. f_B

Simplest 2pt m.e. for B leptonic decay.



$$< 0|A_\mu|B >= p_\mu f_B$$

Has improved a lot with time (QA).



Match A_μ on lattice to continuum. Done to $\mathcal{O}(\alpha_s, 1/m_Q, a)$ for NRQCD and heavy Wilson. Typical error 'budget':

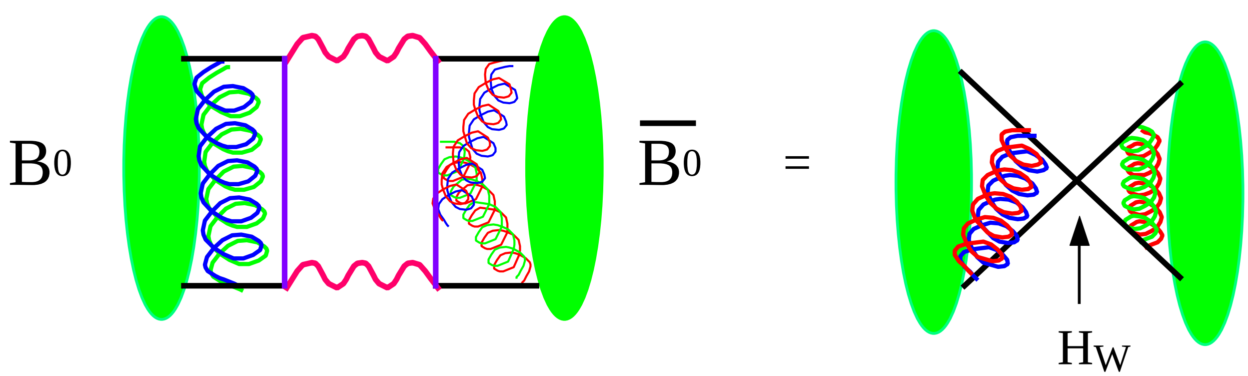
Source	*percent*
statistical + interp.	3
disc. $O((a\Lambda)^2)$	4
pert. $O(\alpha_s^2, \alpha_s^2/(aM))$	7
NRQCD $O((\Lambda/M)^2, \alpha_s\Lambda/M)$	2
light quark mass	+4
$a^{-1}(m_\rho)$	4
Total	**10**

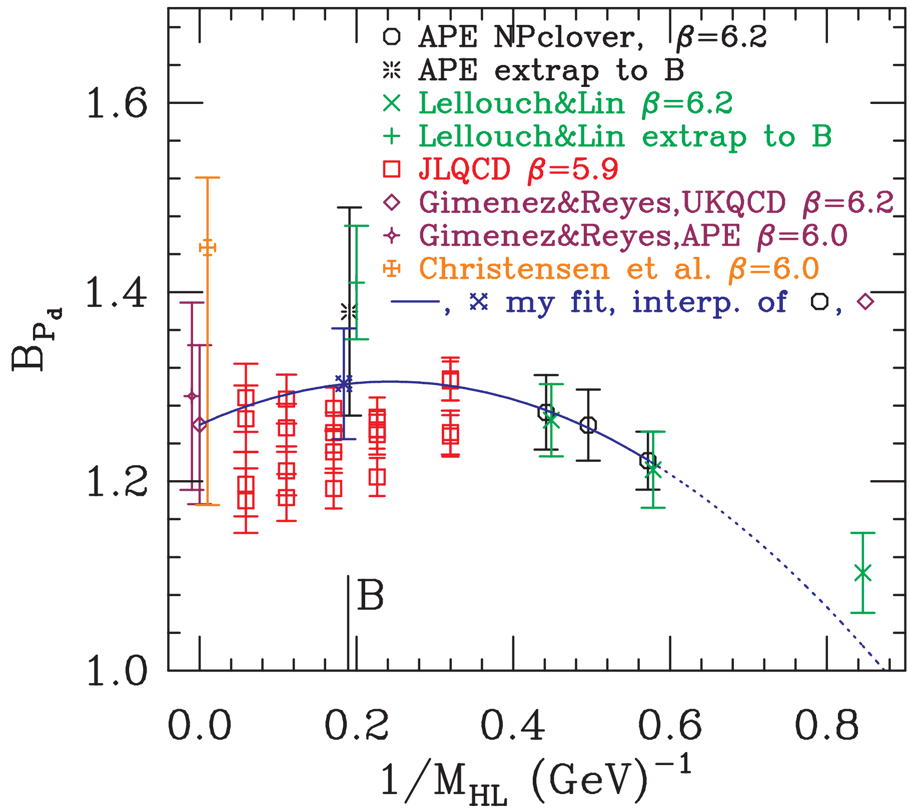
World averages (Ryan, LAT01)
$f_B^{(QA)} = 173 \pm 23$ MeV, 20% larger unq. ?
$f_{D_s}^{(QA)} = 203 \pm 14$ MeV
$f_{B_s}/f_B = 1.15(5)$; $f_{D_s}/f_D = 1.16(4)$
Need to reduce pert. errors by $\mathcal{O}(\alpha_s^2)$ calcs/ non-pert. techniques.

2. B_B

Matrix element for 'box' diagram for $B^0 - \overline{B}^0$ mixing – becomes m.e. of 4-q operator from H_W. Convenient to take ratio to f_B^2, call result B_B.

$\Delta m_B \propto |V_{tb}^* V_{tq}|^2 f_B^2 B_B$.

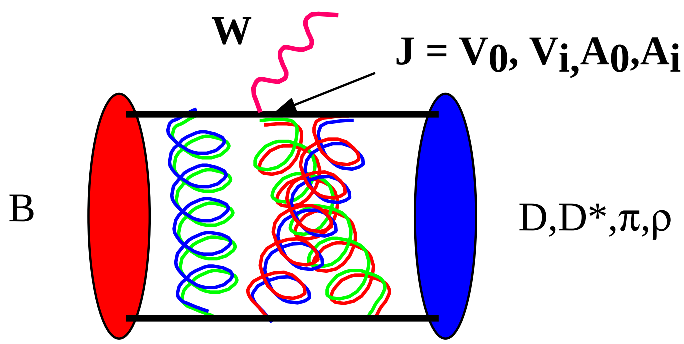




World averages (Ryan, LAT01, Bernard, LAT00):
$\widehat{B}_{B_d} = 1.30(12)(13)$; $f_{B_d}\sqrt{\widehat{B}_{B_d}} = 230(40)$ MeV.
$\widehat{B}_{B_s}/\widehat{B}_{B_d} = 1.01(3)$;

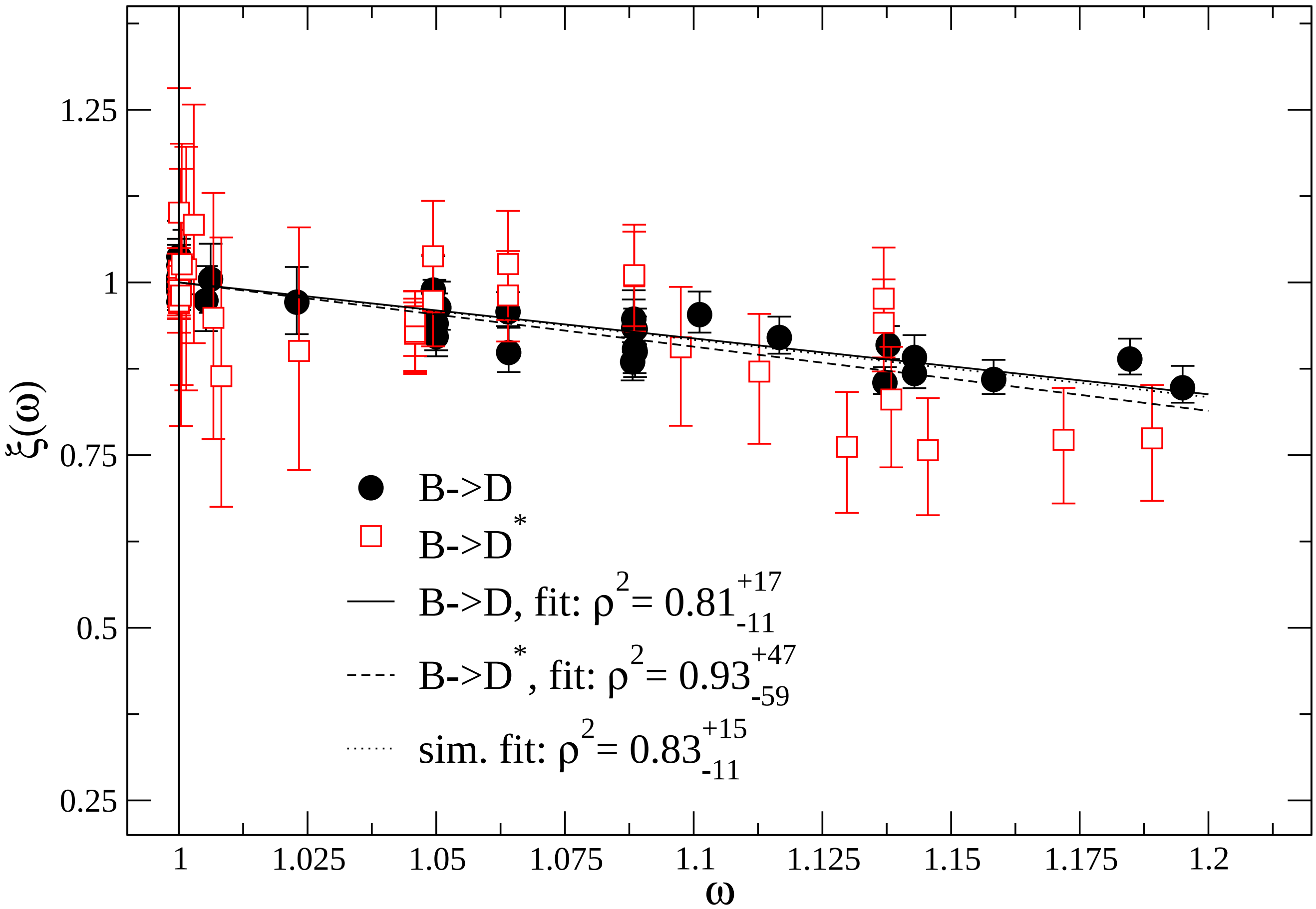
$$\xi \equiv \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}} = 1.16(5)$$

3. B SL decay.



Matrix elements needed for determination of V_{ub}, V_{cb}.

$B \to D^{(*)}$ decay.
Heavy quark symmetry very useful here $\to$ study matrix element as function of $\omega = v_B \cdot v_D$.
Form factor then has universal shape to a good approx. = Isgur-Wise function.

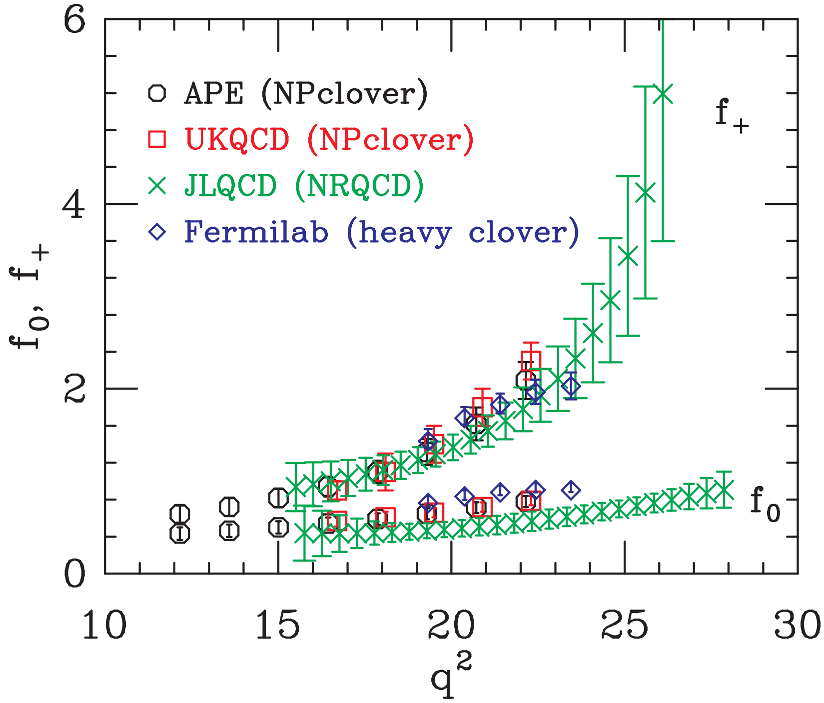


(UKQCD - rel. c quarks)

exptl rate($v_B \cdot v_D = 1$) = $|V_{cb}|^2 \mathcal{F}(1)$.
FNAL LAT01(Hashimoto et al):
$\mathcal{F}_{B\to D^*}(1) = 0.913(30)$. (QA)
gives $V_{cb} \times 10^3 = 38.7 \pm 1.8 \pm 1.5$ (LEP).

$B \to \pi, \rho$ decay.

Lattice calculations (all QA) work at small $\vec{p}_\pi$, far from physical region. Require extrapolation,interpolation, etc. Smooths out v. rough raw data. More work is needed.



Soft pion theorem $f_0(q^2_{max}) = f_B/f_\pi$ doesn't work well. Chiral extrapolations to light quark masses very important.

Future

- MILC collab. using improved staggered quarks $\rightarrow$ can simulate light dyn. quarks with Tflop computers in next few years.

- CLEO-c will determine Υ, ψ, D physics to high precision.

Lattice calcs must improve systematic errors to 2-3%.

- Improve heavy quark actions, higher rel. corrns and better match to continuum.

- Improve matching of matrix elements using automated pert. th. (Trottier, Horgan)

- Simulate with light dynamical quarks and match to chiral pert. theory.