Instantons and baryon dynamics

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1. What are instantons ?

2. How do instantons break chiral symmetry ?

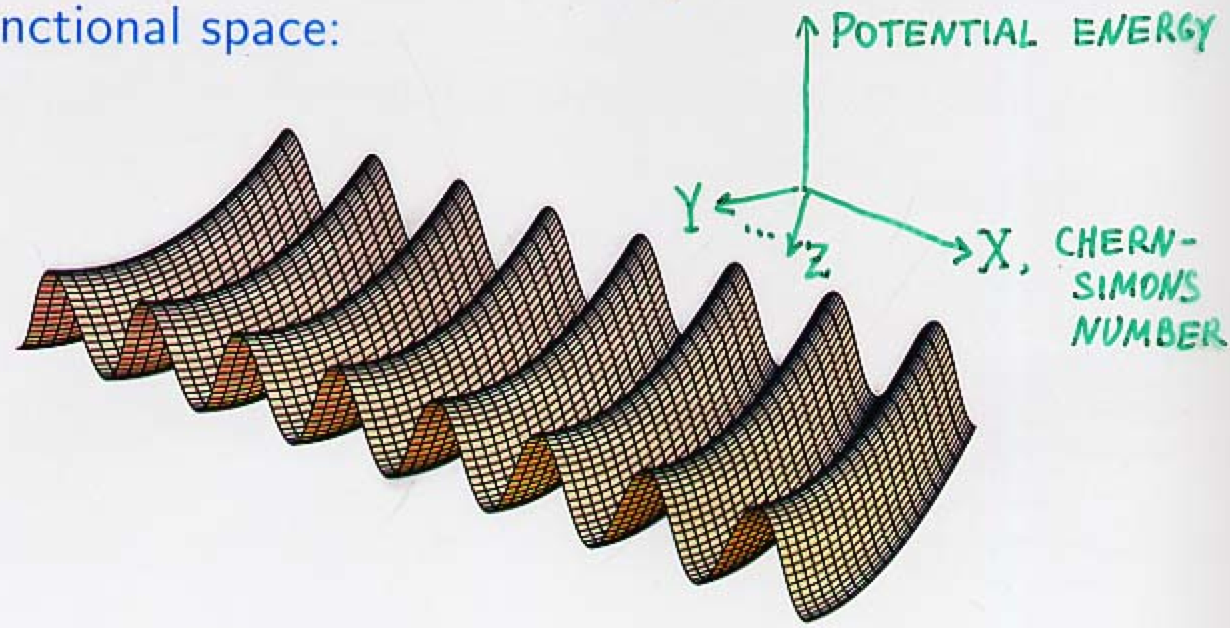
3. Why are baryons so sensitive to chiral symmetry breaking ?

4. Baryon structure (no fitting parameters)

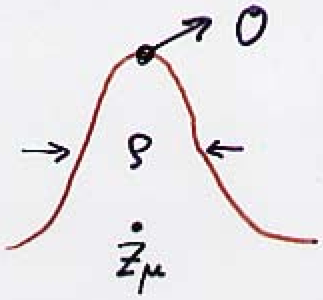
5. Summary

1. What are instantons ?

QCD deals with fluctuating gluon $A_i^a(x)$ and quark $\psi^{\alpha\, f}(x)$ fields. A fundamental fact [Faddeev, Jackiw and Rebbi (1976)] is that the potential energy of gluon fields is a periodic function in one particular direction in the functional space:

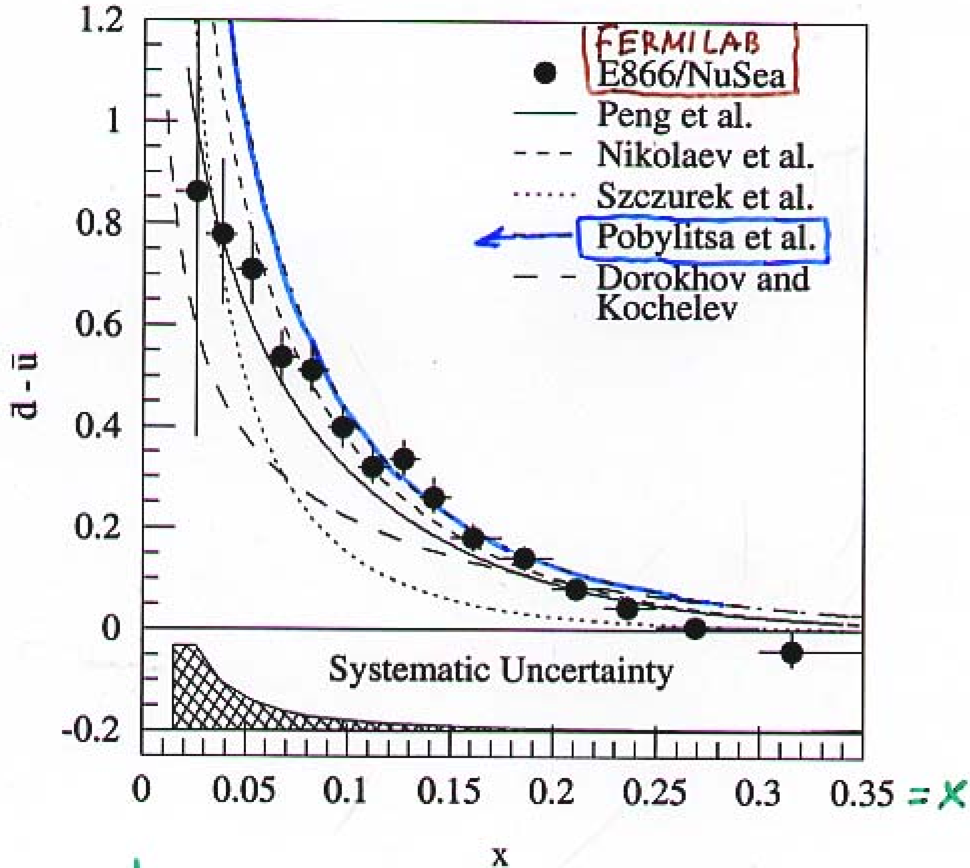


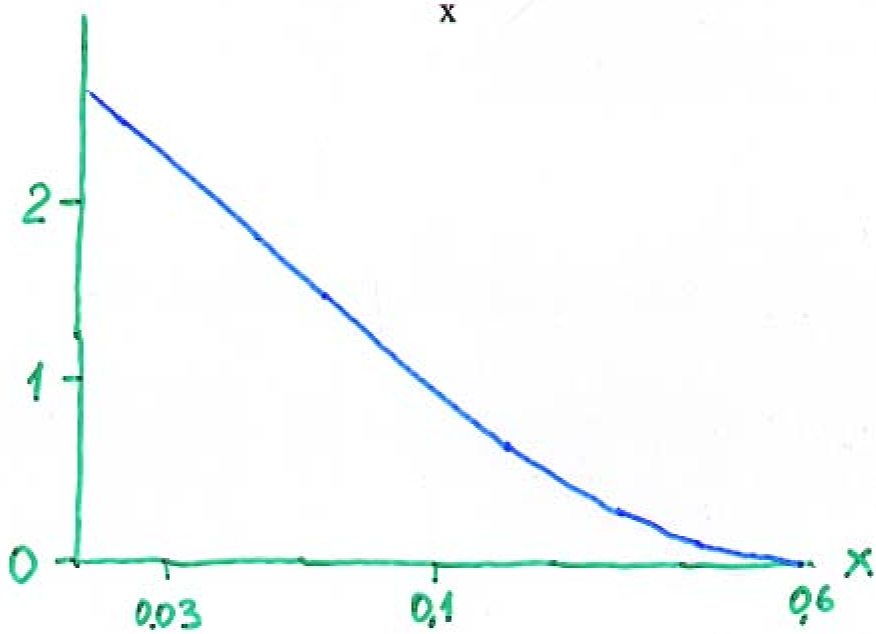
INSTANTON = large fluctuation of the gluon field $A_i^a(x, it)$ which tunnels from one minimum to the neighbour one, with minimal action [Belavin, Polyakov, Schwarz and Tiupkin (1975), Gribov (1976), 't Hooft (1976)]



Instanton fluctuations are characterized by their position in space-time z_μ, its spatial size ρ and its orientation in

ANTIQUARK ASYMMETRY, $\bar{d} - \bar{u}$





PREDICTION FOR $\Delta\bar{u} - \Delta\bar{d}$

DRESSLER, GOEKE, POLYAKOV & WEISS (2000)

ANTI-QUARK DISTRIB'N

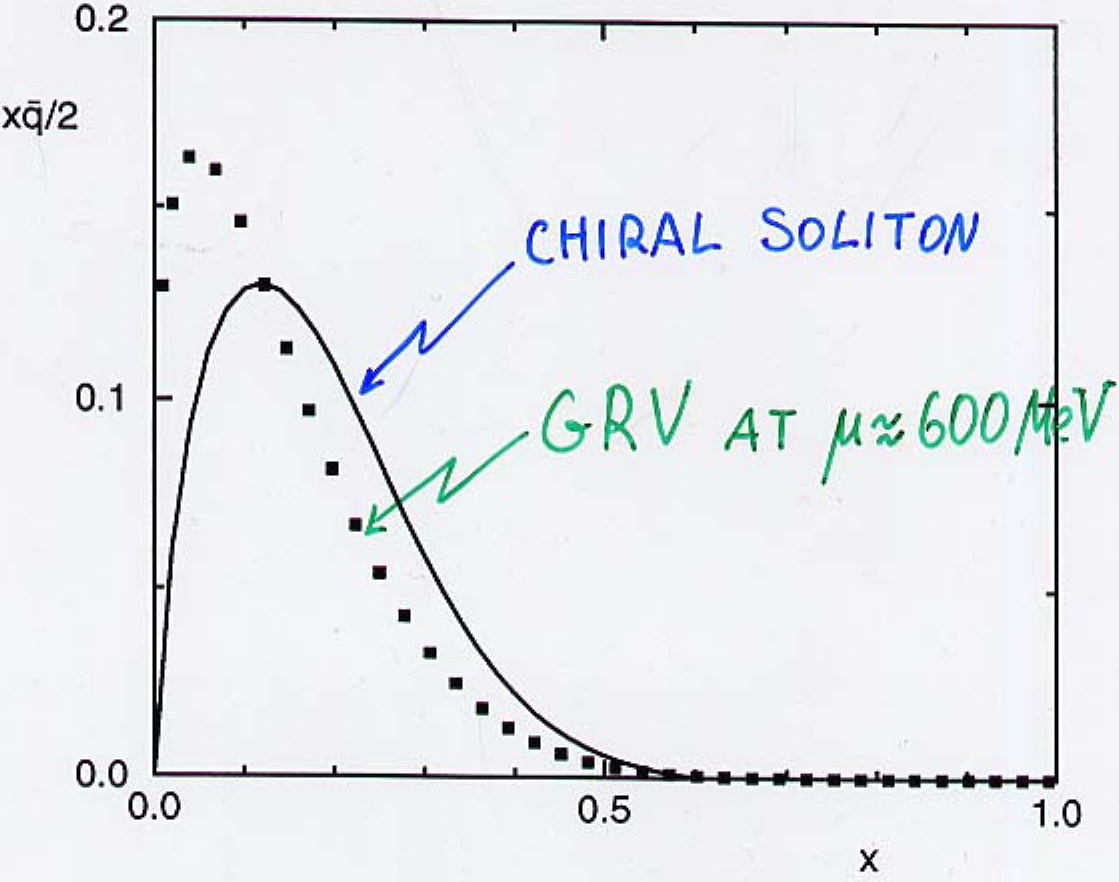
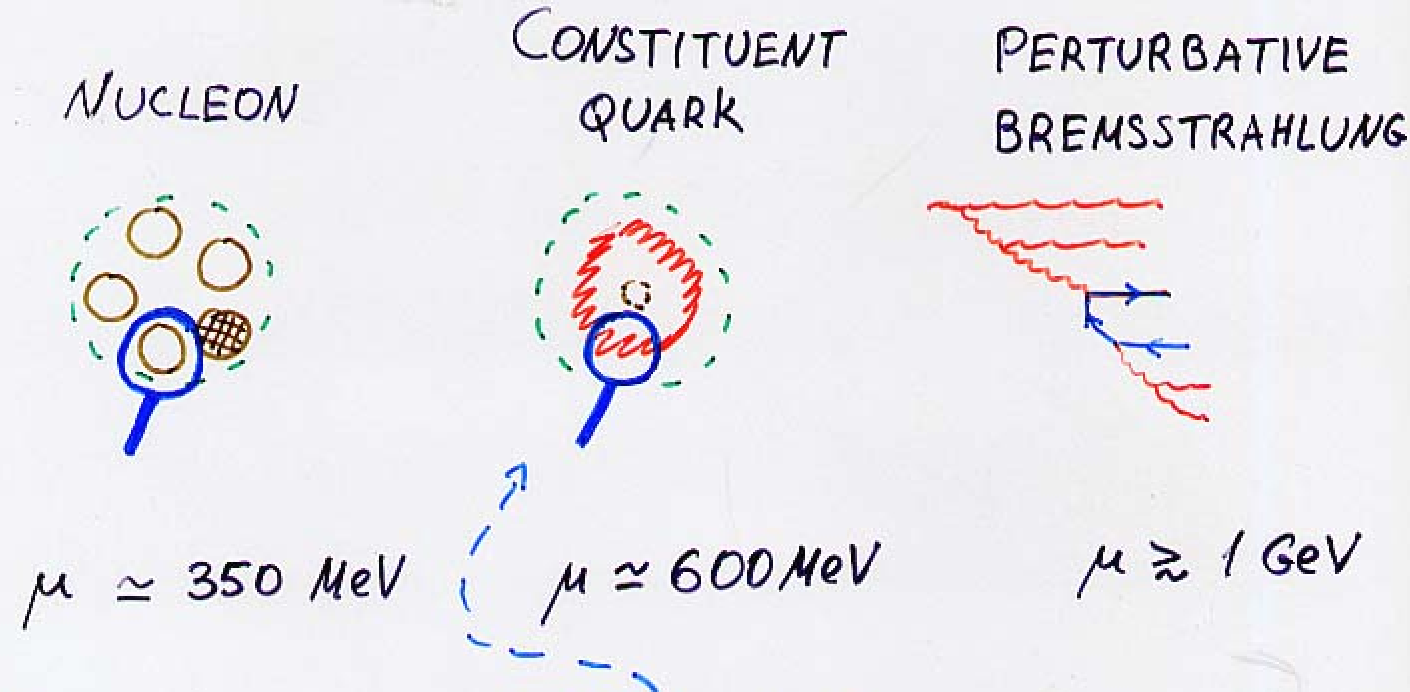


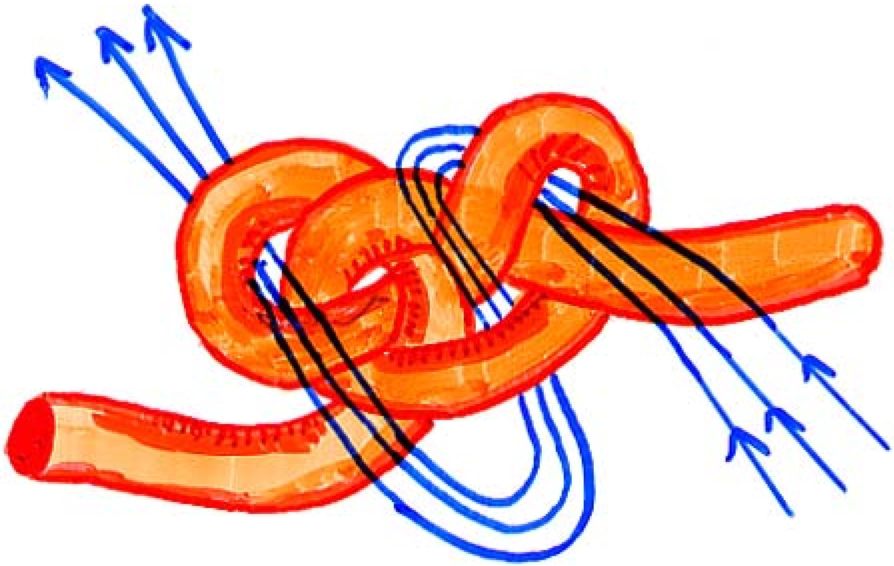
Figure 3: The antiquark distribution, $x[\bar{u}(x) + \bar{d}(x)]/2$. Solid line: theory; squares: the parametrization of ref. [15].

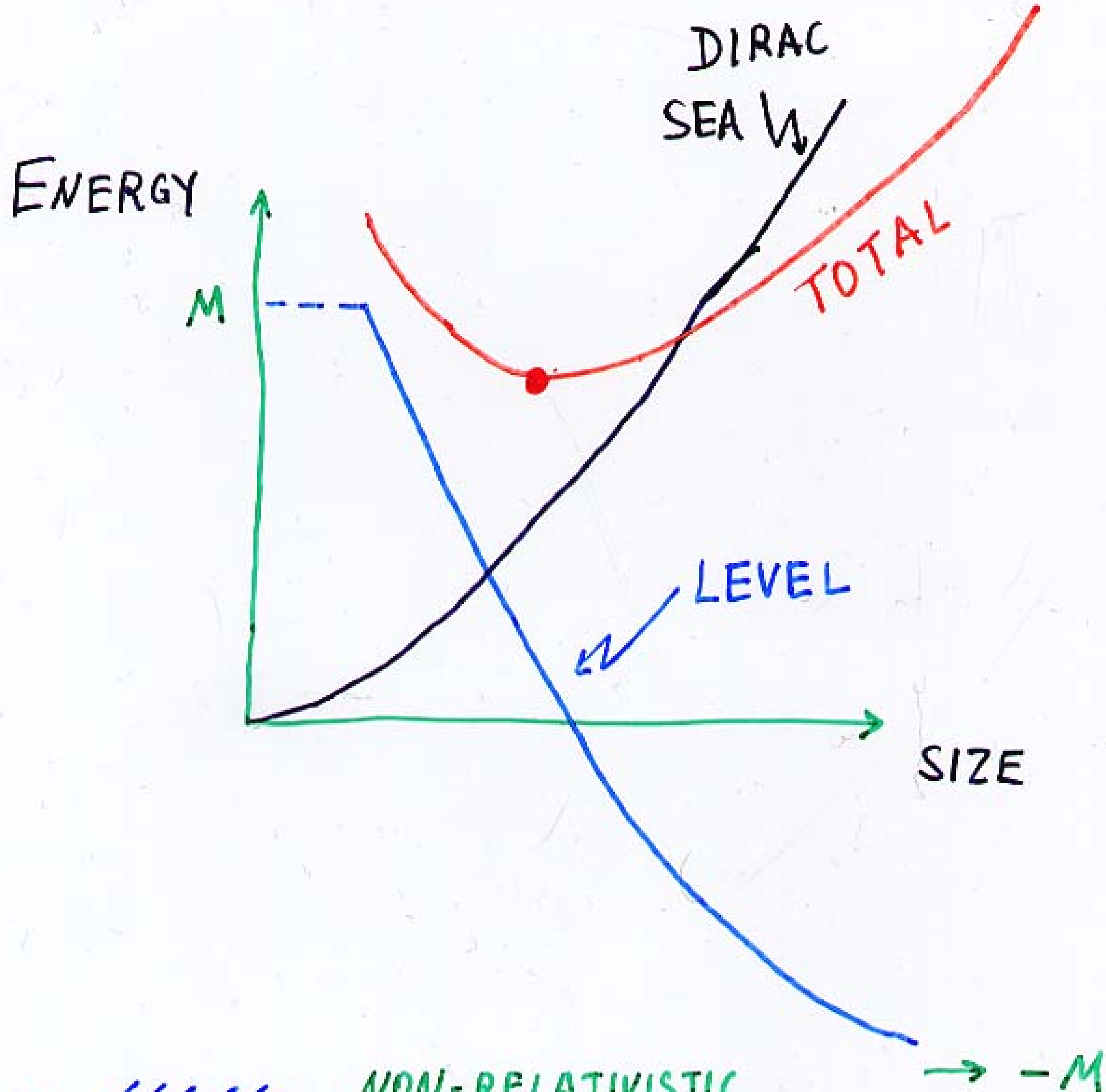
A huge amount of data comes from various parton distributions inside the nucleon. Nucleons, are most sensitive to spontaneous chiral symmetry breaking. Having at hand the microscopic theory one is in a position to calculate parameter-free parton distributions at a low normalization point, from where they are evolved perturbatively to the observable distributions at high momentum transfer in deep inelastic lepton-hadron scattering.

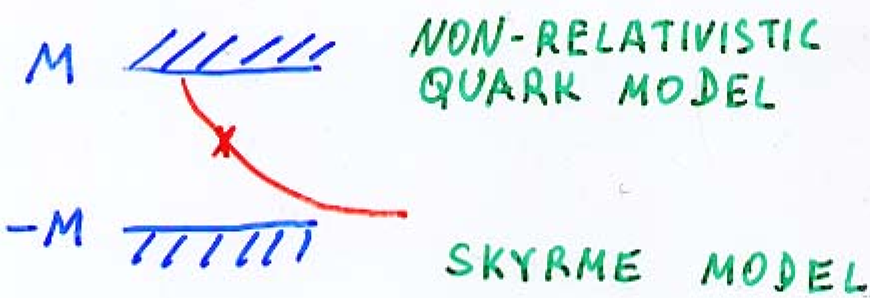


GLUONS EMERGE HERE, AND CARRY

$\simeq (M\rho)^2 \simeq \frac{1}{3}$ OF NUCLEON MOMENTUM







NUCLEON SIZE $\sim \frac{1}{M} \sim 0.7$ fm

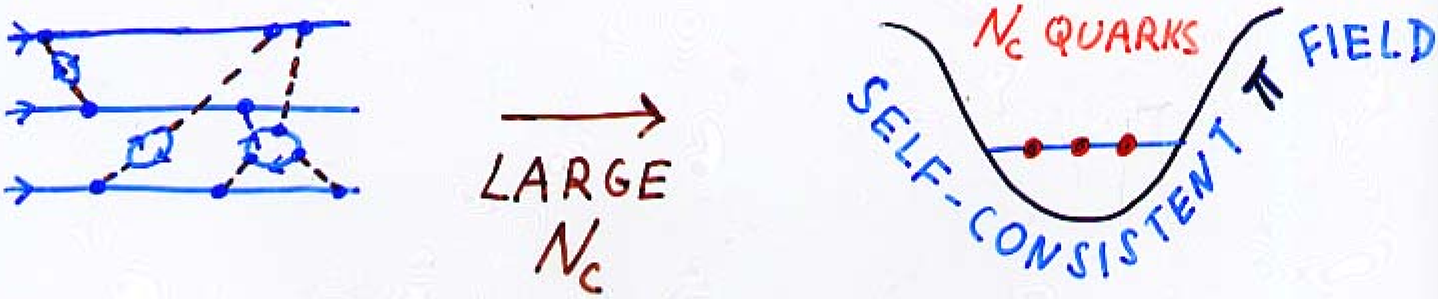
CONST. QUARK SIZE $\sim \rho_{INST} \sim 0.3$ fm

NEW ALGEBRAIC PARAMETRE:

$$(M\rho)^2 \sim \frac{\rho^4}{R^4} \approx 0.3$$

NUCLEON

[BIRSE & BANARJEE (84), KAHANA, RIPKA & SONI (84)]
[V. PETROV + D.D. (86)]



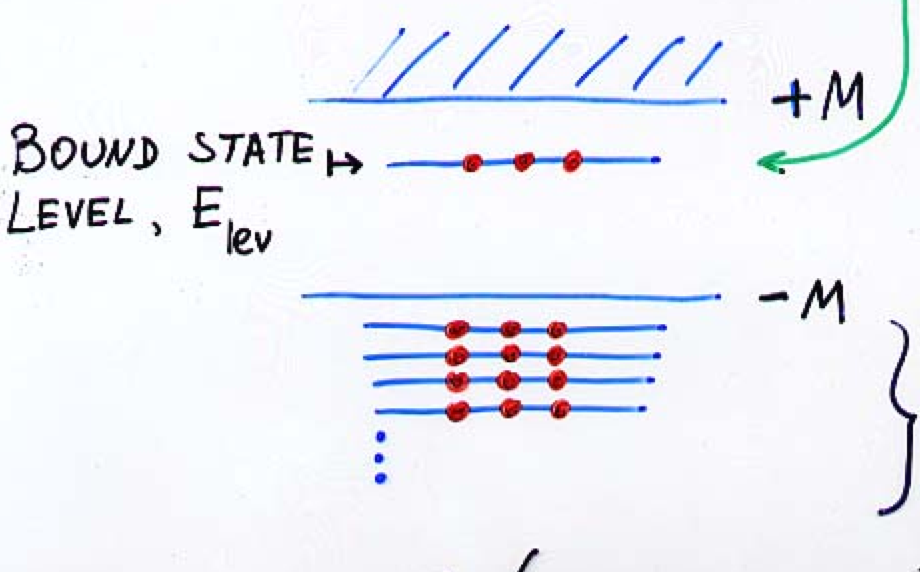
= ANALOG OF LARGE Z FOR THOMAS-FERMI ATOM

ATOM ENERGY = ENERGY OF Z ELECTRONS + COULOMB FIELD ENERGY

$$\vec{E}^2/8\pi$$

NUCLEON ENERGY = ENERGY OF N_c BOUND Q'S + PION FIELD ENERGY

AGGREGATE ENERGY OF LOWER DIRAC SEA



$$m_N = N_c \left(E_{lev}[\pi] + \sum_{E_n<0} \left(E_n[\pi] - E_n[0] \right) \right)$$

HAS A NON-TRIVIAL MINIMUM!

Analogy with solid state physics:

	Metals	QCD
microscopic theory	nuclei, electrons photons	current quarks, gluons
symmetry broken	translational	chiral
Goldstone bosons	phonons	pions
fermions	electrons, $m^*(p)$	'constituent' quarks, $M(p)$
bound states	polaron	nucleons
ground state rearrangement	super-conductivity	"color supercond'ty"

Chiral Quark - Soliton Model of baryons

Saying that chiral symmetry is spontaneously broken we automatically have to acknowledge that quarks get a dynamical mass $M(p)$. Numerically, $M(0) \approx 350\ \mathrm{MeV} \approx m_N/3$.

How to write down the low-energy lagrangian?

$$\bar{\psi}\,(i\partial\!\!\!/ - M)\,\psi$$

is wrong as it is **not invariant** under chiral rotation $\psi \to \exp(i\gamma_5\alpha^A\tau^A)\,\psi$. However

$$\mathcal{L}_{\rm eff} = \bar{\psi}\,\left[i\partial\!\!\!/ - M\exp(i\,\gamma_5\,\pi^A\tau^A/F_\pi)\right]\,\psi$$

is invariant since chiral rotation can be absorbed into the re-definition of the pion field π^A.

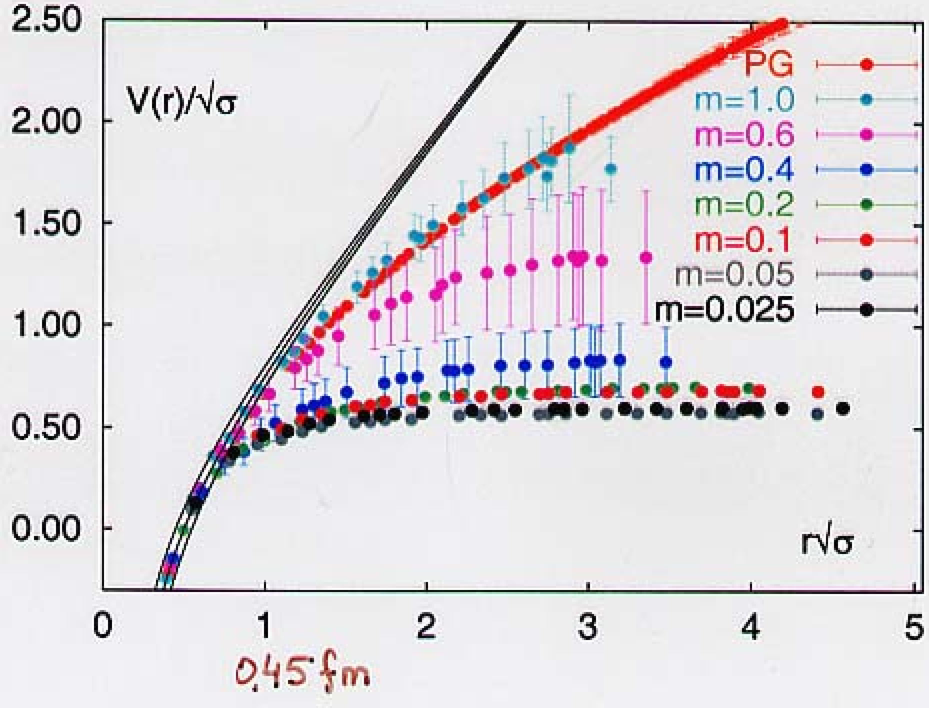
Constituent quarks interact with pions is very strongly:

$$g_{\pi qq} = \frac{M}{F_\pi} \approx 4$$

The ultraviolet cutoff in the effective low-energy theory is provided by the momentum dependence of $M(p)$.

Why are properties of low-lying hadrons (π, ρ N...) insensitive to confinement forces but very sensitive to the spontaneous chiral symmetry breaking?

- π's are Goldstone bosons: they would exist as bound states even without confinement
- From SCSB, quarks get a dynamical mass $M \approx 350\,\mathrm{MeV}$. This alone gives almost correct masses of ρ and N
- Confinement forces operate at large distances, beyond the concentration of the wave function. But in full QCD confinement is screened by meson production:

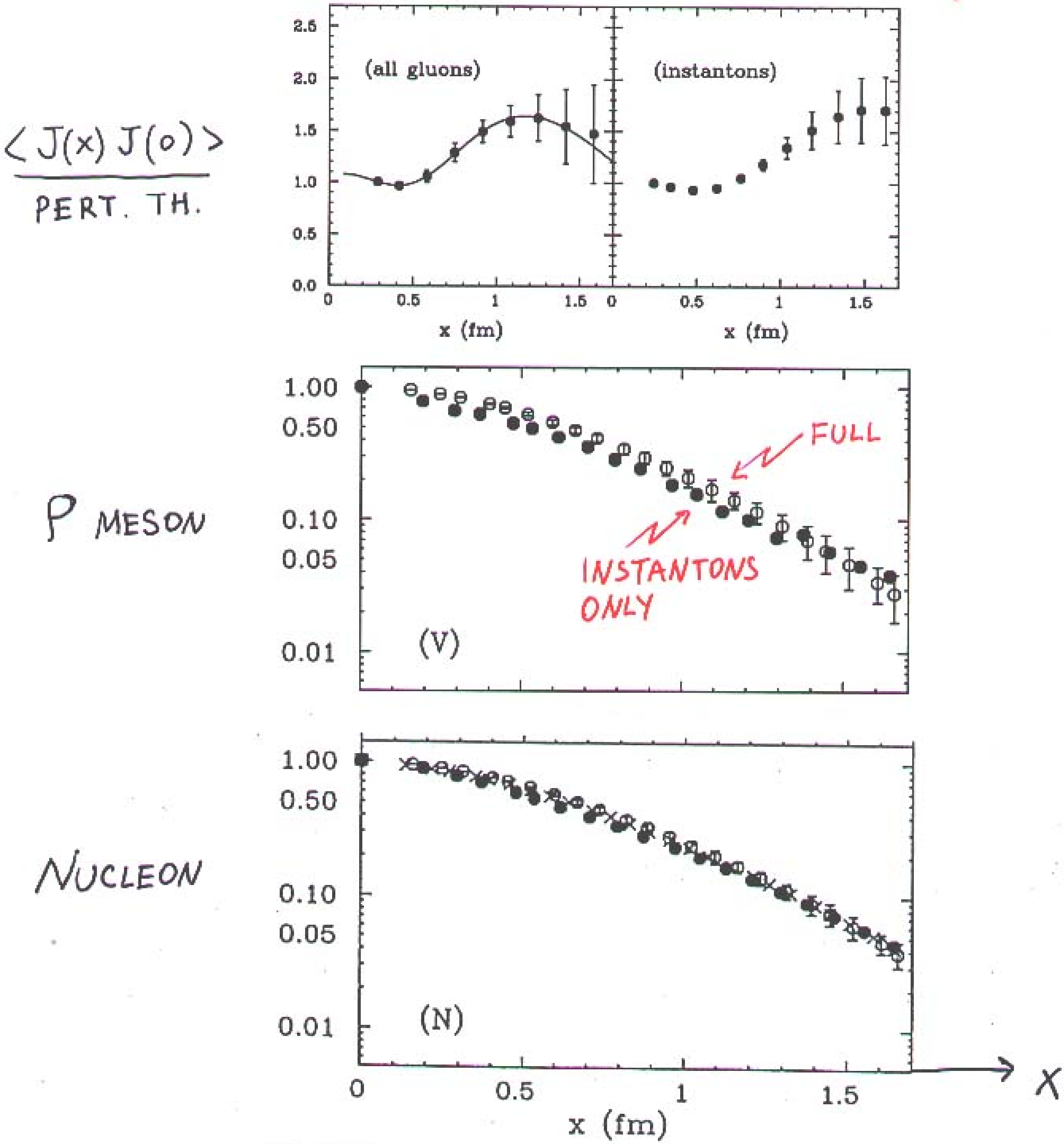


Screening of the confining potential as quark masses get small, $T < T_c$. From F. Karsch, E. Laermann and A. Peikert (2000).

Mass range 1.5 - 2.5 GeV is probably the most interesting for studying the interplay between confinement and SCSB

Verification of the picture and of the formalism.

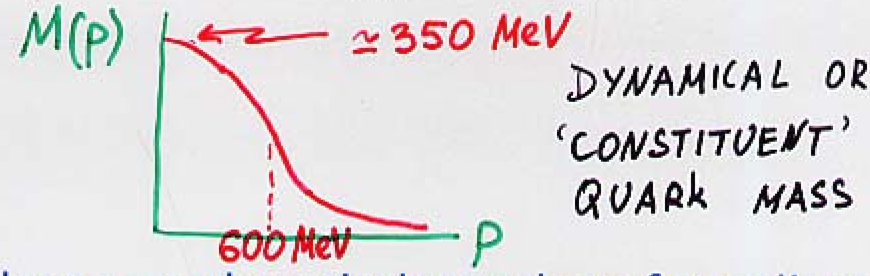
Compare hadron observables computed from the 'full' gluon vacuum, with those from 'instantons only':



Upper plot: current-current correlation function, middle plot: ≈ quark wave function inside ρ **meson**, lower plot: ≈ quark wave function inside **nucleon** (from M.-C. Chu et al.).

Propagating through the random instanton medium quarks get a dynamical mass $M(p)$:

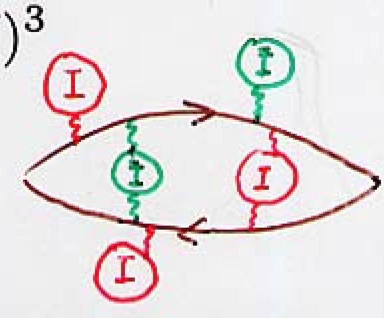
$$G(p) = \frac{Z(p)}{\not{p} - M(p)}$$



Mathematically, there are three independent formalisms yielding identical results [DD and Petrov]:

- Diagonalization of the would-be zero modes
- Averaging of the quark propagator in the medium
- Averaging first over the medium and getting an effective low-energy four-quark interaction

Some results from the calculations:



$$< \bar{\psi}\psi > \;=\; -\frac{\text{const.}}{\bar{R}^2\bar{\rho}} \simeq -(255\,\text{MeV})^3$$

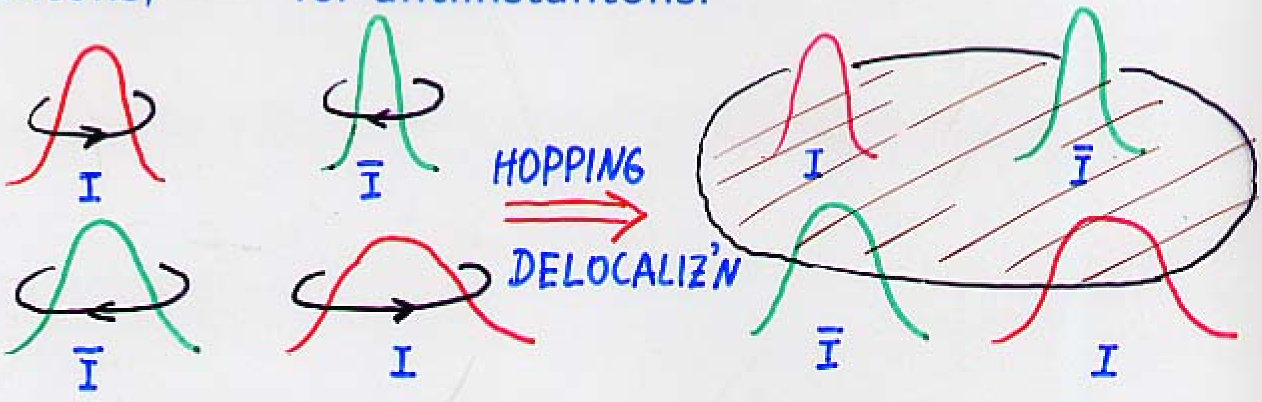
$$M(0) \;=\; \frac{\pi\bar{\rho}}{\bar{R}^2 N_c} \simeq 345\,\text{MeV}$$

$$F_\pi \;=\; \frac{\bar{\rho}}{\bar{R}^2}\sqrt{\log\frac{\bar{R}}{\bar{\rho}}} \simeq 100\,\text{MeV} \text{ vs. } 94\,\text{MeV (exper)}$$

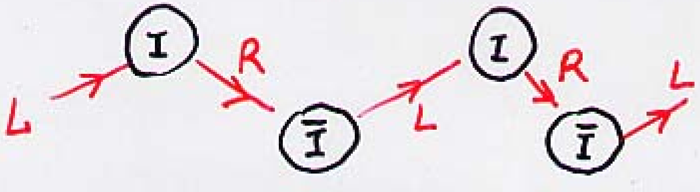
$$m_{\eta'} \;=\; \frac{\text{const.}}{\bar{\rho}} \simeq 980\,\text{MeV} \text{ vs. } 958\,\text{MeV (exper)} \ldots$$

2. How do instantons break chiral symmetry ?

We now switch in light u, d, s quarks into the random instanton ensemble. The basic property is that quarks are bound by instantons with exactly zero 'energy'. These localized states are called quark zero modes; they have definite helicity: "+" for quarks sitting on instantons, "−" for antiinstantons.



Because of the quantum-mechanical overlap, quarks are hopping from one (anti-) instanton to another, and the would-be zero modes get delocalized [DD and Petrov (1986)]. [Similar to the appearance of Anderson conductivity in a material with random impurities]. Each time quarks jump from one instanton to another, they change the helicity. It is a nonperturbative effect.



GENERATES A 'NON-SLASH' TERM IN QUARK PROPAGATOR

Quarks' hopping from one random instanton to another with a helicity flip is the microscopic mechanism of spontaneous chiral symmetry breaking

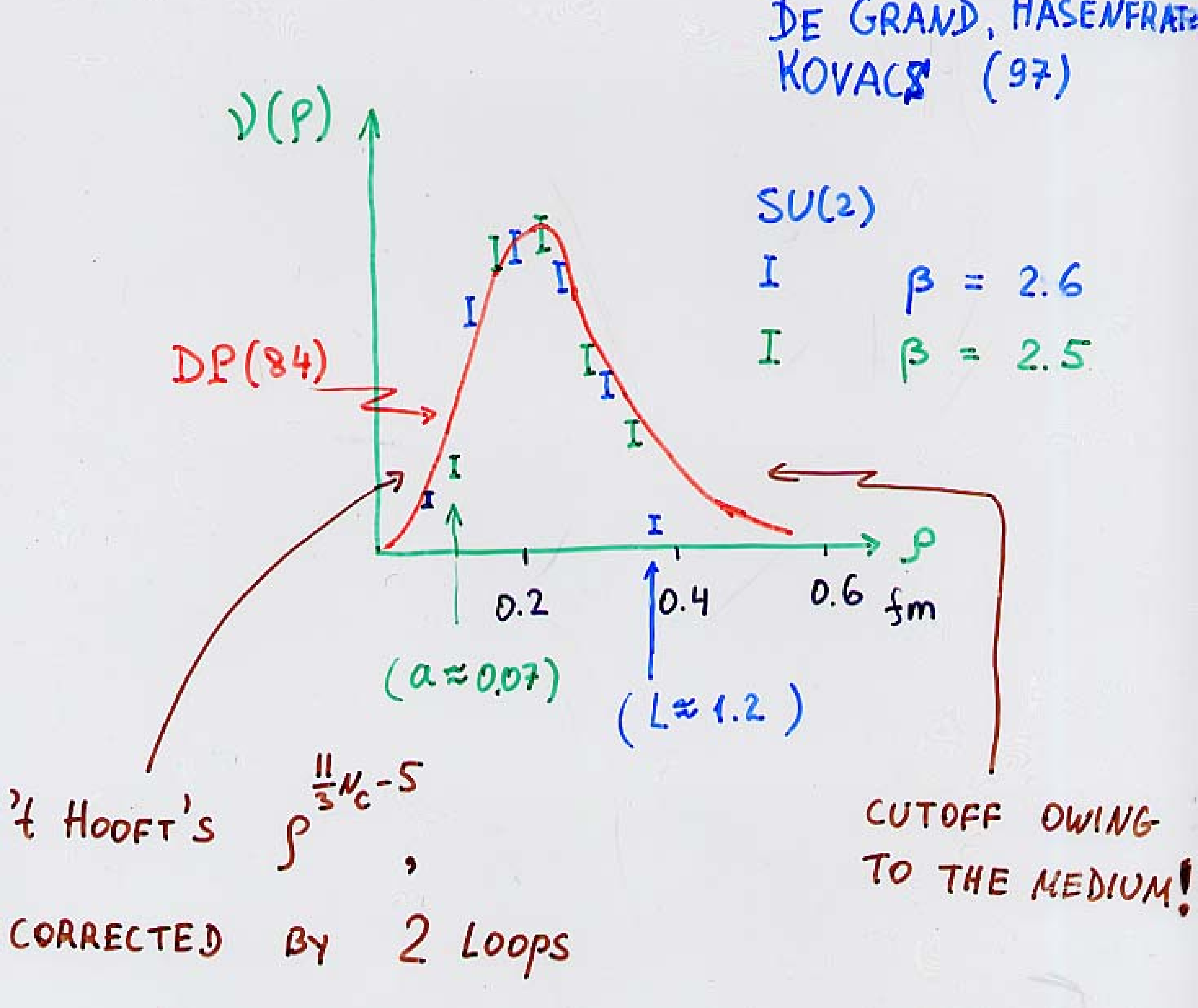
'Instanton vacuuum' is an assumption that the QCD partition function is saturated by large nonperturbative fluctuations of the gluon field (instantons), plus perturbative oscillations about them:

$$\mathcal{Z} = \sum_{N_\pm} \frac{1}{N_+! N_-!} \prod \int d^4z \, dO \, \frac{d\rho}{\rho^5} (\rho\Lambda)^b \exp(-U_{int}).$$

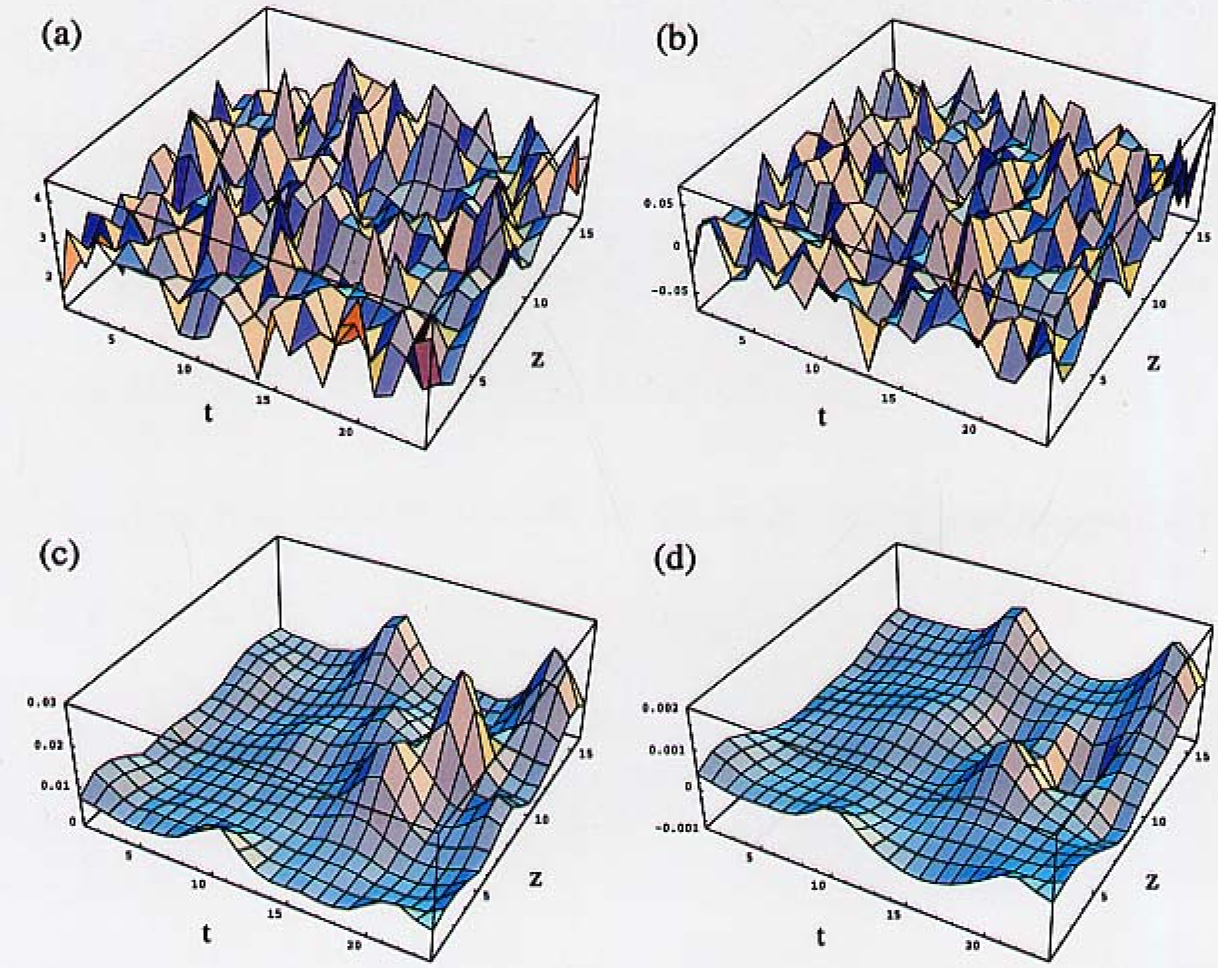
z: instanton centers, ρ: sizes, O: color orientations. It resembles a theory of an interacting liquid (in 4 dimensions). Its basic properties were established from the Feynman variational principle [DP (1984)]

Important: The 'transmutation of dimensions' has already happened at this point $\Longrightarrow$ all dimensionfull quantities, like m_N, $< \bar{\psi}\psi >$ etc. are henceforth expressed through $\Lambda_{\rm QCD}$

INSTANTON SIZE DISTRIBUTION



ACTION DENSITY "TOPOLOGICAL CHARGE" DENSITY



left column: action density in the $t - z$ plane; right column: topological charge density. Upper plots: "true gluon vacuum"; lower plots: same configuration but after smearing.[M.-C. Chu, J. Grandy, S. Huang, J. Negele (1994)]

Average size of instantons: $\bar{\rho} \simeq 0.36$ fm.
Average separation between instantons: $\bar{R} \simeq 0.89$ fm.

Theory [variational principle to 2-loop accuracy, applied to instantons] [D.D. and Petrov (1984), D.D., Polyakov and Weiss (1995)] with $\Lambda_{\overline{MS}} = 280\,\mathrm{MeV}$:

$$\bar{\rho} \simeq \frac{0.48}{\Lambda_{\overline{MS}}} = 0.35\,\mathrm{fm}, \qquad \bar{R} \simeq \frac{1.35}{\Lambda_{\overline{MS}}} = 0.95\,\mathrm{fm}.$$

color space O (all in all, 12 'coordinates').

The probability for an instanton fluctuation to happen is, roughly, given by the WKB tunneling probability:

$$\begin{aligned}\text{Tunneling amplitude} &\sim e^{-\text{Action}} \\ &= \exp\left(-\frac{1}{4g^2}\int d^4x\, F_{\mu\nu}^2\right) \\ &= \exp\left(-\frac{8\pi^2}{g^2}\right)\end{aligned}$$

It is non-analytic in the coupling constant and hence instantons are missed in all orders of the perturbation theory! However, instantons are clearly seen in lattice simulations of the gluon vacuum.

Smearing out zero-point fluctuations of the gluon field in the vacuum reveals much more smooth configurations – instantons and anti-instantons:

Summary

1. Instantons follow from the periodicity of the gluon potential energy. Lattice simulations confirm their presence and properties and, furthermore, indicate that instantons alone are responsible for the properties of lightest hadrons $\pi, \rho, N \ldots$

2. Instantons drive spontaneous chiral symmetry breaking: it is due to 'hopping' of quarks from one randomly situated instanton to another, each time flipping the helicity. The nonperturbative theory is in remarkable agreement with the low-energy phenomenology ($<\bar{\psi}\psi>$, $M(p)$, F_π, $m_{\eta'} \ldots$)

3. The instanton picture leads to the Chiral Quark - Soliton Model, with lowest baryons being constituent quarks strongly bound by a pion field. The model enables one to compute numerous parton distributions, as well as static characteristics of nucleons – with no fitting parameters.

4. Light hadrons seem to be not sensitive to confinement forces but rather to the dynamics of the spontaneous chiral symmetry breaking. For that reason, hadrons in the mass range 1.5 - 2.5 GeV are especially interesting.