Baryons on the Lattice

Robert Edwards
Jefferson Lab

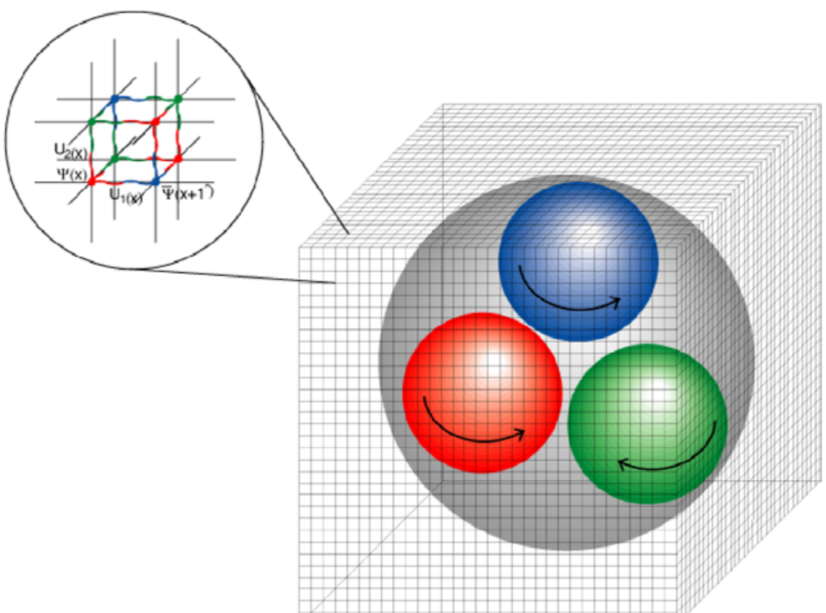
- Review lattice methodology
- Study of confinement mechanisms
- Spectroscopy results:
  - Quenching effects
  - Quenched and dynamical light quark spectroscopy
  - Importance of chiral extrapolations
  - Adding electromagnetic interactions
- Computing resources

Regularization of QCD on a lattice

- Approximate continuous space--time with a 4-dim lattice, and derivatives by finite differences.
- Quarks are put on sites, gluons on links. Gluons represented by 3x3 complex unitary matrices $U_\mu(x) = \exp(iga\, A_\mu(x))$ elements of the group SU(3).

$$\left\langle \mathcal{O}\left(U,\psi,\overline{\psi}\right)\right\rangle = \frac{1}{Z}\int dU_\mu\, d\overline{\psi}\, d\psi\, \mathcal{O}\left(U,\psi,\overline{\psi}\right) e^{-S_G(U)+\overline{\psi}M(U)\psi}$$

$$\rightarrow \frac{1}{Z'}\int dU_\mu\, \mathcal{O}\left(U, M^{-1}(U)\right) \det\left(M(U)\right) e^{-S_G(U)}$$



- Gaussian integration over anti-commuting fermion fields ψ resulted in $\det(M(U))$ and $M^{-1}(U)$ factors.
- Gauge action composed of U fields. Approximates continuum:

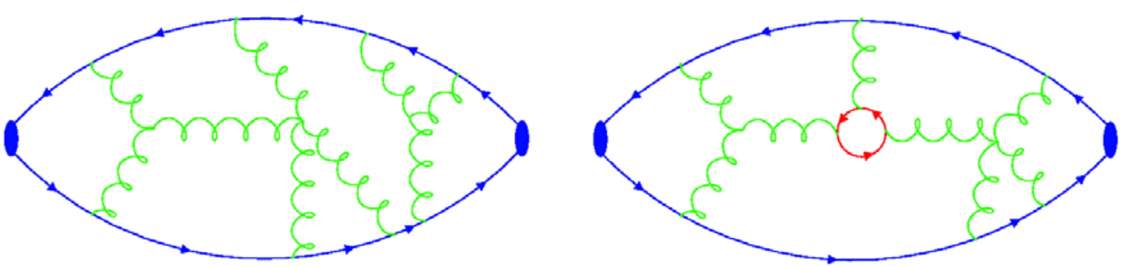
$$S_G = \frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^a_{\mu\nu} + \mathrm{O}\left(a^2\right)$$

Monte Carlo Methods

- On a finite lattice need to compute integral over large, but finite, number U-fields. Can be done numerically, though not by direct integration.
- Stochastic Monte Carlo method: generate series of configurations $U_\mu^{(i)}(x)$ distributed with probability $\exp(-S_G(U))\det(M(U))/Z$ and compute expectation values as averages over those configurations:

$$\left\langle \mathcal{O}\left(U,\psi,\overline{\psi}\right)\right\rangle = \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}\left(U^{(i)}, M^{-1}\left(U^{(i)}\right)\right)$$

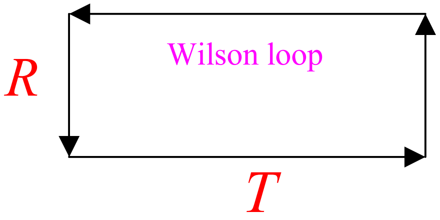
- Statistical errors go like 1/sqrt(N), for N configurations
- The det(M(U)) factor is a big computational cost since the matrix M is order VxV, though it is sparse.
- Quenched approximation: set det(M) = 1, *e.g.*, neglect internal quark loops.



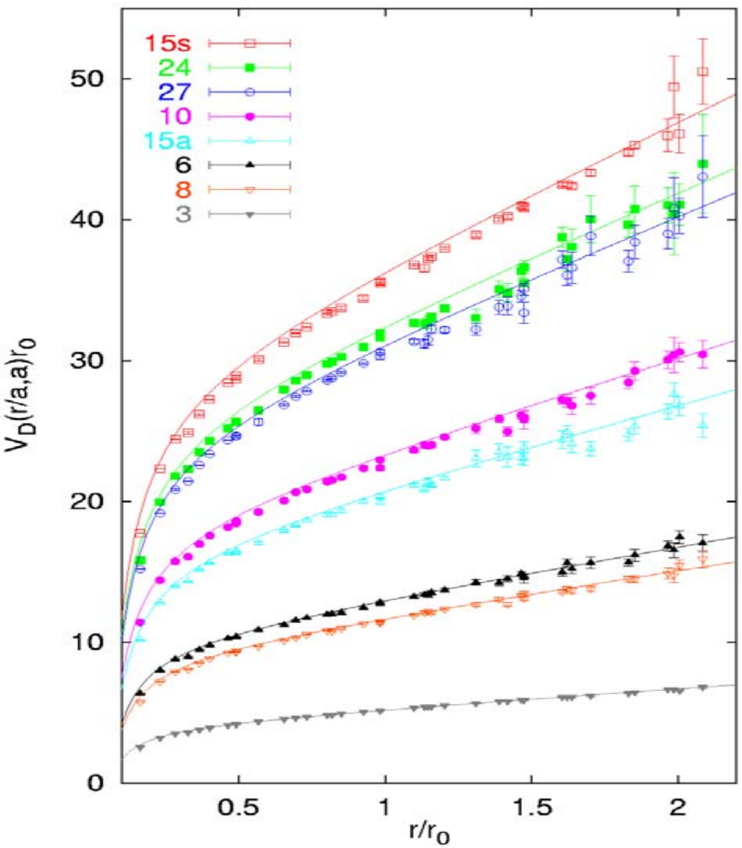
Statistical and Systematic Uncertainties

- Procedure is in principle exact after systematic errors are controlled
- **Statistical uncertainties:**
  - Statistical errors go like 1/sqrt(N), for N configurations.
  - Including determinant, cost of producing each configuration O(100) times more expensive.
- **Systematic uncertainties:**
  - *Finite volume:* lattice box must hold a hadron state, typically $L \sim 2$fm or more. Need $M_\pi L \sim 4$ (several pion Compton wavelengths)
  - *Chiral extrapolations:* calculations with small quark masses expensive – extrapolate observables to physical quark mass region (*delicate!*).
  - *Discretization effects:* inherent $O(a)$ or $O(a^2)$ lattice uncertainty. Must extrapolate to *continuum limit* ($a \to 0$) to recover physical quantities.

Confinement and Model Predictions - Static Quark Potentials



$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log\left(W(R,T)\right)$$
$$\to V_0 + \sigma R - e/R$$



- Many models propose different mechanisms for confinement
- Static quark potential (potential between infinitely massive quarks forming mesons) in different representations can discriminate among the models
- Perturbative Casimir scaling hypothesis well describes non-perturbative lattice data:

$$V(R) = d_D V_F(R), \quad d_D = C_D / C_F$$

for Casimir C_D in representation D=3,6,8,…

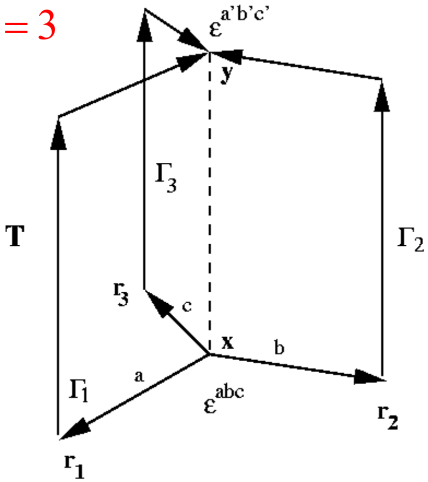
- Claimed to rule out models like Bag and Instanton – scaling different
- Flux tube counting also inconsistent

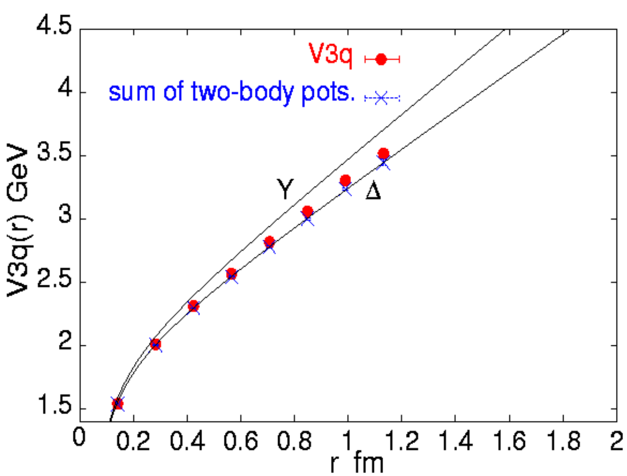
Bali, 99

Static Baryon Potential

$$V_{Nq} = -\lim_{T\to\infty}\frac{1}{T}\log\left(W_{Nq}\right)$$

$N = 3$





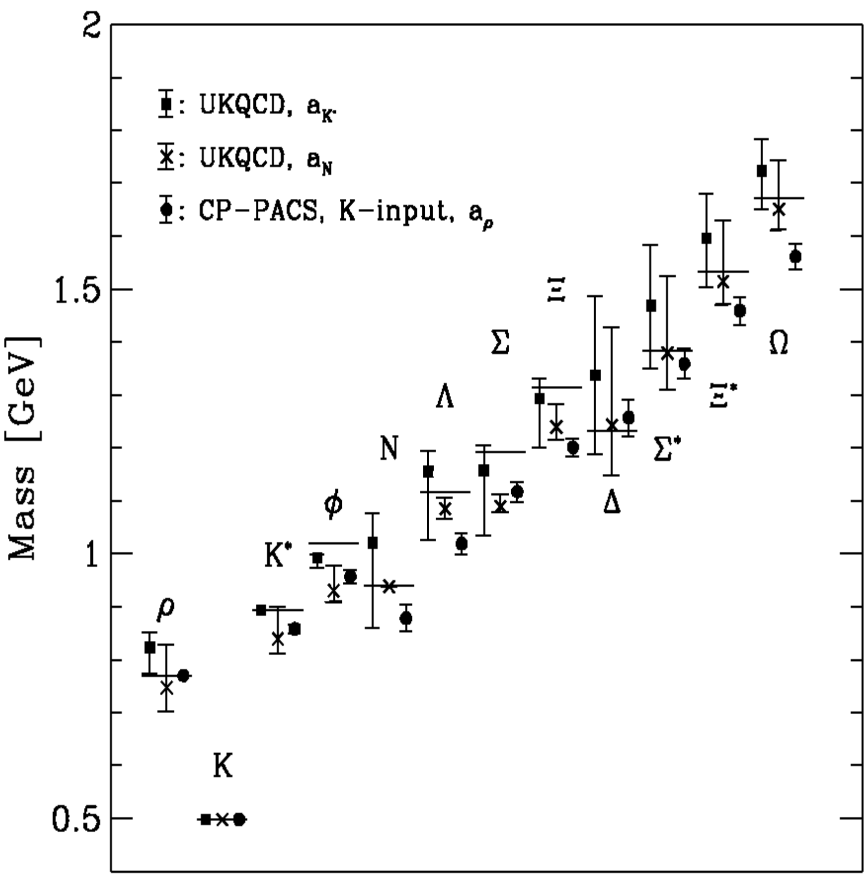
- For SU(N) baryons, form N quarks in a gauge invariant quark state
- What is the area law behavior? Test two ansatze: Y-law and Δ-law
  - Y-law: energy comes from flux tubes of shortest length join quarks. Area looks 3-bladed (N=3) joining at center. Looks like a Y. Length of flux tubes L_Y
  - Δ-law: energy composed of surfaces among all quark line pairs. Looks a delta. Length L_Δ

$$V_{Nq} = \frac{N}{2}V_0 - \frac{1}{N-1}\sum_{j<k} V_{jk} + \sigma\begin{Bmatrix} L_Y \\ \dfrac{1}{N-1}L_\Delta \end{Bmatrix}$$

- Data consistent with Δ-law – sum of 2 body quark potentials. Similar result for N=4
- Simonov argues *impossible*: field strength depleted near Y junction – lowers potential
- Should check using adjoint sources!

Alexandrou, DeForcrand Tsapalis, 2001; Kuzmenko, Simonov, 2002

Hadron Spectrum – Benchmark of Lattice QCD



UKQCD, 99

- Spectrum of lowest lying states is the benchmark of LQCD
- Most extensively pursued lattice calculation
- Quenched spectrum agrees with experiment to 10%
- Inconsistency in meson sector apparently resolved in full QCD
- Systematic uncertainties:
  - *Finite volume*: $V \to \infty$
  - *Continuum extrapolation*: $a \to 0$
  - *Chiral extrapolations:* $M_{PS} \to M_\pi$

- Quenching effects – to some degree controllable/understandable???

Problem of Chiral Symmetry

- Naive lattice discretization of free Dirac operator

$$\sum_\mu \gamma_\mu \partial_\mu \to \frac{1}{2a}\sum_\mu \gamma_\mu \left[\delta_{x+a\hat{\mu},y} - \delta_{x-a\hat{\mu},y}\right]$$

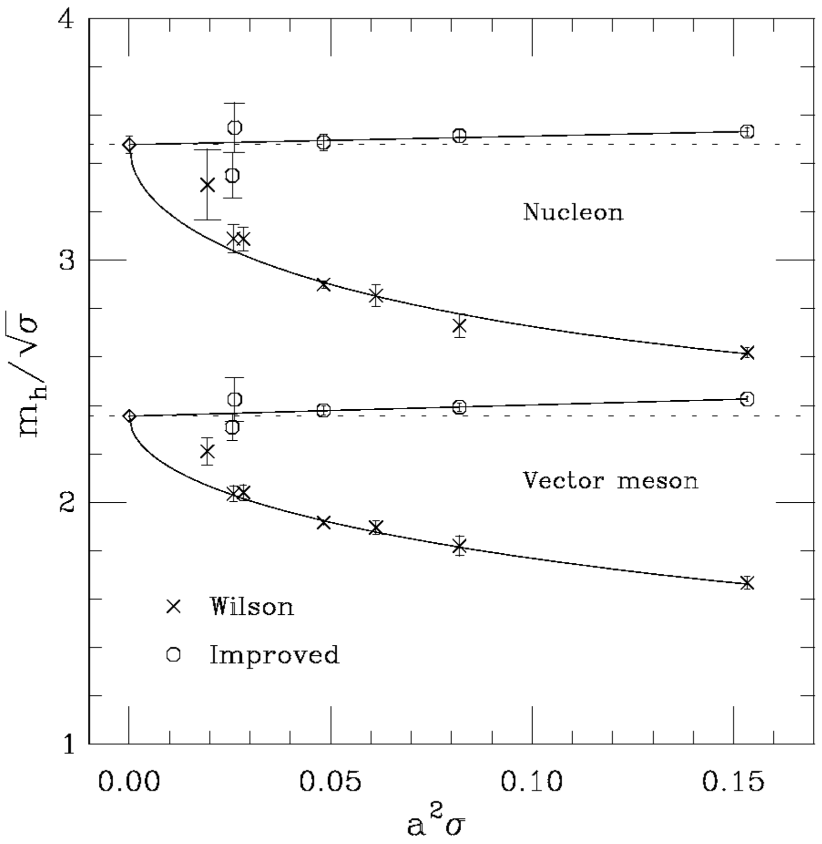
- In momentum space

$$G^{-1}(p) = \frac{i}{a}\sum_\mu \gamma_\mu \sin\left(ap_\mu\right) = i\sum_\mu \gamma_\mu p_\mu \left[1 + O\left(\left(ap_\mu\right)^2\right)\right]$$

- Has additional zeros at all corners of the Brillouin zone, *e.g.* $a * p_\mu = 0, \ldots, \pi$ – infamous doubling problem. Can *lift doublers* – add a Laplacian term that breaks chiral symmetry.
- Nielson-Ninomia no-go theorem – cannot avoid both doubling and chiral symmetry breaking with a local, hermitian action analytic in gauge fields. *Major theoretical problem.*
- Problem has been solved with recent advent of *chiral fermion actions* (e.g., Domain-Wall fermions). Crucial for matrix elements. Needed for spectroscopy??

Scaling – Continuum Limit

$m_{pseudoscalar} / m_{vector} = 0.7$



- Renormalization theory tells us that breaking a symmetry leads to induced quantum terms in an action.
- Wilson fermion action has O(a) scaling from breaking chiral symmetry.
- Can rigorously add a dimension 5 operator (hyper-fine term) to improve scaling from O(a) to $O(a^2)$
- Scaling violations dramatically reduced – mostly from improving chiral symmetry.

Edwards, Heller, Klassen, 97

Quenched Pathologies in Hadron Spectrum

- How well is QCD described by an effective chiral theory of interacting particles (e.g., pions in chiral dynamics)?
- Suppressing fermion determinant leads to well known pathologies as studied in chiral pertubation theory (Bernard, Golterman, Sharpe)
- Missing vacuum contributions to disconnected piece of singlet correlator

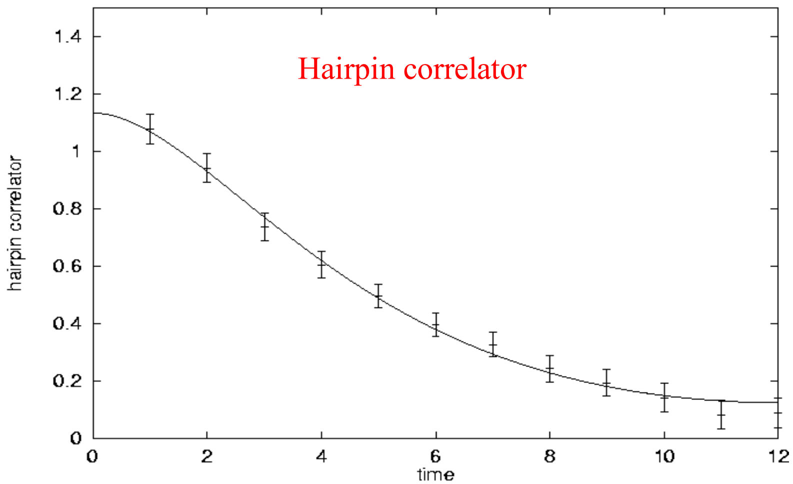
$$\left\langle \overline{\psi}(x)\gamma_5\psi(x)\overline{\psi}(y)\gamma_5\psi(y)\right\rangle = \left\langle \mathrm{Tr}\left[\gamma_5 G(x,y)\gamma_5 G(y,x)\right]_{c,s}\right\rangle$$
$$-N_f\left\langle \mathrm{Tr}\left[\gamma_5 G(x,x)\right]_{c,s}\mathrm{Tr}\left[\gamma_5 G(y,y)\right]_{c,s}\right\rangle$$

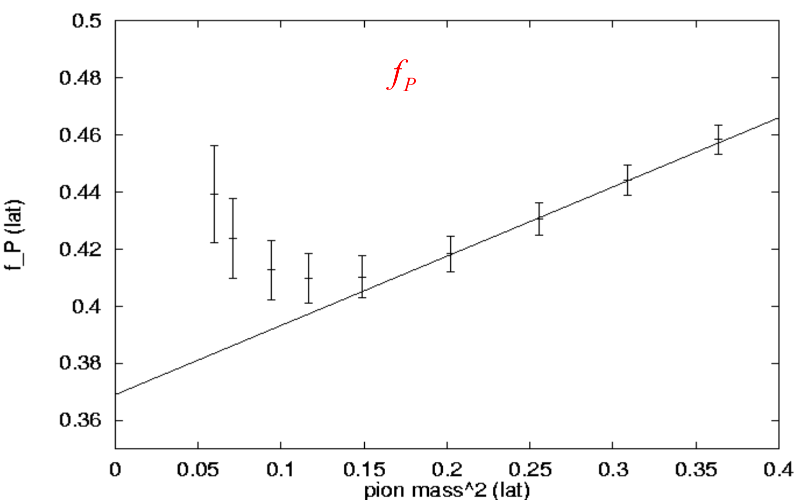
- Manifested in η′ propagator missing vacuum contributions. E.g.,

$$\left\langle \mathrm{Tr}\,\gamma_5 G(x,x)\,\mathrm{Tr}\,\gamma_5 G(0,0)\right\rangle \to f_P \frac{1}{p^2+m_\pi^{\ 2}}\, m_0^{\ 2}\, \frac{1}{p^2+m_\pi^{\ 2}} f_P + \ldots$$

- New divergences arise. One idea is to incorporate knowledge of divergences in calculations and then extract useful information

Anomalous Chiral Behavior





- Compute η′ mass insertion from behavior in QχPT
- Hairpin correlator fit holding m_π fixed - well described by simple mass insertion

$$\langle \mathrm{Tr}\,\gamma_5 G(x,x)\,\mathrm{Tr}\,\gamma_5 G(0,0)\rangle$$

$$\rightarrow f_P \frac{1}{p^2+m_\pi^2} m_0^2 \frac{1}{p^2+m_\pi^2} f_P$$

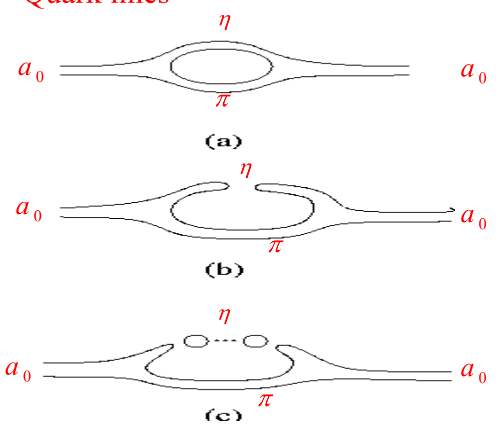
$$f_P^{quenched} = \left(\frac{1}{m_\pi^2}\right)^\delta \tilde{f}_P$$

- f_P shows diverging term. Overall δ=0.059(15)
- m_0 = 680(30) MeV , χPT gives 850MeV. Still O(a) errors

Bardeen, Duncan, Eichten, Thacker, 2000

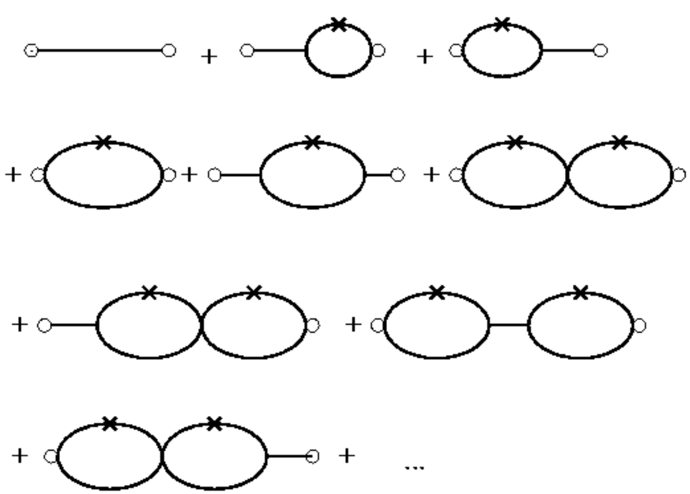
"Decay" in Quenched Approximation

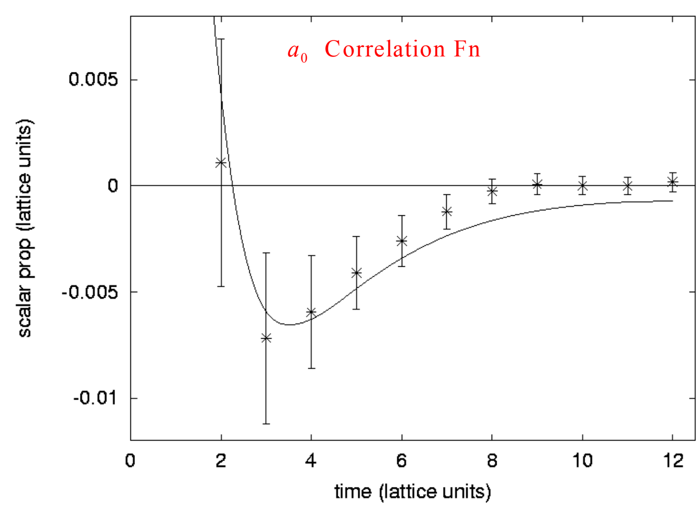
Quark lines



- Dramatic behavior in Isotriplet scalar particle a_0 — η-π intermediate state
- Construct a_0 propagator from chiral lagrangian including couplings between η-π states and rescattering states which can be resummed
- Lightest a_0 propagator fairly well described by 1-loop resummed bubble term with η mass insertion fixed
- Find mass $a_0 = 1.34(9)$ GeV. Still O(a) errors

χPT Scalar and multiple bubble graphs

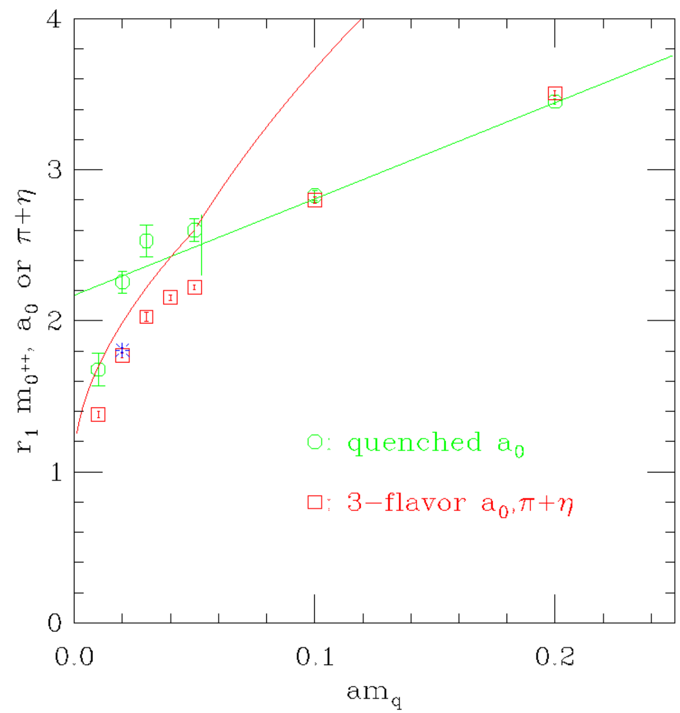




Bardeen, Duncan, Eichten, Thacker, 2000

Decay in Full QCD

- MILC: evidence of (S-wave) decay in $a_0 \rightarrow \eta\pi$ in a N_f=2+1 calculation
- 3-flavor mass follows quenched then drops below. *Decay* not computed
- What is a decay? *Subtle*
  - Most straightforward way is for mass exactly at threshold
  - Compute all 2-point correlators $C_{H\text{-}H'}$ for $H=a_0$, $\eta\pi$
  - For $m(a_0) = m(\eta\pi)$, can compute transition amplitude $< a_0 | \eta\pi >$ for large time separations from ratios of $C_{H\text{-}H'}$

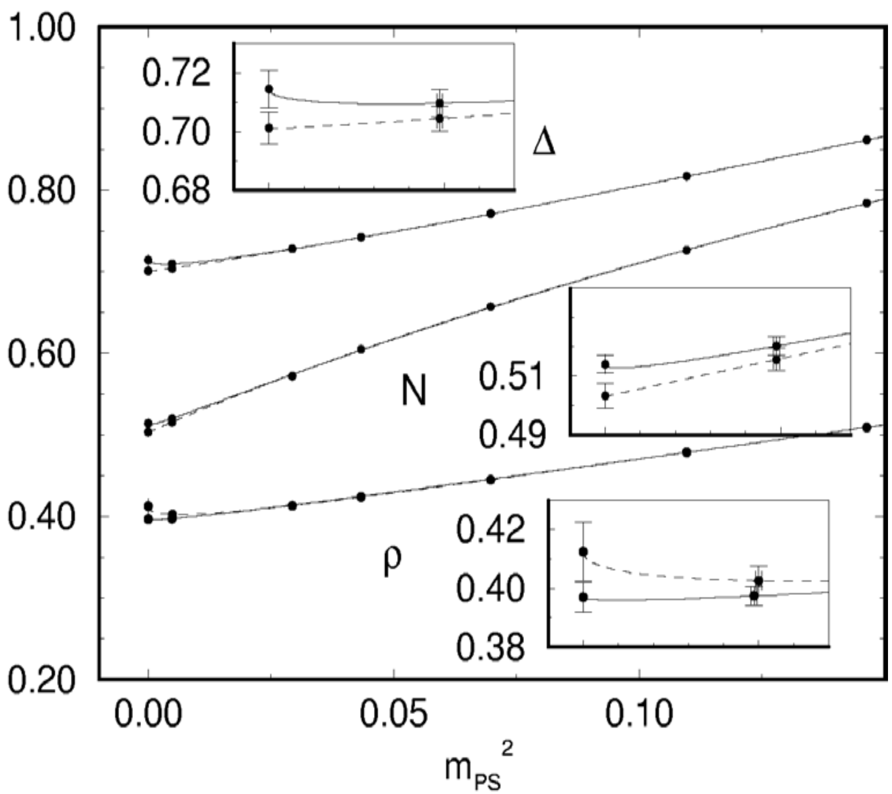


Quenched Spectroscopy

Quenched XPT predictions for pseudoscalar, vector mesons, and decuplet baryons

$$m_{PS,12}^2 = A(m_1+m_2)\left\{1-\delta\left[\ln\left(2Am_1/\Lambda_X^2\right)\right]+m_2/(m_2-m_1)\ln(m_2/m_1)\right\}+B(m_1+m_2)^2+O(m^3)$$

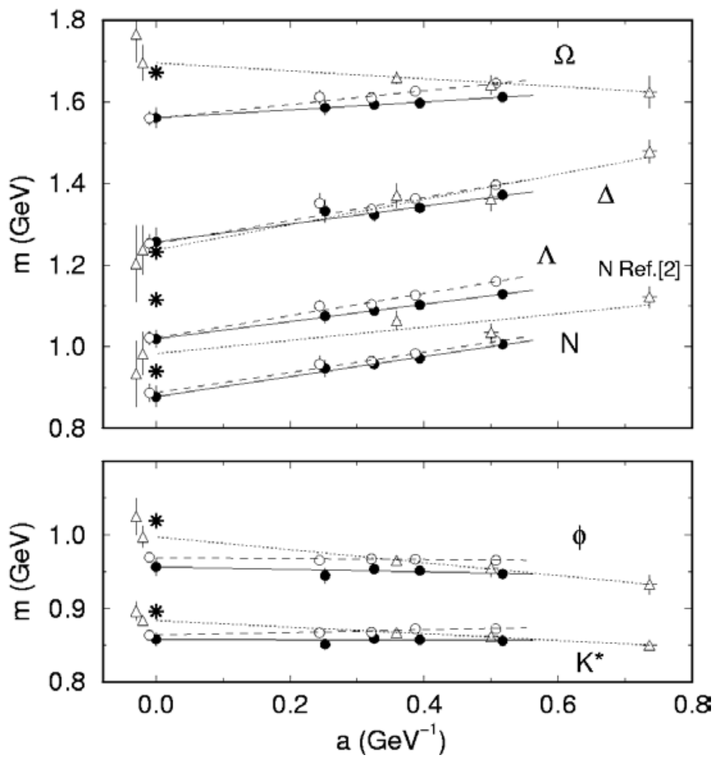
$$m_H(m_{PS}) = m_0 + C_{1/2}m_{PS} + C_1 m_{PS}^2 + C_{3/2}m_{PS}^3\ , \qquad C_{1/2} \propto \delta$$

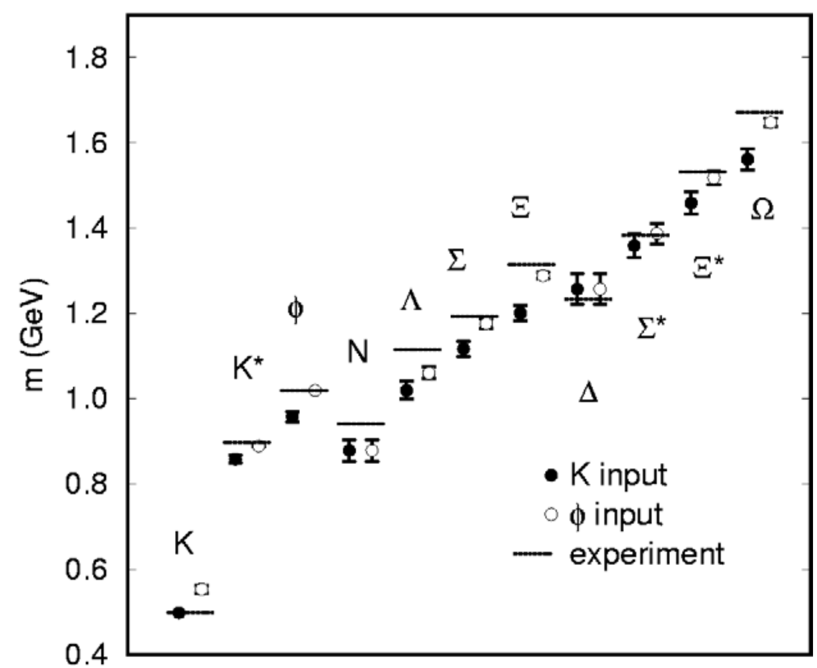


- Large Wilson fermion calculation by CPPACS (Tsukuba) (99)
- Some evidence for quenching effects: more clearly seen in pseudoscalar channel
- Masses computed at 4 lattice spacings. Lattice sizes ranging up to 64^3x112 for a 3.2fm box.

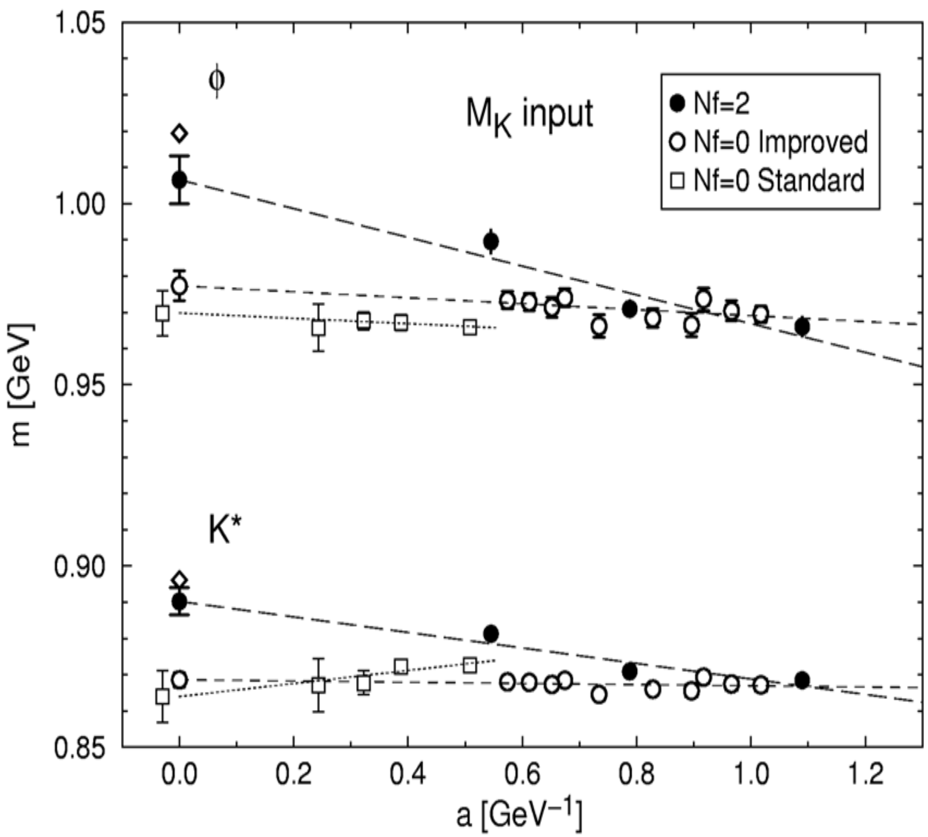
Mass Predictions from Quenched Spectroscopy

- After chiral extrapolation, another extrapolation to continuum limit
- Fix scale at each coupling from experimental π, ρ, and K (or ϕ) masses
- Quenched spectrum systematically deviates from experiment. Typically 5% too small.
- Calculation ~ 50 Gigaflop-year





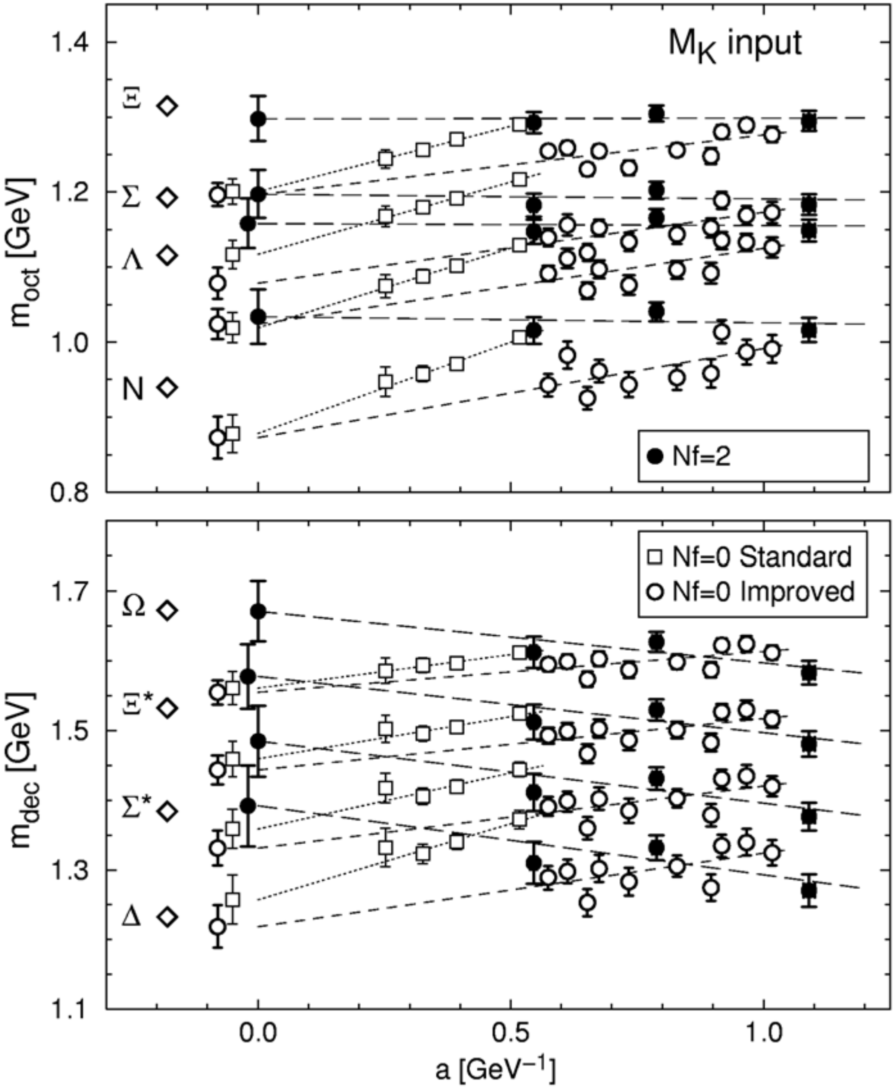
Meson Mass Predictions from Dynamical Fermions



CPPACS, 2000

- CPPACS: Nf=2 dynamical calculation. 4 quark masses at 3 couplings. Box sizes about 2.5fm
- Consistent results between original quenched calculations. Systematic deviation from experiment.
- Nf=2 calculation agrees within 1% of experiment – sea quark effects important.
- Increased hyperfine splitting consistent with suppressed spin-spin coupling in quenched from faster running of coupling.
- Calculation ~ 1 Teraflop-year

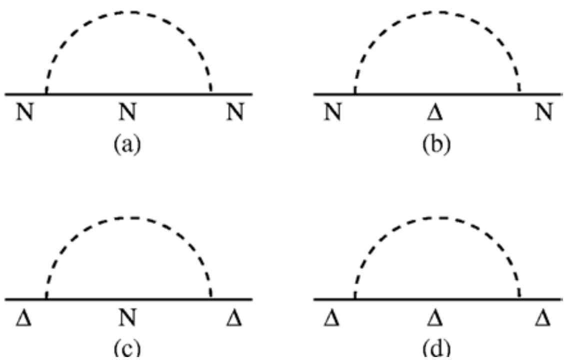
Baryon Mass Predictions from Dynamical Fermions



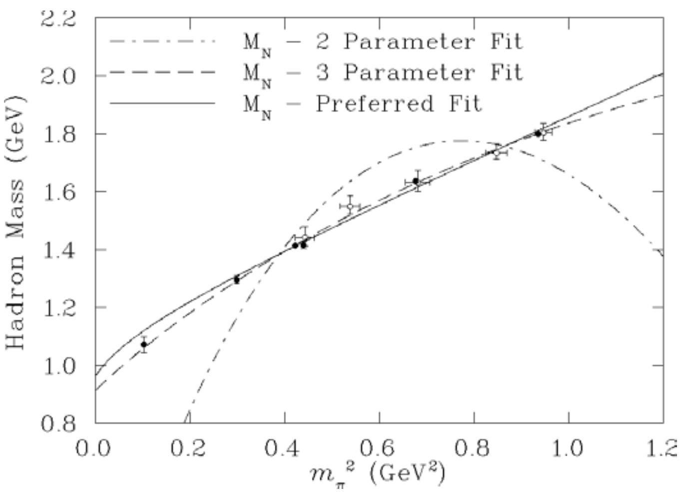
- CPPACS: Nf=2 dynamical sea quark effects not as apparent
- N and Δ mass high, but other masses consistent.
- Box sizes about 2.5fm. Worry finite-volume effects large.
- Octet and decuplet chiral extrapolations have many parameters.

Improved Chiral Extrapolations

Self-energy contributions



$M_N = \alpha + \beta m_\pi^2 + \gamma m_\pi^3$



Leinweber, Thomas, Tsushima, Wright, 99

- Adelaide group – extensively studying higher order chiral PT effects on hadronic quantities
- Basic upshot – naïve chiral extrapolations just too naïve!!
- Incorporate leading non-analytic behavior from heavy baryon χPT: B→B′ π→B for B=N,Δ

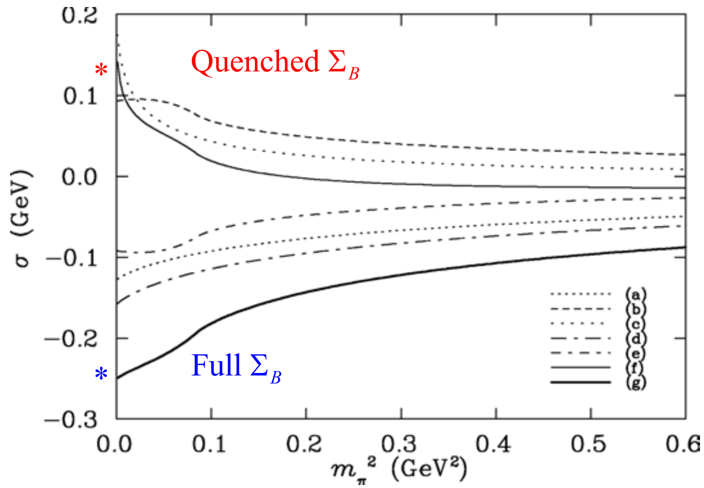
$$M_B = \alpha_B + \beta_B + \Sigma_B(m_\pi, \Lambda)$$

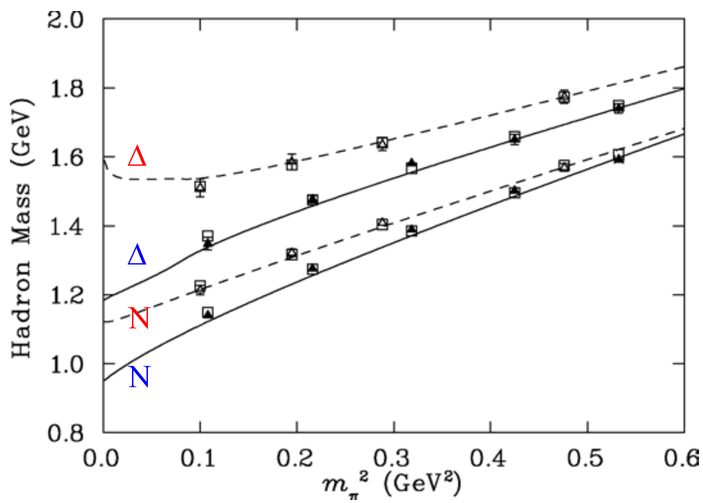
- Leading order:

$$M_B = M_B^0 + c_1^B m_\pi + c_2^B m_\pi^2 + c_3^B m_\pi^3 + c_4^B m_\pi^4 + c_{4L}^B m_\pi^4 \log(m_\pi) + \dots$$

- Bad approx at moderate m_π. Use form-factor

Comparing Quenched and Full QCD Chiral Extrapolations





Young, Leinweber, Thomas, Wright, 2001

- Compare MILC quenched and N_f=2+1 staggered spectrum:

$$M_B = \alpha_B + \beta_B + \Sigma_B(m_\pi, \Lambda)$$

- Note: Σ_B not 0 in chiral limit. Fit parameters for quenched and full. Here α is not chiral limit mass. In chiral limit, full N mass still near 1GeV

	α_N (GeV)	β_N	α_Δ (GeV)	β_Δ
Full	1.24(2)	0.92(5)	1.43(3)	0.75(8)
Quenched	1.23(2)	0.85(6)	1.45(4)	0.71(11)

- Supports claim dominant effects of quenching attributed to first order meson loop corrections

(Quenched) Electromagnetic Splittings

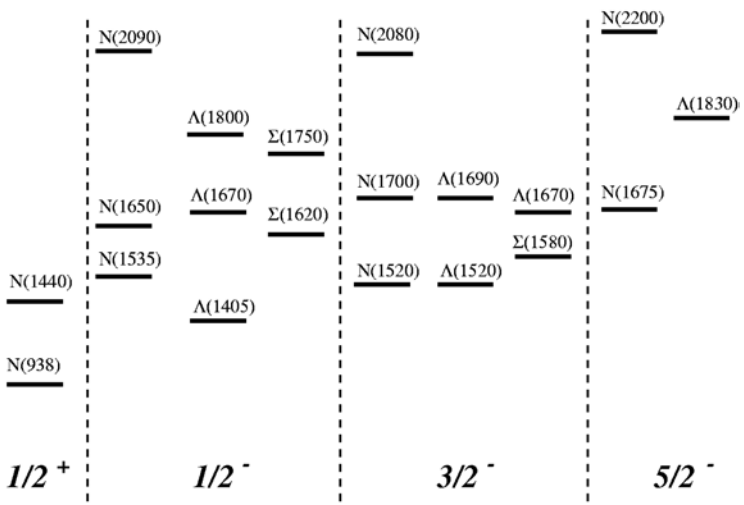
- Determination of N-P splitting long standing problem.
- Virtual photon effects mass splittings within isomultiplets comparable to up-down quark mass difference.
- Accurate computations of isospin splitting must include EM effects.
- A first generation calculation (quenched) including U(1) gauge fields.
  - Assign electric charges to quarks
  - Use χPT in both SU(3) and U(1). Scale electric charge large(r)
  - Estimate final volume and meson cloud effects from χPT
- Results surprisingly reasonable. Finite volume corrections large.

Level splitting	Raw lattice	Finite volume	Meson cloud	Total lattice	Physical splitting
N - P	2.83(56)	-0.75	-0.53	1.55(56)	1.293
$\Sigma^0 - \Sigma^+$	3.43(39)	-0.80	-0.16	2.47(39)	3.18(10)
$\Sigma^- - \Sigma^0$	4.04(36)	+0.86	-0.27	4.63(36)	4.88(10)
$\Sigma^+ + \Sigma^- - 2\Sigma^0$	0.61(19)	+1.66	-0.11	2.16(19)	1.70(15)
$\Xi^- - \Xi^0$	4.72(24)	+0.86	+0.10	5.68(24)	6.4(6)

MeV

Duncan, Eichten, Thacker, 96

Excited Baryons



- Describing N* spectrum gives vital clues about dynamics of QCD and hadronic physics
  - Role of excited glue
  - Quark-diquark picture
  - Quark interactions
- Open mysteries:
  - Nature of *Roper*?
  - Λ(1405) mass?
  - Missing resonances?
- History of lattice studies of excited baryons quite brief. Recent work using improved gauge and fermion actions

Lattice Representations

Continuum spin reducible under three irreducible ray representations of the cubic group

Rep.	Continuum spin reps
G_1	1/2, 7/2, …
H	3/2, 5/2, 7/2, …
G_2	5/2, 7/2, …

- As spin increases, baryon spin rep. occurs in multiple lattice representations.

Parity

Negative parity interpolation operators:

- Measure three local interpolating operators for the proton:

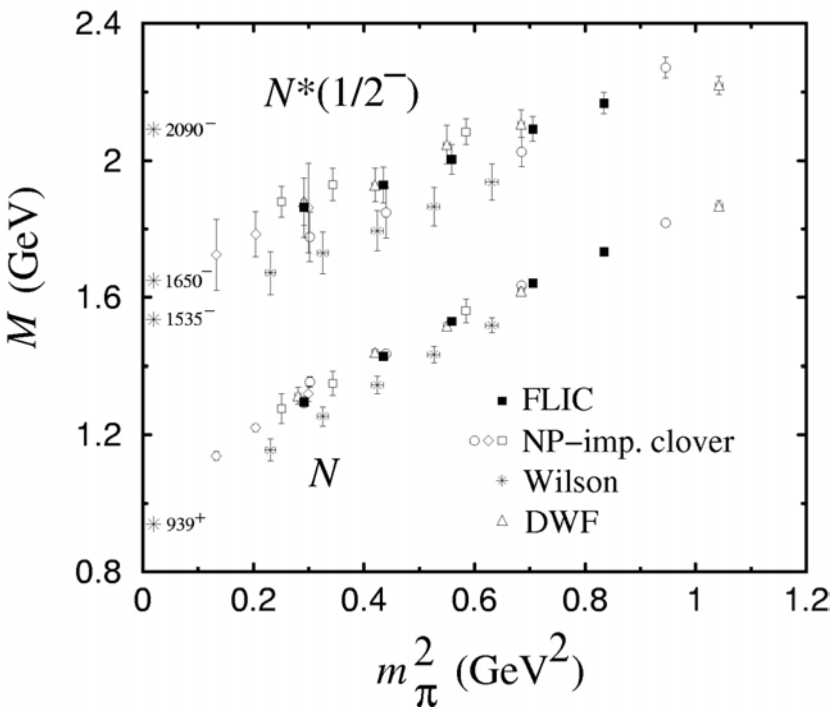
$$\left\{\begin{aligned} N_1^{1/2+} &= \varepsilon_{ijk}\left(u_i^T C\gamma_5 d_j\right)u_k \\ N_2^{1/2+} &= \varepsilon_{ijk}\left(u_i^T C d_j\right)\gamma_5 u_k \\ N_3^{1/2+} &= \varepsilon_{ijk}\left(u_i^T C\gamma_4\gamma_5 d_j\right)u_k \end{aligned}\right\}$$

- N_1 (N_2) connects upper (lower) spinor components in diquark piece - N_2 vanishes in NR limit
- $\Delta^{3/2,1/2} = \varepsilon_{ijk}(u_i^T C\gamma_\mu u_j)\, u_k$ - Spin projection
- *Quark model suggests a better interpolation operator for $N^{1/2-}$ have a covariant derivative in the valence quark*

- On lattice (anti)-periodic in time, have both positive and negative parity states. Fit proton correlation function at each end of lattice to obtain the respective masses.

How Crucial is Chiral Symmetry?

Quenched $N^{1/2+}$ and $N^{1/2-}$



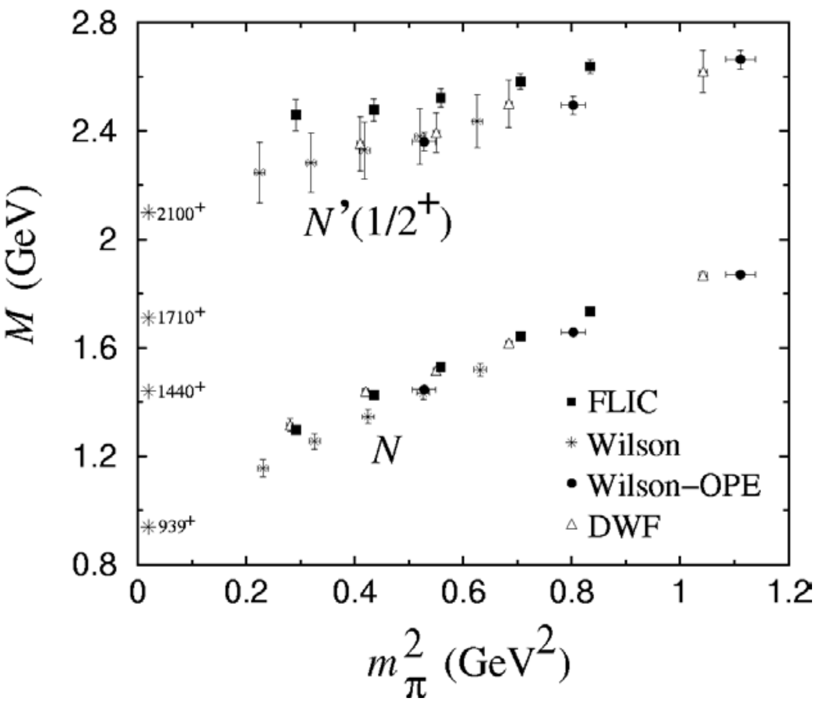
N_1 interpolating field

- If we had unbroken chiral summetry, $N^{1/2+}$ and $N^{1/2-}$ would be *degenerate*
- Chiral symmetry crucial: *Do we require "lattice" chiral symmetry?*
- *Apparently no* – compare Wilson with O(a) χ breaking terms; Non-Perturbatively imp. Clover with O(a^2) χ breaking terms; Domain Wall (almost) no χ breaking terms, but O(a^2) discretization errors.
- Some mixing of results with discretization errors here – similar mass splitting of 400 MeV for each action.
- In large N_c, mass splitting comes from l=1 to l=0 (S-P splitting) – reproduced by O(a) Wilson action.

Various Adelaide, Jlab, UKQCD, 2001

Roper

$N^{1/2+}$ and excitation



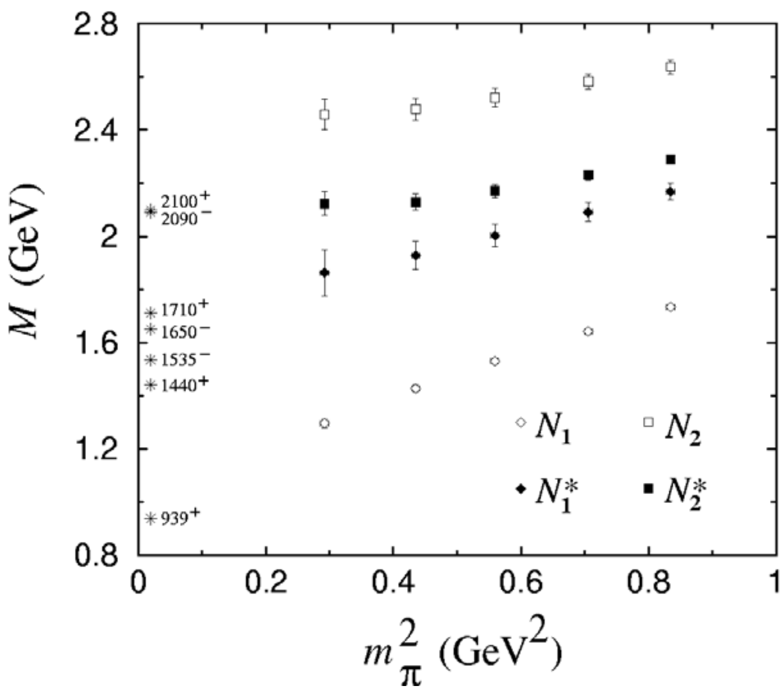
N_2 interpolating field

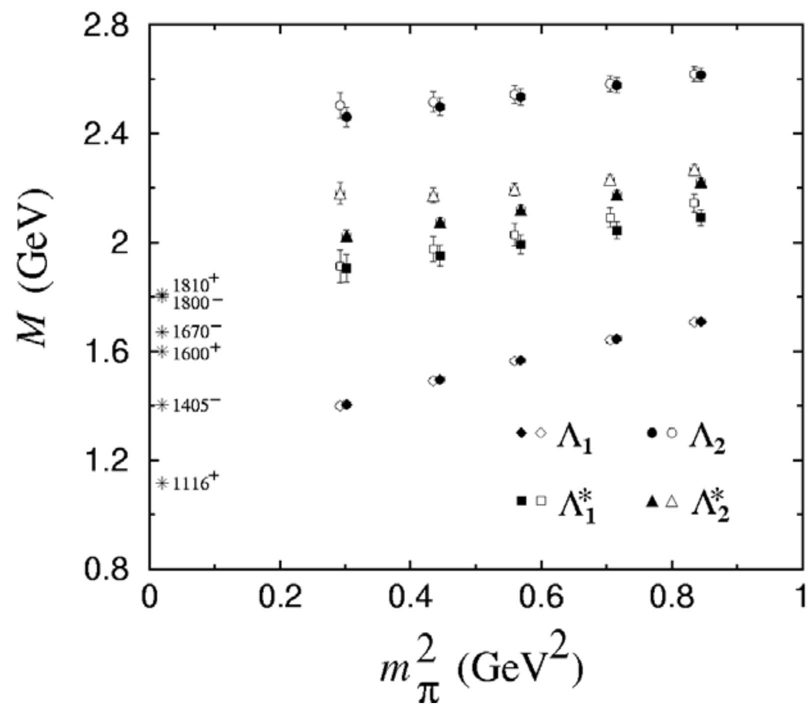
- Compare $N^{1/2+}$ with first (radially) excited state (Roper)
- Large mass splitting. Excited state higher than $N^{1/2-}$. Other channels similar.
- Possible (but unlikely) strong m_π dependence. More likely bad overlap of N_2 with excited state. Excited state masses notoriously difficult. *Need anisotropic lattices – greatly improves signal*
- *Concern*: small physical volume of 1.6fm for DWF and 2.0fm for others – can squeeze up excited state since larger in size

Various Adelaide, Jlab, UKQCD, 2001

Other Excited States

- In recent works, generically see too large splitting among pos. parity – excited state and too small in neg. parity. Quite possibly too small volume (2fm) – by quark model *l=1* twice as large as *l=0*.
- Splitting not as large in Λ channel

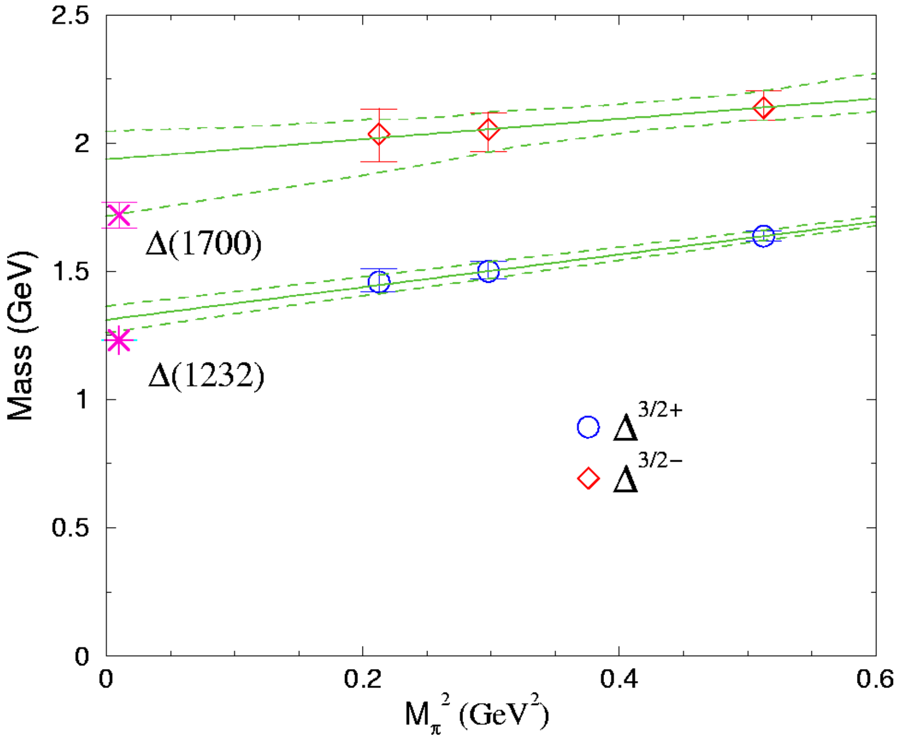




Adelaide, 2001

Delta

- I=3/2, J=3/2 Delta (2.1fm). Splitting probably also high – new chiral extrapolation??!!



Jlab/UKQCD, 2001

Exotics and Hybrids

- Exotics: big focus of JLab (and lattice group!)
  - Spin exotic mesons are J^{PC} states not accessible in quark model
  - Characterized by excited glue or perhaps four-quark states
- Several lattice calculations of heavy hybrid and exotic meson states
- Lattice calculations of light exotic meson states still first generation (*noisy*)!
  - Lightest 1^{-+} exotic roughly 2GeV
  - Considerably higher than experimental candidates 1.4, 1.6 GeV

- No baryon exotics!
- Baryon hybrids? Model questions.
  - Gluonic versions of baryons one of many states induced
  - Study of baryon potentials might provide good insight

Lattice Hadron Physics Collaboration (LHPC)

- JLab/MIT and 8 other universities. Over 20 senior physicists
- Four identified physics goals:
  - Nucleon structure
  - Spectroscopy – N^* , Hybrids, glueballs
  - Hadron-Hadron interaction
  - Fundamental aspects of QCD (e.g., mechanisms of confinement)
- Computing resources:
  - Small clusters of workstations and a QCDSP
  - Currently awaiting purchase of 200 node box dual Pentium 4 cluster – expect to sustain > 200 Gigaflop/sec

SciDAC

Scientific Discovery through Advanced Computing

- DOE program supporting national effort by US lattice community to develop software and hardware infrastructure for next generation computers
- Physics efforts centered around JLab (hadron physics), FNAL (weak matrix elements), BNL (high temperature)
- Current funding only supports software developers – around $2M to Jlab over 3 years
- Currently awaiting purchase of 200 node box dual Pentium 4 cluster – expect to sustain > 200 Gigaflop/sec
- Goal is a coordinated three 10 Terflop/sec computing facilities for national community by 2005.

- US lattice resources greatly lagging other countries!

Conclusions

- First generation lattice calculations of excited baryon spectroscopy
- Precise calculations commmensurate with experimental program require:
  - Measure of large number of correlators
  - Sufficiently light pions to resolve pion cloud
  - Large physical volumes
  - Continuum extrapolation
  - Full QCD
- State-of-the-art calculations require ~ 100 Gigaflop-year in Quenched QCD and ~ 1 Teraflop-year in full QCD.
  - Required resources not available to US lattice community
  - Focus of interest on weak-matrix element calculations