Quark-Hadron Duality

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- What is Duality?
- Examples & Applications
- Search for the Origins of Duality

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What is Duality?

Quark-Hadron Duality implies that in a certain kinematic regime, properly averaged hadronic observables can be described by perturbative QCD.

We observe duality in many reactions:
$e^+e^- \rightarrow$ hadrons, semileptonic decays of heavy quarks, hadronic decay of τ, inclusive inelastic electron scattering (Bloom-Gilman duality), . . .

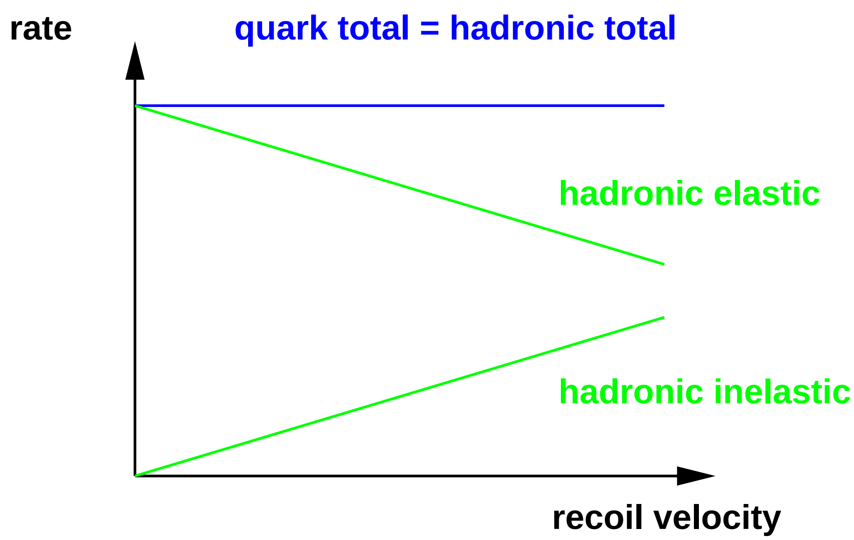
Where does duality hold?

Duality has interesting and useful applications! New duality experiments have been completed at JLab, and there will be a large duality program at CEBAF @ 12 GeV.

Semileptonic decays with infinitely heavy quark masses:

$b \to c\, l\, \bar{\nu}_l$ **quark process,**

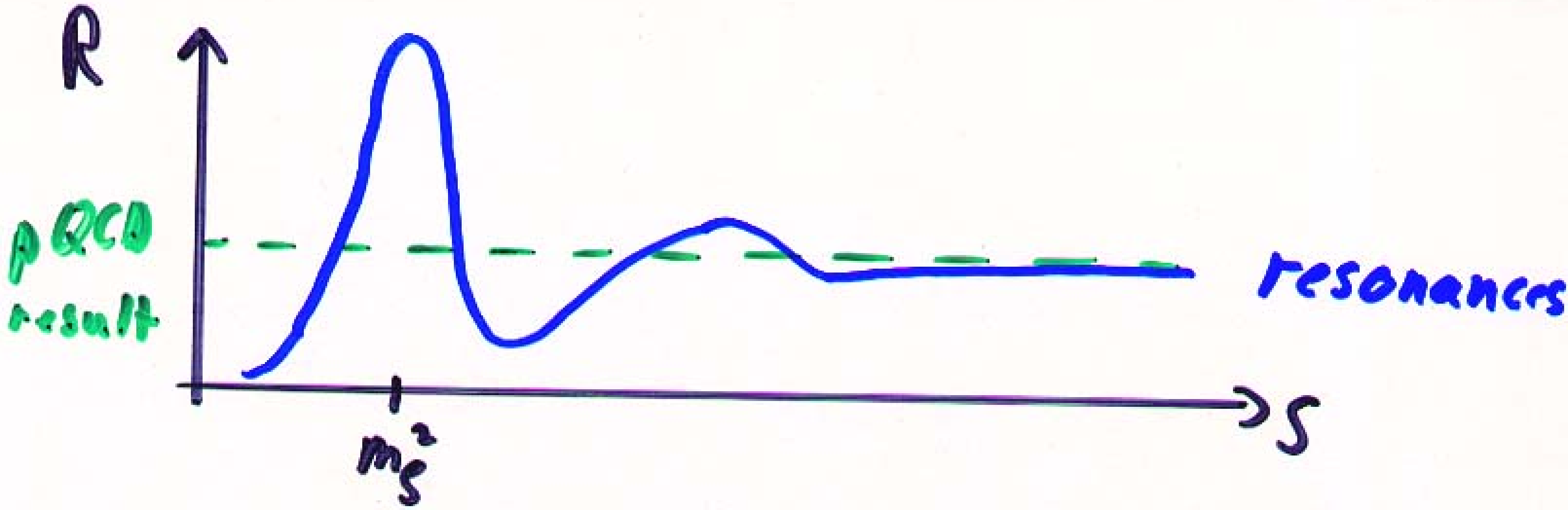
$\bar{B} \to X_c\, l\, \bar{\nu}_l$ **hadronic process** X_c: **any charmed meson,** $X_c = D, D^*, D_1, \ldots$

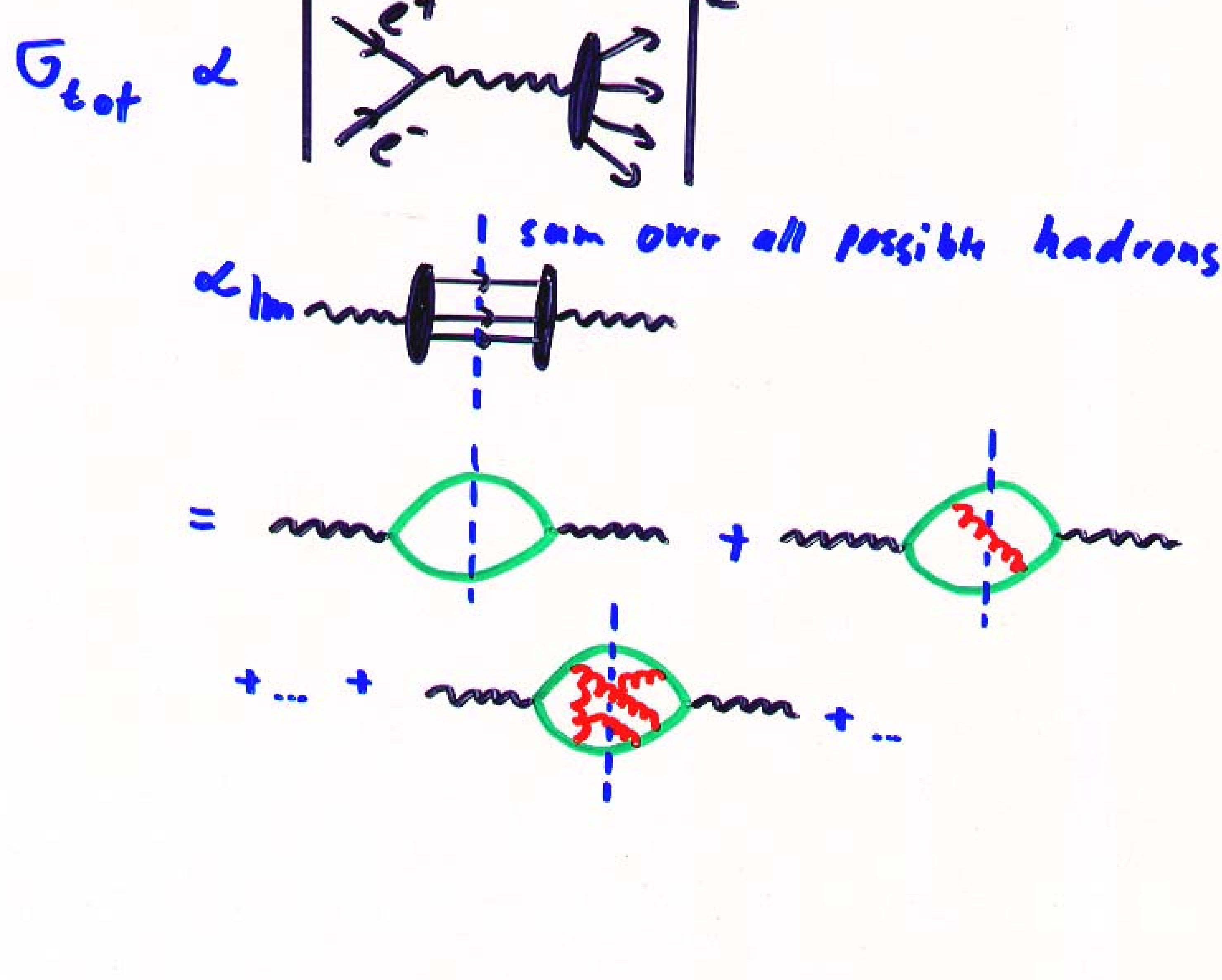


Isgur PLB 448, 111 (1999), Isgur & Wise, PRD 43, 819 (1991)

Goal: determine V_{cb} and V_{ub} from hadronic data

For finite masses, $1/M_q$ or $1/M_q^2$ corrections appear depending on the observable and the averaging process. See e.g. Lebed & Uraltsev, PRD 62, 094011 (2000)


R
pQCD
result
resonances
s
m_ρ^2


$\sigma_{tot} \propto$
e^+
e^-
2
sum over all possible hadrons
$\propto \mathrm{Im}$
=
+
+ ... +
+ ...

$e^+e^- \rightarrow hadrons$

classic example for duality, E. C. Poggio, H. R. Quinn, S. Weinberg, PRD 13, 1958 (1976)

$$R = \frac{\sigma^{tot}(e^+e^- \rightarrow hadrons)}{\sigma^{tot}(e^+e^- \rightarrow \mu^+\mu^-)}$$

At large s, we find perfect duality: $R = N_c \sum_q e_q^2$.

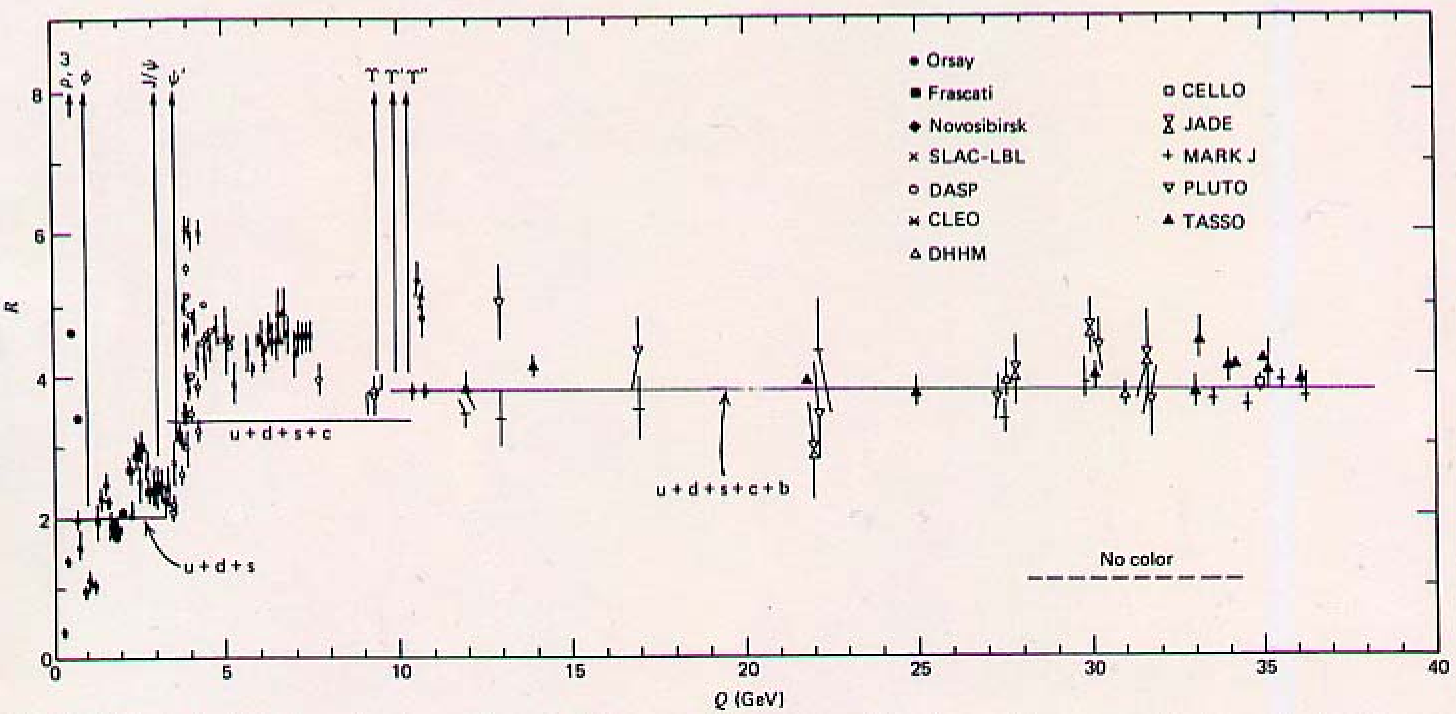


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

Halzen Martin

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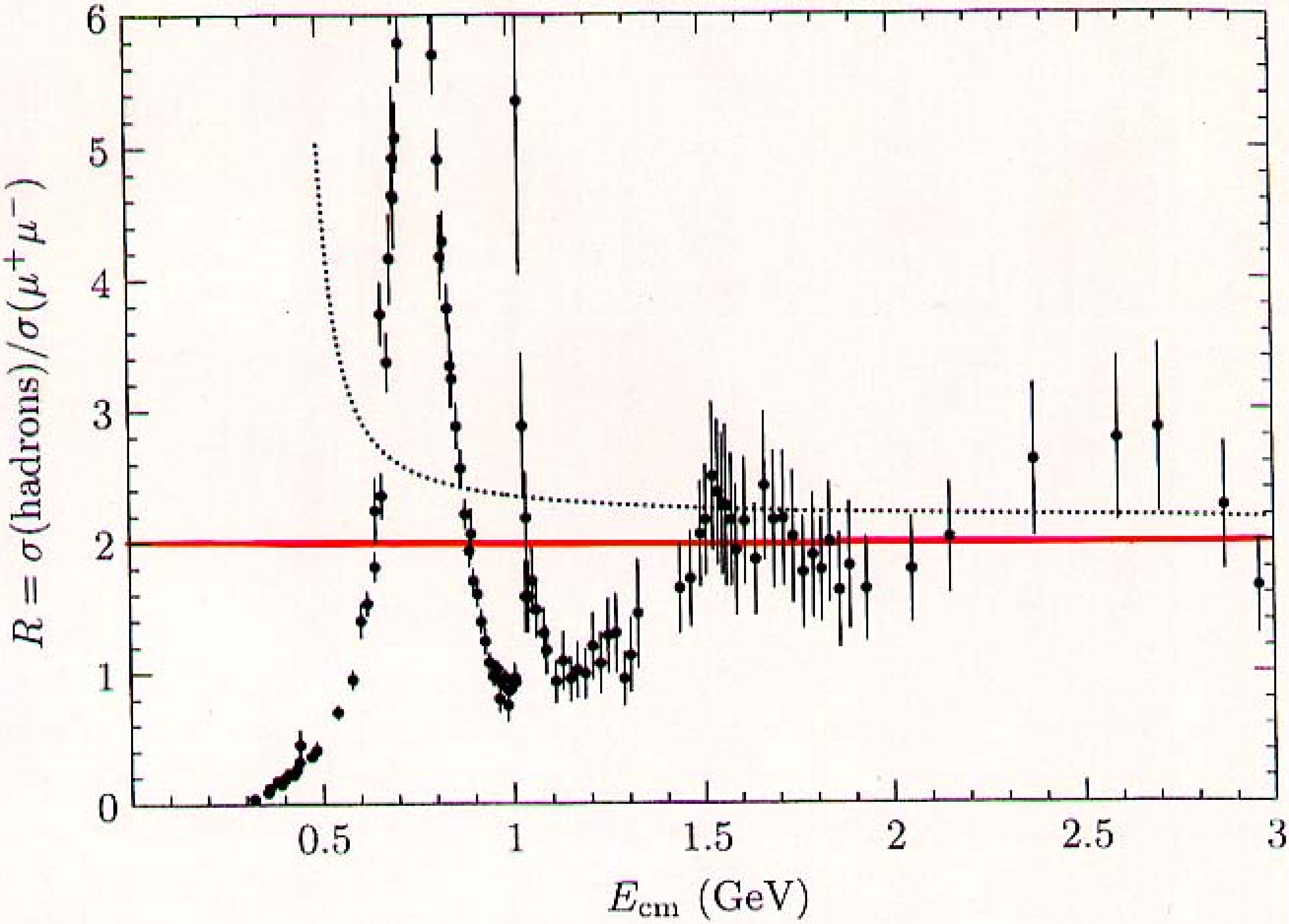


Figure 18.8. Experimental measurements of the total cross section for the reaction $e^+e^- \to$ hadrons at energies below 3 GeV, compared to the prediction of perturbative QCD for 3 quark flavors. The data are taken from the compilation of M. Swartz, *Phys. Rev.* **D53**, 5268 (1996). Complete references to the various results are given there.

functions, which reflect the form of the proton wavefunction and are determined by soft QCD dynamics. However, we saw in Section 17.5 that effects of QCD perturbation theory cause the parton distributions to change their form as a function of the momentum transfer Q^2. We will now show that much of this picture can be reconstructed from our new viewpoint, using the operator product expansion.

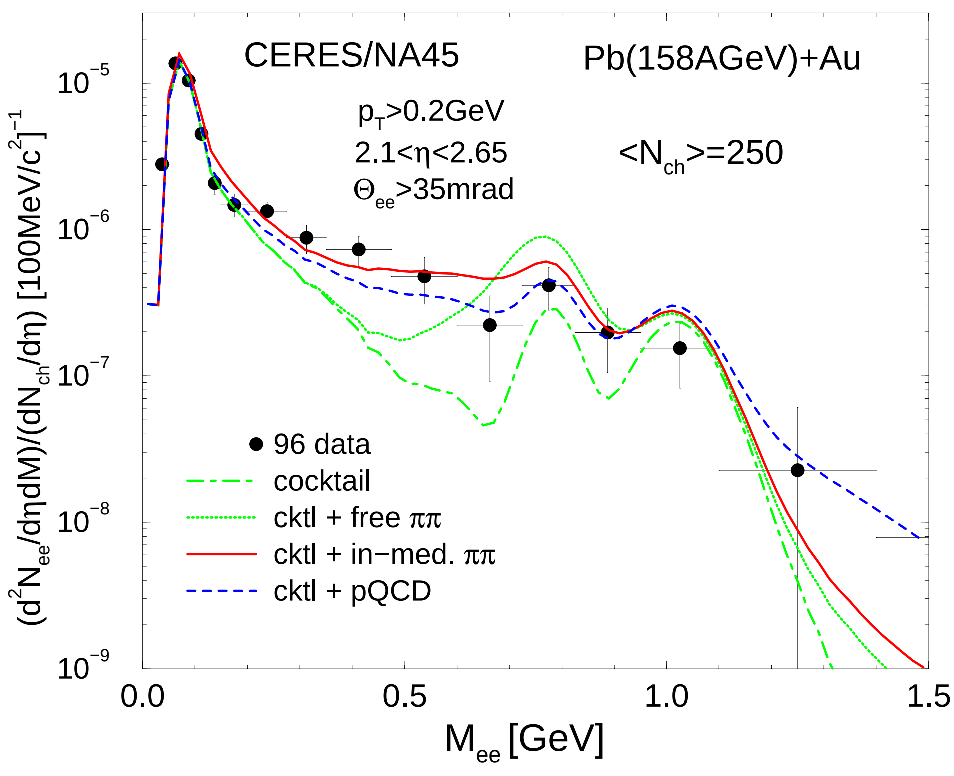
In the previous section, we derived the OPE relations for the e^+e^- annihilation cross section in three steps. First, we used the optical theorem to relate this cross section to a matrix element of a product of currents. Second, we applied the operator product expansion to the product of currents. Unfortunately, this expansion could be used only in an unphysical kinematic region.

Duality in heavy ion reactions:

Dilepton production: inverse reaction to $e^+e^- \rightarrow hadrons$

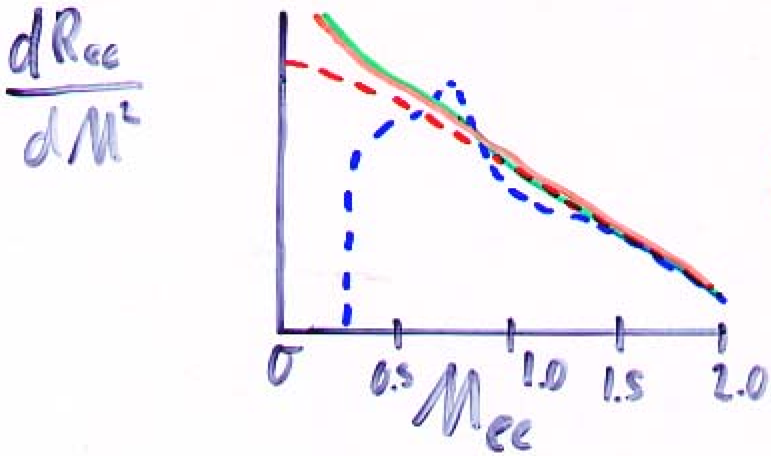
duality threshold in vacuum: $M_{l\bar{l}} \approx 1.5 GeV$

duality threshold in medium: $M_{l\bar{l}} \approx 0.5 GeV$



R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000)

Dilepton Production Rates



free hadron gas

in-medium hadron gas

free quark-gluon plasma

in-medium quark-gluon plasma

R. Rapp, hep-ph/0201101

"Overall emissivities from the hadronic and quark gluon phase look surprisingly similar... which has been interpreted as an in-medium reduction of the quark-hadron duality scale"

Kinematics & Observables

e p $\longrightarrow$ e X

Inelastic electron scattering from a nucleon:

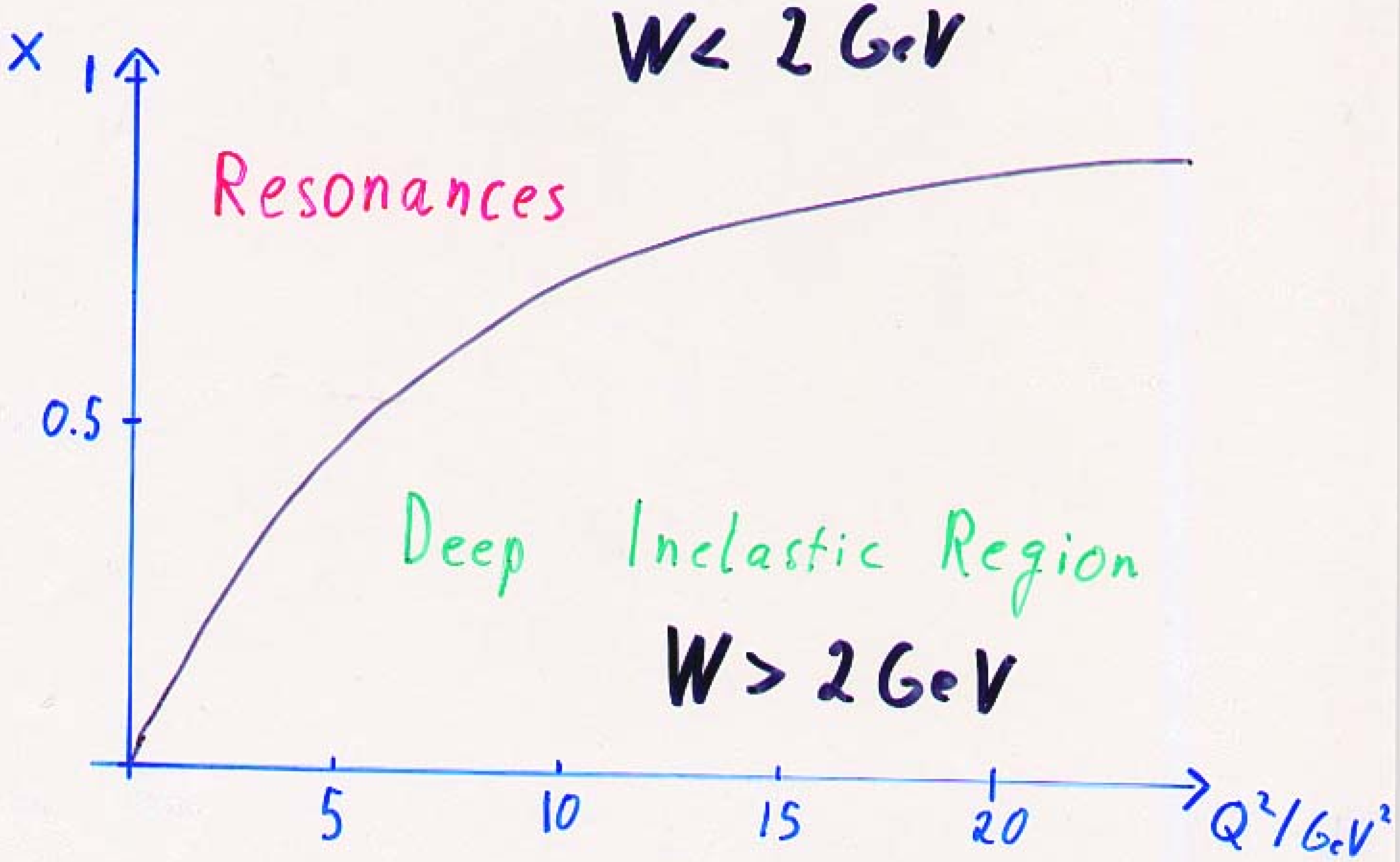
$$\frac{d\sigma}{d\Omega dE_f} = \sigma_{\text{Mott}} \left(W_2 + 2\, W_1 \tan^2 \frac{\vartheta_e}{2} \right)$$

The structure functions W_1, W_2 depend on the transferred energy ν and the negative transferred four-momentum, $Q^2 = \vec{q}^{\,2} - \nu^2$.

In the Bjorken limit, $Q^2 \to \infty, \dfrac{\nu}{Q^2} = \text{const.}$, we have scaling:

$\nu W_2(\nu, Q^2) \to F_2(x)$ [**actually,** $F_2(x, \log Q^2)$]

where $x_{Bj} = \dfrac{Q^2}{2 M_N \nu}$.



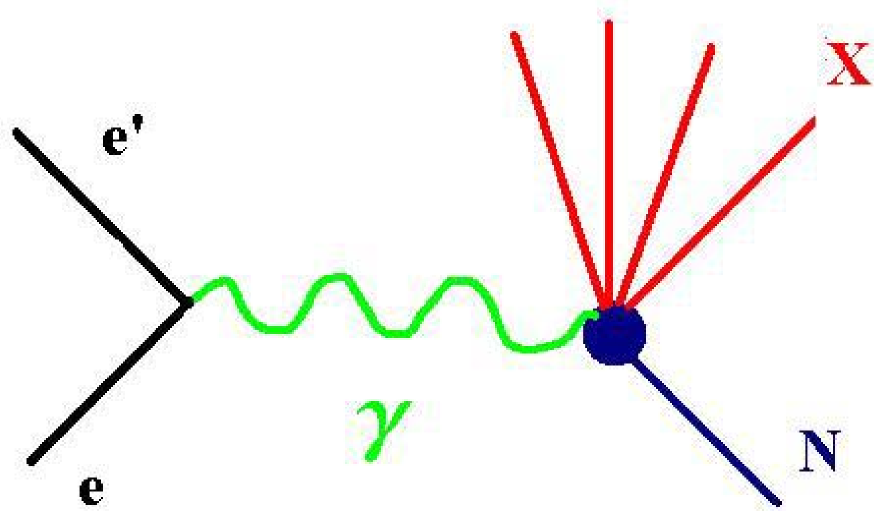
$$\sigma_{Mott} \propto \frac{1}{Q^4}$$

$$W^2 = M^2 + 2M\nu - Q^2$$
$$= M^2 + Q^2\,\frac{1-x}{x}$$

$$x = \frac{Q^2}{2M\nu}$$

Where does Duality hold?

Consider $eN \to e'X$



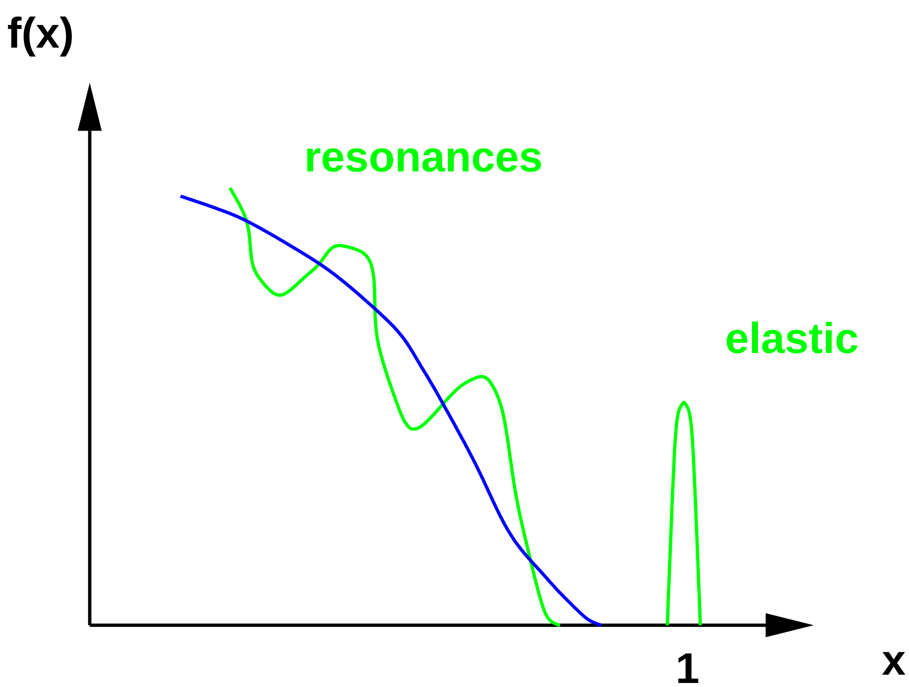
It must hold in the scaling region, as perturbative QCD = full QCD in this case.

It must break down at very low Q^2, $Q^2 \to 0$, where QCD is strong.

Interesting question: How does the transition from high Q^2 to low Q^2 take place?

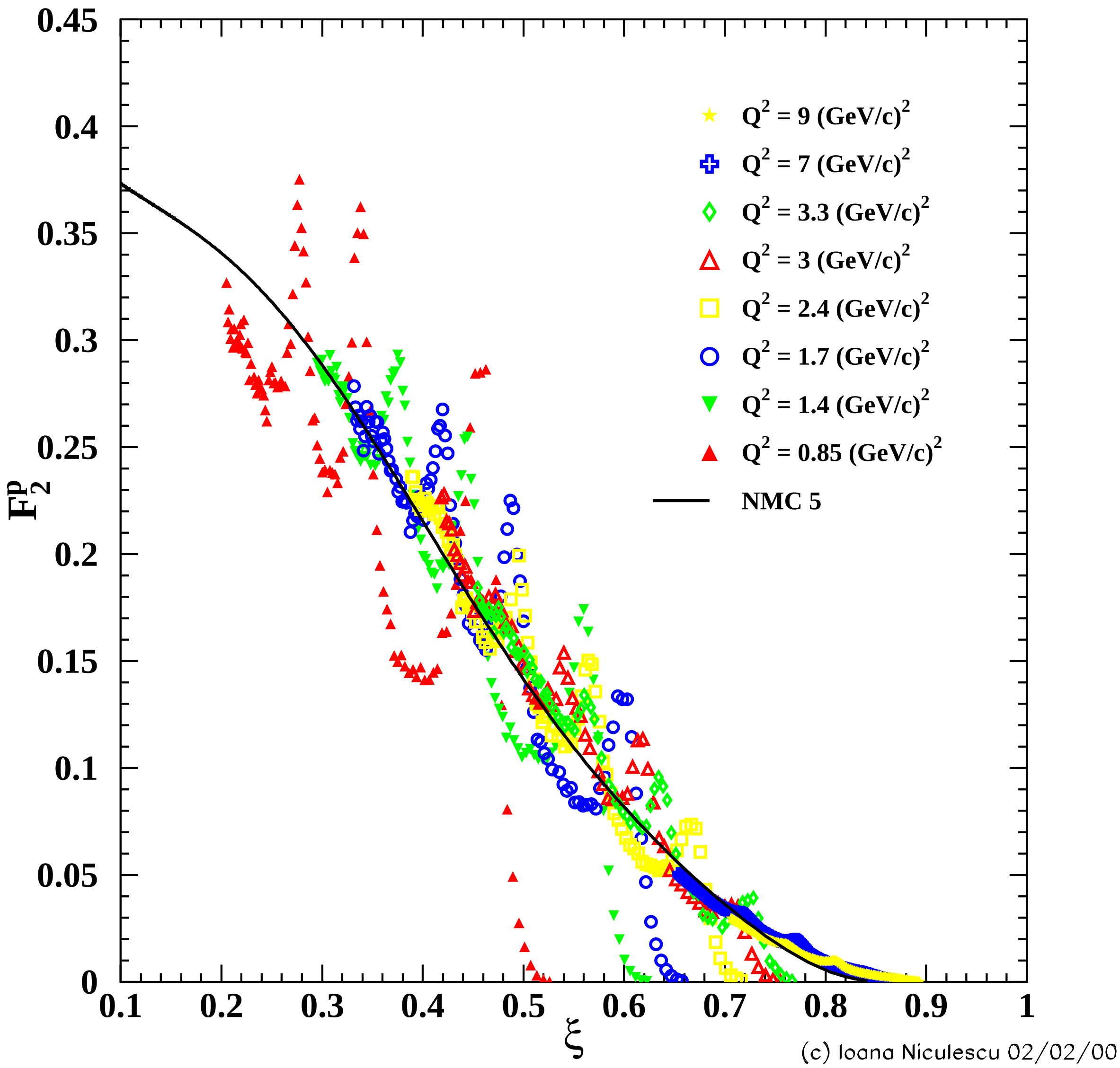
Where does Duality hold in inelastic inclusive electron scattering?

Answer from experiment: Duality works well down to low $Q^2 \approx 0.5$ GeV^2!



New Data from Jefferson Lab

Data from I. Niculescu et al., PRL 85 (2000)



scaling variable:

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}} = \frac{1}{M}(\sqrt{\nu^2 + Q^2} - \nu)$$

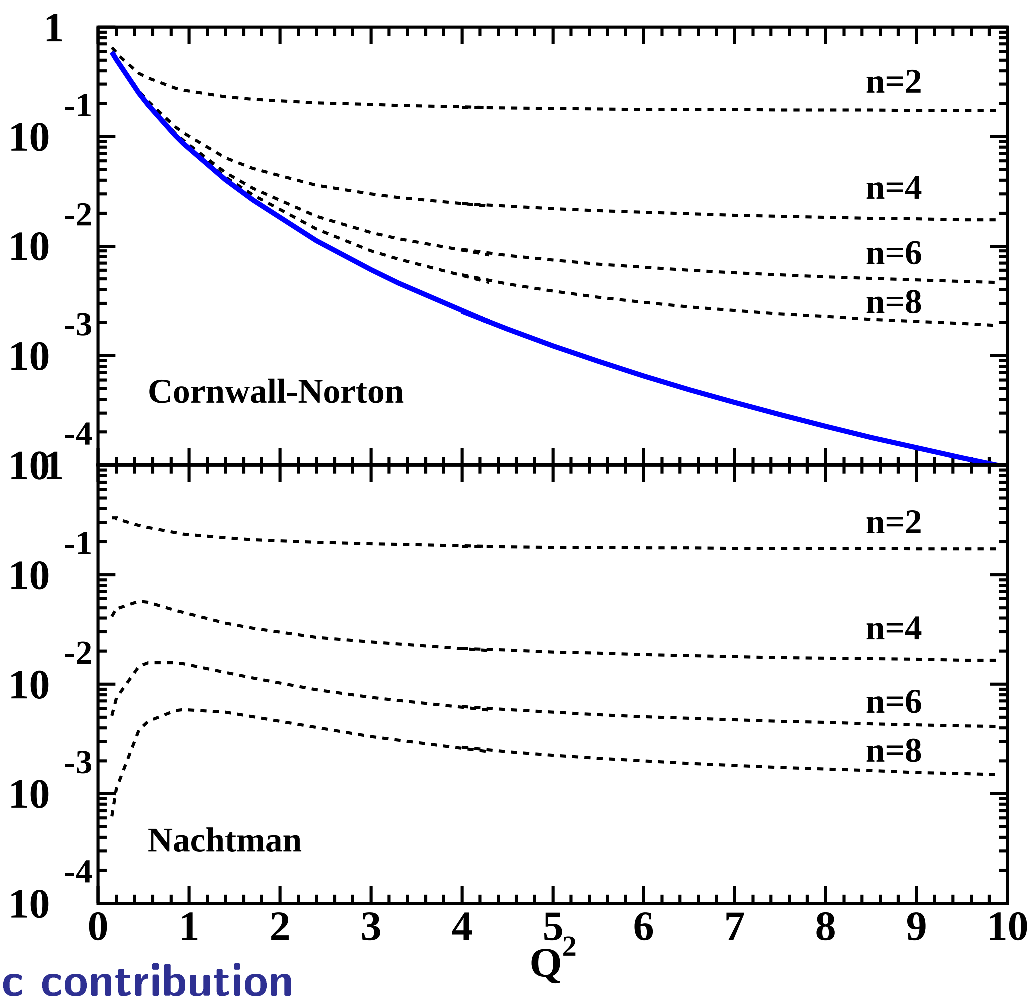
New Data from JLab: Moments

Data from C. S. Armstrong et al., PRD 63, 094008 (2001)

$M_n(Q^2) = \int dx\, x^{n-2}\, F(x, Q^2)$

Cornwall–Norton Moments: x = x_{Bj}

Nachtmann Moments: x = ξ



elastic contribution

Coherent vs. Incoherent Scattering

Low Q^2 vs. Scaling Region

$(\sum_q e_q)^2$ **vs.** $\sum_q e_q^2$

Two quarks with charges e_1, e_2 and masses m_1, m_2 in a non-relativistic system:

Coulomb Sum Rule

$$S(\vec{q}) = e_1^2 + e_2^2 + 2e_1e_2\mathcal{F}(\vec{q})$$

$$\mathcal{F}(\vec{q}) = \Re \int d^3r \; \psi_0^\dagger(\vec{r}) \, e^{i\vec{q}\cdot\vec{r}} \, \psi_0(\vec{r})$$

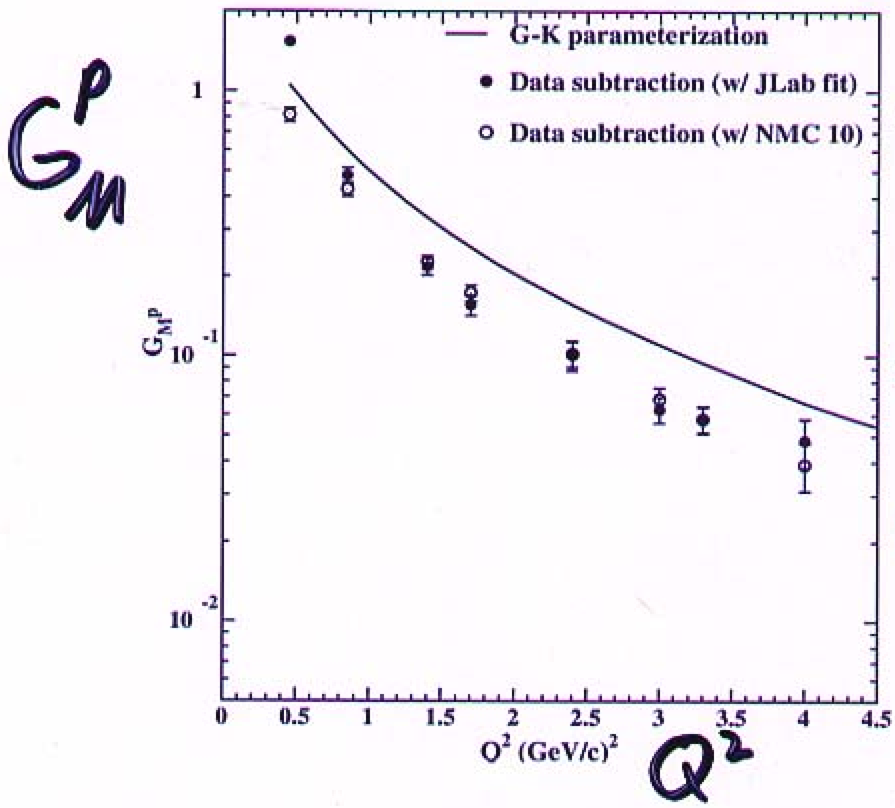
(SJ & J. W. Van Orden, hep-ph/0201113)

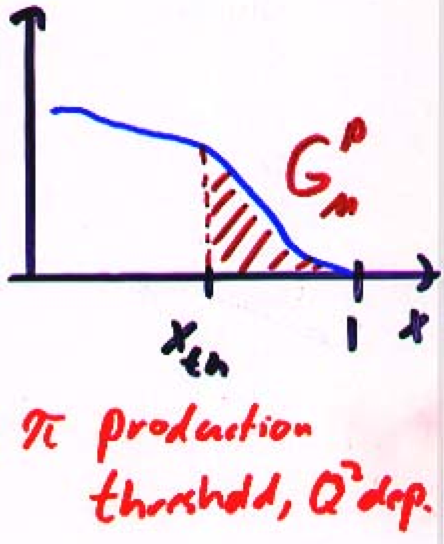
Necessary condition for duality: cancellations due to summation over resonances of different symmetry types, Close & Isgur, PLB 509, 81 (2001).

Applications of Duality

Duality establishes a relation between the resonance region and the deep inelastic region.

Nice examples in X. Ji & P. Unrau, PRD 52, 72 (1995), A. DeRujula, H. Georgi, and H. D. Politzer, Ann. Phys. 103, 315 (1977)





Duality may allow us to learn about the deep inelastic region at high x by measuring the resonances at high x with CEBAF @ 12 GeV.

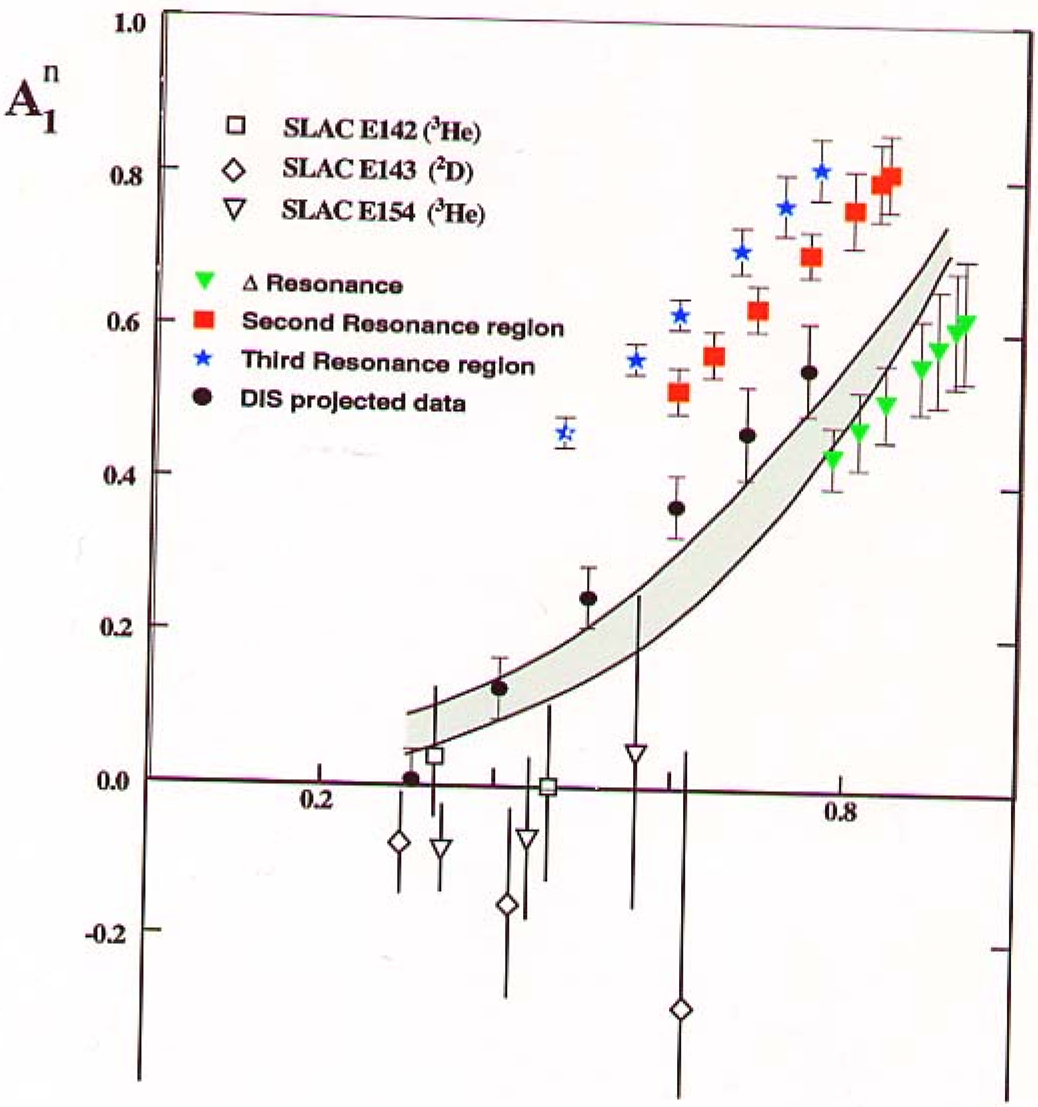
Applications of Duality: A_1^n

Polarization asymmetry of the neutron,
$A_1^n(x)$ at high x:

- many, widely differing theoretical predictions, ranging from 0 to 1
- available data have large error bars, are taken at lower x
- Physics we can learn: valence quark spin distribution functions

use known elastic form factor data to extract A_1^n,
Melnitchouk, PRL 86, 35 (2001)

Projections for 11 GeV beam



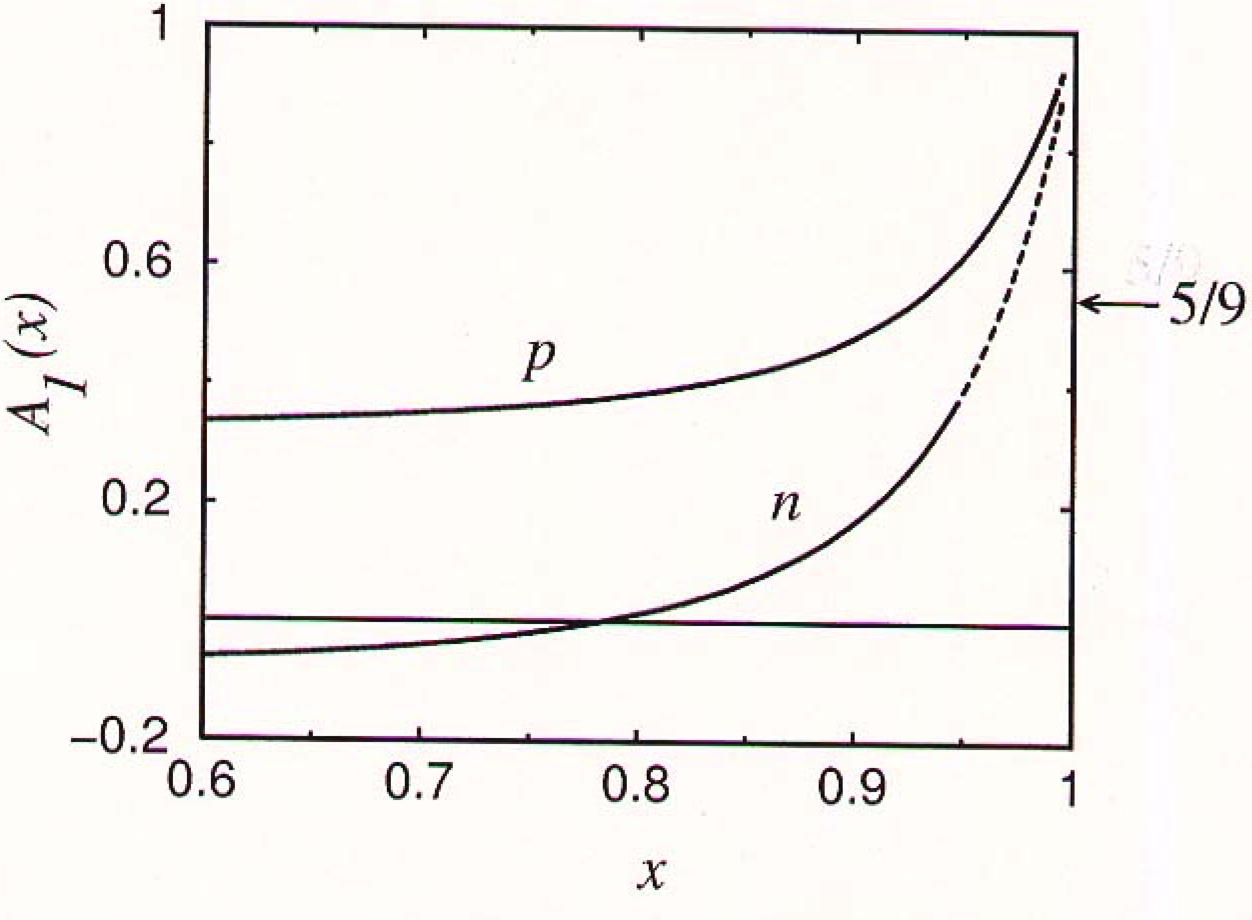
Nilanga Liyanage et al.,

Jefferson Lab Proposal 01-012

unpolarized case Bloom & Gilman, PRD 4, 2901 (1971)

$$F_2(x_{th}) = -\frac{Q^4}{W_{th}^2}\frac{d}{dQ^2}\left(\frac{\left[G_e^2 + \frac{Q^2}{4M_N^2}G_M^2\right]^2}{1+\frac{Q^2}{4M_N^2}}\right)$$

polarized case Melnitchouk, PRL 86, 35 (2001)



Open Questions

- **Why do we observe duality? How can we see precocious scaling in a region where interactions are strong?**

- **For which observables in which kinematic regimes can we apply quark-hadron duality, and how precise are our results going to be?**

Current Efforts

- **Refine analysis: scaling variables, Cornwall-Norton vs Nachtmann moments, target mass corrections, averaging** . . . e.g. C. Armstrong, R. Ent, C. Keppel, S. Liuti, and I. Niculescu PRD 60, 094001 (1999); PRD 63, 094008 (2001); hep/ph/0111063; S. Simula, M. Taiuti et al, PRD 65, 034017 (2002); PRC 57, 356 (1998)

- **Model duality: What is necessary to obtain duality? How is duality realized? When and how precisely does duality work?**

Towards Understanding Duality

Goal: develop a qualitative understanding of duality

How to tackle this complicated problem?

Try to make it as simple as possible!

Take a fully solvable model for hadrons, and compare the results to the free quark results.

Disclaimer: at the present stage, no intention to quantitatively describe any data

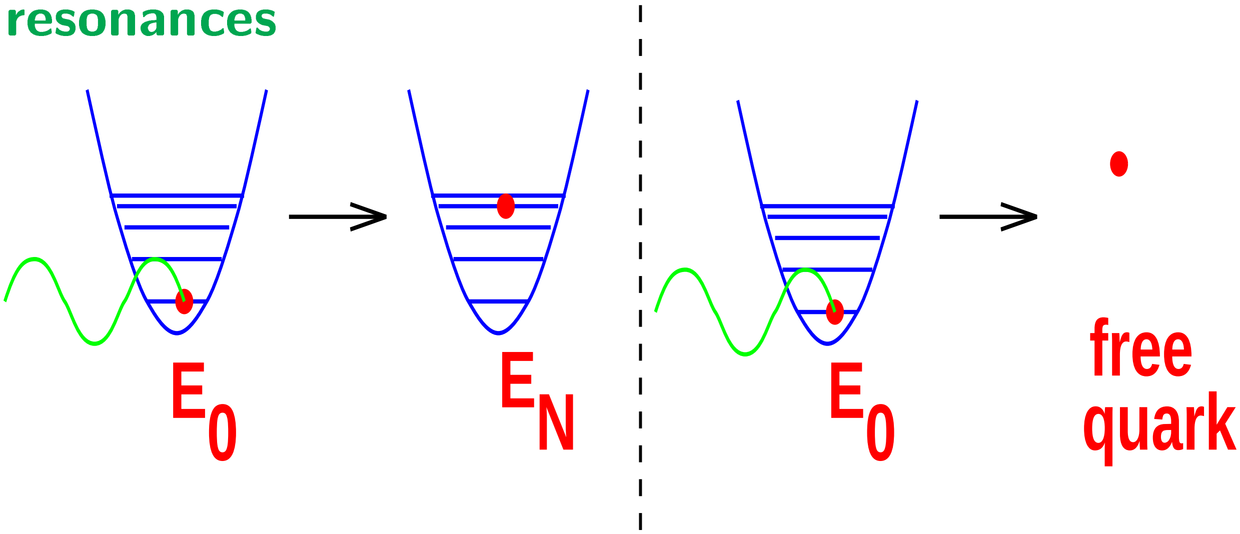
Isgur, SJ, Melnitchouk, Van Orden, PRD64, 054005 (2001)
SJ & Van Orden, hep-ph/0201113

Paris & Pandharipande, PLB514, 361 (2001); PRC65, 035203 (2002)

F. E. Close & Qiang Zhao, hep-ph/0202181

Duality: Requirements for Models

Current models describe the excitation of resonances



no decays yet: $\nu = E_N - E_0 \Rightarrow \delta(\nu - E_N - E_0)$

A model must reproduce:

- **scaling: scaling curve for the bound-bound transition must be the same as for the bound-free transition**
- **moments flatten out with large enough** Q^2
- **resonance region results oscillate around the scaling curve**

A Model for Duality

basic ingredients:

- relativity
- valence quarks only, treat them as scalars
- confinement: linear potential ("relativistic harmonic oscillator")

simplify: reduce from the nucleon target to a one-body problem: light quark bound to an infinitely heavy diquark or antiquark

solve KG equation, obtain relativistic energy spectrum $E_N \propto \sqrt{N}$; wave functions are identical to the NR HO case

parameters: constituent quark mass $m = 0.33$ GeV, string tension $= 0.16$ GeV^2

all scalar case Isgur, SJ, Melnitchouk, Van Orden, PRD64, 054005 (2001)

electromagnetic current & scalar quarks SJ & Van Orden, hep-ph/0202113

Scaling Variables & Scaling Functions

$$x_{Bj} \text{ and } F_2(x_{Bj}) = \nu W_2(\nu, Q^2)$$

appropriate for the Bjorken limit where $Q^2 \gg M_T^2, m_q^2$

Duality is observed at much lower $Q^2 \approx M_T^2, Q^2 > m_q^2$, scaling variable and scaling function need to reflect that:

ad hoc variable of Bloom & Gilman: $x' = \frac{Q^2}{W^2+Q^2}$

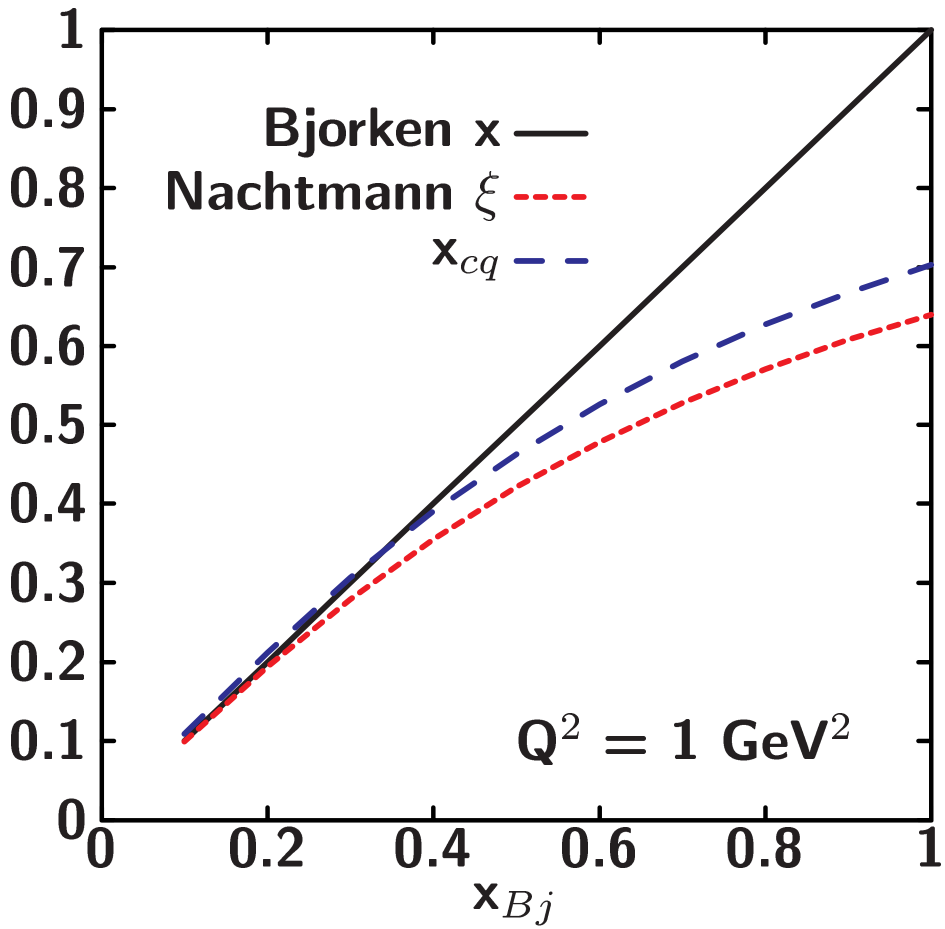
derive Barbieri et al, PLB 64, 171 (1976)

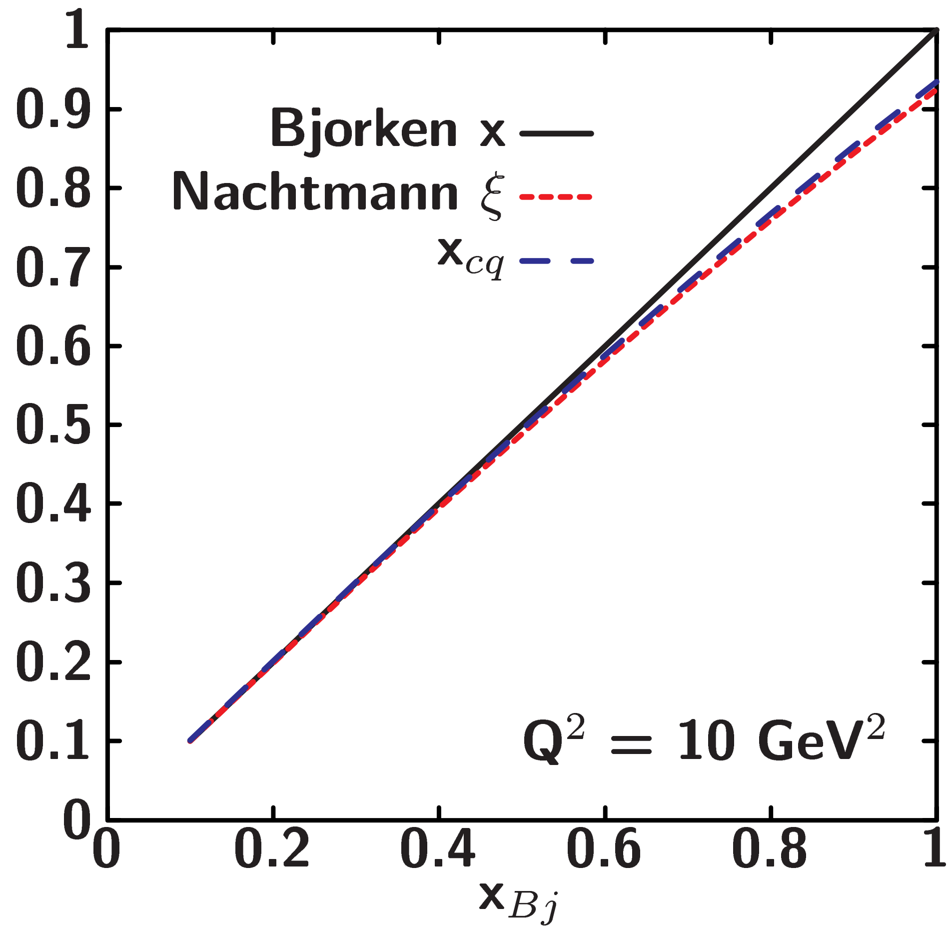
$$x_{cq} \equiv \frac{1}{2M_T}\left(\sqrt{\nu^2+Q^2}-\nu\right)\left(1+\sqrt{1+\frac{4m^2}{Q^2}}\right)$$

$$\mathcal{S}_{2cq} \equiv |\vec{q}|W_2 = \sqrt{\nu^2+Q^2}W_2$$

In the Bjorken limit, all versions of scaling variable and scaling function must converge to x_{Bj} and F_2.

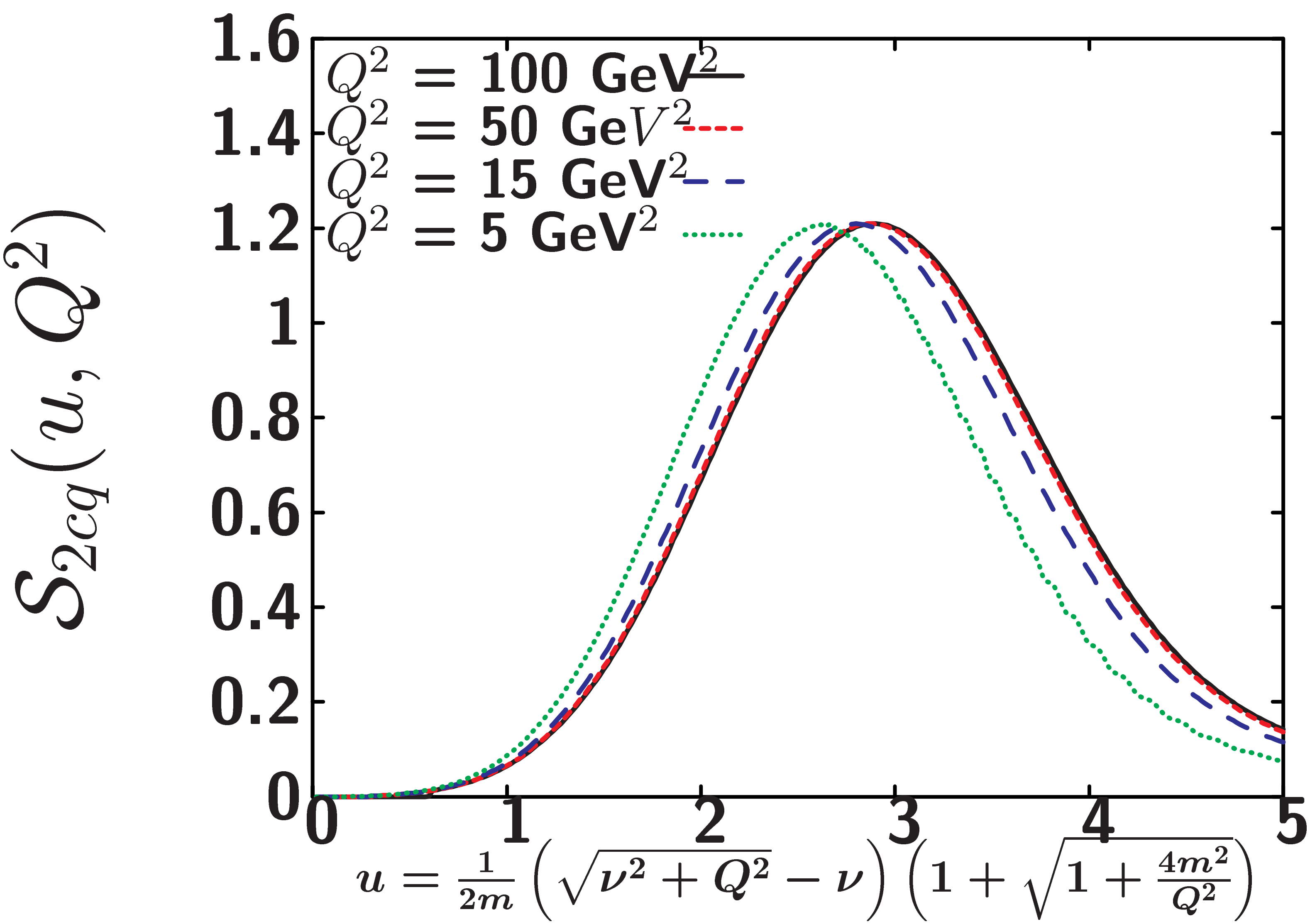
Comparison of Scaling Variables





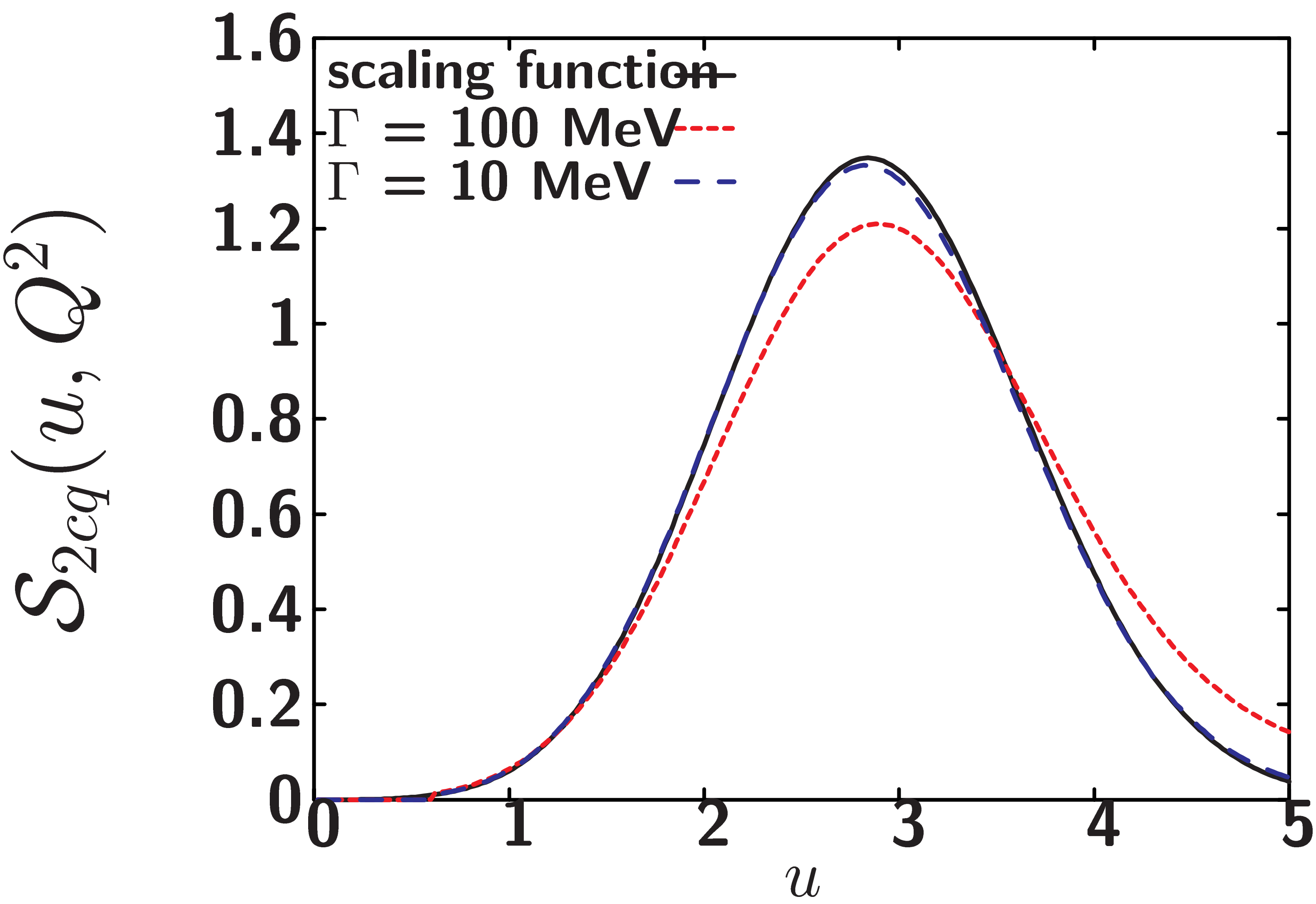
Scaling: Bound-Bound Transition

electromagnetic current; $\mathcal{S}_2cq = |\vec{q}|W_2 = \sqrt{\nu^2 + Q^2}W_2$

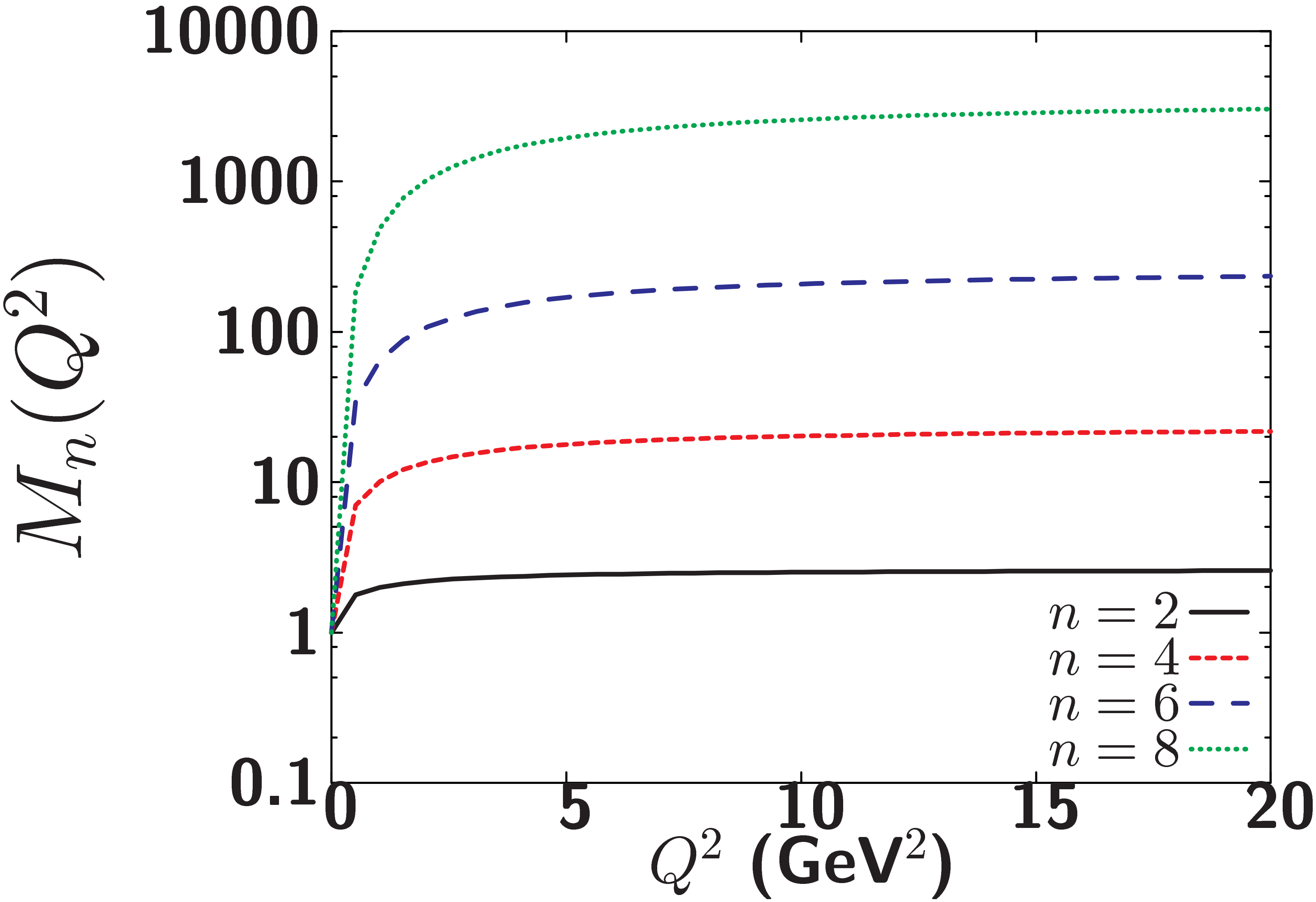


Scaling: Bound-Bound vs Bound-Free

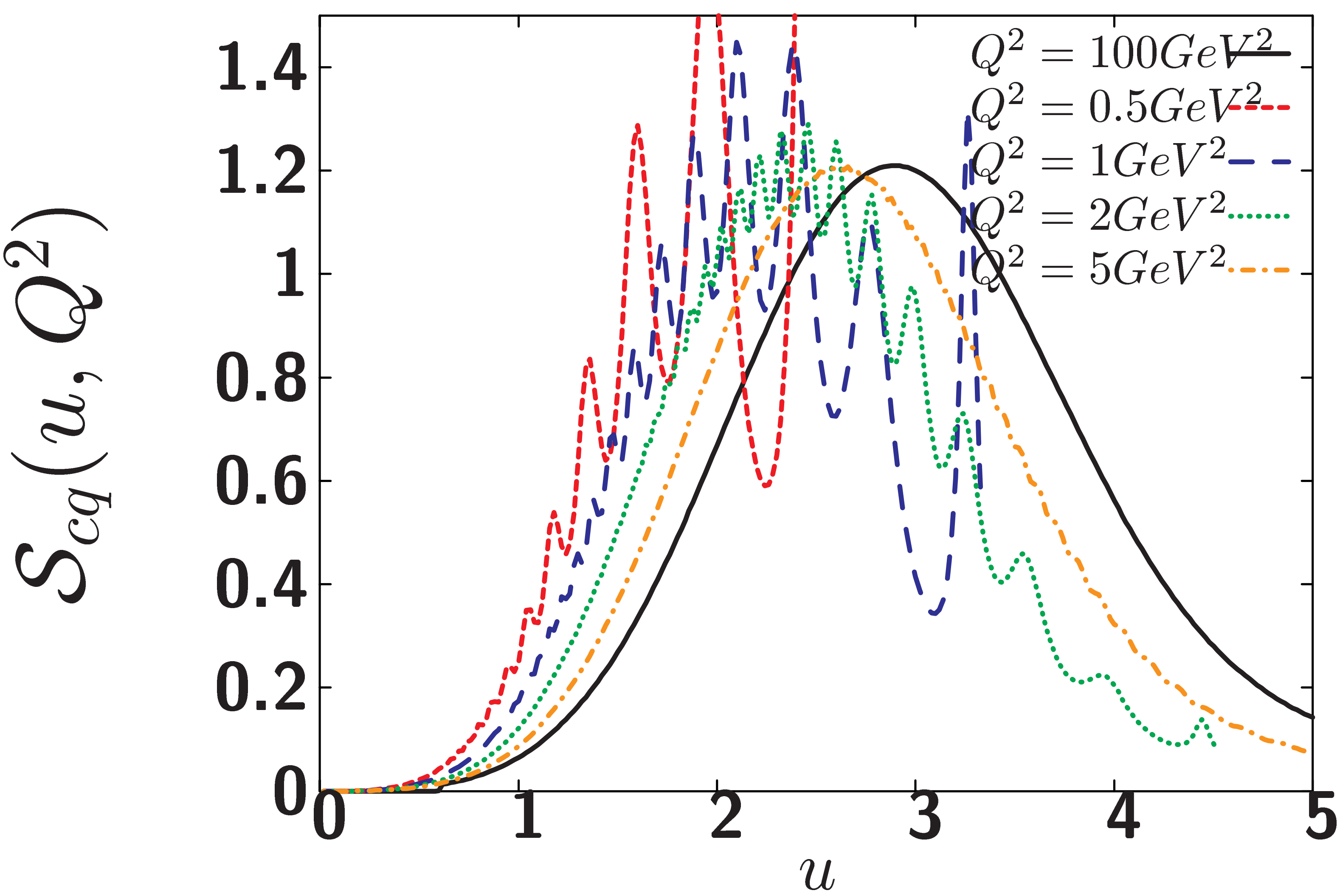
we can **prove analytically** that bound-bound and bound-free have the same scaling curve



Moments



Local Duality



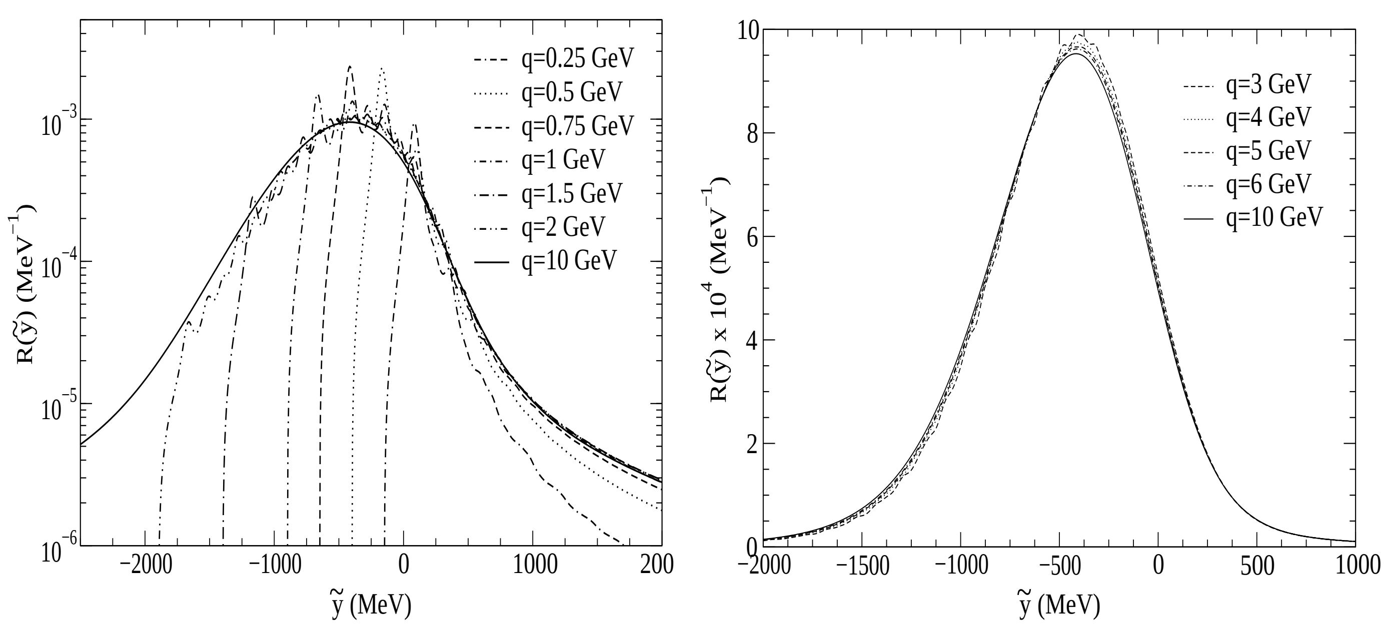
Another Model

Paris & Pandharipande, PLB514, 361 (2001); PRC65, 035203 (2002)

scalar probe, scalar quarks ; numerical solutions

$$H = \sqrt{|\vec{p}|^2} + \sqrt{\sigma r} \quad \textbf{(massless quarks)}$$

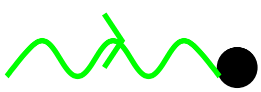
They find scaling and local duality; address contributions to sum rules from the time-like region.

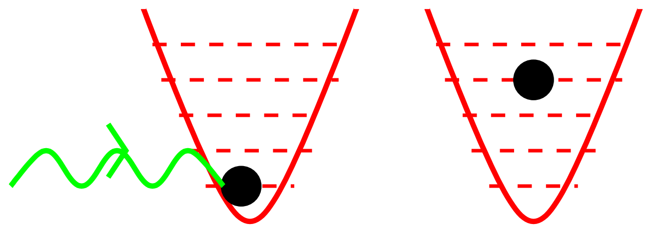


Duality in Meson Production

$N(e, e'\pi)$:

free quark picture (resonances) **hadronic picture**





jet

single resonance with quantum numbers l, m: angular distribution of the pion $\propto |Y_{l,m}|^2$ no jet!

sum over all resonances: interference $\rightarrow$ jet!

Where can we see this best? see Carlson et al for the most suitable kinematic regimes, e.g. Afanasev, Carlson, Wahlquist, PRD 62, 074011 (2000)

Summary & Outlook

Duality

- appears in many processes
- is well established experimentally
- has useful applications
- in a simplified version, it can be modelled (and understood?) successfully

The future holds

- more data!
- development of more realistic models
  - produce mesons
  - use spin 1/2 quarks
- interesting things e.g. connection to Deeply Virtual Compton Scattering Close & Zhao, hep-ph/0202181