Hadronic Structure from Lattice QCD

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This talk summarizes recent work done by the QCDSF Collaboration:

S. Capitani, M. Göckeler, R. Horsley, B. Klaus, W. Kürzinger, H. Perlt, D. Petters, D. Pleiter, P. Rakow, S. Schaefer, A. Schäfer, G. Schierholz, A. Schiller

Some of the results including dynamical quarks involve gauge field configurations which were generated by the UKQCD Collaboration.

Wisdom from Frank Wilczek

" ... the Higgs particle (or the doublet) is certainly *not* – depite much loose talk to the contrary – the Origin of Mass. (Still less is it the God Particle, whatever that means.) Most of the mass of ordinary matter is concentrated in protons and neutrons. It arises from an entirely different, and I think more profound and beautiful, source. Numerical simulation of QCD shows that if we built protons and neutrons in an imaginary world with no Higgs mechanism – purely out of quarks and gluons with zero mass – their masses would not be very different from what they actually are. Their mass mostly arises from pure energy, associated with the dynamics of confinement in QCD, according to relation $m = E/c^2$. This profound account of the origin of mass is a crown jewel in our Theory of Matter."

hep-ph/0101187

Outline

Introduction

Lattice QCD

Structure Functions (Highlights)

Higher-Twist Contributions

Form Factors

Conclusions

Introduction

Much of our knowledge about QCD and the structure of hadrons has been derived from DIS experiments and measurements of elastic form factors.

Perturbative QCD allowed us to extract quark and gluon distribution functions from the experimental data. A quantitative understanding of these distribution functions and, in particular, how quarks and gluons provide the mass (binding) and spin of the nucleon, is still missing.

Continuing advances in computing power, and recent theoretical developments, such as

- $O(a)$ Improvement of the action and the operators, to reduce finite cut-off effects and to facilitate the extrapolation to the continuum limit,
- (Non)-perturbative renormalization and matching of the (bare) lattice operators,
- Chiral perturbation theory, to extrapolate reliably from the masses where the lattice calculations are performed to the physical pion mass,

have now brought Lattice QCD to the point that definitive quantitative calculations of a host of hadron observables are becoming possible.

Key quantities: Moments of unpolarized & polarized structure functions

Higher-twist contributions

Form factors

Lattice QCD

Lattice QCD has improved in several respects in the last couple of years. The major improvements are:

We consider Wilson-type fermions only

$\mathcal{O}(a)$ Improvement

Cut-off effects can be reduced to $\mathcal{O}(a^2)$ by adding an irrelevant operator to the fermion action and the operators

$$S_F \to S_F - \frac{a}{4} c_{SW}\, g \sum_x \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

$$\mathcal{O} \to (1 + c_0\, am)\mathcal{O} + a \sum_{i \geq 1} c_i\, \mathcal{O}_i$$

with c_{SW}, c_0, c_1, $\cdots$ to be determined with non-perturbative precision.

Symanzik, Lüscher et al.

Example: (local) vector current

$$\bar{\psi}\gamma_\mu\psi \to (1 + c_0\, am)\bar{\psi}\gamma_\mu\psi - c_1\, a\frac{1}{2}\bar{\psi}\overleftrightarrow{D}_\mu\psi$$

Status of $\mathcal{O}(a)$ improvement

All coefficients of all quark-bilinear operators with up to one covariant derivative (e.g. $\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi$) are known in one-loop tadpole improved perturbation theory. In the quenched approximation several of them are known non-perturbatively, while in full QCD only some of them are known non-perturbatively.

ALPHA
Bhattacharya et al.
QCDSF
⋮

Renormalization

The lattice operators/matrix elements are in general divergent and need to be renormalized:

$$\mathcal{O}^{\mathcal{S}}(\mu) = Z_{\mathcal{O}}^{\mathcal{S}}(a\mu)\mathcal{O}(a) \qquad \mathcal{S}: \text{ scheme}$$

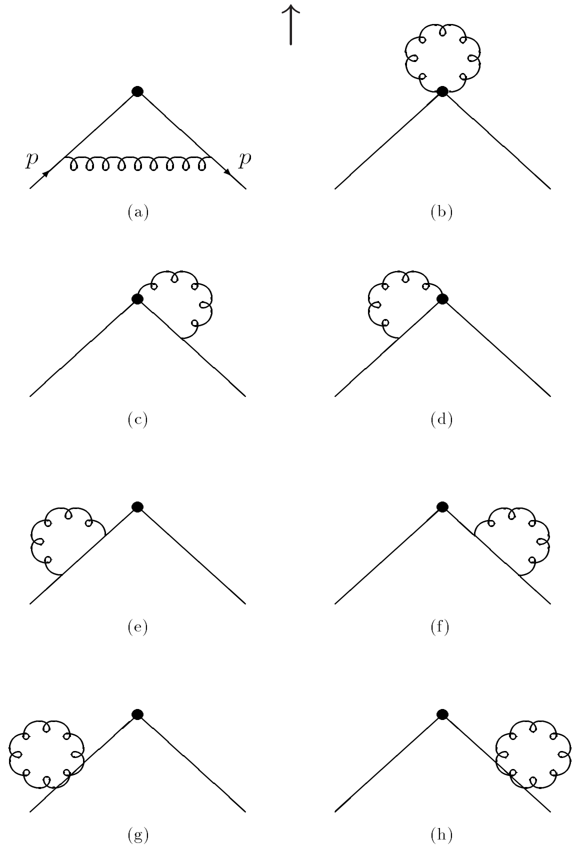
For example

$$\langle p|\mathcal{O}^{\mathcal{MOM}}(\mu)|p\rangle = Z_{\mathcal{O}}^{\mathcal{MOM}}(a\mu)\langle p|\mathcal{O}(a)|p\rangle \underset{p^2=\mu^2}{=} \langle p|\mathcal{O}(a)^{\text{tree}}|p\rangle$$

In lattice perturbation theory

$$Z = 1 + \mathcal{O}(g^2)$$

$$\uparrow$$



Even for a modest calculation one needs to determine $\mathcal{O}(30)$ parameters.

(improvement & renormalization)

Status of perturbative renormalization (one-loop)

Wilson fermions (unimproved)

All Z's of all relevant operators (quark-bilinear and four-fermi) are known.

Martinelli & Zhang
Okawa
Capitani & Rossi
Beccarini et al.
Gupta et al.
QCDSF

Improved fermions

All Z's of all quark-bilinear operators with up to one covariant derivative (e.g. $\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi$) are known.

Heatlie et al.
Gabrielli et al.
Borrelli et al.
QCDSF

Cannot account for mixing with lower-dimensional operators !

Status of non-perturbative renormalization

$N_f = 0$

Method	Observable		
SF	V, A	√	ALPHA
SF	$\mathcal{O}_{14}$		Guagnelli et al.
MOM	S, P, V, A, $S(p)$, $(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)$		Martinelli et al.
MOM ms	S, P, V, A, $S(p)$, $\mathcal{O}_{\mu\nu}$, $\mathcal{O}_{\mu\nu\rho}$, $\mathcal{O}_{\mu\nu\rho\sigma}$, $\mathcal{O}^5_{\mu\nu}$, $\mathcal{O}^5_{\mu\nu\rho}$		QCDSF
WI	S, P, V, A	√	Bhattacharya et al.
WI	V, A, $S(p)$	√	QCDSF

$N_f = 2$

WI	V, A, $S(p)$	√	QCDSF

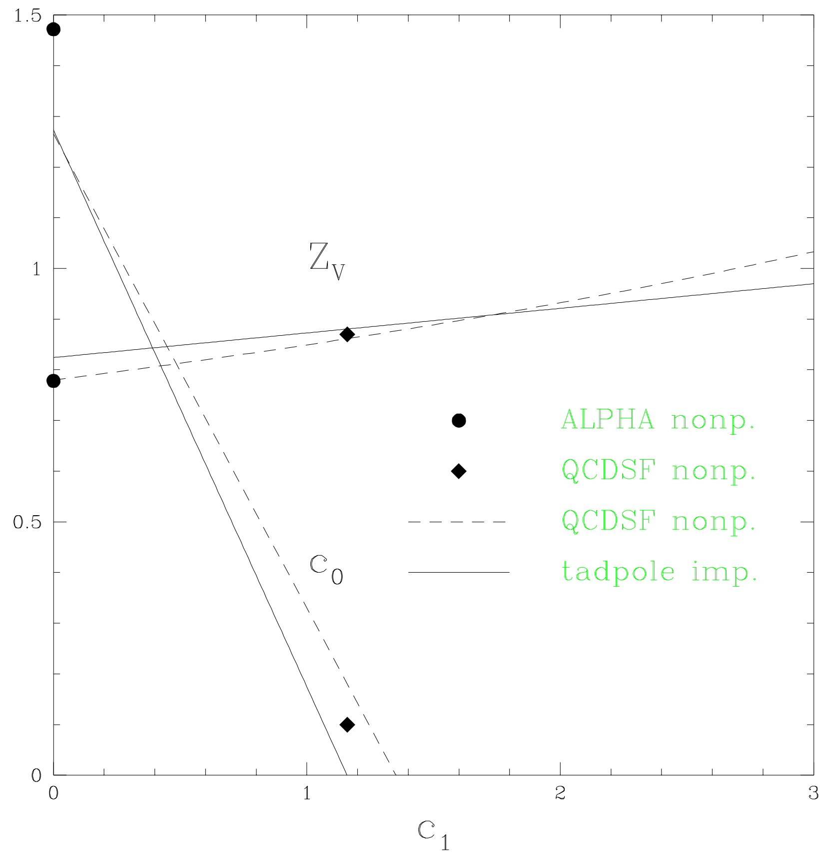
√ Improved fermions

How reliable are results obtained from one-loop tadpole improved perturbation theory ?

In many cases the agreement with the non-perturbative numbers is surprisingly good !

Local vector current

Quenched, $\beta = 6.0$



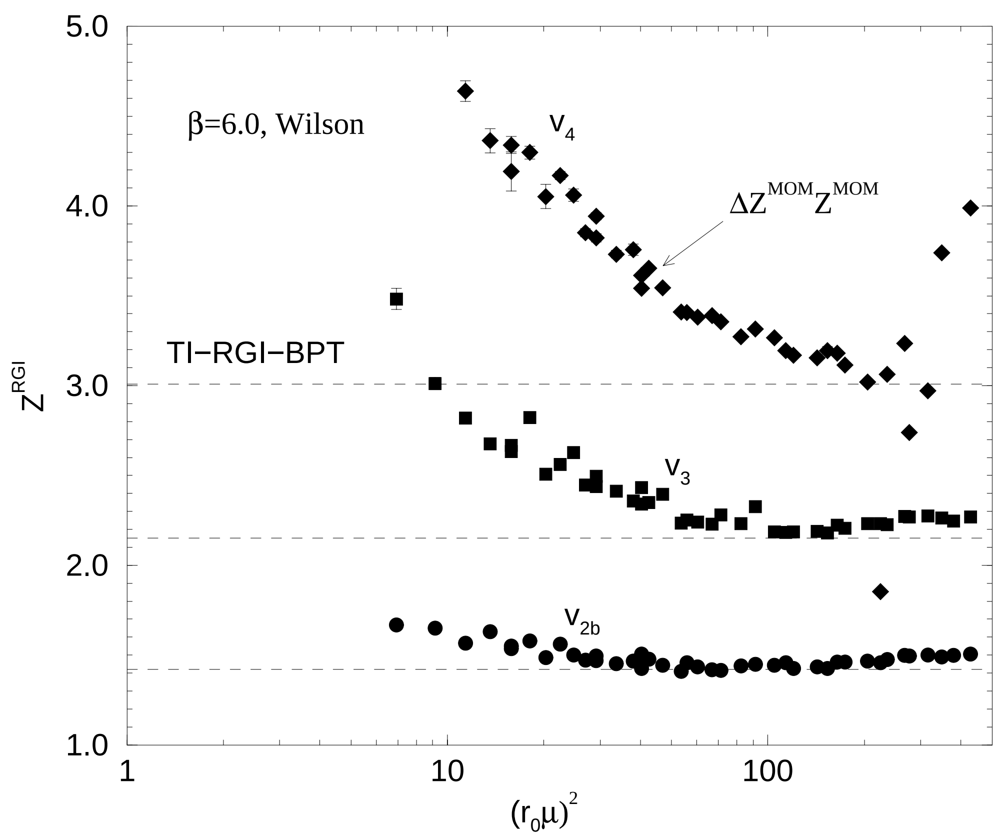
QCDSF

Moments

Quenched, $\beta = 6.0$

$$\begin{array}{ccccccc} v_{2b} & \longrightarrow & \langle x \rangle & \longleftarrow & \bar{\psi}\gamma_\mu D_\nu \psi & = & \mathcal{O}_{\mu\nu} \\ v_3 & \longrightarrow & \langle x^2 \rangle & \longleftarrow & \bar{\psi}\gamma_\mu D_\nu D_\rho \psi & = & \mathcal{O}_{\mu\nu\rho} \\ v_4 & \longrightarrow & \langle x^3 \rangle & \longleftarrow & \bar{\psi}\gamma_\mu D_\nu D_\rho D_\sigma \psi & = & \mathcal{O}_{\mu\nu\rho\sigma} \end{array}$$

QCDSF



● ■ ♦ MOMms

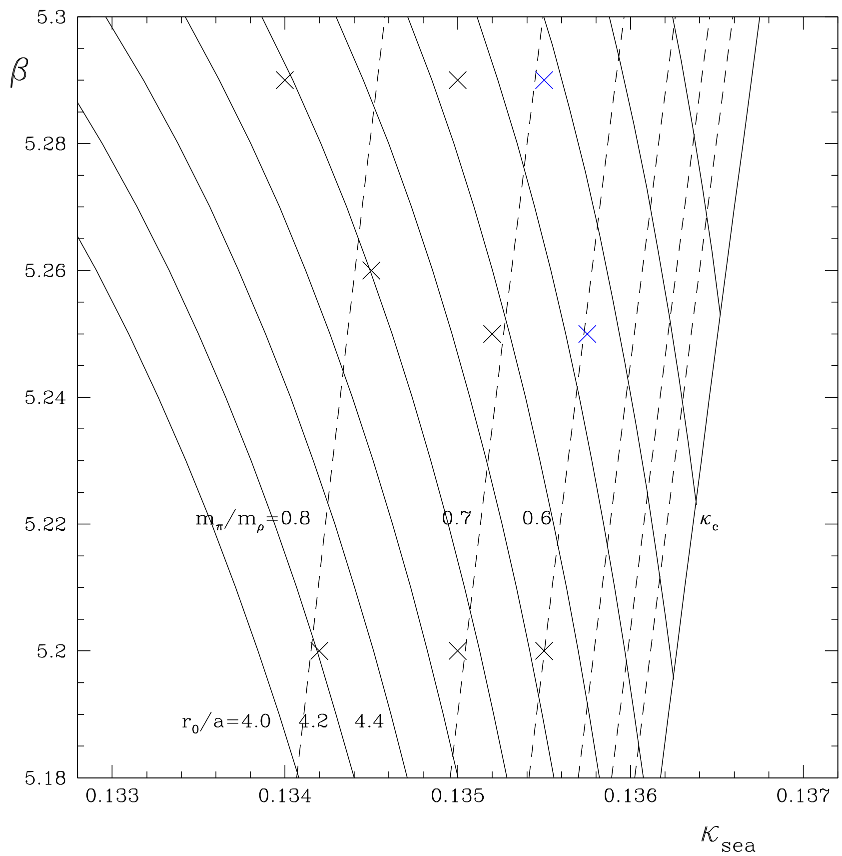
Martinelli et al.
QCDSF

TI-RGI-BPT: Tadpole Improved, Renormalization Group Improved, Boosted Perturbation Theory

Inclusion of Dynamical Fermions

Where are we ?

$N_f = 2$



$\times$ $16^3\,32$ $V \approx (1.7\,\text{fm})^3$

$\times$ $24^3\,48$ $V \approx (2.2\,\text{fm})^3$

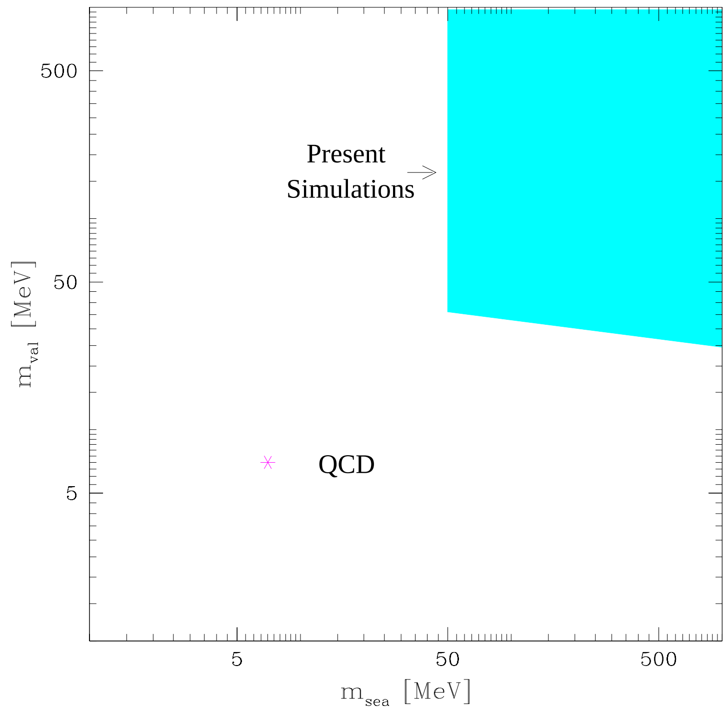
QCDSF–UKQCD

$m_\pi \gtrsim 500$ MeV $\longleftarrow$ dynamical

3–5 years behind quenched calculation.

Chiral Extrapolation

Results have to be extrapolated to the physical quark masses ($*$):



This needs theoretical guidance, which is provided by chiral perturbation theory.

Extrapolation formulae are now available for:

$\langle x^n \rangle^{(q)}$	Thomas, Melnitchouk & Steffens Arndt & Savage Chen & Ji
$\langle x^n \rangle^{(g)}$	Arndt & Savage
$\Delta^n q$, $\delta^n q$	Chen & Ji

Structure Functions

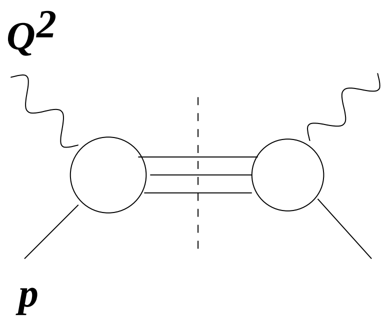
OPE

Renormalization & matching

Mixing

Renormalons

Nucleon



Hadronic tensor

$$W_{\mu\nu} = \mathrm{i} \int dx \mathrm{e}^{\mathrm{i}qx} \langle \vec{p}, \vec{s} | \left[J_\mu(x), J_\nu(0) \right] | \vec{p}, \vec{s} \rangle$$

$$= \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1 + \left(p_\mu - \frac{pq}{q^2} q_\mu \right) \left(p_\nu - \frac{pq}{q^2} q_\nu \right) F_2$$

$$+ \mathrm{i}\epsilon_{\mu\nu\rho\sigma} \frac{q^\rho s^\sigma}{pq} g_1 + \mathrm{i}\epsilon_{\mu\nu\rho\sigma} \frac{q^\rho (pq\, s^\sigma - sq\, p^\sigma)}{(pq)^2} g_2$$

$$Q^2 = -q^2$$

Drell-Yan: $\longrightarrow h_1$

Tensor charge

OPE

Unpolarized

$$2\int_0^1 d\mathsf{x}\, \mathsf{x}^{n-1} F_1(\mathsf{x}, Q^2) = c_{1,n}^{(2)}(Q^2/\mu^2) v_n^{(2)}(\mu) + c_{1,n}^{(4)}(Q^2/\mu^2) \frac{v_n^{(4)}(\mu)}{Q^2} + O(1/Q^4)$$

$$\int_0^1 d\mathsf{x}\, \mathsf{x}^{n-2} F_2(\mathsf{x}, Q^2) = c_{2,n}^{(2)}(Q^2/\mu^2) v_n^{(2)}(\mu) + c_{2,n}^{(4)}(Q^2/\mu^2) \frac{v_n^{(4)}(\mu)}{Q^2} + O(1/Q^4)$$

Leading twist

$$\frac{1}{2} \sum_{\vec{s}} \langle \vec{p}, \vec{s} | \mathcal{O}_{\{\mu_1 \cdots \mu_n\}} | \vec{p}, \vec{s} \rangle = 2 v_n^{(2)} [p_{\mu_1} \cdots p_{\mu_n} - \text{traces}],$$

$$\mathcal{O}_{\mu_1 \cdots \mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{\psi} \gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n} \psi - \text{traces}$$

In parton model language

$$v_n^{(2)} = \langle \mathsf{x}^{n-1} \rangle = \int_0^1 d\mathsf{x}\, \mathsf{x}^{n-1} q(\mathsf{x}, \mu^2) = \int_0^1 d\mathsf{x}\, \mathsf{x}^{n-1} \Big(q_\uparrow(\mathsf{x}, \mu^2) + q_\downarrow(\mathsf{x}, \mu^2) \Big)$$

Polarized (leading twist only)

$$2\int_0^1 d\mathsf{x}\,\mathsf{x}^n g_1(\mathsf{x},Q^2) = \frac{1}{2}e_{1,n}(Q^2/\mu^2)a_n(\mu) + O(1/Q^2)$$

$$2\int_0^1 d\mathsf{x}\,\mathsf{x}^n g_2(\mathsf{x},Q^2) \overset{n\geq 2}{=} \frac{n}{2(n+1)}\Big(e_{2,n}(Q^2/\mu^2)d_n(\mu) - e_{1,n}(Q^2/\mu^2)a_n(\mu)\Big) + O(1/Q^2)$$

$$\langle\vec{p},\vec{s}|\mathcal{O}^5_{\{\sigma\mu_1\cdots\mu_n\}}|\vec{p},\vec{s}\rangle = \frac{1}{n+1}a_n[s_\sigma p_{\mu_1}\cdots p_{\mu_n} + \cdots - \text{traces}]$$

$$\langle\vec{p},\vec{s}|\mathcal{O}^5_{[\sigma\{\mu_1]\cdots\mu_n\}}|\vec{p},\vec{s}\rangle = \frac{1}{n+1}d_n[(s_\sigma p_{\mu_1} - s_{\mu_1}p_\sigma)p_{\mu_2}\cdots p_{\mu_n} + \cdots - \text{traces}]$$

$$\mathcal{O}^5_{\sigma\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^n \bar{\psi}\gamma_\sigma\gamma_5\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}\psi - \text{traces}$$

In parton model language

$$a_0^{(u)} = 2\Delta u,\ a_0^{(d)} = 2\Delta d$$

$$a_n = 2\int_0^1 d\mathsf{x}\,\mathsf{x}^n\Big(q_\uparrow(\mathsf{x},\mu^2) - q_\downarrow(\mathsf{x},\mu^2)\Big) = 2\Delta^n q$$

while

$$d_2 = \text{twist-3}$$

has no parton model interpretation ($\sim$ transverse momentum).

Focus on:

- $\langle x^n \rangle_{NS}$
  - Quenched and full QCD
  - Extrapolation to chiral limit
- Evolution of nucleon

  $m_\pi \longrightarrow 0$

- $g_A = \Delta u - \Delta d$
  - Quenched and full QCD
  - Extrapolation to chiral limit
  - Benchmark calculation

$\updownarrow$

$\int_0^1 dx\, g_1^{p-n}(x, Q^2) = \frac{1}{6}\, g_A\, (1 - \frac{\alpha_s(Q^2)}{\pi} + \cdots)$

Bjorken sum rule

- $\int_0^1 dx\, x^2 g_2(x, Q^2)$
  - Mixing problem
  - d_2
- Higher twist & renormalons

  $\longrightarrow$ Next section

  Requires new approach !

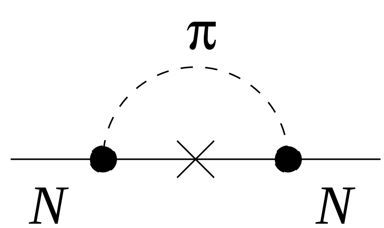
Heavy baryon chiral perturbation theory: Jenkins & Manohar

$$\langle \mathrm{x}^n \rangle_{NS} = A\Big(1 - \frac{3g_A^2+1}{(4\pi f_\pi)^2}\, m_\pi^2 \ln\Big(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\Big)\Big) + \text{analytic terms}$$

$$\Delta^n q_{NS} = A\Big(1 - \frac{2g_A^2+1}{(4\pi f_\pi)^2}\, m_\pi^2 \ln\Big(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\Big)\Big) + \text{analytic terms}$$

Thomas, Melnitchouk & Steffens
Arndt & Savage
Chen & Ji

↑

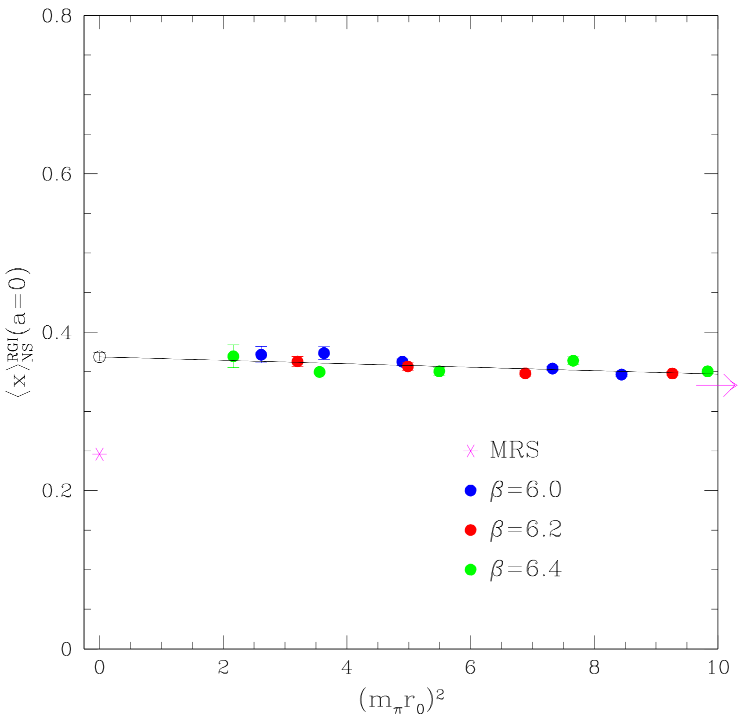


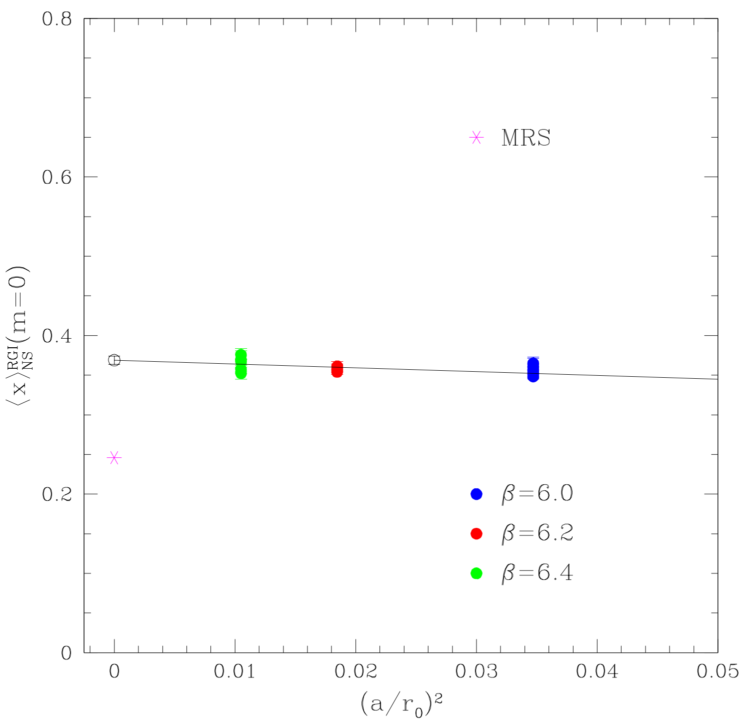
Quenched

$$\langle \mathrm{x}^n \rangle_{NS} = A\Big(1 - 0.28\, r_0^2\, m_\pi^2 \ln\Big(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\Big)\Big) + \text{analytic terms}$$

$r_0 = 0.5$ fm Chen & Savage

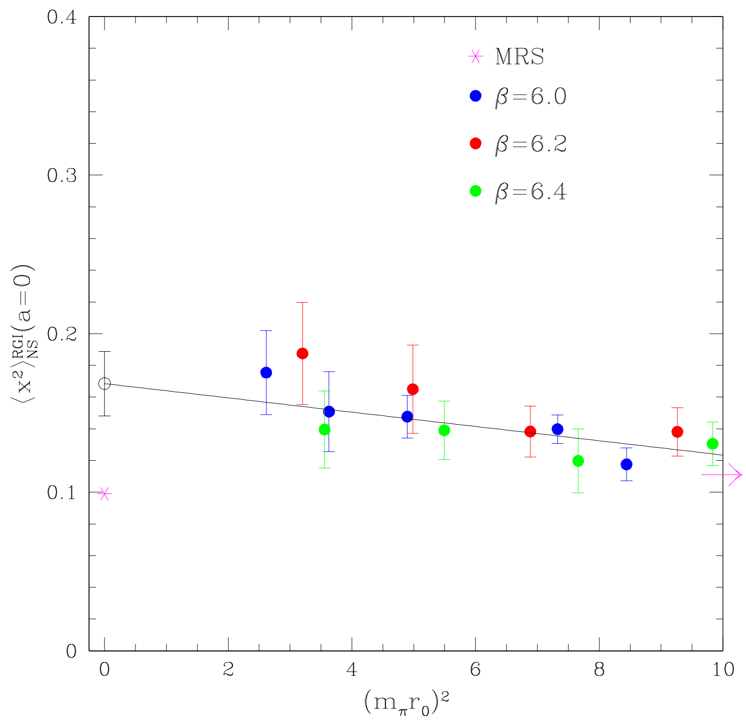
$\langle x \rangle$ Quenched NS, RGI

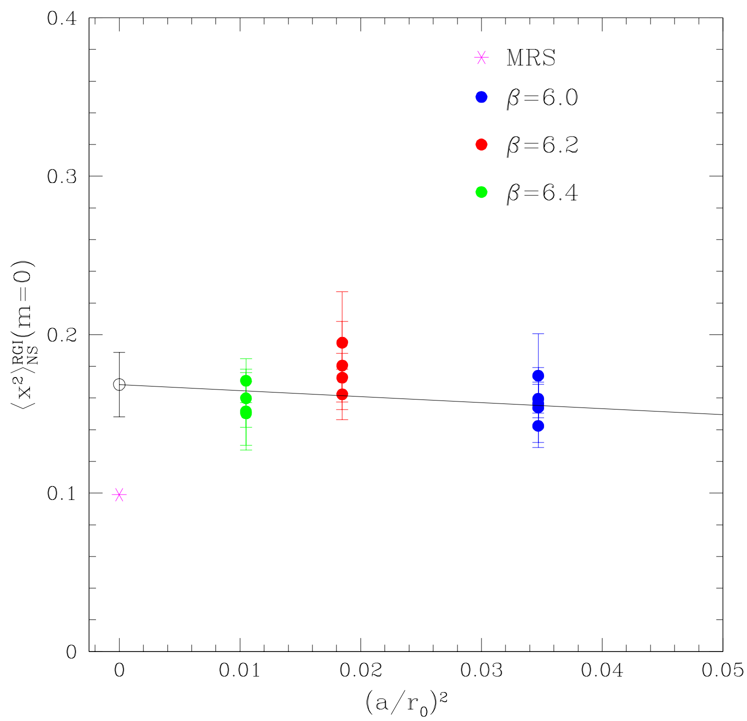




Fit ansatz: $\langle x \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$, $r_0 = 0.5$ fm
analytic

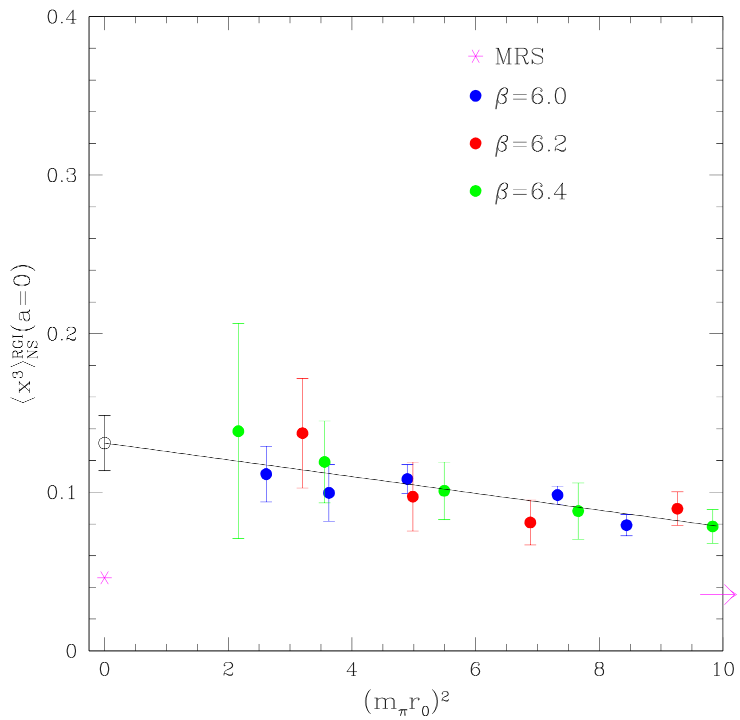
$\langle x^2 \rangle$ Quenched

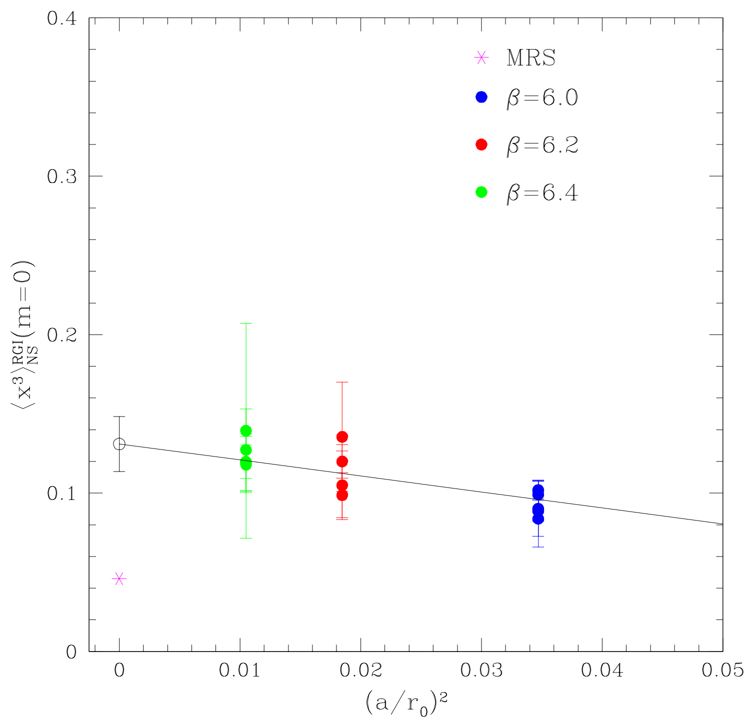




Fit ansatz: $\langle x^2 \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$

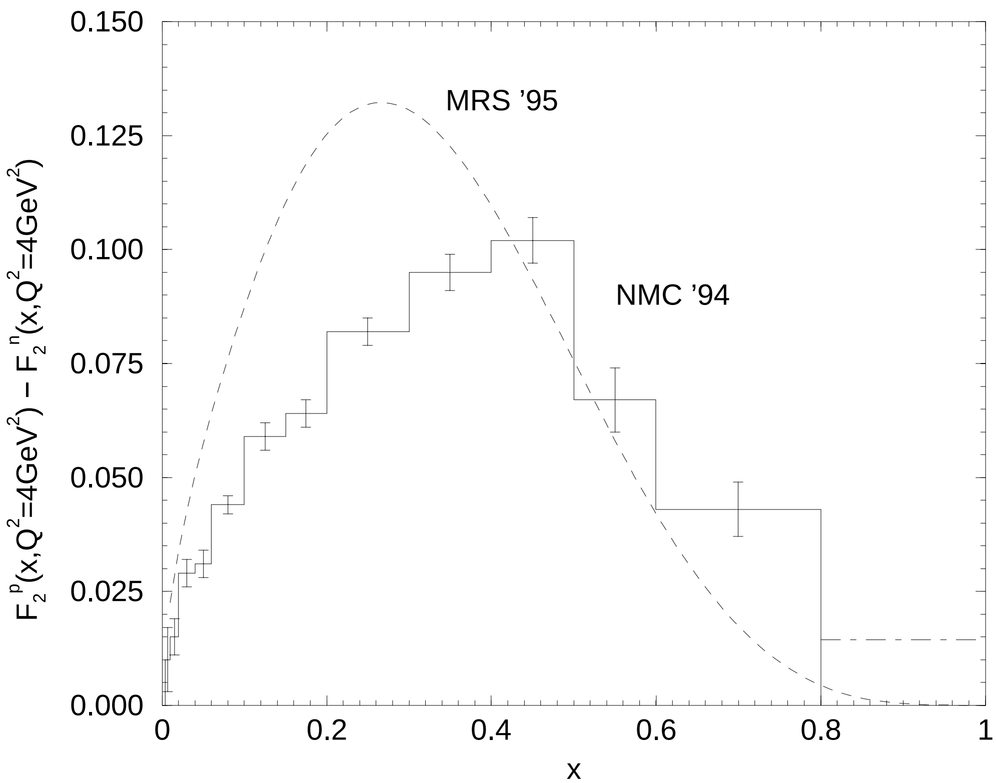
$\langle x^3 \rangle$ Quenched





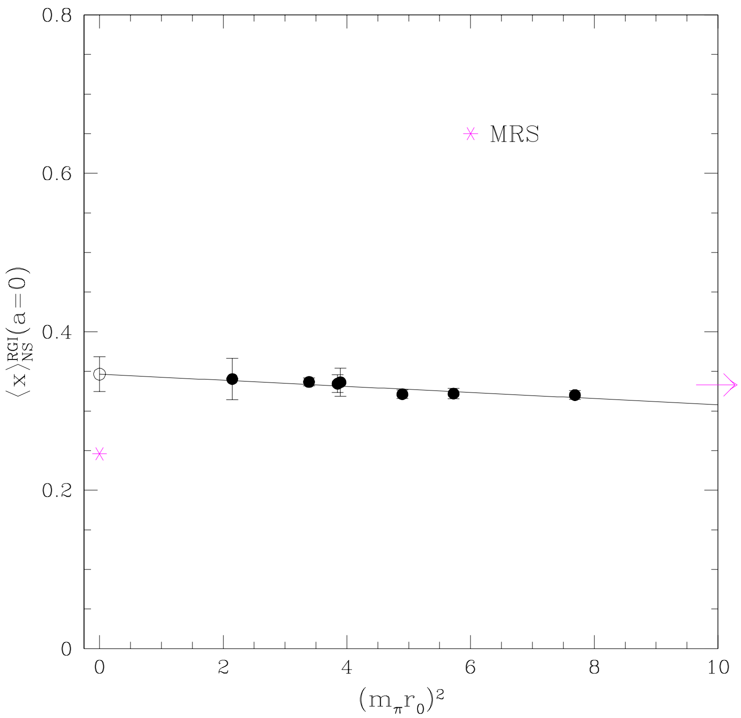
Fit ansatz: $\langle x^3 \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$

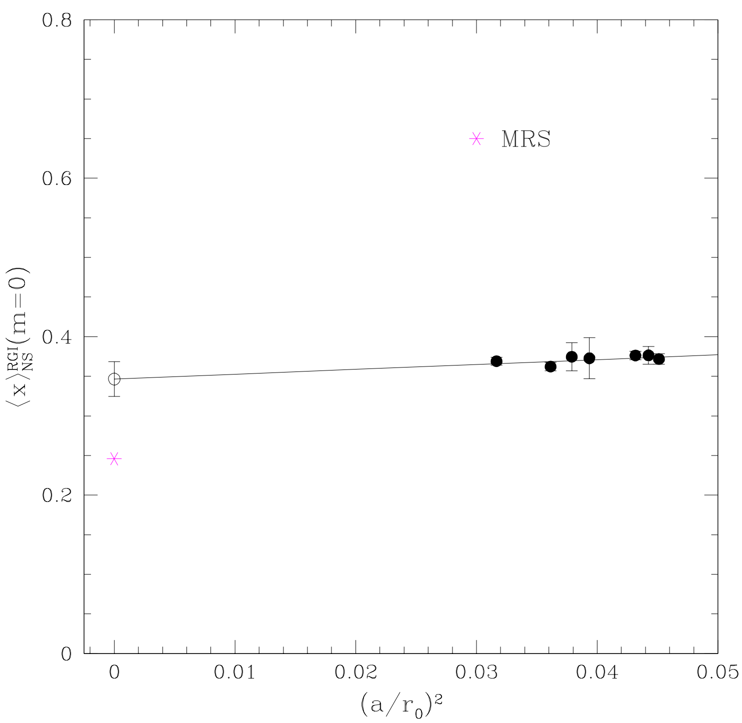
But be aware that MRS does not reproduce $\langle x^2 \rangle_{NS}$ and $\langle x^3 \rangle_{NS}$ correctly:



Need more accurate phenomenological quark and gluon distribution functions, if possible with error bars !

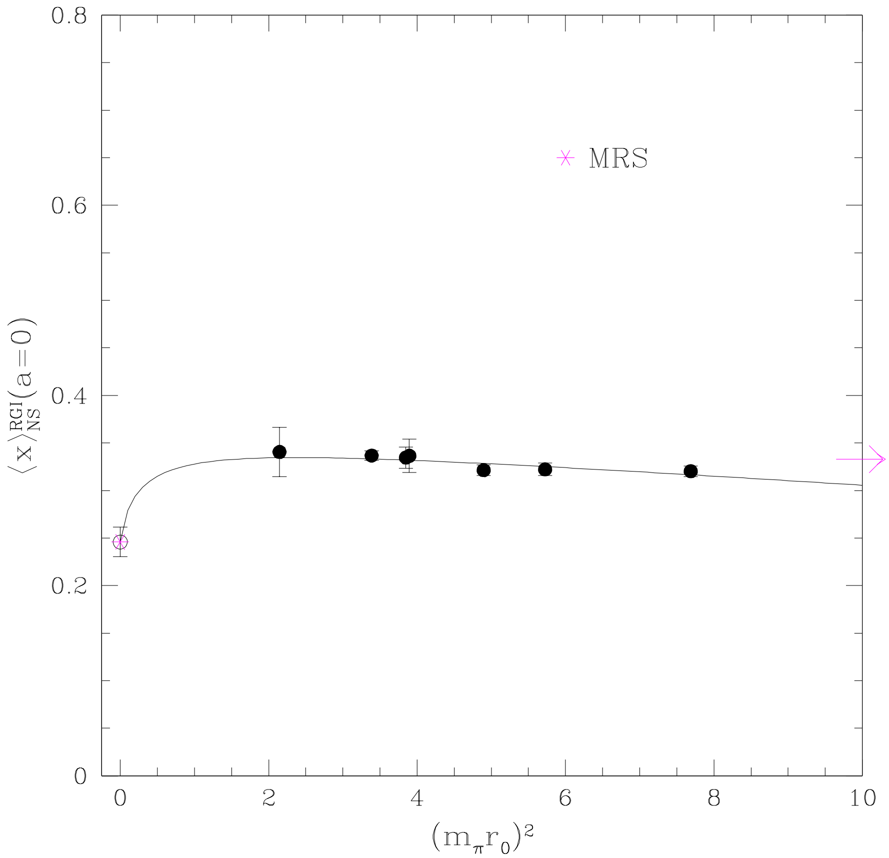
$\langle x \rangle$ $N_f = 2$ NS, RGI





Fit ansatz: $\langle x \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$

$\langle x \rangle$ $N_f = 2$ (revisited)



Fit ansatz:
non-analytic

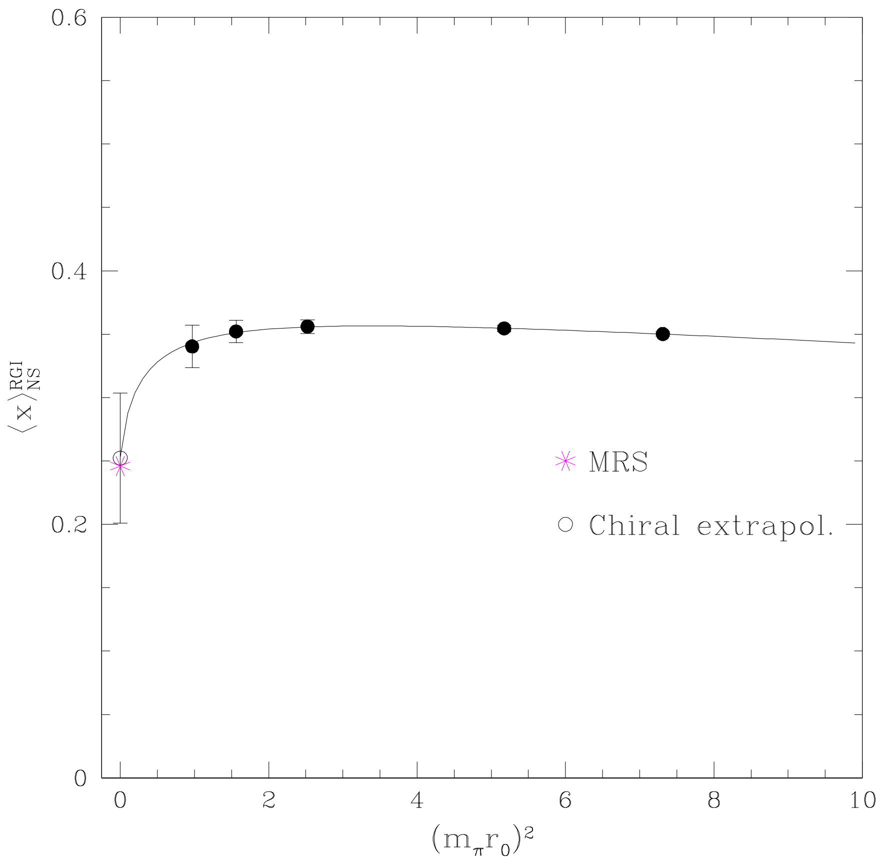
$$\langle x \rangle = A\left(1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2}\, m_\pi^2 \ln\left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2}\right)\right) + B(m_\pi r_0)^2 + C(a/r_0)^2$$

Thomas et al., Arndt & Savage, Chen & Ji

A forced fit through the experimental point ($*$) gives $\Lambda \approx 350$ MeV.

True chiral behavior

Quenched, $\beta = 6.0$, small m



A fit to

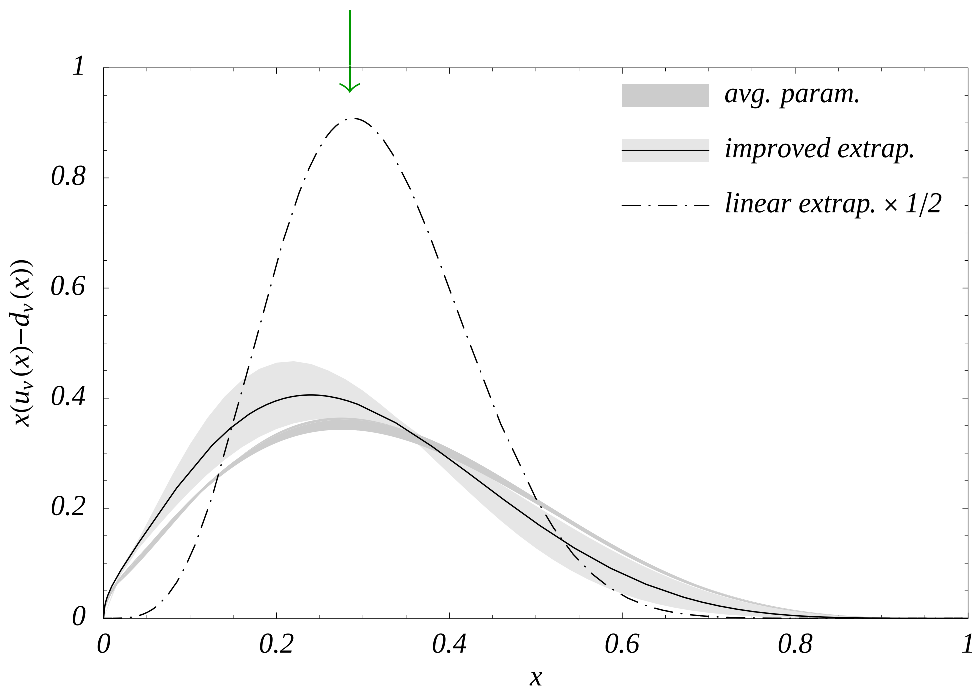
$$\langle x \rangle = A\Big(1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln\Big(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2}\Big)\Big) + B(m_\pi r_0)^2$$

including the (wrong) non-analytic term is possible and gives $\Lambda \approx 350$ MeV. $\surd$

Lesson: To make contact with chiral perturbation theory, one will in general have to do simulations at dynamical quark (pion) masses of $m \approx 20$ MeV ($m_\pi \approx 300$ MeV), so that the parameters of the non-linear expansion are well determined by the lattice calculation.

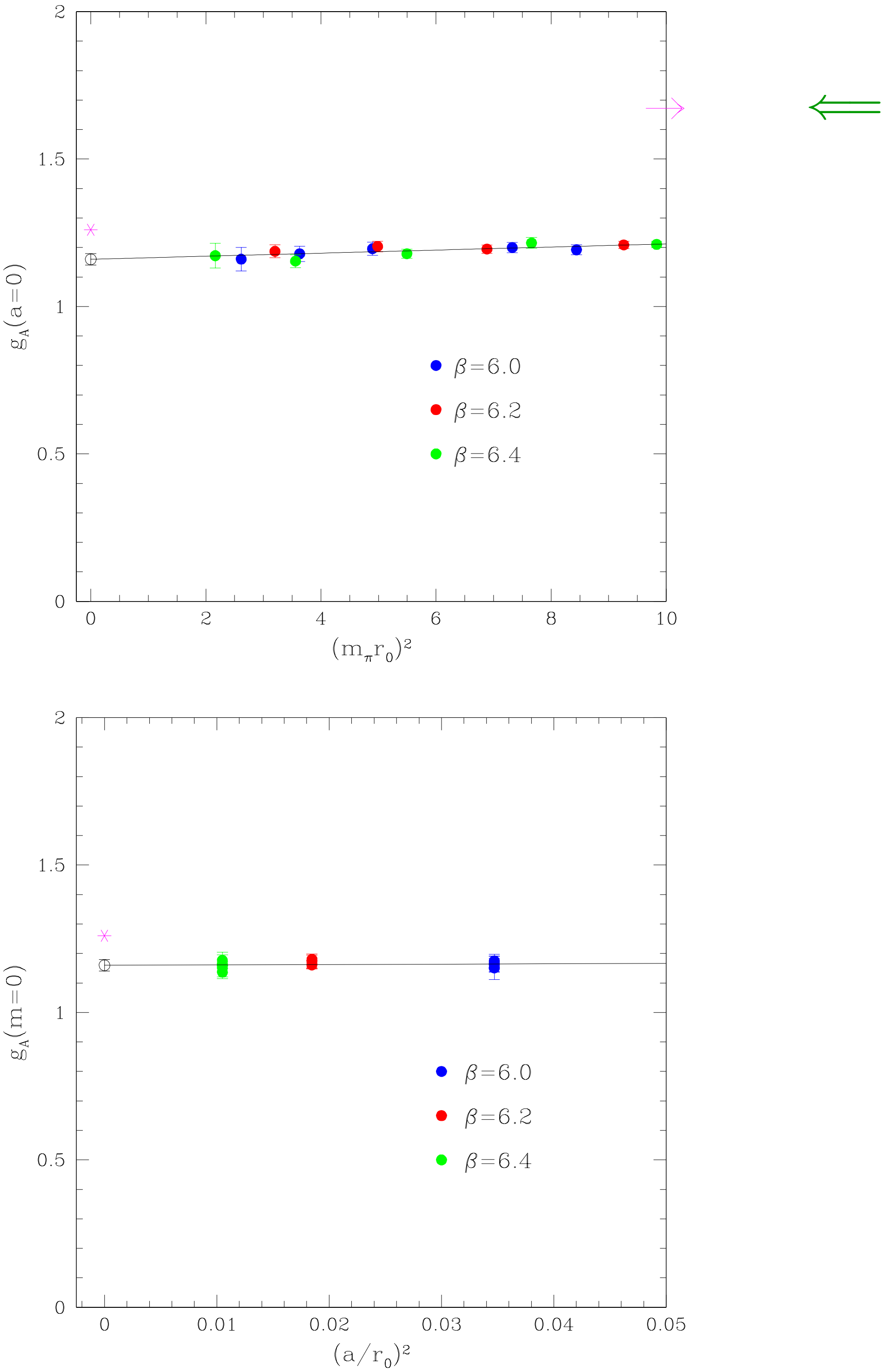
Evolution of nucleon's momentum distribution

$m_\pi \geq 500$ MeV



Detmold, Melnitchouk & Thomas

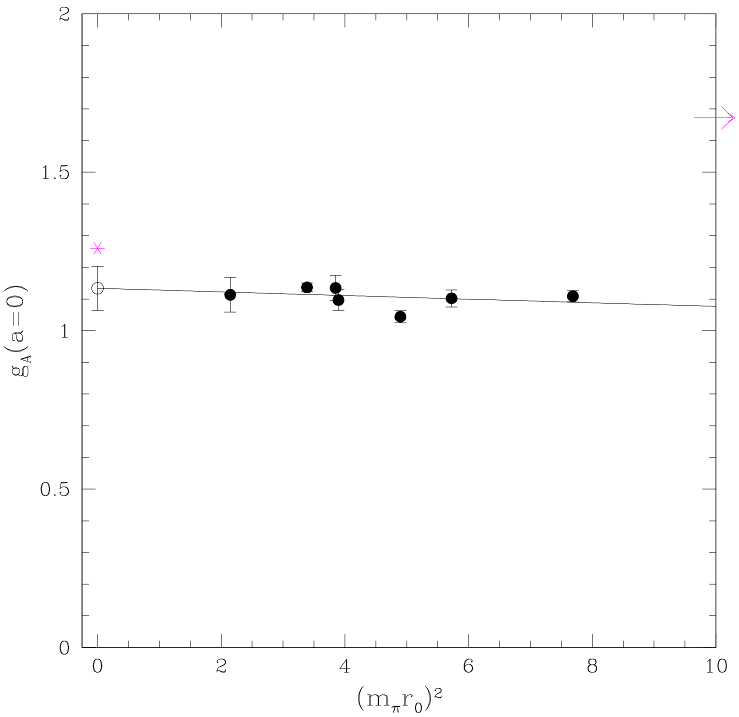
g_A Quenched

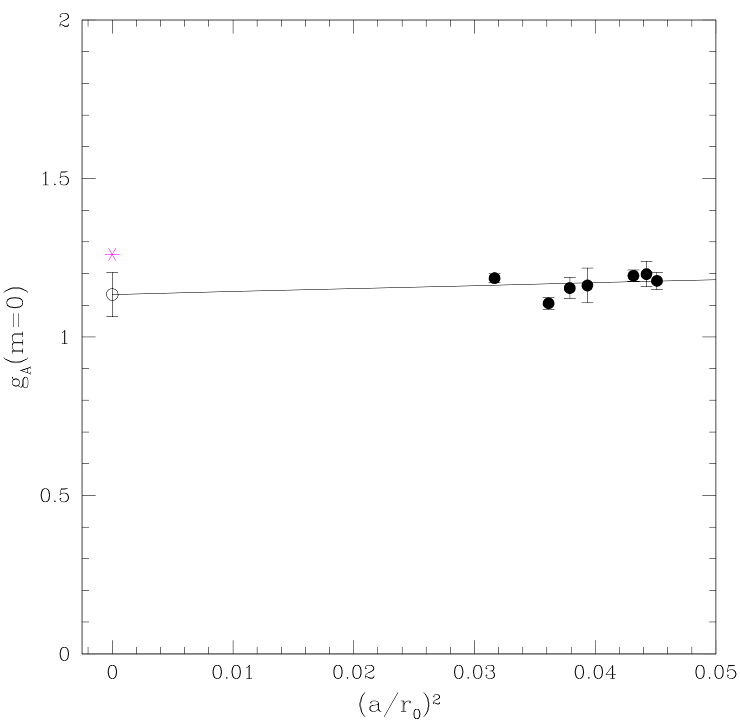


Fit ansatz: $g_A = A + B(m_\pi r_0)^2 + C(a/r_0)^2$

analytic

$g_A \quad N_f = 2$





Fit ansatz: $g_A = A + B(m_\pi r_0)^2 + C(a/r_0)^2$

Non-analytic behavior ?

$\Updownarrow$

In the chiral limit, $1/3$ of g_A is to be found at infinite distance from the nucleon, says Jaffe

Λ is not a universal parameter but may depend on the process !

$\int_0^1 dx\, x^2 g_2(x, Q^2) \qquad \longleftrightarrow \qquad \int_0^1 dx\, x^2 g_1(x, Q^2)$

Need to know

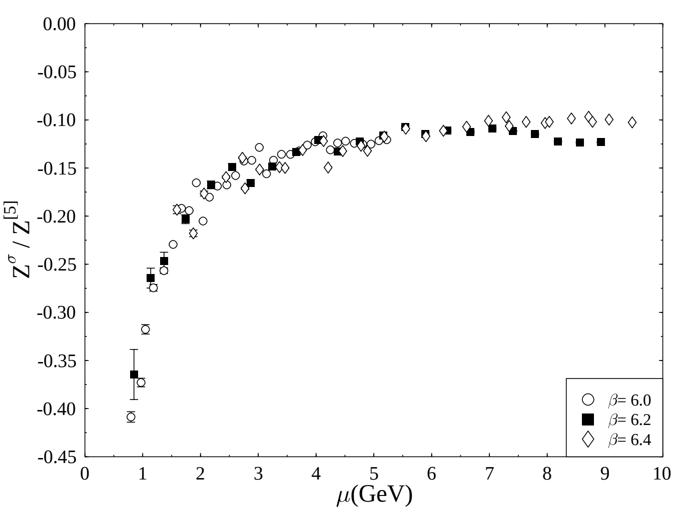
$$\begin{aligned}
&\mathcal{O}^5_{\{214\}}(\mu) && =: \mathcal{O}^{\{5\}}(\mu) \\
&\mathcal{O}^5_{[2\{1]4\}}(\mu) = \tfrac{1}{3}\big(2\mathcal{O}^5_{2\{14\}} - \mathcal{O}^5_{1\{24\}} - \mathcal{O}^5_{4\{12\}}\big)(\mu) && =: \mathcal{O}^{[5]}(\mu)
\end{aligned}$$

The operator $\mathcal{O}^5_{[2\{1]4\}}(a)$ mixes with an operator of lower dimension $(d = 4)$

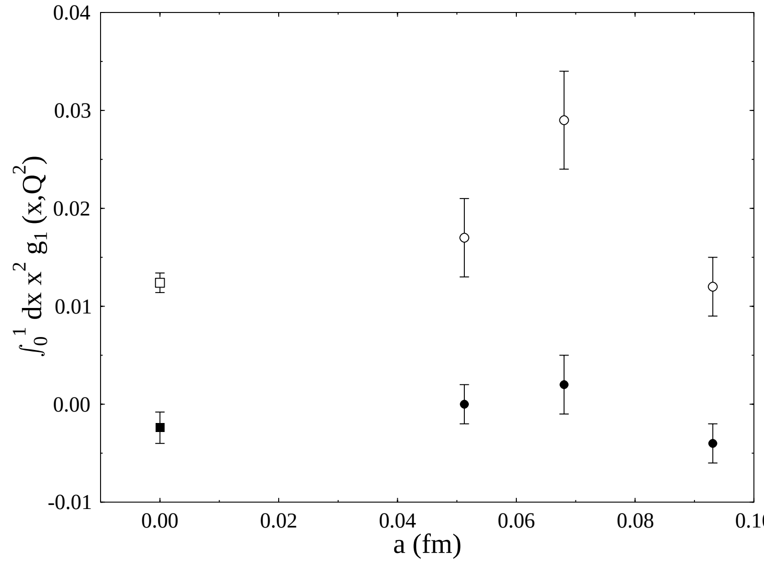
$$\frac{1}{12}\mathrm{i}\bar{\psi}\big(\sigma_{13}\overleftrightarrow{D}_1 - \sigma_{43}\overleftrightarrow{D}_4\big)\psi =: \mathcal{O}^\sigma$$

so that

$$\begin{aligned}
\mathcal{O}^{\{5\}}(\mu) &= Z^{\{5\}}(a\mu)\mathcal{O}^{\{5\}}(a) \\
\mathcal{O}^{[5]}(\mu) &= Z^{[5]}(a\mu)\mathcal{O}^{[5]}(a) + Z^\sigma(a\mu)\frac{1}{a}\mathcal{O}^\sigma(a) \\
&= Z^{[5]}(a\mu)\Big(\mathcal{O}^{[5]}(a) + \frac{Z^\sigma(a\mu)}{Z^{[5]}(a\mu)}\frac{1}{a}\mathcal{O}^\sigma(a)\Big)
\end{aligned}$$

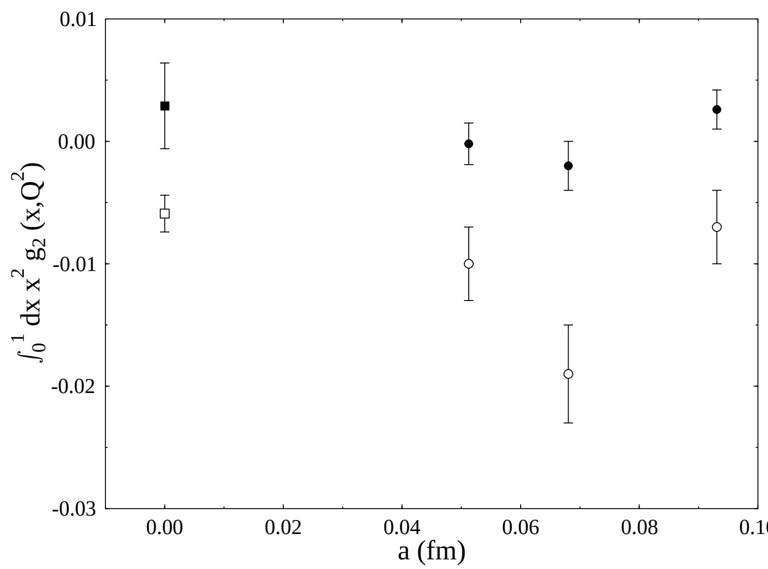


g_1, g_2 Quenched



⇑ Experiment

○ proton
● neutron



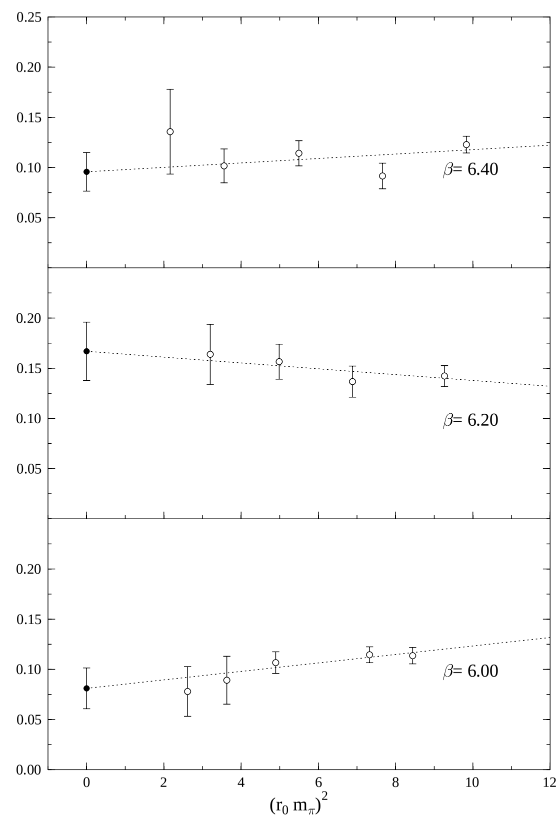
⇑ Experiment

Pure Wilson

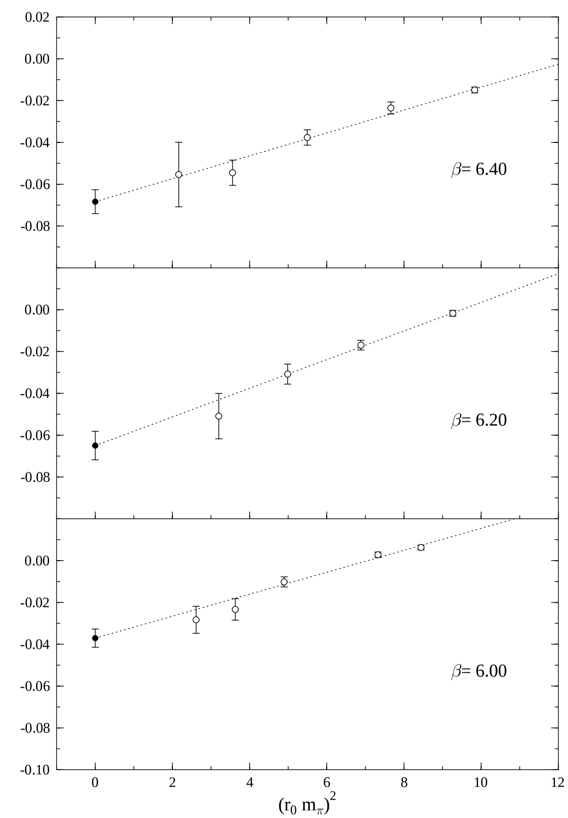
Sea contribution ($x \lesssim 0.1$) can be neglected here.

Chiral extrapolation

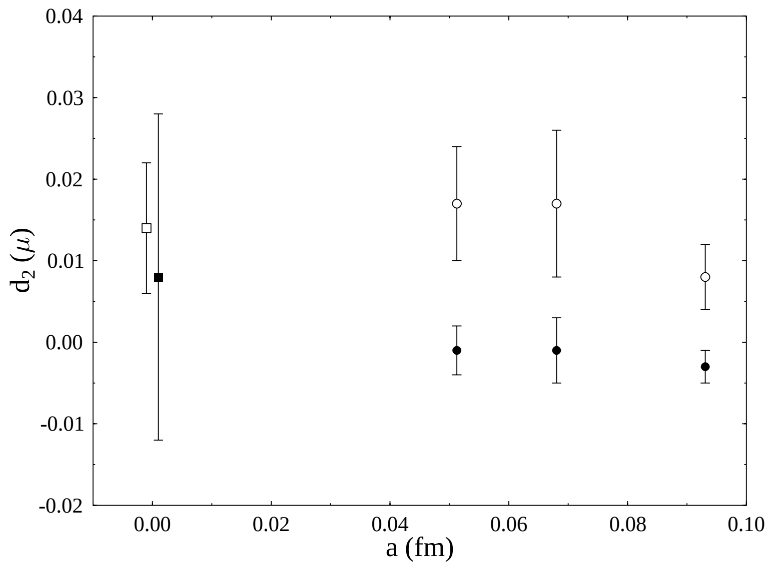
$a_2^{(u)}$



$d_2^{[5]\,(u)}$



d_2 Quenched



⇑
Experiment

○ proton
● neutron
$\mu^2 = 5$ GeV^2

$Q^2 \gtrsim 5$ GeV^2:

$\int_0^1 dx\, x^2 g_2(x, Q^2) \approx -\frac{3}{2} \int_0^1 dx\, x^2 g_1(x, Q^2)$ Wandzura-Wilczek

Expect deviations at smaller Q^2.

Higher-Twist Contributions

$$\mathcal{M}_2(Q^2) = \int_0^1 dxF_2(x, Q^2)$$
$$= c_2^{(2)}(Q^2/\mu^2)v_2^{(2)}(\mu) + c_2^{(4)}(Q^2/\mu^2)\frac{v_2^{(4)}(\mu)}{Q^2} + \ldots$$

IR UV

Renormalon Ambiguity

$$\mathcal{M}_2(Q^2) = c_2^{(2)}(a^2Q^2)v_2^{(2)}(a) + c_2^{(4)}(a^2Q^2)\frac{v_2^{(4)}(a)}{Q^2} + \ldots$$

↑ Need to know Wilson coefficient to all orders

↑ Mixing with lower-dimensional operator

Evaluate

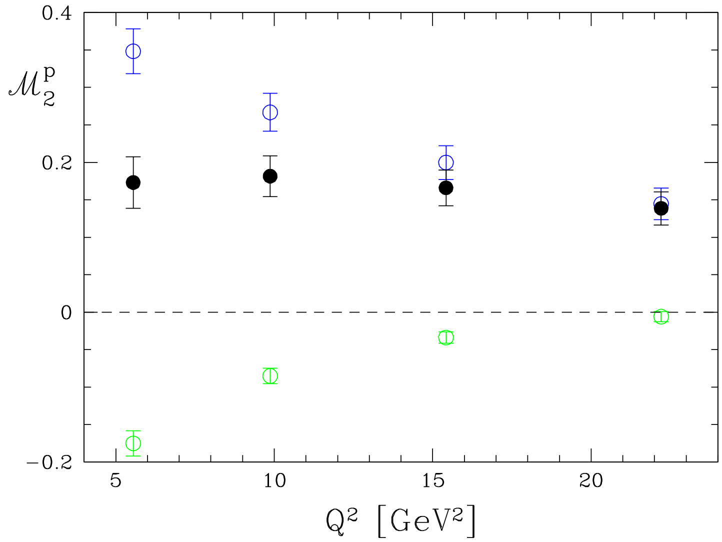
$$\langle p|J_\mu(q)J_\nu(-q)|p\rangle + \cdots = \sum_{m,n} c^m_{\mu\nu\mu_1\cdots\mu_n}(aq)\ \langle p|\mathcal{O}^m_{\mu_1\cdots\mu_n}|p\rangle$$

input input

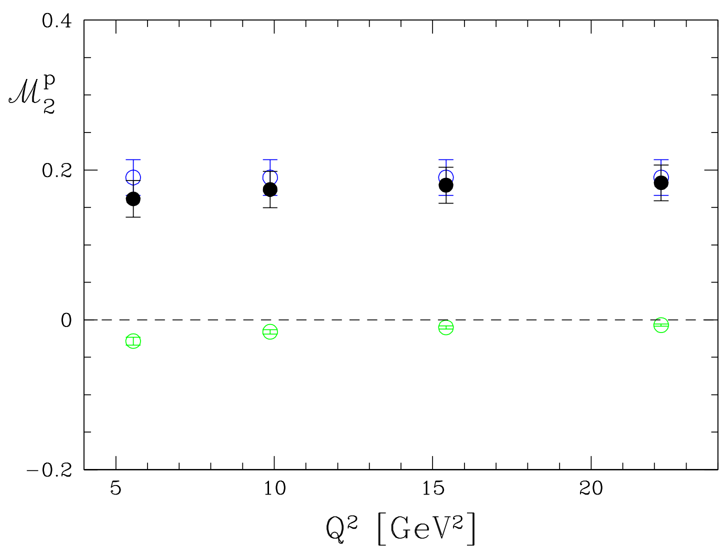
between quark states $|p\rangle$ and solve for Wilson coefficients $c^m_{\mu\nu\mu_1\cdots\mu_n}(aq)$.

So far only done for quark-bilinear $\mathcal{O}$'s.

$\mathcal{M}_2$ = twist-2 + twist-4 Quenched $\beta = 6.0, m \to 0$



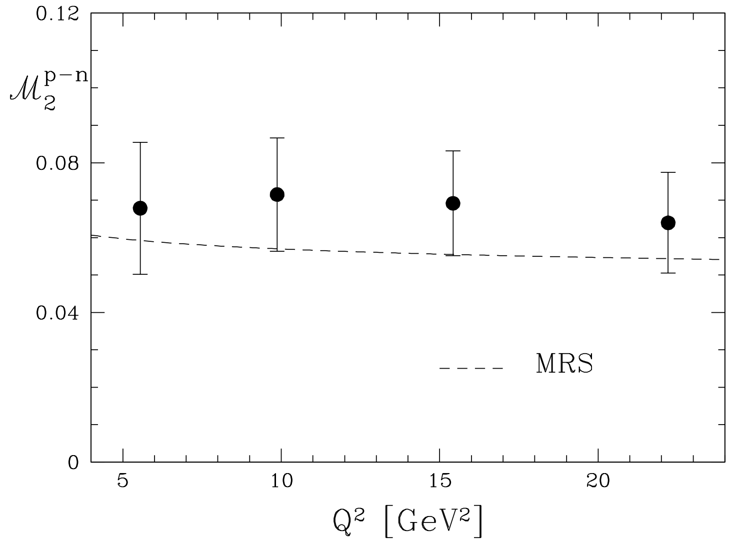
$\mathcal{O}_{14}$



$\mathcal{O}_{44} - \frac{1}{3}\sum_{i=1}^{3} \mathcal{O}_{ii}$

Pure Wilson

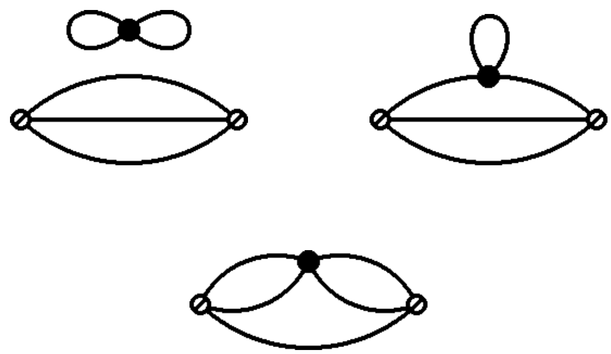
NS



Power corrections $(\propto 1/Q^2)$ are small down to Q^2 values of a few GeV^2.

cf. Armstrong et al.

Four-fermi operators



Quenched $\beta = 6.0, m \to 0$

$$\int_0^1 d\mathsf{x} F_2(\mathsf{x}, Q^2)\big|_{\mathbf{27}, I=1} = -0.0005(5)\,\alpha_s(Q^2)\,\frac{m_p^2}{Q^2}$$

↑ To avoid mixing with lower-dimensional operators

Individual contributions to **27**-plet, divided by m_p^4:

Spin-two operators $\mathcal{O}_{44} - 1/3 \sum_{i=1}^{3} \mathcal{O}_{ii}$

operator	
$A^c_{\{\mu\nu\}}$ − trace	$(-0.9 \pm 0.8) \cdot 10^{-4}$
$V^c_{\{\mu\nu\}}$ − trace	$(-4.5 \pm 1.0) \cdot 10^{-4}$
$T^c_{\{\mu\nu\}}$ − trace	$(+4.9 \pm 0.8) \cdot 10^{-4}$
$A_{\{\mu\nu\}}$ − trace	$(-0.0 \pm 1.3) \cdot 10^{-4}$
$V_{\{\mu\nu\}}$ − trace	$(+8.4 \pm 1.4) \cdot 10^{-4}$
$T_{\{\mu\nu\}}$ − trace	$(-4.6 \pm 1.9) \cdot 10^{-4}$

Diquark operators of spin zero $\mathcal{O}_{44}$ and spin one $\sum_{i=1}^{3} \mathcal{O}_{ii}$

operator	color	spin 0	spin 1
$\frac{1}{10}(\bar{u}\gamma_\mu\gamma_5\gamma_5 C\bar{u}^{\mathrm{T}})(u^{\mathrm{T}}C^{-1}\gamma_5\gamma_\nu\gamma_5 u)$	$\overline{\mathbf{3}}$	$(-6.8 \pm 2.4) \cdot 10^{-4}$	$(-76 \pm 7) \cdot 10^{-4}$
$\frac{1}{10}(\bar{u}\sigma_{\mu\alpha}\gamma_5 C\bar{u}^{\mathrm{T}})(u^{\mathrm{T}}C^{-1}\gamma_5\sigma_{\nu\alpha} u)$	$\overline{\mathbf{3}}$	$(-5.7 \pm 6.1) \cdot 10^{-4}$	$(+48 \pm 18) \cdot 10^{-4}$
$\frac{1}{10}(\bar{u}\gamma_\mu\gamma_5 C\bar{u}^{\mathrm{T}})(\bar{u}^{\mathrm{T}}C^{-1}\gamma_5\gamma_\nu u)$	$\mathbf{6}$	$(+15.0 \pm 3.6) \cdot 10^{-4}$	$(+30 \pm 7) \cdot 10^{-4}$

⇑

Form Factors

Elastic form factors describe the overall distribution of charge, magnetism and axial charge in hadrons.

Nucleon

$$\langle \vec{p}, \vec{s} | J_\mu | \vec{p}, \vec{s} \rangle = \bar{u}(\vec{p}, \vec{s}) \Big(\gamma_\mu F_1(q^2) + \mathrm{i}\sigma_{\mu\nu} \frac{q_\nu}{2m} F_2(q^2) \Big) u(\vec{p}, \vec{s})$$

$q^2 = -Q^2$

$$\begin{aligned} F_1(0) &= 1 & \text{CVC} \\ F_2(0) &= \mu - 1 & \text{anomalous magnetic moment} \end{aligned}$$

Sachs form factors

$$\begin{aligned} G_e(q^2) &= F_1(q^2) + \frac{q^2}{4m^2} F_2(q^2) \\ G_m(q^2) &= F_1(q^2) + F_2(q^2) \end{aligned}$$

$$\begin{aligned} \langle \vec{p}, \vec{s} | A_\mu^{p-n} | \vec{p}, \vec{s} \rangle = \bar{u}(\vec{p}, \vec{s}) \Big(& \gamma_\mu \gamma_5 \, g_A(q^2) \\ & + \mathrm{i}\gamma_5 \frac{q_\mu}{2m} h_A(q^2) \Big) u(\vec{p}, \vec{s}) \end{aligned}$$

Quenched $\beta = 6.2,\ m \to 0$

($\beta = 6.0$ and 6.4 give very similar results)

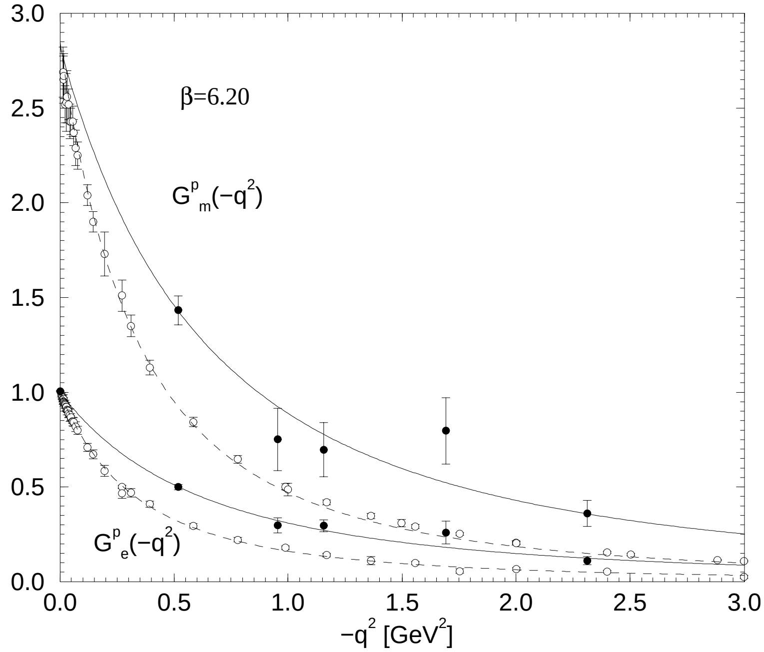
$N_f = 2$ $\beta = 5.25$ $m_\pi \approx 750$ MeV

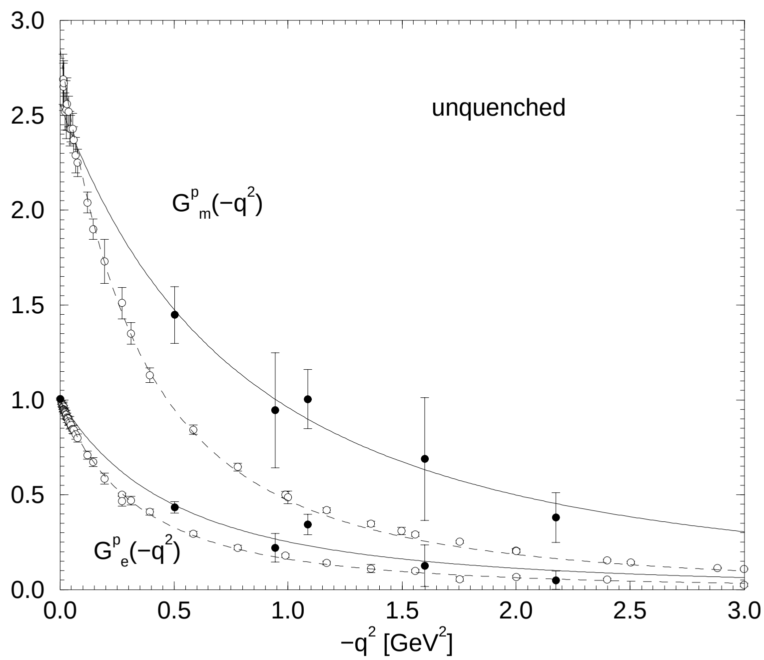
Extrapolation using chiral perturbation theory is in progress.

Meißner et al.

Fit ansätze:

$$
\begin{aligned}
G_e(q^2) &= \frac{1}{(1+q^2/m_e^2)^2} \\
G_m(q^2) &= \frac{\mu}{(1+q^2/m_m^2)^2} \\
g_A(q^2) &= \frac{g_A}{(1+q^2/m_A^2)^2}
\end{aligned}
$$

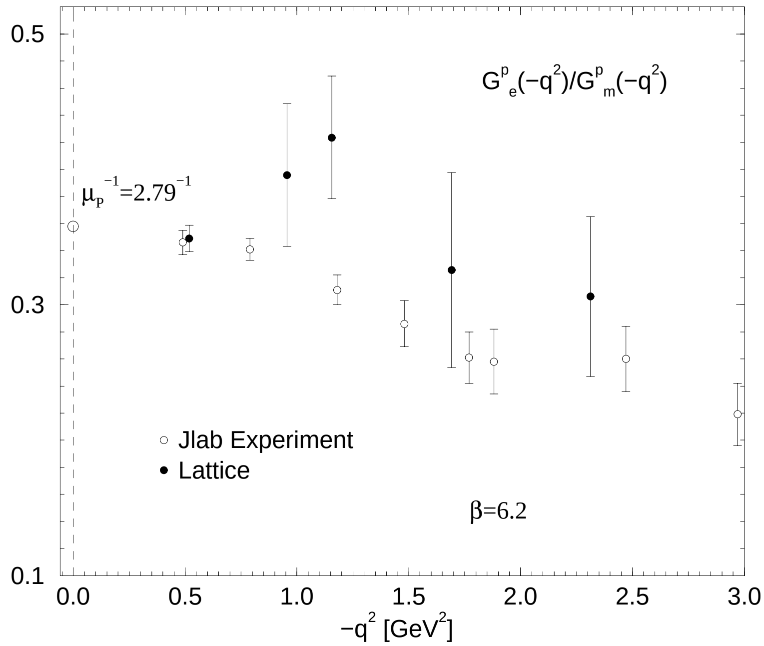


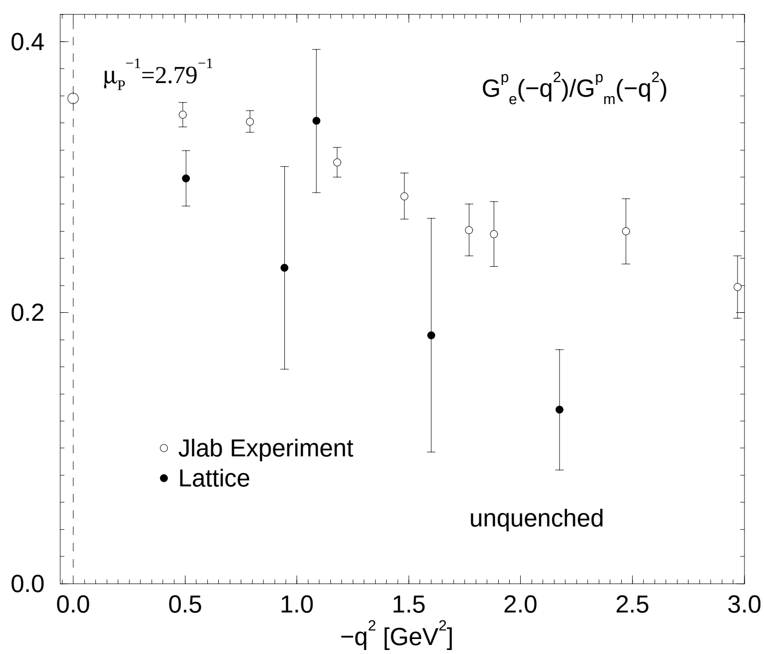


Quenched: $r^e_{\rm rms} = \sqrt{12}/m_e \approx 0.7$ fm $< r^{exp}_{\rm rms} = 0.83$ fm

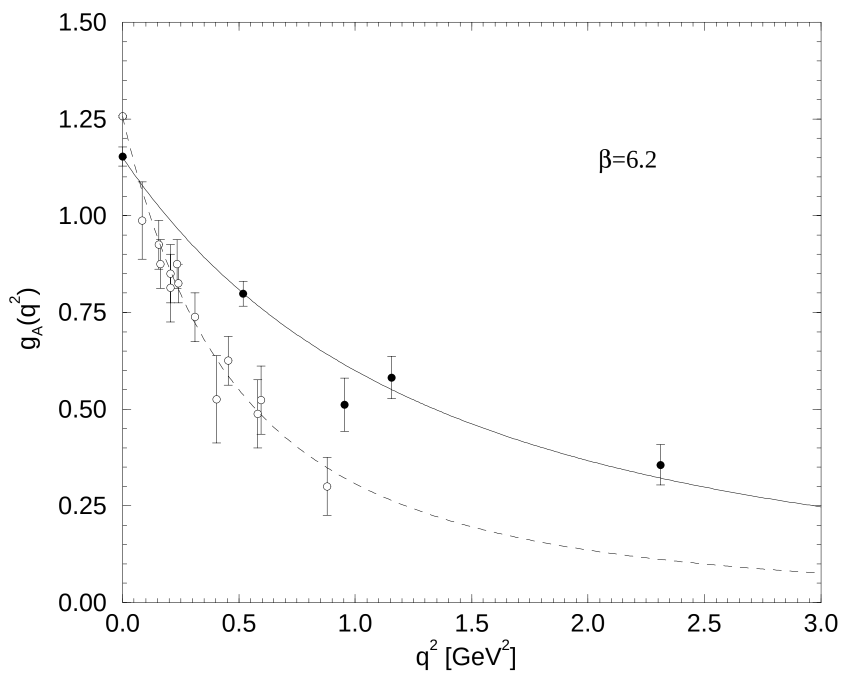
$r^m_{\rm rms} \approx r^e_{\rm rms}$

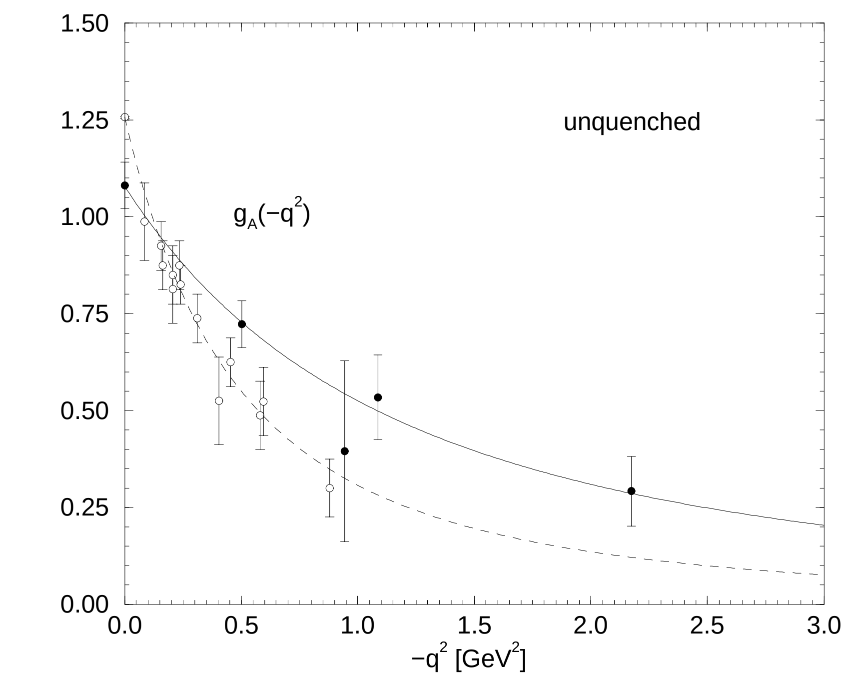
$\mu_p,\ \mu_n$ ✓





$$r^e_{\rm rms} \overset{?}{>} r^m_{\rm rms}$$





Quenched: $r_{\rm rms}^A < r_{\rm rms}^{exp}$

Conclusions

- Precision of numerical results is steadily improving
  - Computer power
  - Improved action
  - Improved operators
  - Renormalization
  - Chiral perturbation theory

- The price is high

  $\mathcal{O}(30)$ parameters to be computed to non-perturbative precision

- Improvement has paid off: Discretization (cut-off) errors are small and proportional to a^2!

- Serious simulations including dynamical quarks have just begun

  3–5 years behind quenched calculation

- First results on nucleon structure including dynamical quarks are available

  QCDSF–UKQCD Collaboration

- Results (of quenched and full QCD) are in qualitative agreement with experiment, except for $\langle \mathrm{x}^n \rangle$

- To safely extrapolate to the chiral limit, need to do simulations at $m_\pi \lesssim 300$ MeV

  Requires terascale computers

- Higher-twist contributions are generally small: $\approx \Lambda^2/Q^2$ with $\Lambda \ll m_p$

  Important prediction !

- Challenge: Sea quark and gluon distribution functions

  For example $\langle \mathrm{x}^n \rangle_{\mathrm{sea}}$, Δs, δs, ΔG, δG, $\cdots$

  Computationally very demanding

- Need
  - better data for nucleon structure functions, in particular at large x
  - phenomenological parton distribution functions (like MRS) with error bars