Baryon spectrum and Magnetic Moments in Nonperturbative QCD

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Contents:

- Introduction
- Baryon operators. Y or Δ ?
- Baryon Green's function
- Field distribution in baryon
  Confinement and string shape
- Hamiltonian for baryon
- How to calculate constituent quark mass and magnetic moments from string tension alone.
- Spectrum spin-averaged.
- Spin forces in baryon
- Hybrid baryons, 5q states etc.
- Conclusions

- Introduction

The goal is — starting from known QCD vacuum properties derive baryon spectrum and structure.

Basics: Vacuum is described by correlators of gluonic field

$$\underbrace{g^2\langle F_{\mu\nu}(x)\Phi F_{\lambda\sigma}(0)\Phi\rangle}_{\parallel},\ \ldots \underbrace{g^n\langle F\Phi F\Phi F\Phi \ldots F\Phi\rangle}_{\text{contributes} \sim 1\%}$$

$$(\delta_{\mu\lambda}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\lambda})D(x)+ + \partial_\mu(x_\lambda D_1 \delta_{\nu\sigma} + \text{perm})$$

99% of pert. and nonpert. contents is given by $D(x)$, $D_1(x)$. (lattice data: G. Bali, Deldar + ...)

String tension

$$\boxed{\sigma = \frac{1}{2}\int d^2x\, D(x)}$$

all spin-averaged spectrum defined by $\sigma = 0.18\ GeV^2$

Spin splittings depend also on D, D_1.

D, D_1 fixed by lattice measurements

$\alpha_s(Q^2)$ – " "

current quark masses – fixed + theory.

No fitting parameters!

5q baryons

Heavy - light
4q meson

In the same way as for hybrids

$M(4q) = M_1 + 2.4$ GeV $\qquad (\sigma = 0.16$ GeV$^2)$

$$\boxed{\Delta M_{5q} = 2.4 - 0.72 + 0.3 = 2 \text{ GeV}}$$

Thus 5q states of this configuration should be in the region $M_B \sim 3$ GeV.

Color Coulomb correction and spin interaction can contribute around -0.5 GeV

$$M(5q) \geq 2.5 \text{ GeV}$$

Expected Z^+ $(\bar{q} = \bar{s})$ with decay $Z^+ \to NK$

From soliton models much lower mass.

(Diakonov et al., Weigel, Polyakov et al, ..)

$$M_{Z^+} \sim 1.5 \text{ GeV}$$

But confinement is not implemented in chiral soliton models.

Hybrid baryons

+ ΔE

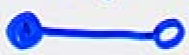
heavy-light hybrid

energy flow to spectator quarks.

$$H_0 = M_1 + \frac{\mu_2 + \mu_3}{2} + \frac{\bar{p}_2^{\,2}}{2\mu_2} + \frac{\bar{p}_3^{\,2}}{2\mu_3} + \sigma(r_{12} + r_{23})$$

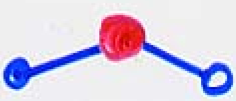
$$\Delta H_{self.} = -\frac{2\sigma}{\pi\mu}$$ (for any $q, \bar{q}$)

Heavy-light



$$M = M_0 + \Delta H_{self} = 0.938 - 0.217 = 0.721 \text{ GeV}$$

$\sigma = 0.16$



$$M = M_0 + \Delta H_{self} = 1.355 \text{ GeV} \quad (L=0)$$

$$\begin{array}{r} 1.355 \\ +\ .373 \\ \hline 1.728 \text{ GeV} \end{array}$$

HL Hybrid radial excit.

$$M_n = 1.728 \text{ GeV} + 0.74 \cdot n, \quad n = 0, 1, 2.$$

For Hybrid baryon : add ΔE

ΔE = E (excitation in 3q) − E (exc. in HL)

e.g. radial. $n=1$ 0.87 GeV   0.563 GeV

$\Delta E \sim 0.3$ GeV

Total excitation energy of hybrid

$$\Delta M = 1.728 - 0.721 + 0.3 \cong \boxed{1.3 \text{ GeV}}$$

Comp. to $\Delta M \sim 0.93 \pm 0.1$ GeV Capstick, Page (flux tube)

For $q\bar{q}$ lattice $\Delta M \sim 1.3$ GeV!

Spin forces   Perturbative

Known since '75 (De Rujula, Georgi, Gl.).

To lowest order: $O(\alpha_s)$

$$\frac{1}{r}\frac{dV_1}{dr} \equiv 0$$

$$V_{SD}^{(\varepsilon)} = \frac{2\alpha_s}{3}\sum_{i>j}\left[\frac{(\vec{R}_{ij}\times\vec{p}_i)\vec{\sigma}_i}{4\mu_i^2 R_{ij}^3} + \frac{(\vec{R}_{ji}\times\vec{p}_j)\vec{\sigma}_j}{4\mu_j^2 R_{ij}^3}\right]$$

$$V_{SD}^{(2)} = -\frac{2\alpha_s}{3(N_c-1)}\sum_{i>j}\frac{(\vec{R}_{ij}\times\vec{p}_j)\vec{\sigma}_i + (\vec{R}_{ji}\times\vec{p}_i)\vec{\sigma}_j]}{\mu_i\mu_j\ R_{ij}^3}$$

$$V_4(R_{ij}) = \frac{32\pi\alpha_s}{3}\ \delta^{(3)}(\vec{R}_{ij})$$

$$V_3(R_{ij}) = \frac{4\alpha_s}{R_{ij}^3}$$

$$V_{SD}^{(pert)} = V_{SD}^{(\varepsilon)} + V_{SD}^{(2)} + \sum_{i>j}\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j\, V_4 + S'_{ij}\ V_3}{12\mu_i\mu_j(N_c-1)}$$

$$S_{ij} = 3\vec{\sigma}_i\cdot\vec{n}\ \vec{\sigma}_j\cdot\vec{n} - \vec{\sigma}_i\cdot\vec{\sigma}_j$$

$$\vec{n} = \frac{\vec{R}_{ij}}{R_{ij}}$$

Spin forces (cont'd) Nonperturbative.

$$\boxed{\frac{1}{r}\frac{dV_1}{dr}} = -\int_0^r \frac{d\lambda}{r}\left(1-\frac{\lambda}{r}\right)\int_{-\infty}^{\infty} d\nu\left[D(\lambda,\nu)+D_1(\lambda,\nu)+\lambda^2\frac{\partial D_1}{\partial\lambda^2}\right] \approx -\frac{\sigma}{r}$$

$$-\int_{-\infty}^{\infty}\nu^2 d\nu\int_0^r\frac{d\lambda}{r}\frac{\partial D_1}{\partial\lambda^2}$$

$$\boxed{\frac{1}{r}\frac{dV_2}{dr}} = \int_0^1\beta d\beta\int_{-\infty}^{\infty}d\nu\left[D(r_{ij},\nu)+D_1+(r_{ij}^2+\nu^2)\frac{\partial D_1}{\partial\nu^2}\right]$$

$$\boxed{\vec{r}_{ij} = \vec{r}_i - \beta\,\vec{r}_j}$$

$$\boxed{\frac{1}{r}\frac{d\varepsilon}{dr} = \frac{1}{r}\sigma}$$

$$\boxed{V_3(u)} = -\int_{-\infty}^{\infty}d\nu\, u^2\frac{\partial D_1(u,\nu)}{\partial u^2}$$

$$\boxed{V_4(u)} = \int_{-\infty}^{\infty}d\nu\left[3D(u,\nu)+3D_1(u,\nu)+2u^2\frac{\partial D_1}{\partial u^2}\right]$$

$$\boxed{V_5} = -\int_0^1\beta d\beta\int_{-\infty}^{\infty}d\nu\,\frac{\partial D_1(r_{ij},\nu)}{\partial\nu^2},$$

$$D = D\left(\sqrt{r^2+\nu^2}\right),\quad D_1(r,\nu) = D_1\left(\sqrt{r^2+\nu^2}\right)$$

$$D, D_1 = D_0, D_{1,0}\exp\left(-\frac{\sqrt{r^2+\nu^2}}{T_g}\right)$$

Spin forces in a baryon

nonperturbative were not known (except for asymptotics of one spin-orbit term – Thomas precession.)

New results (Yu.S. to be published

$$\hat{V}_{SD} = \left(\frac{v|\,n}{n\,w}\right)\left(\frac{v|\,n}{n\,w}\right) + \left(\frac{v|\,n}{n\,w}\right)\left(\frac{v|\,n}{n\,w}\right) + \left(\frac{v|\,n}{n\,w}\right)\left(\frac{v|\,n}{n\,w}\right)$$

The only approximation: lowest correlator $\langle \sigma F \sigma F \rangle$ is retained. Expected accuracy: 1–2%

The dominant part is green (special case is possible for some states with red important)

$$V_{SD} = \sum_{i=1}^{3} \frac{\vec{\sigma}^{(i)}\vec{L}^{(i)}}{2\mu_i^2}\left(\frac{1}{r_i}\frac{dV_1}{dr_i} + \frac{1}{(1+\frac{m_i}{\mu_i})r_i}\frac{d\varepsilon}{dr_i}\right) +$$

$$+ \frac{1}{N_c-1}\sum_{i<j} \frac{\vec{\sigma}^i\vec{L}^j + \vec{\sigma}^j\vec{L}^i}{2\mu_i\mu_j}\frac{1}{r_j}\frac{dV_2}{dr_j} +$$

$$+ \frac{1}{N_c-1}\sum_{i<j}\frac{1}{12\mu_i\mu_j}\left[\vec{\sigma}^i\vec{\sigma}^j V_4 + (3\vec{\sigma}^i\vec{n}^i\cdot\vec{\sigma}^j\vec{n}^j - \vec{\sigma}^i\vec{\sigma}^j\cdot\vec{n}^i\vec{n}^j)V_3\right]$$

$$+ \sum_{i>j} V_5(\vec{r}_{ij})\frac{\vec{\sigma}^i\vec{r}_{ij}\cdot\vec{L}^j\vec{r}_{ij}}{(N_c-1)\,2\mu_i\mu_j}$$

Spectrum (cont'd)

Ground state $\frac{N+\Delta}{2} = 1.086$ GeV (Exp).

$$M_0 = 6\mu_0 = 2.16 \text{ GeV} \quad (\sigma = 0.15 \text{ GeV}^2)$$

$$\mu_0 = 0.36 \text{ GeV}$$

$$\Delta H_{self} = -\frac{6\sigma}{\pi\mu_0} = -0.795 \text{ GeV}$$

$$\Delta H_{coul} = -0.274 \text{ GeV} \quad (\alpha_s = 0.4).$$

$$\Delta H_{string} \equiv 0$$

$$M = M_0 + \Delta H_{self} + \Delta H_{Coul} + \Delta H_{string}$$

$$M(\mathcal{K}=0) = 1.08 \text{ GeV}$$

Excitations:

	M	$(M_N + M_\Delta)$ exp
$\mathcal{K}=n=0$	1.08	1.08
$\mathcal{K}=0, n=1$ br. m	1.91	1.8
$\mathcal{K}=0, n=2$ br. m.	2.62	?
$\mathcal{K}=1, n=0$	1.63	1.6
$\mathcal{K}=2, n=0$	2.04	?

Spectrum spin-averaged

restoring μ_i :

$$\boxed{H_0 = \sum_i \left(\frac{\bar{p}_i^2}{2\mu_i} + \frac{\mu_i}{2} + \sigma r_i \right)} \quad ; \quad H_0 \Psi = M_0 \Psi$$

Hyperspherical method

(Yu.S., A.Badalian + Yu.S. '66)

Hyperradius $\rho^2 = \sum_{i<j} \beta_{ij} (\bar{r}_i - \bar{r}_j)^2$

Hyper momentum

$\mathcal{K} \geq L \quad ; \quad \mathcal{K} = 0, 1, 2, \ldots$

$$\Psi = \frac{1}{\rho^{5/2}} \sum u_{\mathcal{K}}^{\nu} \chi_{\mathcal{K}}^{\nu}(\rho)$$

$$\frac{d^2}{d\rho^2} \chi_{\mathcal{K}}^{\nu} + \left(2\mu M_0 - \frac{(\mathcal{K}+\frac{3}{2})(\mathcal{K}+\frac{5}{2})}{\rho^2} \right) \chi_{\mathcal{K}}^{\nu} = 2\mu \sum U_{\mathcal{K}\mathcal{K}'} \chi_{\mathcal{K}'}$$

For confining interaction convergence is excellent : $M_0 (\mathcal{K}=0)$ is accurate within 1% !

(By comparison to Green-function MC method)

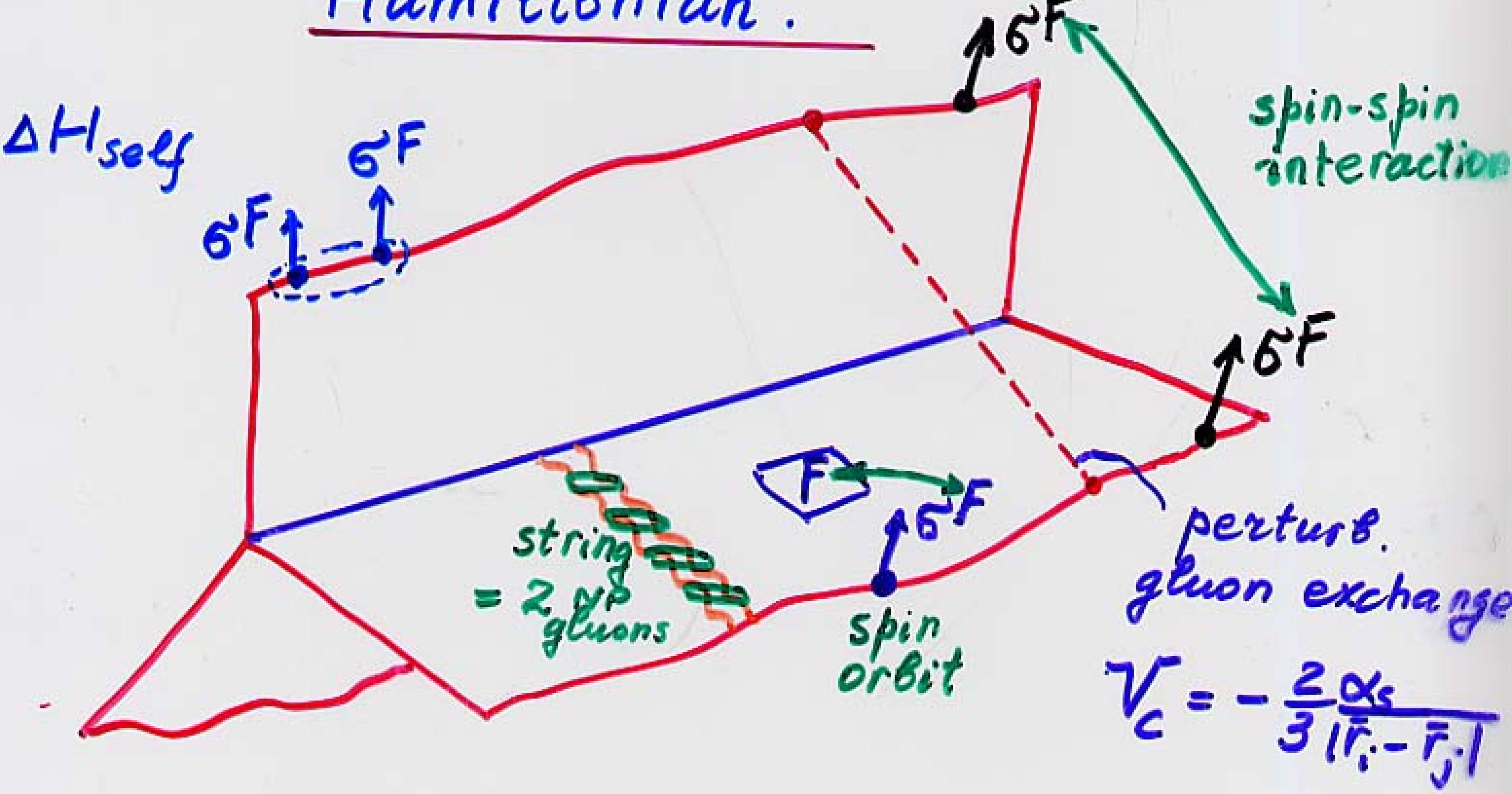
Fabre + Yu.S. Ann. Phys. 212 (91) 235

$$M_B^{(0)} = \frac{4}{2^{1/4}\sqrt{3}} (\lambda_{\mathcal{L}})^{3/4}$$

$$\lambda_{\mathcal{L}} = 3 \, (0.56\sigma)^{2/3} \left(1 + \frac{1}{\sqrt{3\mathcal{L}(\mathcal{L}+1)}} \right) \left[\mathcal{L} (\mathcal{L}+1) \right]^{1/3}$$

$$\mathcal{L} = \mathcal{K} + \frac{3}{2} \; ;$$

Schematic derivation of 3q Hamiltonian.



$$\boxed{H = H_0 + \Delta H_{string} + \Delta H_{Coul.} + \Delta H_{self.} + \Delta H_{spin}.}$$

$$H_0 = \sum_{i=1}^{3}\left\{\sqrt{\bar p_i^2 + m_i^2} + \sigma r_i\right\}; \quad \Delta H_{Coul} = -\sum_{i>j}\frac{2/3\,\alpha(r_{ij})}{r_{ij}}$$

$$\Delta H_{self} = -\sum_{i=1}^{3}\frac{4\sigma\,\eta_i}{\pi\cdot 2\mu_i}, \quad \eta_i(\sigma, \mu_i, m_i)$$

$$\eta_i = 1, \quad m_i \to 0$$

Yu. S. PLB 515 (2001) 137.

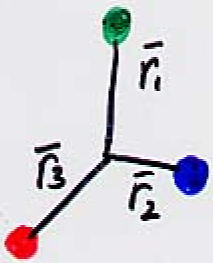
$$\eta_i = 0, \quad m_i \to \infty$$

Baryon magnetic moments

B. Kerbikov
Yu.S. $\boxed{\text{PRD}}$

Hamiltonian for the 3q baryon:

$$H = \sum_{k=1}^{3} \left\{ \left(\frac{m_k^2}{2\mu_k} + \frac{\mu_k}{2} \right) + \frac{\vec{p}_k^{\,2}}{2\mu_k} + \sigma \sum |\vec{r}^{\,k}| \right\}$$



m_1, m_2, m_3 – current masses!
($m_k \to 0$ for N)

$$M_n(\mu_k) = \frac{m_k^2}{2\mu_k} + \frac{\mu_k}{2} + E_n(\mu_k)$$

$$\sum_k \left(\frac{\bar{p}_k^{\,2}}{2\mu_k} + \sigma |\bar{r}_k| \right) \Psi_n = E_n \Psi_n$$

$$\Psi_n = \sum_{K, \nu_i,} \mathcal{Y}_{K\nu_i}(\Omega_k)\, \chi_{K\nu_i}(\rho)$$

Lowest $K = 0$, $\mathcal{Y}_0 = \text{const}$, $\rho^2 = \sum |\bar r_k - \bar r_0|^2$
yields 98% of E_n.

$$\frac{\partial M_n(\mu_k)}{\partial \mu_k} = 0, \quad \mu_k = \mu_k^{(0)} := \sqrt{\sigma}\, C_k$$

The only input

$$\boxed{\sigma = 0.15 \text{ GeV}^2; \; m_s = 0.175 \text{ GeV}, \; m_u = m_d = 0}$$

$$\mu_{magn} = \sum \frac{e_k \sigma_z^{(k)}}{2\mu_k}; \quad \langle \mu_{magn} \rangle = \langle \sum \cdots \rangle_{\Psi}$$

Baryon	p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	Ω^-	Δ^{++}
theory	2.54	−1.69	−0.69	−0.9	0.8	2.48	−0.63	−1.49	−2.04	4.36
exper.	2.79	−1.91	−0.61	−1.16		2.46	−0.65	−1.25	−2.02	4.52

Constituent mass and magnetic moments.

$$\frac{\delta H}{\delta \mu_i} = 0 \rightarrow \mu_i = \sqrt{\bar{p}_i^2 + m_i^2} \quad \text{- operator.}$$

$$H\Psi = M_B \Psi$$

approximate method (accuracy checked ~ 5%)

$$\frac{\delta M_B}{\delta \mu_i} = 0 \; ; \qquad \mu_i = \sqrt{\sigma}\, C\,(\text{quantum number}).$$

For $\underline{u, d}$ quarks

$$\underline{\mu_u} = \mu_d = \underbrace{2\sqrt{\frac{2\sigma}{\pi}}\left[\frac{2}{3\cdot 5^{1/3}}\left(1+\frac{2}{3\sqrt{5}}\right)\right]^{3/4}}_{C\sqrt{\sigma}} = \underline{0.37\,\text{GeV}}$$

for $\sigma = 0.15\,\text{GeV}^2$

For s quark, $m_s = 0.245$ GeV

$$\underline{\mu_s} = \underline{0.46\ \text{GeV}}.$$

Magnetic moments

$$\mu_z^B = \sum_{k=1}^{3} \frac{e_k \sigma_z^{(k)}}{2\mu_k} \; ; \qquad \mu^B \equiv \langle \Psi_B | \mu_z | \Psi_B \rangle$$

$$\mu^{\text{proton}} = \frac{M_P}{\mu_u} \cong \underline{2.54}\ (\text{Nucl. magn.}) \; ; \; \mu^{\text{Neutr.}} = -\frac{2}{3}\mu^{Pr} = \underline{-1.69}$$

$$\mu^{\Omega^-} = -\mu^{Pr}\cdot\left(1 + \frac{3}{4}\frac{m_s^2}{C^2\sigma} - \frac{15}{32}\frac{m_s^4}{C^4\sigma^2}\right)^{-1} = \underline{-2.04}$$

$$C = 0.957.$$

Properties of Hamiltonian.

- H depends on σ and <u>current</u> masses only.

- $\hat{L}_i = 0$

$$\frac{\delta H}{\delta \mu_i} = 0 \;, \to \; \mu_i = \sqrt{p_r^2 + m_i^2}$$

$$\frac{\delta H}{\delta \nu_i} = 0 \to \; \nu_i = \sigma r_i$$

$$\boxed{H_0 = \sum_{i=1}^{3} \left\{ \sqrt{\vec{p}_{ir}^{\,2} + m_i^2} + \sigma r_i \right\}}$$

$$\vec{r}_i = (\vec{z}_i - \vec{Y})\,;$$

Familiar rel. quark model Hamiltonian.

- $L_i > 0$, $L_i < 4$

Take string rotation as a correction:

$$H = \sum_{i=1}^{3} \left\{ \sqrt{\vec{p}_i^{\,2} + m_i^2} + \sigma r_i + \Delta h_i^{(string)} \right\}$$

$$\Delta h_i^{(string)} = - \frac{2\sigma^2 L_i (L_i + 1)}{M_0^4} \left(1 + \frac{4}{3(L_i + 1)} \right)$$

Derivation of Hamiltonian

$$\int (\mathcal{D}r)_{xy}\, e^{i\int_0^T L(\dot r, r)\,dt} = \langle x | e^{-iHT} | y \rangle$$

$$H = H(p, r), \quad p = \frac{\partial L}{\partial \dot r}$$

$$H(p, r, \nu, \mu) = \sum_{i=1}^{3} \Big\{ \frac{(\vec p_r^{(i)})^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2} + \frac{\hat L_i^2 / r_i^2}{2[\mu_i + \int \beta_i^2 \nu_i(\beta_i)\, d\beta_i]} + \int_0^1 \frac{\sigma^2 d\beta_i \vec r_i^{\,2}}{2\nu_i(\beta_i)} + \frac{1}{2}\int_0^1 \nu_i\, d\beta_i \Big\}$$

$$\mu_i, \nu_i \; : \quad \frac{\delta H}{\delta \mu_i} = 0, \quad \frac{\delta H}{\delta \nu_i} = 0$$

$$p_r^2 = \frac{(\vec p \vec r)^2}{r^2}, \quad p_{it}^2 \equiv \frac{(\vec p_i \times \vec r_i)^2}{r_i^2} =$$

$$= \Big(\mu_i + \underbrace{\int \beta_i^2 \nu_i\, d\beta_i}_{\text{string moment of inertia}/r_i^2}\Big)^2 \frac{(\dot{\vec r}_i \times \vec r_i)^2}{\vec r_i^{\,2}}$$

$$\hat L_i^2 = (\vec p_i \times \vec r_i)^2 = p_{it}^2 r_i^2$$

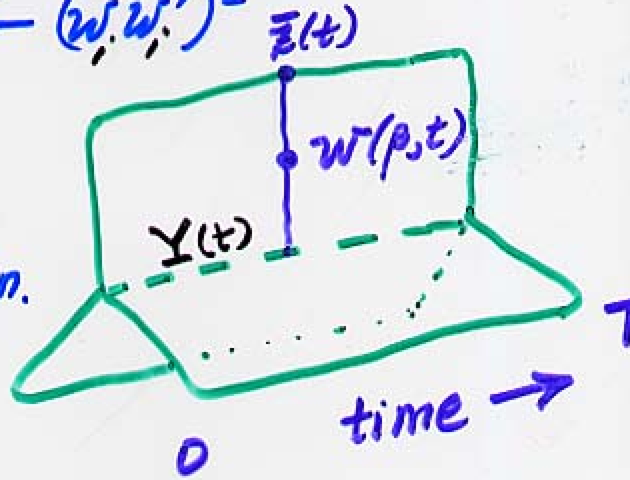
Transformation of the action

$$\text{Action} = \sum_{i=1}^{3} \left(\mathcal{K}_i + \sigma \left(S_i + \delta_{int} \right) \right)$$

← attenuation of simple area law due to lobe interference

$$\mathcal{K}_i = \frac{m_i^2}{2\mu_i(t)} + \frac{\mu_i(t)}{2}\left(1 + \dot{\vec{z}}_i^{\,2}\right)$$

$$\sigma S_i = \sigma \int_0^T dt_i \int_0^1 d\beta_i \sqrt{\dot{w}_i^2 w_i'^2 - (\dot{w}_i w_i')^2}$$



approximation:

Disregard string excitation since $\Delta M_{exc.} \gtrsim 1$ GeV

Take string straight.

$$\vec{w}_i(\beta, t) = \vec{z}_i(t)\,\beta_i + \vec{Y}(t)\,(1-\beta_i)$$

↑ string-junction trajectory.

Einbein variable:

$$e^{-\sigma \iint dt\, d\beta \sqrt{\dot{w}^2 w'^2 - \dot{w} w'}} = \int D\nu(t)\, e^{-\frac{\sigma}{2} \iint dt\, d\beta\, a}$$

$$a_i = \nu_i \left(\dot{w}^2 - \frac{(\dot{w} w')^2}{(w')^2} \right) + \frac{\sigma^2 w'^2}{\nu_i}$$

ν_i (at stat. point) will be energy density of string.

Transformation of the Green's function

$$G_B \sim \prod_{i=1}^{3} (m_i - \hat{D}_i) \int \underbrace{ds_i \, Dz^{(i)}} e^{-K_i} \langle W_3 e^{g\sigma F} \rangle$$

$$\underbrace{ds \, Dz_4} \, D\vec{z}$$

Introducing new mass variable!
(To be constituent mass at stationary point)

real time $\rightarrow$

$$\boxed{\frac{dz_4}{d\tau} = 2\mu(\tau) = 2\mu(z_4)}$$

proper time

$$Dz_4 = \prod_{n=1}^{N\to\infty} \frac{d\Delta z_4(n)}{(4\pi\varepsilon)^{1/2}} \, \delta\left(\sum_{n=1}^{N} \Delta z_4(n) - T\right)$$

$$\Delta z_4(n) = z_4(n) - z_4(n-1), \quad N\varepsilon = s, \quad n\varepsilon = \tau$$

$$\delta\left(\sum \Delta z_4(n) - T\right) = \delta\left(\int_0^s d\tau \left(2\mu(\tau) - \frac{T}{s}\right)\right) \rightarrow$$

$$\rightarrow \delta\left(2\bar{\mu} - \frac{T}{s}\right)$$

$$ds \, Dz_4(t) = D\mu(t) \, ds \, \delta\left(2\bar{\mu} - \frac{T}{s}\right) =$$

$$= \text{const } D\mu(t)$$

— Action

$$G_B \sim \prod (m - \hat{D}) \int D^3 \bar{z}(t) \, D\mu(t) \, e$$

3d dynamics!

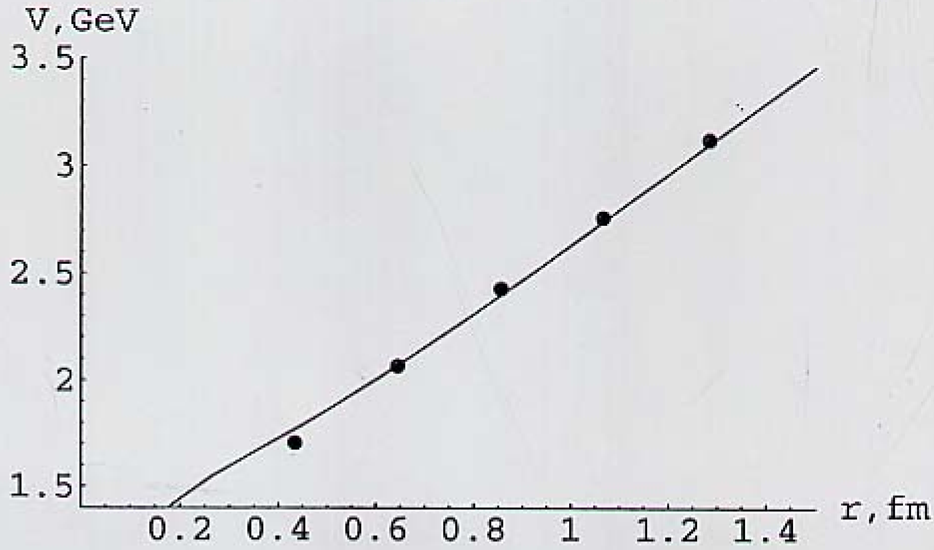


Figure 3: The lattice potential from [7] with $\beta = 5.8$ (points) and the potential $V^{\text{pert}}(r) + V_B(r)$ (solid curve) with $T_g = 0.12$ fm, $\sigma = 0.188$ GeV^2 and $\alpha_s = 0.4$.

Points : from Alexandru, De Forcrand et al.

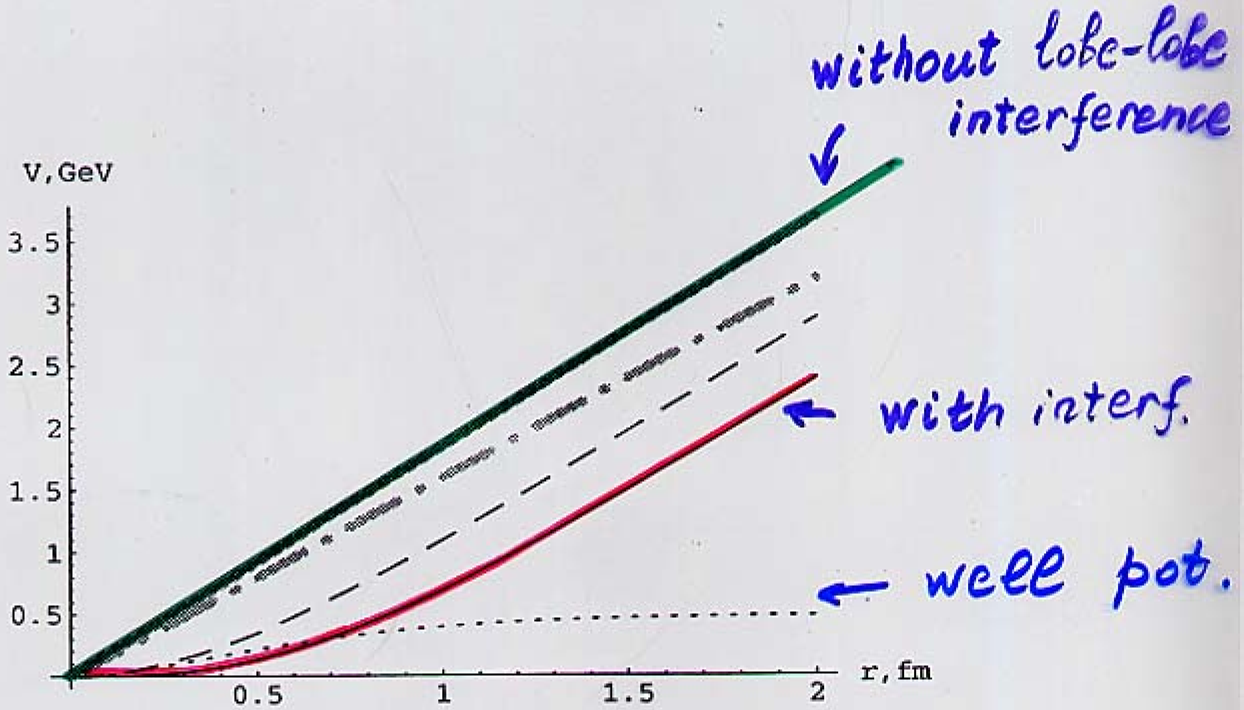
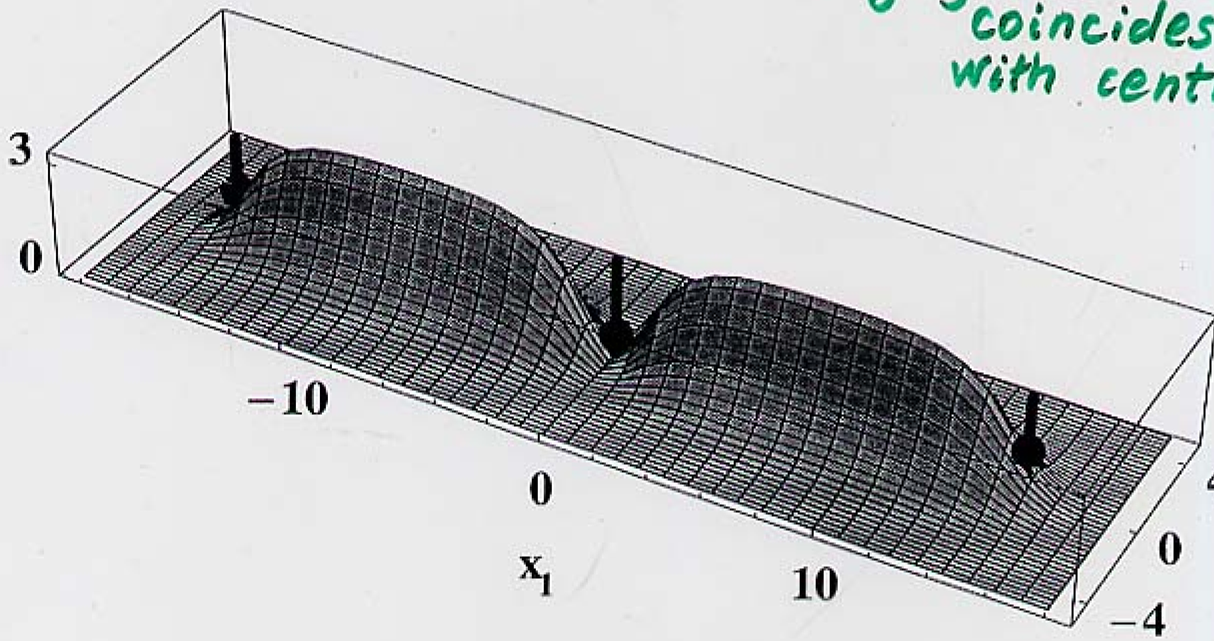


Figure 3: $V_Y^{(B)}(r)$ (solid curve) along with its constituents $3V^{(M)}(r)$ (dashed curve) and $V^{(\mathrm{hole})}(r)$ (dotted curve) in comparison with linear potentials $3\sigma R = \sqrt{3}\,\sigma r$ (solid gray curve) and $3/2\,\sigma r$ (dashed-dotted gray curve). σ=0.21 GeV^2 and $T_g = 0.2$ fm.

Kuzmenko
+ Yu.S.

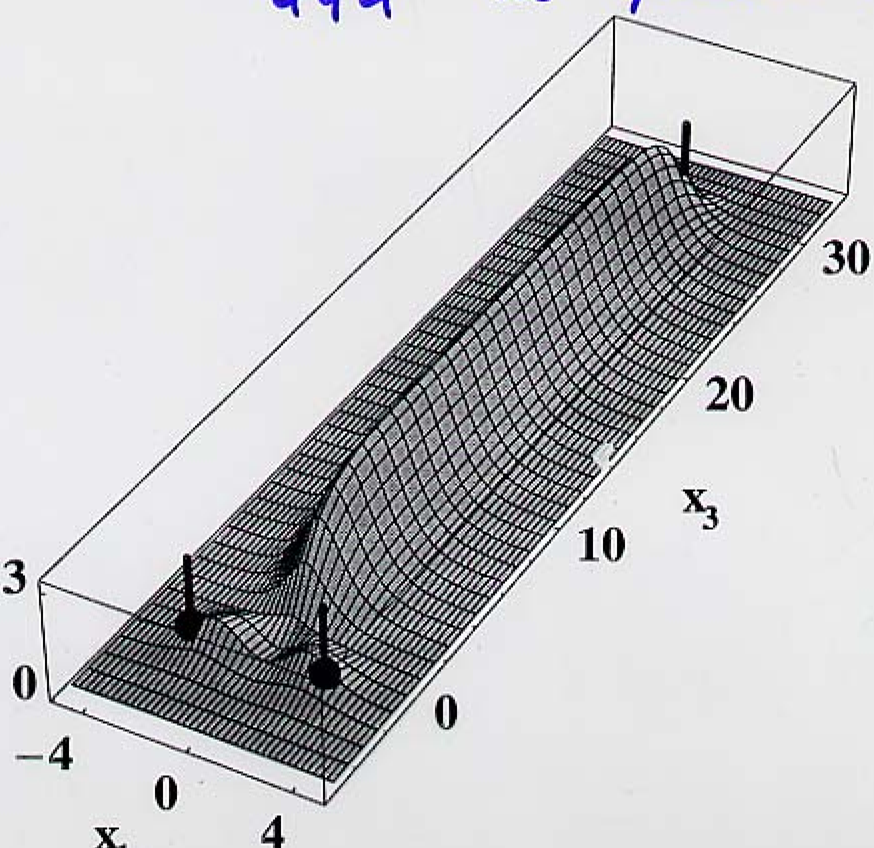
QQQ on one line

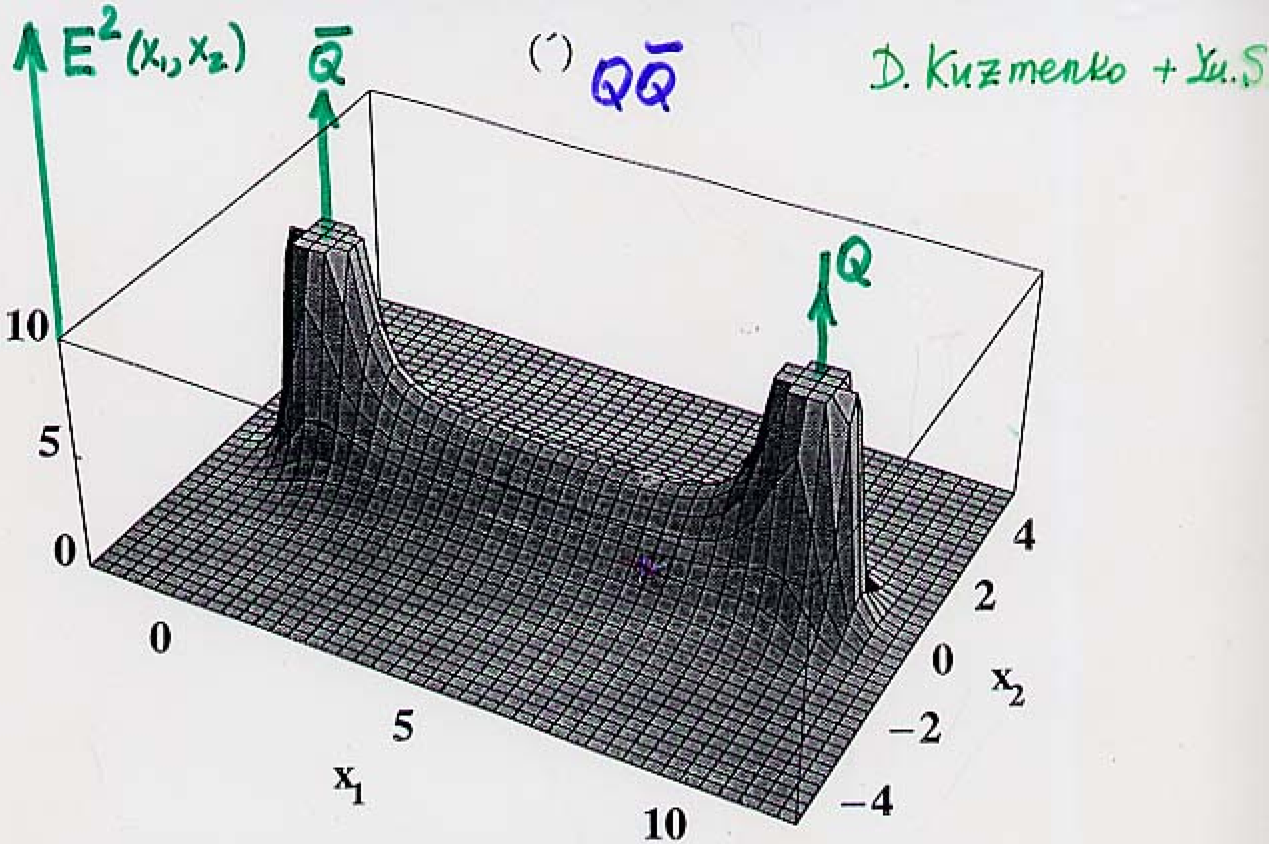
(a) (string junction coincides with central Q)



(b)

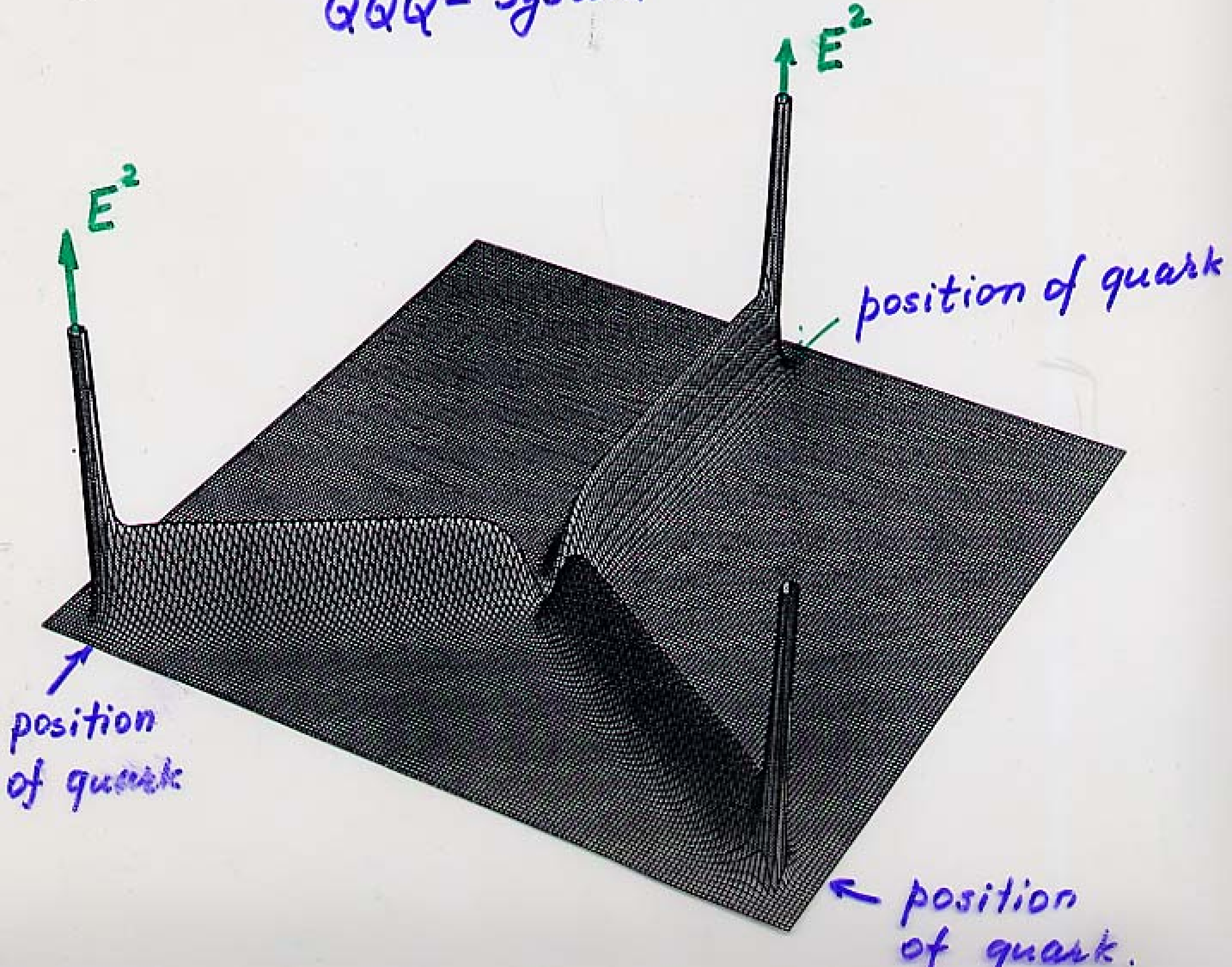
QQQ ( ) as quark - dyquark.





$E^2(x_1, x_2)$
$\bar{Q}$
$Q\bar{Q}$
D. Kuzmenko + Yu.S
Q
10
5
0
0
5
x_1
10
4
2
0
x_2
−2
−4

QQQ- system



E^2
E^2
position of quark
position of quark
position of quark.

Computing field structure of the string.

$$\rho_{\mu\nu}(x) = \frac{\langle \mathrm{tr}\,(W L P_{\mu\nu}(x) L^{+} \rangle}{\langle \mathrm{tr}\, W \rangle} - 1 \to a^2 \langle F_{\mu\nu}(x) \rangle_{Q\bar{Q}}$$

Using Stokes theorem, cluster theorem and keeping only bilocal corr.

$$\boxed{\rho_{\mu\nu}(x) = a^2 \int_S d\sigma_{\rho\lambda}(x')\, D_{\rho\lambda,\mu\nu}(x', x) + O(a^4)}$$

$$\frac{g^2}{N_c} \mathrm{tr} \langle F_{\rho\lambda}(x') \Phi(x', x) F_{\mu\nu}(x) \Phi(x, x') \rangle$$

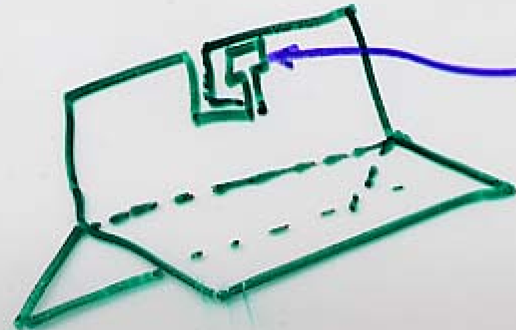
$$\parallel$$

$$(\delta\delta - \delta\delta) D(x - x') + \frac{1}{2}[\partial \ldots] D_1$$

$$D = D(0) \exp\left(-\frac{|x|}{T_g}\right); \; D_1 = \frac{1}{3} D(0) \exp\left(-\frac{|x|}{T_g}\right).$$

$$\boxed{\langle E_1(x_1, x_2) \rangle_{Q\bar{Q}} = \int_0^R dx_1' \int_{-\frac{T}{2}}^{\frac{T}{2}} dx_4' \left[D(0) + D_1(0) - \frac{1}{T_g} \frac{h_4^2 + h_1^2}{2h} D_1(0) \right] e^{-h/T_g}}$$

$$h = \sqrt{h_4^2 + h_1^2 + h_2^2} \; ; \; h_\mu = x_\mu - x_\mu'$$



probing plaquette $P_{\mu\nu}(x)$.

$$\langle W_3 \rangle = tr_Y \exp \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int_{\sum S_i} \langle\langle F(1) \ldots F(n) \rangle\rangle d\sigma(1) \ldots d\sigma(n)$$

approximation (supported by lattice data): keep only lowest correlator
$\langle\langle F(1) F(2) \rangle\rangle \to D(x), D_1(x)$.

Then also include spins: $\exp g \int_0^s \sigma F d\tau$:

$$\langle W_3 \exp(g\sigma F) \rangle = tr_Y \exp \left[\sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int \langle\langle F(1) .. F(n) \rangle\rangle d\rho(1) \ldots d\rho(n) \right]$$

$$\Rightarrow tr_Y \exp\left\{ -\frac{g^2}{2} \int \langle\langle F(1) F(2) \rangle\rangle d\rho(1) d\rho(2) \right\}$$

$$d\rho^{(i)} = d\sigma^{(i)}_{\mu\nu}(u) + \frac{1}{i} \sigma^{(i)}_{\mu\nu} d\tau^{(i)}$$

surface element

spin matrix of i-th quark.

proper time of i-th quark.

New phenomenon: integrals over three lobes of minimal surface $\sum_{i=1}^{3} S_i$

$$\frac{g^2}{2} \iint_{S_i} d\sigma(u)\, d\sigma(v) \langle\langle F(u) F(v) \rangle\rangle = \int d\sigma\left(\frac{u+v}{2}\right) \int d\sigma(u-v) \frac{D(u-v)}{2}$$

$$= S_i \cdot \sigma \qquad ; \quad D(x) = D(0) \exp\left(-\frac{|x|}{T_g}\right)$$

$\mathrm{size}(S_i) \gg T_g$.

Baryon Green's function

Define $|in\rangle$, $|out\rangle$ states

$$\Psi_{in,out}(x,y,z,Y) = \Gamma_{in,out}\, B_Y(x,y,z,Y).$$

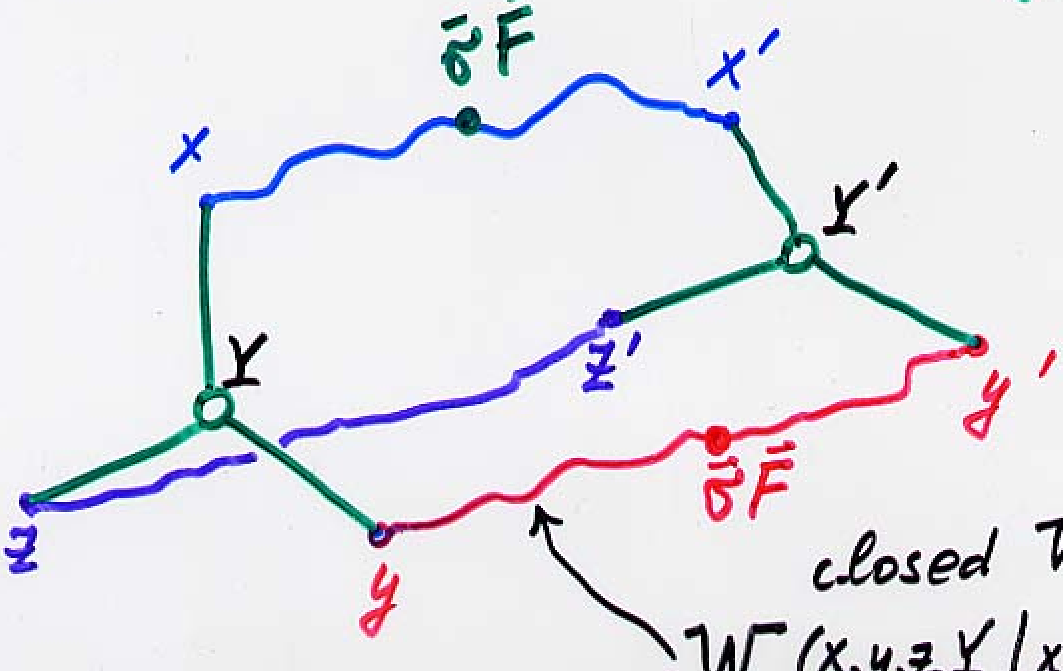
$$G_B(x\,y\,z\,Y\,|\,x'y'z'Y') = \left\langle tr_Y\, \Gamma_{out}\, \underset{\alpha\alpha'}{S}(x,x')\, \underset{\beta\beta'}{S}(y,y')\, \underset{\gamma\gamma'}{S}(z,z')\, \Gamma_{in} \right\rangle$$

$$tr_Y = \frac{1}{6}\varepsilon_{\alpha\beta\gamma}\,\varepsilon_{\alpha'\beta'\gamma'}$$

$$S(x,x') = (m-\hat{D})\int_0^\infty ds\,(\mathcal{D}z)_{xx'}\, e^{-K}\, \underset{z}{\Phi}(x,x')\, e^{g\int_0^s \sigma_{\mu\nu}F_{\mu\nu}\,d\tau}$$

$$\underset{z}{\Phi}(x,x') = P\exp ig\int_{x'}^{x} A_\mu\, dz_\mu$$

$$K = m^2 s + \frac{1}{4}\int_0^s \dot{z}_\mu^2\, d\tau\,, \quad \sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \vec\sigma\vec B & \vec\sigma\vec E \\ \vec\sigma\vec E & \vec\sigma\vec B \end{pmatrix}$$



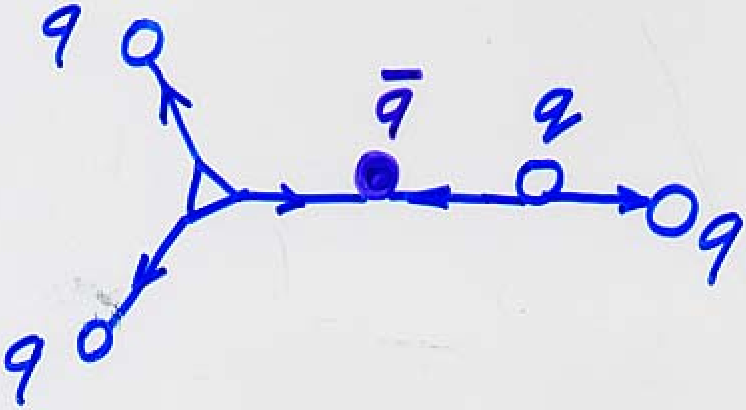
closed Wilson loop

$W_3(x,y,z,Y\,|\,x',y',z',Y')$

gauge invariant !

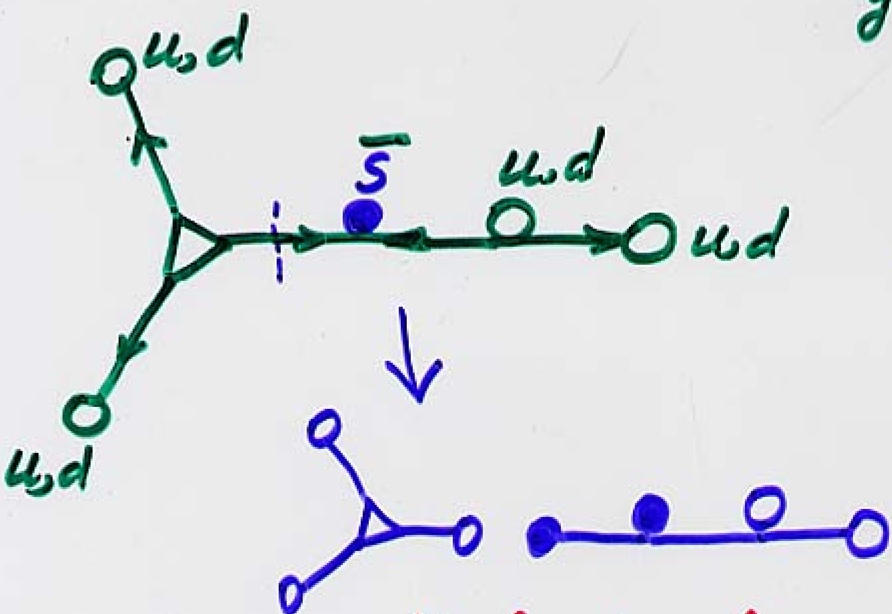
Other possible gauge-invariant baryons.

$$q_{\alpha\beta}(x) \equiv \varepsilon_{\alpha\beta\gamma}\, q^{\gamma}(x)$$



5q baryons

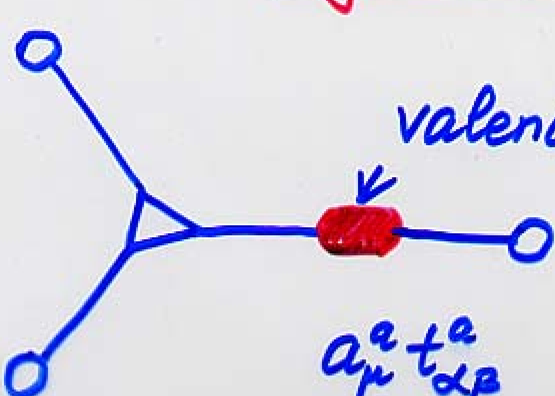
For $\bar{q} = \bar{s}$ one gets Z^+



$Z^+ \to K\, N$

high threshold.

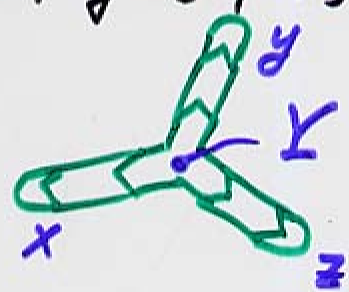
Hybrid baryon:



• Baryon OPERATORS

More carefully: extended quark $q^\beta(x,Y) = q^\alpha(x)\Phi_\alpha^\beta(x,Y)$

Baryon: $B_Y(x,y,z,Y) = \varepsilon_{\alpha\beta\gamma}\, q^\alpha(x,Y)\, q^\beta(y,Y)\, q^\gamma(z,Y)$.

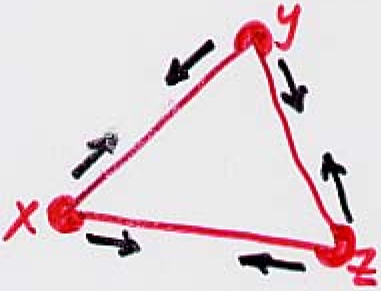


New object:

$$q_{\alpha\beta}(x) = \varepsilon_{\alpha\beta\gamma}\, q^\gamma(x)$$

Try to construct B_Δ:

$$B_\Delta(x,y,z) = q_{\alpha\beta}(x)\Phi^\beta_\gamma(x,y)\, q_{\gamma\delta}(y)\Phi^\delta_\varepsilon(y,z)\, q_{\varepsilon\rho}(z)\Phi^\delta_\alpha(z,x)$$



Not gauge-invariant!

$$q^\alpha(x) \to V^\alpha_\beta(x)\, q^\beta(x)$$

Physical reason: electric fluxes are not connected continuosly

For gluons (3g glueball) both Y-type and Δ-type possible (Kuzmenko + Yu.S., 2002)

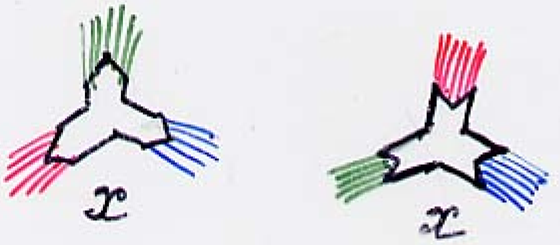
For baryons: only Y-type is possible.

Physical states

Elements:

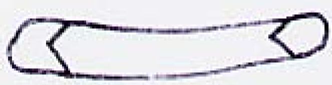
q $\bar{q}$ g

parallel transporter
$$\mathcal{P}(x,y) \equiv P\exp ig\int_y^x A_\mu dz_\mu$$



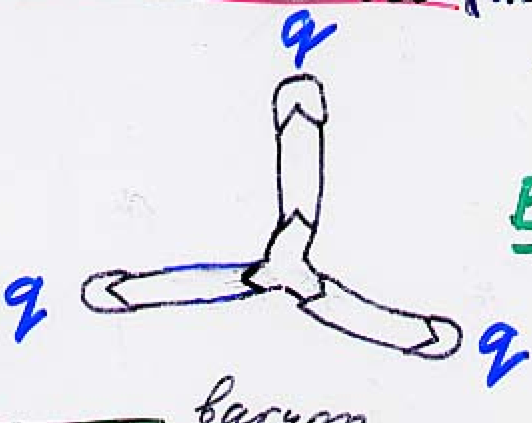
$e_{\alpha\beta\gamma}$ string junction antistring junction

Hadrons: **Principle of gauge invariance:** connect elements in a white set (no color light seen!)



meson

$$\Psi_{as}^{M}(x,y) = \bar{\psi}(x)\,\mathcal{P}(x,y)\,\psi(y)$$



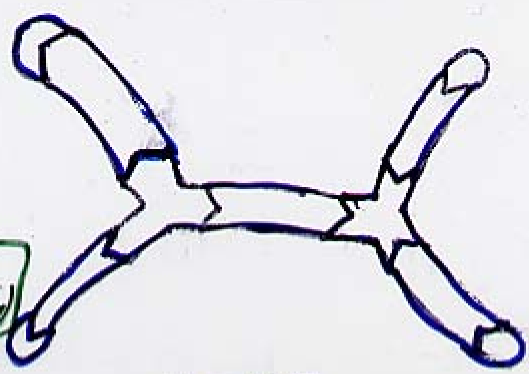
Baryon

baryon

$$\Psi_{as}^{B}(x,y,z) = e^{abc}\,\psi^a(x)\,\mathcal{P}(x,x_0)\,\psi^b(y)\,\mathcal{P}\,\psi^c(z)\,\mathcal{P}$$

hybrid

$$\Psi_{as}^{H}(x,y,z) = \bar{\psi}(x)\,\mathcal{P}(x,z)\,F_{\mu\nu}(z)\,\mathcal{P}(z,y)\,\psi(y)$$



$qq\bar{q}\bar{q}$

Mystery of confinement: how parallel transp. become strings connecting $q\bar{q}$ with force of 14 ton!!

This program: prediction of spin-averaged spectrum through σ only, and spin-splittings via α_s and D.D. was done:

For heavy-light mesons:

Yu.S. '91, Yu. Kalashnikova + Nefediev + Yu.S. '01

For heavy quarkonia

A. Badalian et al. Phys.Rev.D '99 - '01

For light mesons

Yu.S. '91 A. Badalian et al. hep-ph/0202246

For glueballs

A. Kaidalov + Yu.S. '00 PLB

For gluelumps

Yu.S. '00 Nucl. Phys. B.

For hybrids

Yu.S. '91, Yu. Kalashnikova + D. Kuzmenko '99 - '02

For baryons without spin and string, selfenergy corrections

Yu.S. '90, M. Fabre + Yu.S., Ann. Phys. '91.

Agreement with exper. and lattice $5 \div 10\%$

QCD vacuum and Local Hamiltonian

Only pair correlations $\langle F(x) F(0) \rangle$ are essential

Casimir scaling (Bali '00, Deldar)

Size of such „dipoles" is small

$T_g \approx 0.2$ fm. $\quad \langle F(x) F(0) \rangle = D(x) = e^{-\frac{|x|}{T_g}}$

Lattice (Di Giacomo et al, Bali et al.)

analytic (Dosch et al. Yu. S.)

Hybrid excitation energy is large –

$\Delta M_h \approx 1 \div 1.3$ GeV

Lattice + analytic

This means that string excitation can be neglected (in first approxim).

Small T_g and large ΔM_h

$\downarrow$

Local effective Hamiltonian

$\downarrow$

spin-av. spectrum dep. on σ

Spin splitting dep. on α_s, T_g

Conclusions

- Nonperturbative QCD approach predicts spin-averaged baryon spectra without fitting parameters through σ only.
- Nonpert. spin interaction is now available, expressed through D, D_1
- Spin interaction appears much more complicated, than expected and used. Exact computations are needed.
- Origin of constituent quark and gluon masses is understood and they are computed through σ only together with magnetic moments.
- Negative energy states of quarks are strongly coupled by spin operators and may show up in some states (Roper?)
- Pions are emitted by quarks at the end of strings (Yu. S. hep-ph/0201170) and pion exchange forces should be added.