

# Recent Successes and Open Issues in Baryon Structure

A. W. Thomas

CSSM, Adelaide



QCD



Confinement



Vacuum



Test and Refine Models



Baryon Structure



Lattice QCD



Baryon Properties



Spectroscopy



Dense Matter



Nuclear Structure



Bound Nucleon



Baryons In-Medium

## QCD

$$\mathcal{L} = \bar{\psi} i \hat{D} \psi - \frac{G^2}{4}$$

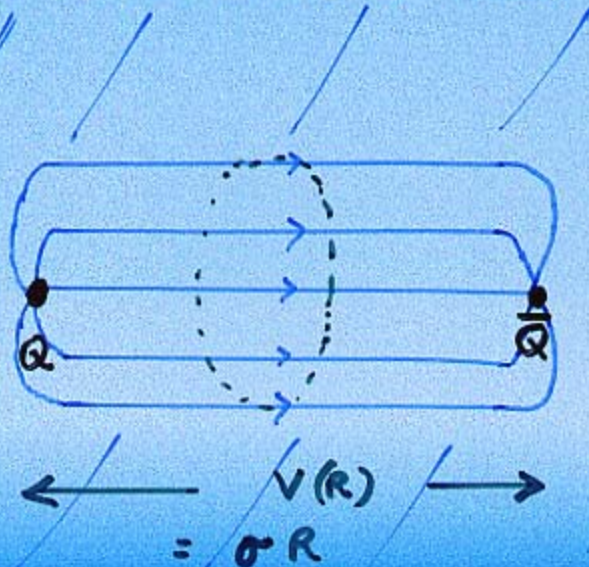
● Bizarre Properties:

● Asymptotic Freedom

i.e. Quarks feel almost no strong force when **close** together

● Confinement

i.e. Quarks can **never** be separated  
— Restoring Force of **10 TONNES**  
no matter how far apart they are!



## QCD Vacuum

- Non-zero gluon energy density

$$\begin{aligned}\epsilon_{\text{vac}} &= -\frac{9}{32} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \\ &\approx -0.5 \text{ GeV}^4 / \text{fm}^3\end{aligned}$$

Shifman, Vainstein & Zakharov

- This is large!

c.f. MIT bag:  $\frac{B}{|\epsilon_{\text{vac}}|} \sim 0.1$

⇒ little change from **inside** to **outside** hadron ?

- **ALSO** important **topological** structure:  
e.g. **instantons** ....
- linked to chiral symmetry breaking (?)

Slides: Leinweber, Bonnet, Thompson, Detmold.....

## Covariant Models

### ● Schwinger-Dyson Equations

(Roberts, Williams, Maris, Tandy, Alkofer, Cahill....)

### ● Very impressive phenomenology

● truncate infinite set integral equations

● pion form factor & structure function

● vector mesons

● nucleon form factors

● Can now test and refine against lattice data

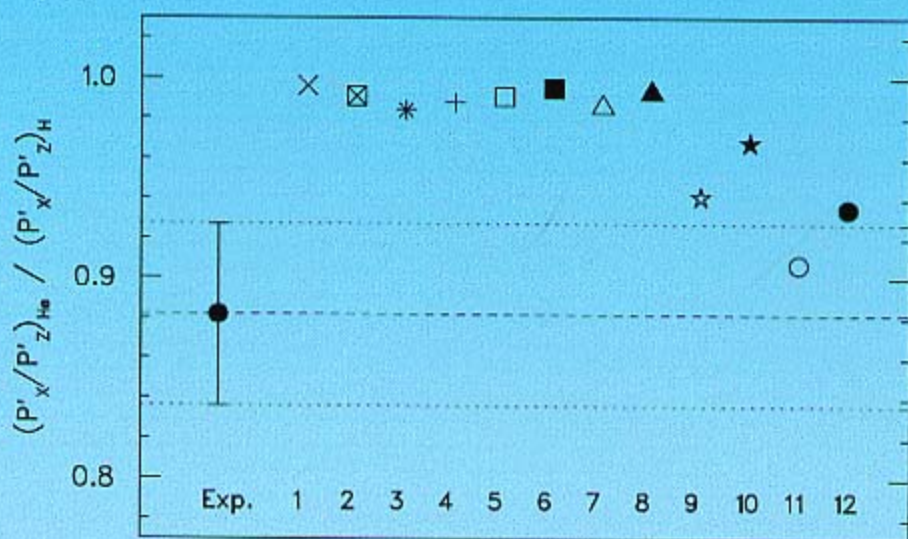
● gluon propagator

● quark propagator

● eventually quark-gluon vertex

## Theoretical Uncertainties

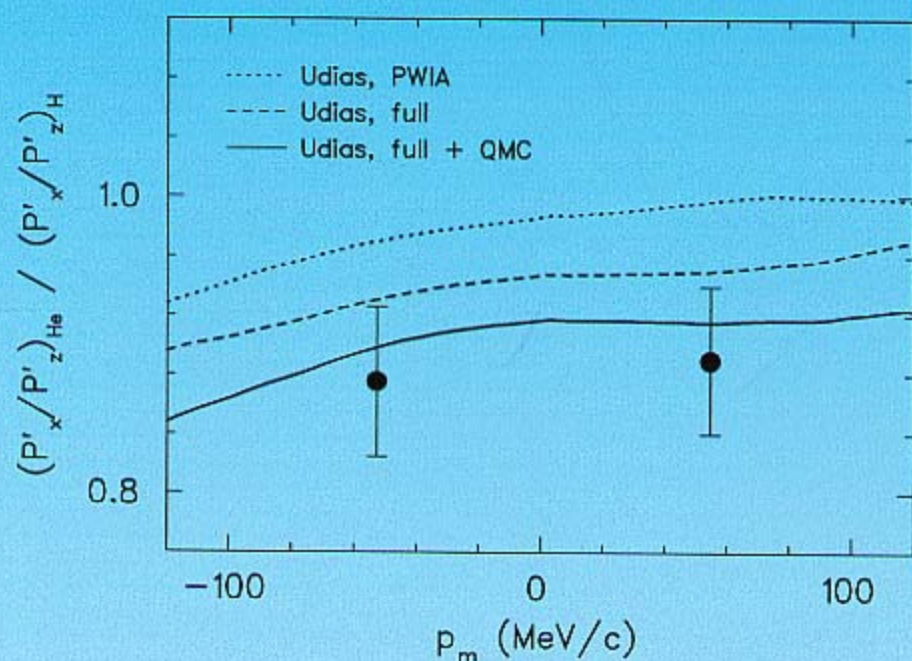
- Sensitivity to MEC, spin-orbit effects, distorted waves etc. — **very slight** in ratio:



Theoretical analysis of Udias et al., Laget etc.

## First Evidence for Change of Bound Nucleon Structure

- Ratio of electric to magnetic form factor of the proton in  ${}^4\text{He}$



S. Dieterich et al., Phys. Lett. B500 (2001) 47

- Full distorted wave analysis by Udias et al.

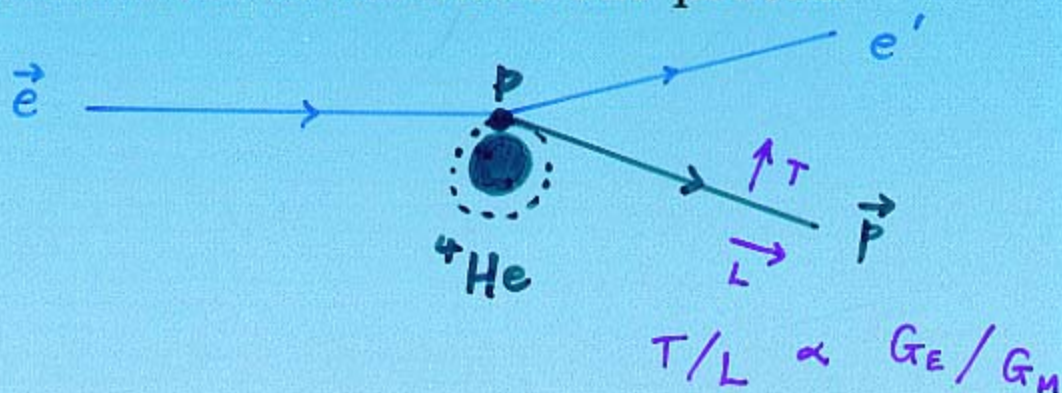
## Major Experimental Advance: $(\vec{e}, e'\vec{p})$

### ● Mainz/Jefferson Laboratory:

High intensity, polarised, 100% duty factor electrons

### ● Quasi-free scattering:

Use polarised beam and measure polarisation of knocked-out proton



### ● Measure $G_E/G_M$ :

Using  $(\vec{e}, e'\vec{p})$ , measure ratio of electric to magnetic proton form factor, compared to free-space ratio, in a specific shell-model state!

## Finite Nuclei

(P. Guichon, K. Saito, E. Rodionov and A.W. Thomas,  
Nucl. Phys. A601 (1996) 349)

### ● **Born-Oppenheimer Approximation:**

Composite particles in relativistic, external fields is **very complicated**

Expect good to about 3% to use Born-Oppenheimer

### ● **Obtain Shell Model:**

“Nucleon” internal structure

**self-consistently** adjusts to local mean-field, for each single-particle state

### ● **MAJOR CONCEPTUAL CHANGE:**

What occupies shell-model states are **NOT NUCLEONS** but “nucleon-like”

quasi-particles with **DIFFERENT** mass, magnetic moment, charge radii.....

### ● **How to Test?**



## ATOMIC NUCLEI

### ● **Traditional picture:**

“Point-like” protons and neutrons moving in **strong** Lorentz scalar and vector mean-fields (e.g. QHD)

### ● **Role of Internal Structure?**

Energy to excite nucleon, 300-500 MeV: **same** as scalar mean-field!

### ● **QMC: Quark-Meson Coupling model**

P. Guichon, Phys. Lett. B200 (1988) 235

K. Saito and A.W. Thomas, Phys. Lett. B327 (1994) 9

**Self-consistently** solve for the change of the internal structure of the nucleon – which is in turn the source of the scalar mean-field

### ● **Nuclear Matter:**

Natural Saturation Mechanism

At present requires hadron model

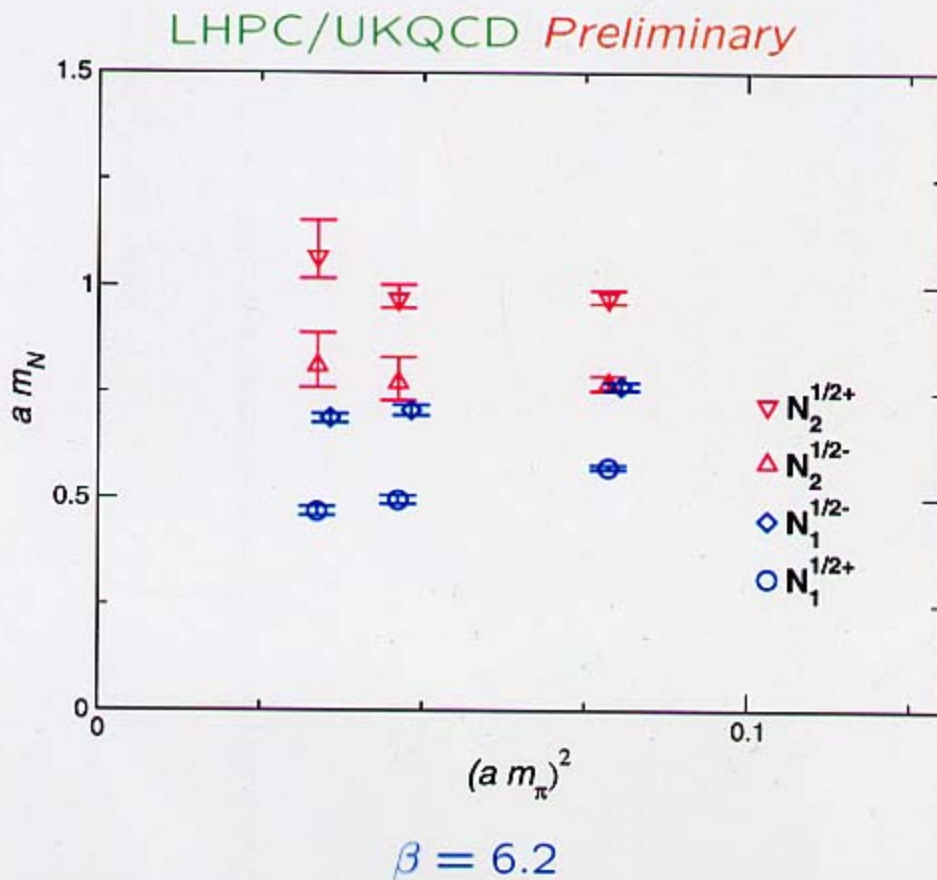
In principle could calculate **scalar response** of nucleon and hence nuclear structure **directly from QCD!**

## "Roper"

Most models predict the first radial excitation of the nucleon around 1600 MeV - *What is the Roper (1440)?*

"Bad" operator... Recall that the "bad" operator vanishes in NR limit.

Overlap on radial excitation



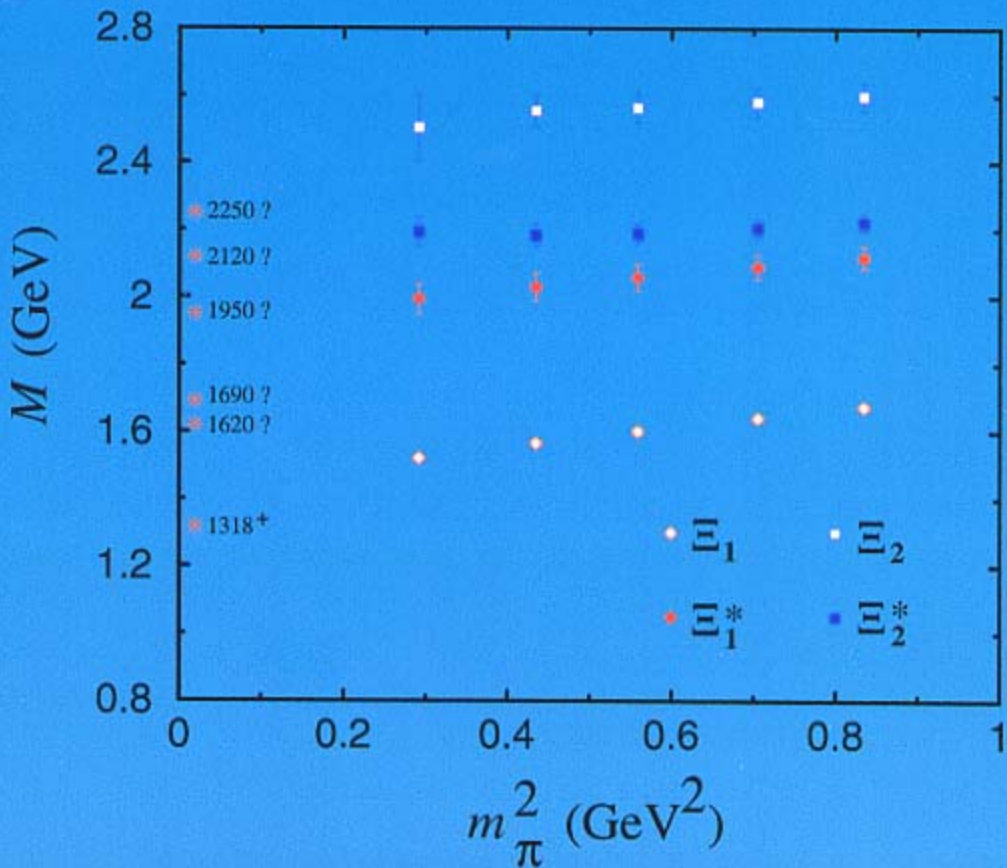
Poor signal at lightest masses

## Hadron Spectroscopy : SUMMARY

- Low lying, **+ve parity** states **NOT** found by Sasaki et al. or Melnitchouk et al. **BUT** c.f. Richards et al.....different source?
- Similarity to **constituent quark model** in large mass regime is clear
  - e.g. equal spacing of major shells.....
- Splitting of -ve parity states clear
  - State with  $S=0$  di-quark lower in mass
- Major worry: lattices are all small
  - Sasaki et al. and Richards et al. 1.6 fm
  - Melnitchouk et al. 2.0 fm
  - c.f. QCD proton radius about 0.65 fm
  - .....
- Of course, issue of **chiral extrapolation** remains an **open problem**.....

## Strangeness -2

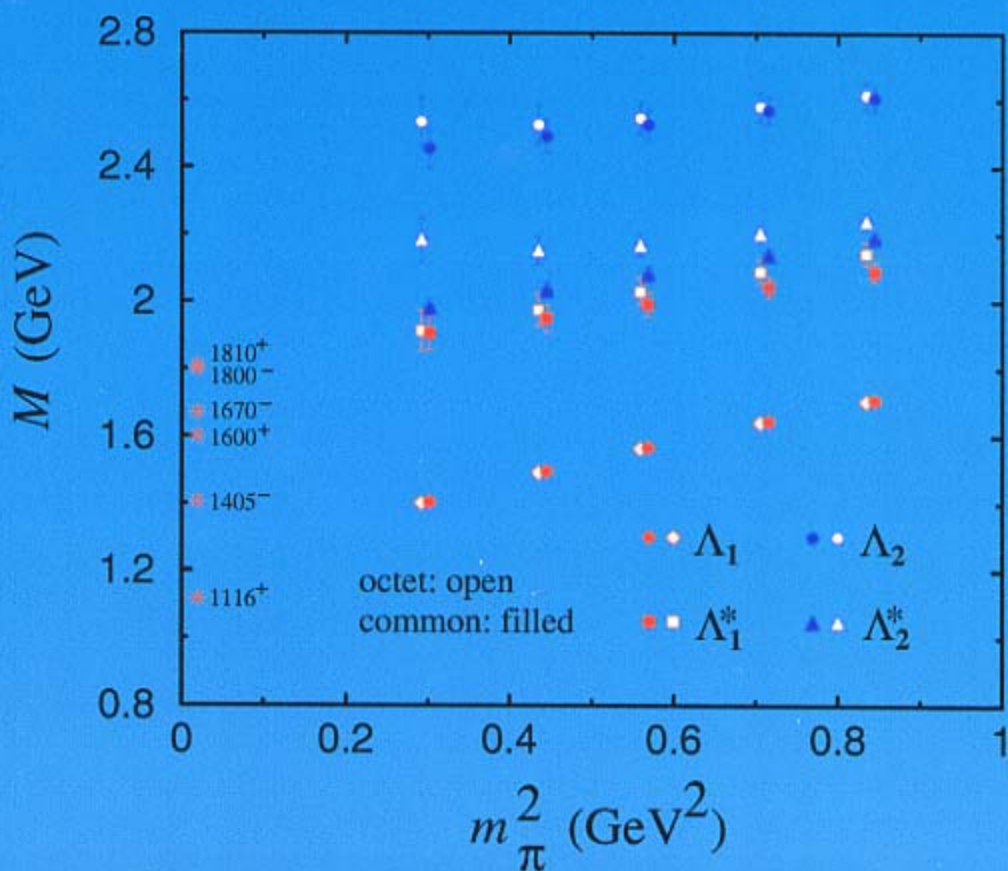
- Same shell structure as  $N, \Sigma$  and  $\Lambda$



From Melnitchouk et al., hep-lat/0202022

## $\Lambda$ Excited States

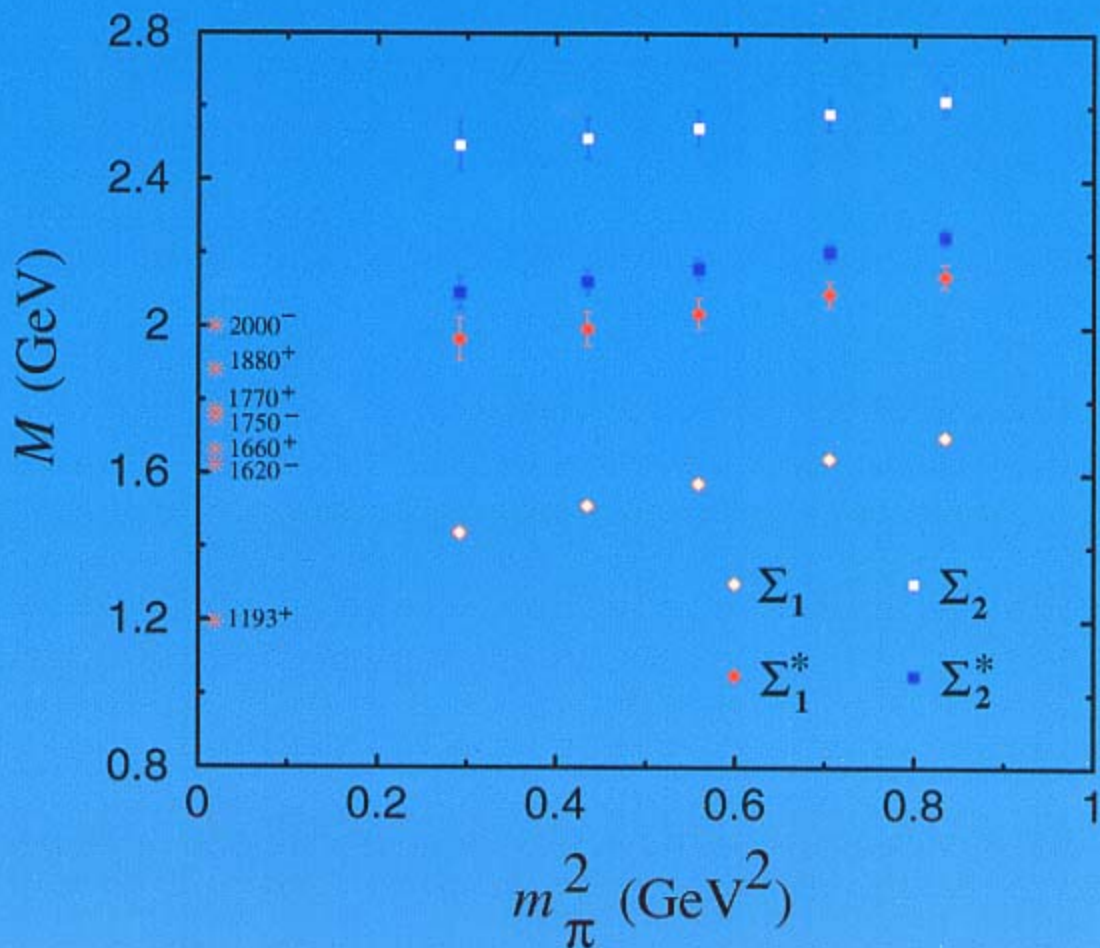
- Little overlap with  $\Lambda(1405)$



From Melnitchouk et al., hep-lat/0202022

## Strangeness -1

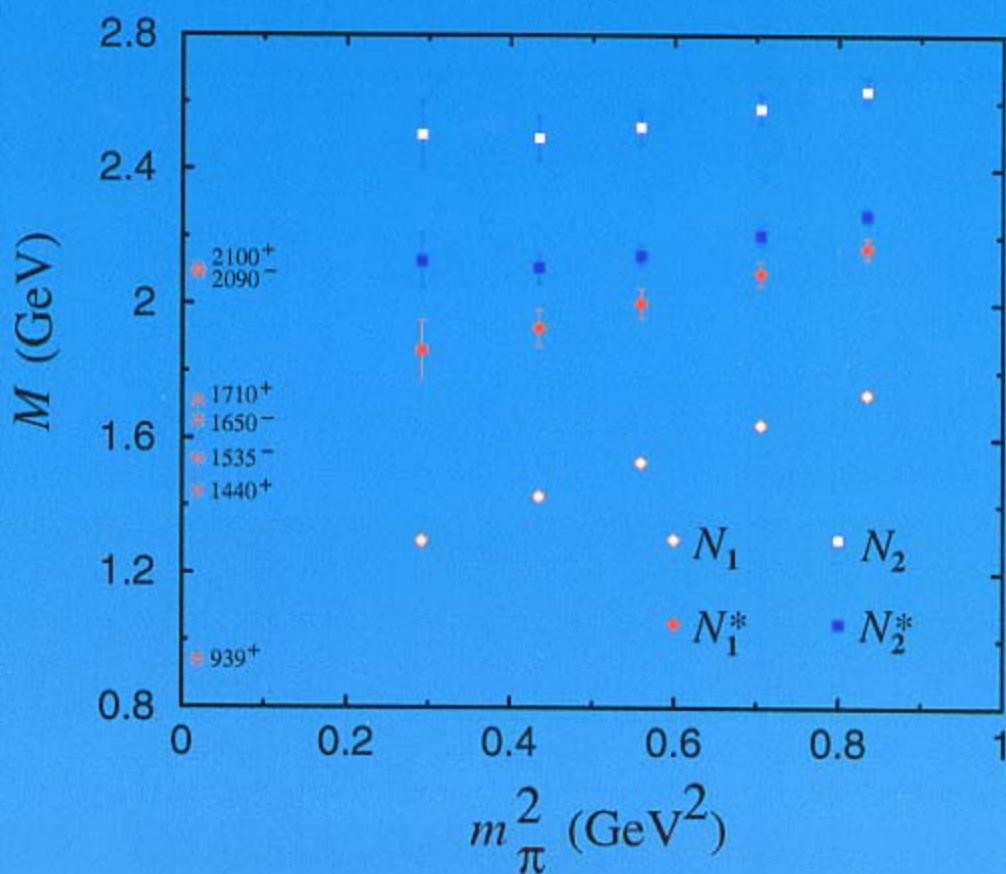
- Excited  $\Sigma$  similar to  $N$ :
- As for  $N$ , baryon with  $S = 0$  di-quark lower in mass



From Melnitchouk et al., hep-lat/0202022

# NUCLEON EXCITED STATES

- Note similarity to oscillator model
- Splitting of  $\frac{1}{2}^-$  states clear



From Melnitchouk et al., hep-lat/0202022;

very similar to Sasaki et al., hep-lat/0102010

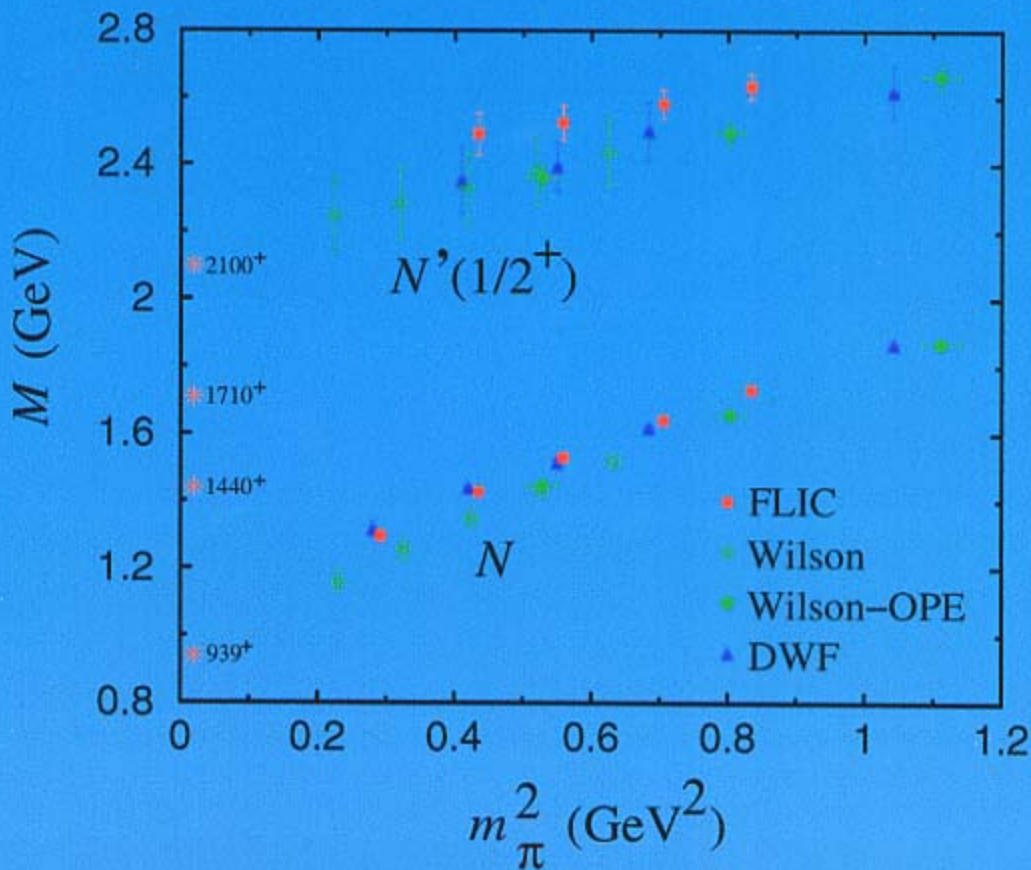
## Hadron Spectroscopy on the Lattice

- There are **new techniques** to produce information on **excited baryons**
- Leinweber: Phys. Rev. D51 (1995) 6383 :  
 $N'(\frac{1}{2}^+)$ , Wilson fermions
- Lee & Leinweber, Nucl. Phys. B(PS) 73 (1999) 258:  
 $N^*(\frac{1}{2}^-)$ ,  $N^*(\frac{3}{2}^-)$ , D $\chi$ 34 fermions
- Sasaki, Blum & Ohta, hep-lat/0102010 :  
 $N'(\frac{1}{2}^+)$ ,  $N^*(\frac{1}{2}^-)$ , Domain Wall Fermions
- Richards et al., JLAB-THY-01-38 :  
 $N^*(\frac{1}{2}^-)$ ,  $\Delta^*(\frac{3}{2}^-)$ , non-perturbative clover
- Melnitchouk et al., hep-lat/0202022 :  
Octet  $\frac{1}{2}^+$  and  $\frac{1}{2}^-$ , FLIC action
- So far, all calculations based on QCD



## Nucleon and $N'(1/2^+)$

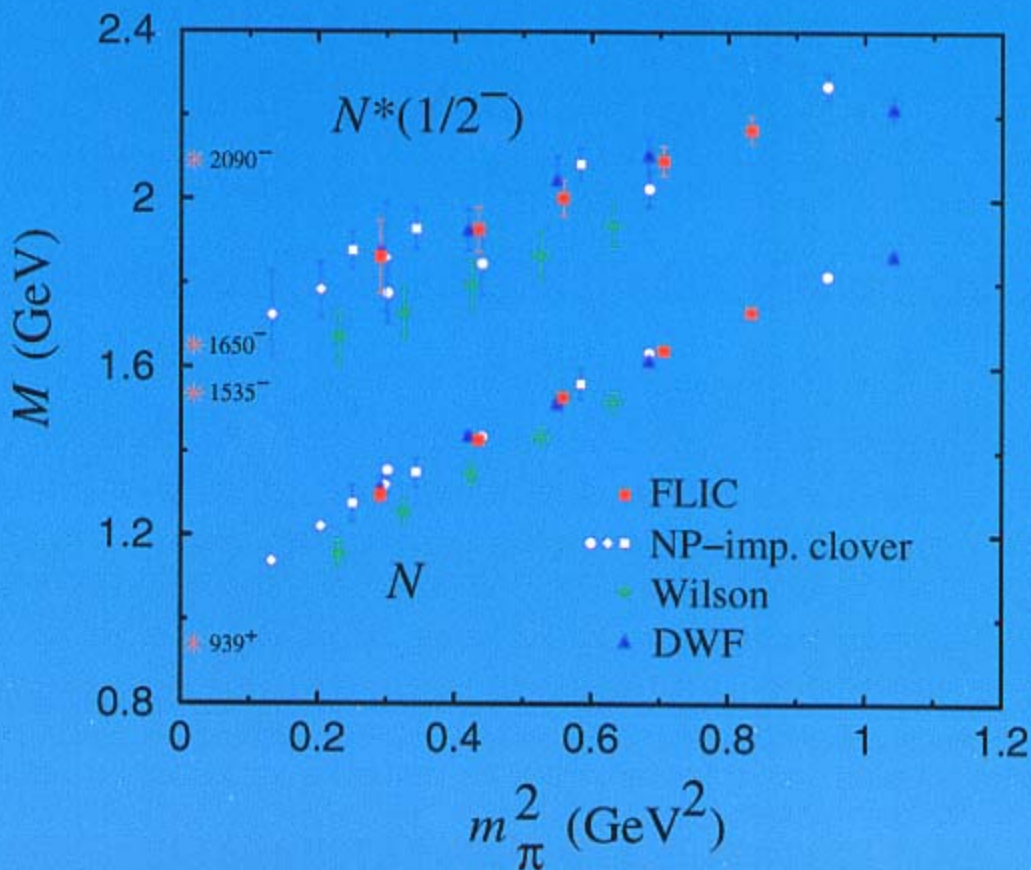
- Summary of various actions:



From Melnitchouk et al., hep-lat/0202022.

## Nucleon and $N^*(\frac{1}{2}^-)$

- Summary of various actions:



From Melnitchouk et al., hep-lat/0202022.

## Hadron Spectroscopy – II

● Are some baryons NOT primarily 3-quark states?

● e.g. Roper (1440):

●  $2h\omega$  BUT below  $1h\omega$  !

● Is it a breathing mode?

Guichon....

● Is it channel coupling effect?

Speth, Hanhart...Afnan....

● Similarly  $\Lambda(1405)$

● Is it  $\bar{K}N$  bound state?

Dalitz, Yuan.....

● OR  $\Sigma\pi - \bar{K}N$  coupled channel effect?

Veit, Jennings.... Weise.....

● There has been tremendous progress on such questions on the Lattice.....

## HADRON SPECTROSCOPY

- Exciting new prospects at JLab
- Are there states of **pure glue**?
- Are there **hybrid states**
  - e.g. BNL group

Phys. Rev. Lett. 81 (1998) 5760

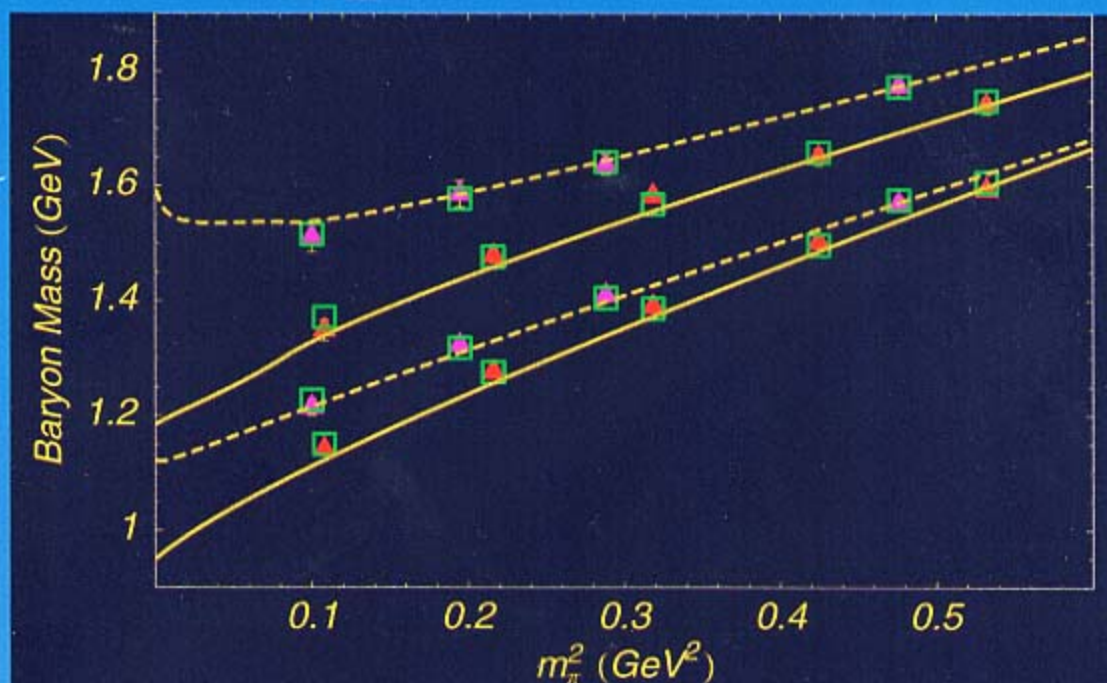
- $\pi_1(1370) \rightarrow \eta\pi, \eta'\pi$
- $\pi_1(1640) \rightarrow \eta'\pi, \rho\pi, f_1\pi$

## Connection Between QQCD and QCD?

● Values of  $\alpha, \beta \sim$  same as  $\tilde{\alpha}, \tilde{\beta}$ :

**N:** [1.24(1), 0.91(3)] **c.f.** [1.23(2), 0.85(6)]

**$\Delta$ :** [1.44(3), 0.75(7)] **c.f.** [1.45(4), 0.71(11)]



Common  $\Lambda = 0.8$  GeV

Data: Bernard et al., Phys Rev D64 054506 (2001)

(Analysis: Young et al., hep-lat/0111041)

## Connection of Quenched to Full QCD

- Suggests **LNA** and **NLNA** loops are the major difference between FULL and Quenched QCD
- Thus QQCD is NOT an “uncontrolled approximation” BUT may be a very **cost efficient** source of information on hadron **structure** and **spectroscopy**
- Need to test on other octet baryons
- Need better QQCD data at low mass  
eg. FLIC actions (Zanotti et al., hep-lat/0110216)

## Baryon Masses in QQCD –

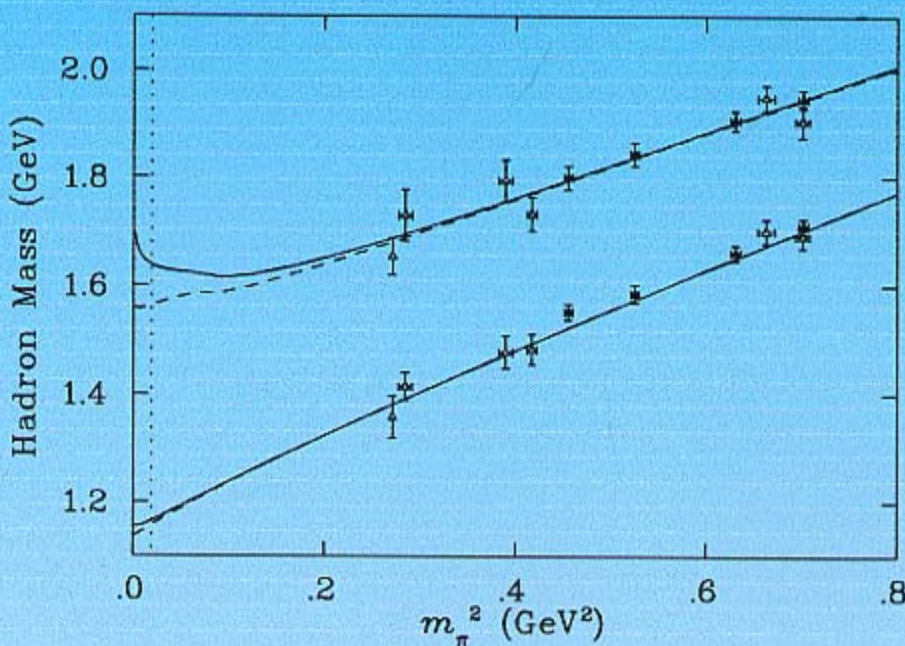
5

- Now fit QQCD data using:

$$M_B = \tilde{\alpha}_B + \tilde{\beta}_B m_\pi^2 + \tilde{\Sigma}_B(m_\pi, \Lambda)$$

- Use same  $\Lambda$  as in full QCD

(Consistent with QQCD data on rms radii)



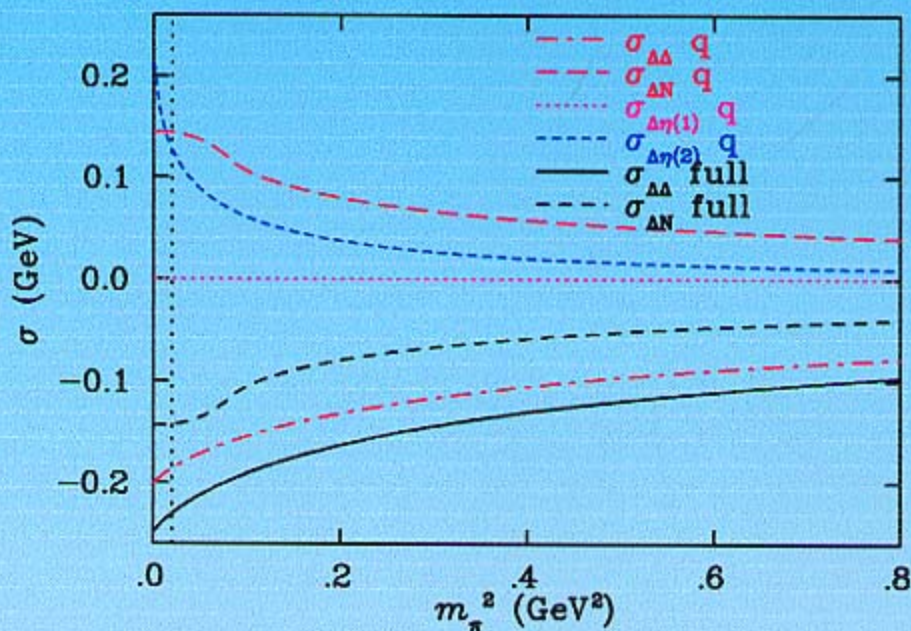
NOTE: major difference in behaviour for  $N$  and  $\Delta$

(From Young et al., hep-lat/0111041)

## Baryon Masses in QCD –

4

- For the  $\Delta$  major change
- Several contributions are **repulsive** rather than attractive – as in full QCD



Self-energy contributions to the  $\Delta$

(From Young et al., hep-lat/0111041)



## Baryon Masses in QQCD –

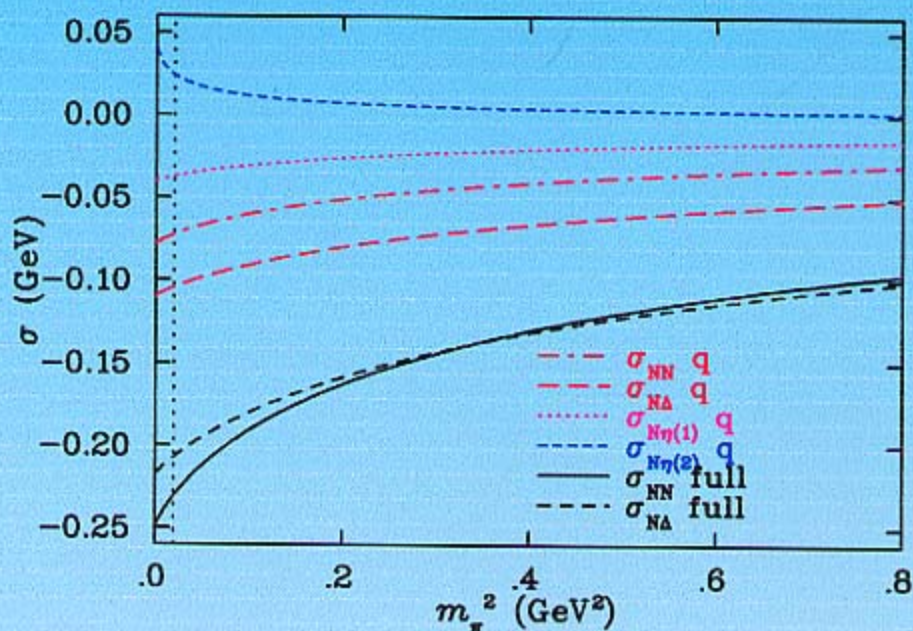
3

- Adopt same approach with QQCD data

$$M_B = \tilde{\alpha}_B + \tilde{\beta}_B m_\pi^2 + \tilde{\Sigma}_B(m_\pi, \Lambda)$$

- $\tilde{\Sigma}_B$  sum of QQCD self-energy terms yielding **LNA** and **NLNA** behaviour

from Labrenz & Sharpe Phys. Rev. D54 (1996) 4595



Self-energy contributions to the nucleon

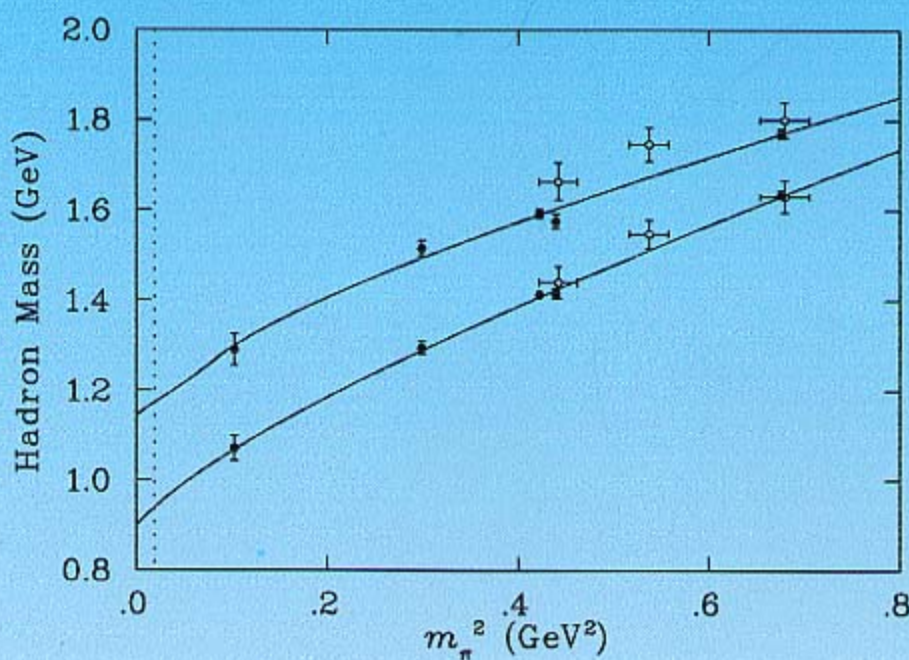
(From Young et al., hep-lat/0111041)

## Baryon Masses in QQCD – 2

● Success of describing  $N$  and  $\Delta$  masses in full QCD using:

$$M_B = \alpha_B + \beta_B m_\pi^2 + \Sigma_B(m_\pi, \Lambda)$$

●  $\Sigma_B$  is sum of the self-energy terms yielding **LNA** and **NLNA** behaviour



From Young et al., hep-lat/0111041.  $\Lambda = 0.92$  GeV

## Baryon Masses in Quenched QCD

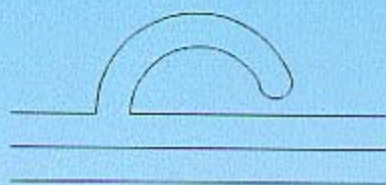
● Chiral behaviour of baryon masses is quite different in QQCD

●  $\eta'$  is also a Goldstone boson  $\Rightarrow$

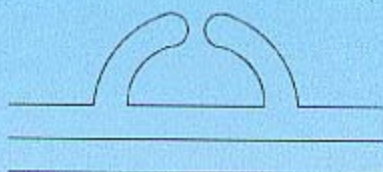
$$M_B = M_B^{(0)} + c_1^B m_\pi + c_2^B m_\pi^2 + c_3^B m_\pi^3 + \dots$$

● Coefficients of  $m_\pi$ ,  $m_\pi^3$  and  $m_\pi^4 \ln m_\pi$  are **model independent**

●  $m_\pi$  term **unique to QQCD**: arises from (b):



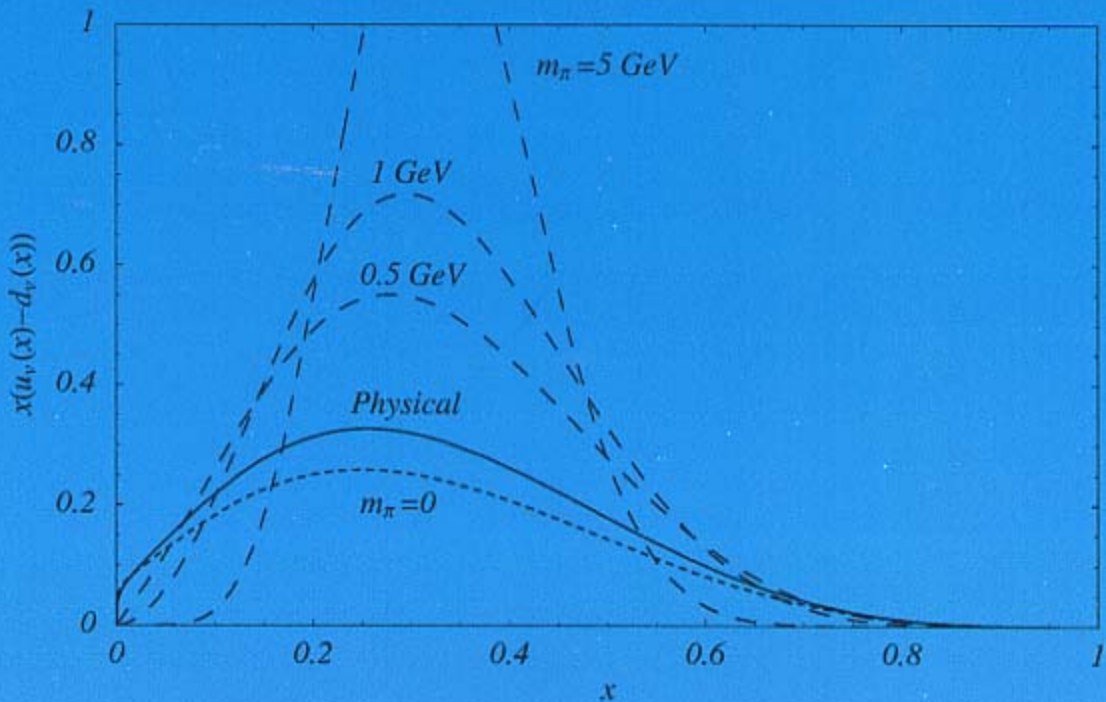
(a)



(b)

## Parton Distributions vs Mass

- Can use fit to plot  $x(u_V - d_V)$  as a function of quark mass:

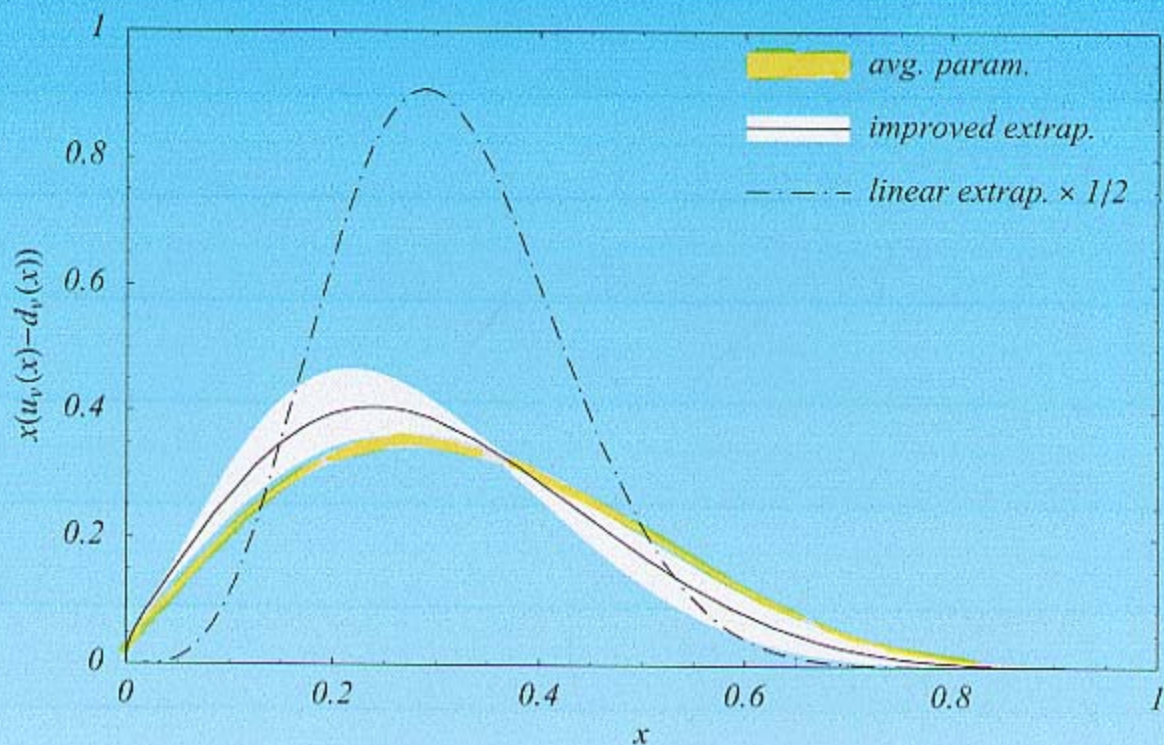


- Note **constituent-quark** behaviour as mass passes strange quark mass

Detmold et al., Eur. Phys. J. direct C13, 1 (2001)

## Comparison with Data

- Can use lowest 4 moments to reconstruct  $x$ -dependence of  $u_v(x) - d_v(x)$



Detmold, Melnitchouk....

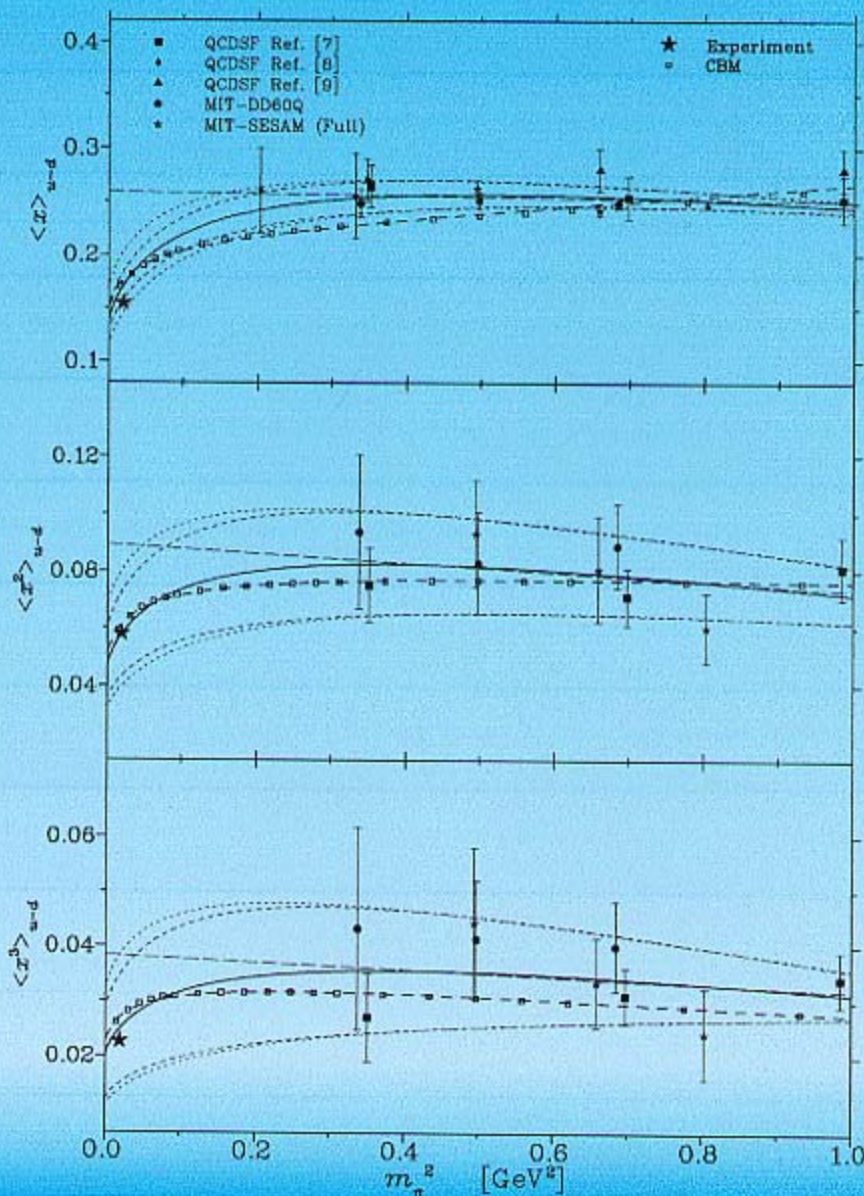
- Dark shading is world data
- Light shading **chiral extrapolation**

## Moments of $u - d$

### ● Chiral extrapolation is essential:

Detmold, Melnitchouk, Negele, Renner & Thomas,

hep-latt/0103006:  $a + bm_\pi^2 + ac_{\text{LNA}} m_\pi^2 \ln(m_\pi^2 / (m_\pi^2 + \mu^2))$



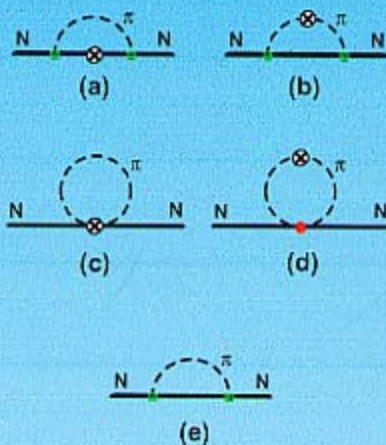
## Chiral Behaviour

- For Gottfried sum rule violation:

$$\bar{d} - \bar{u}|_{\text{LNA}} \sim c_{\text{LNA}} m_\pi^2 \ln m_\pi$$

Thomas, Melnitchouk & Steffens, PRL 85 (2000) 2892

- For  $u - d$ :



- $\langle x^n \rangle = a_n + b_n m_\pi^2 + a_n c_{\text{LNA}} m_\pi^2 \ln \frac{m_\pi^2}{m_\pi^2 + \mu^2}$

- $c_{\text{LNA}}$  from chiral perturbation theory

Ji & Chen and Arndt & Savage

- reproduces CBM calculation over full range of  $m_\pi^2$

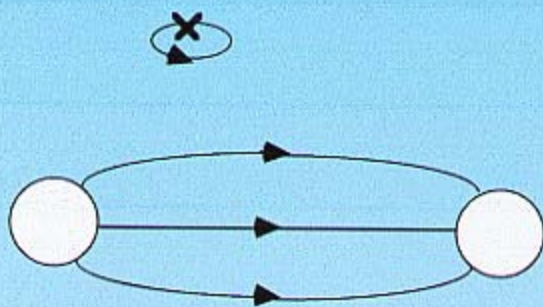
- $\mu$  is scale at which chiral behaviour turns off: expect  $\sim 500$  MeV

## Moments of Parton Distributions

- Can only compute low moments on lattice:

$$\begin{aligned} \langle x^n \rangle &= \int_0^1 dx x^n [q(x) + (-)^{n+1} \bar{q}(x)] \\ &\sim \langle N | \bar{\psi} \gamma_{\{\mu_1} D_{\mu_2} \dots D_{\mu_{n+1}} \} \psi | N \rangle (1) \end{aligned}$$

- Compute  $\langle x^n \rangle$  for  $u - d$
- $\Rightarrow$  NO “disconnected diagrams”



- Until now, discrepancies with data of 50 % or more.....



$$\langle r^2 \rangle_i =$$

$$\frac{c_1 + \chi_i \ln m_\pi}{1 + c_2 m_\pi^2}$$

Table 1

Baryon electric charge radii and the quark sector contributions. The latter are defined on the left-hand sides of Eqs. (6)–(11). One-loop corrected estimates of  $\alpha_i^{(\pi)}$  (in Eq. (1)) and  $\chi_i$  (in units of fm<sup>2</sup>) for each octet baryon are indicated. For each extrapolation, the fit parameters,  $c_1$  and  $c_2$ , and the predicted value of  $\langle r^2 \rangle$  at the physical pion mass are reported. Asterisks denote the squared charge radii reconstructed from the sum of separate quark sector extrapolations. (The units are such that the pion mass is in GeV and the squared charge radius in fm<sup>2</sup>)

Baryon or quark sector	$\alpha_i^{(\pi)}$	$\chi_i$	$c_1$	$c_2$	$\langle r^2 \rangle$	Experiment
$p$	$\frac{1}{6} \frac{5}{6}(D+F)^2$	0.174	0.34	0.50	0.68(8)	0.740(15) [14]
$u_p$	$\frac{2}{3} \left[ \frac{1}{6} \frac{5}{6}(D+F)^2 \right]$	0.116	0.52	0.73	0.74(11)	
$d_p$	$\frac{1}{3} \left[ \frac{1}{6} \frac{5}{6}(D+F)^2 \right]$	0.058	0.18	1.38	0.06(5)	
$*p$					0.68(10)	0.740(15) [14]
$n$	$\frac{1}{6} + \frac{5}{6}(D+F)^2$	0.174				
$u_n$	$\frac{2}{3} \left[ \frac{1}{6} + \frac{5}{6}(D+F)^2 \right]$	0.116	0.35	1.38	– 0.12(10)	
$d_n$	$\frac{1}{3} \left[ \frac{1}{6} + \frac{5}{6}(D+F)^2 \right]$	0.058	0.26	0.73	– 0.37(6)	
$*n$					– 0.25(8)	– 0.113(4) [15]
$\Lambda$	0	0				
$l_\Lambda$	0	0	0.15	0.97	0.14(3)	
$s_\Lambda$	0	0	0.07	0.10	0.07(1)	
$*\Lambda$					0.07(3)	
$\Sigma^+$	$\frac{1}{3} \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right)$	0.138	0.68	2.03	0.92(11)	
$l_{\Sigma^+}$	$\frac{1}{3} \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right)$	0.138	0.58	0.93	0.83(8)	
$s_{\Sigma^+}$	0	0	0.06	0.17	0.06(1)	
$*\Sigma^+$					0.77(8)	
$\Sigma^0$	0	0				
$l_{\Sigma^0}$	0	0	0.19	1.48	0.18(2)	
$s_{\Sigma^0}$	0	0	0.06	0.17	0.06(1)	
$*\Sigma^0$					– 0.12(2)	
$\Sigma^-$	$\frac{1}{3} + \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right)$	0.138	0.25	0.08	– 0.52(3)	– 0.60(16) [16]
						– 0.91(72) [17]
$l_{\Sigma^-}$	$\frac{1}{3} + \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right)$	0.138	0.21	0.37	– 0.47(3)	
$s_{\Sigma^-}$	0	0	0.06	0.17	– 0.06(1)	
$*\Sigma^-$					– 0.54(3)	– 0.60(16) [16]
						– 0.91(72) [17]

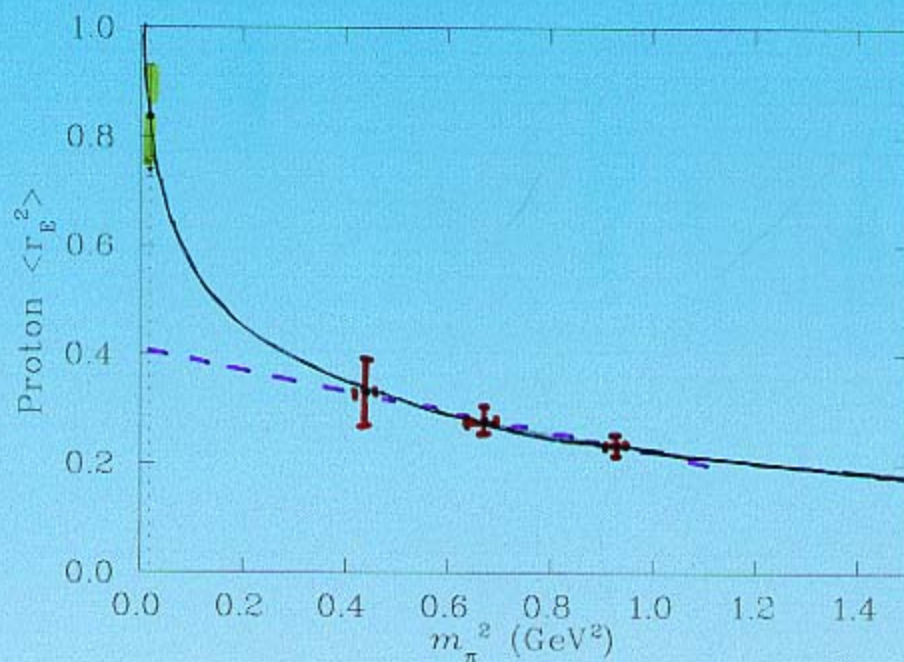
(continued on next page)

## Analogy to Charge Radius in Lattice QCD

### ● Same asymptotic behaviour

(Hackett-Jones, Leinweber & Thomas)

$$\langle r^2 \rangle_E = \frac{c_1 \pm \frac{\chi N}{2} \log \frac{m_\pi^2}{\mu^2 + m_\pi^2}}{1 + \bar{c}_2 m_\pi^2}$$

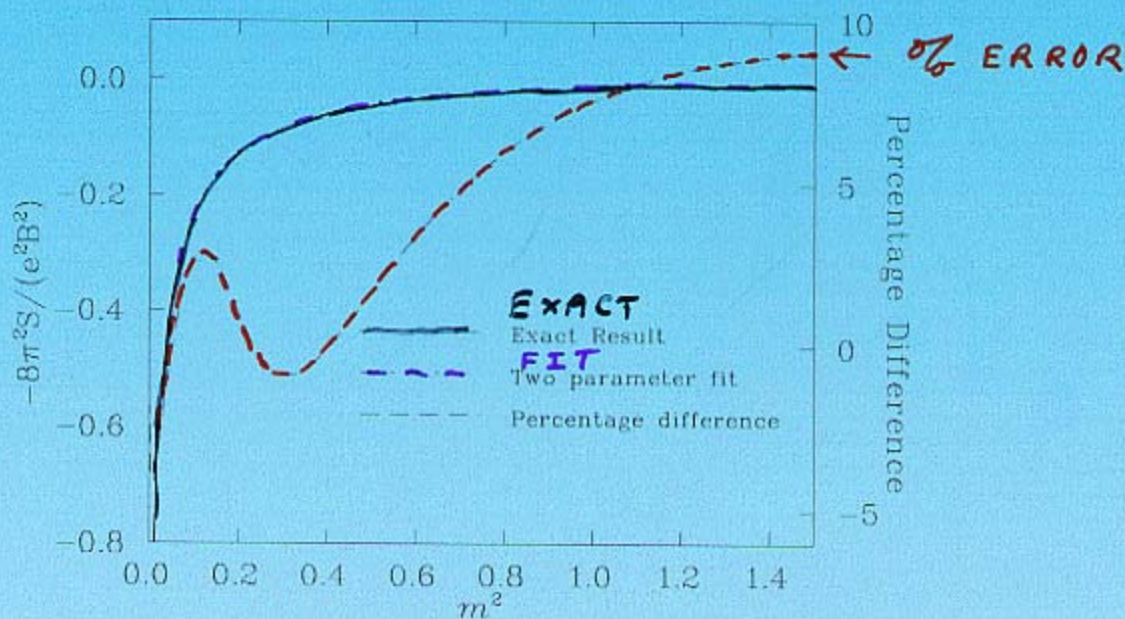


Fit to lattice QCD data for charge radius of the proton

## Euler-Heisenberg (cont.)

- To build a very effective approximation  
Ensure exact **large and small** mass limits

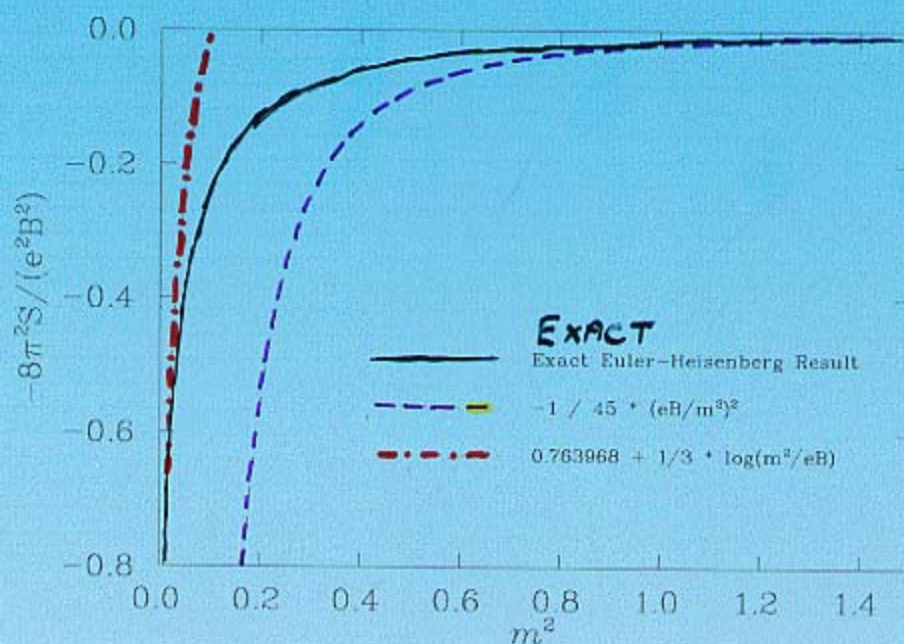
$$S_{\text{int}} = -\frac{e^2 B^2}{8\pi^2} \left( \frac{d_1 + \frac{1}{3} \log\left(\frac{m^2}{m^2 + eB}\right) - d_2 \frac{m^2}{eB}}{1 + 45d_2 \left(\frac{m^2}{eB}\right)^3} \right)$$



Two parameter representation of the exact result:  
accurate to better than ~~10~~ % for all  $m$

## Euler-Heisenberg (cont.)

- Either asymptotic limit is a poor approximation to exact solution



## Euler-Heisenberg Problem

- Exactly soluble EFT

(Dunne, Thomas, Wright: hep-th/0110155)

- $S = -i \ln \det(i\mathcal{D} - m)$

- Exact solution:**

$$S = -\frac{e^2 B^2}{8\pi^2} \int_0^\infty \frac{ds}{s^2} \left( \coth s - \frac{1}{s} - \frac{s}{3} \right) e^{-m^2 s/(eB)}$$

- Asymptotic expansion for large  $m$ :**

$$S = -\frac{e^2 B^2}{8\pi^2} \left[ -\frac{1}{45} \left( \frac{eB}{m^2} \right)^2 + \frac{4}{315} \left( \frac{eB}{m^2} \right)^4 \dots \right]$$

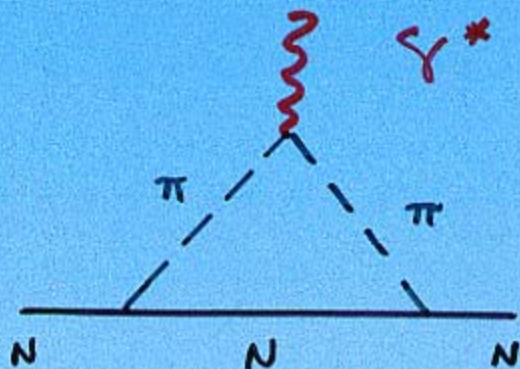
- Non-analytic small  $m$  expansion:**

$$S = -\frac{e^2 B^2}{8\pi^2} \left\{ \frac{1}{3} \log \frac{m^2}{eB} + 0.763969 + O\left(\frac{m^2}{eB}\right) \right\}$$

## Charge Radii

- **Diverge in chiral limit:**

As pion mass  $m_\pi \sim m_q^{1/2} \rightarrow 0$ ,  
charge radii must diverge



- **Pion loop:**

gives contribution that behaves like:

$$\ln m_\pi \rightarrow \infty \quad \text{as} \quad m_\pi \rightarrow 0$$

- **Such behaviour can never arise in constituent quark model.....**

## Future Developments

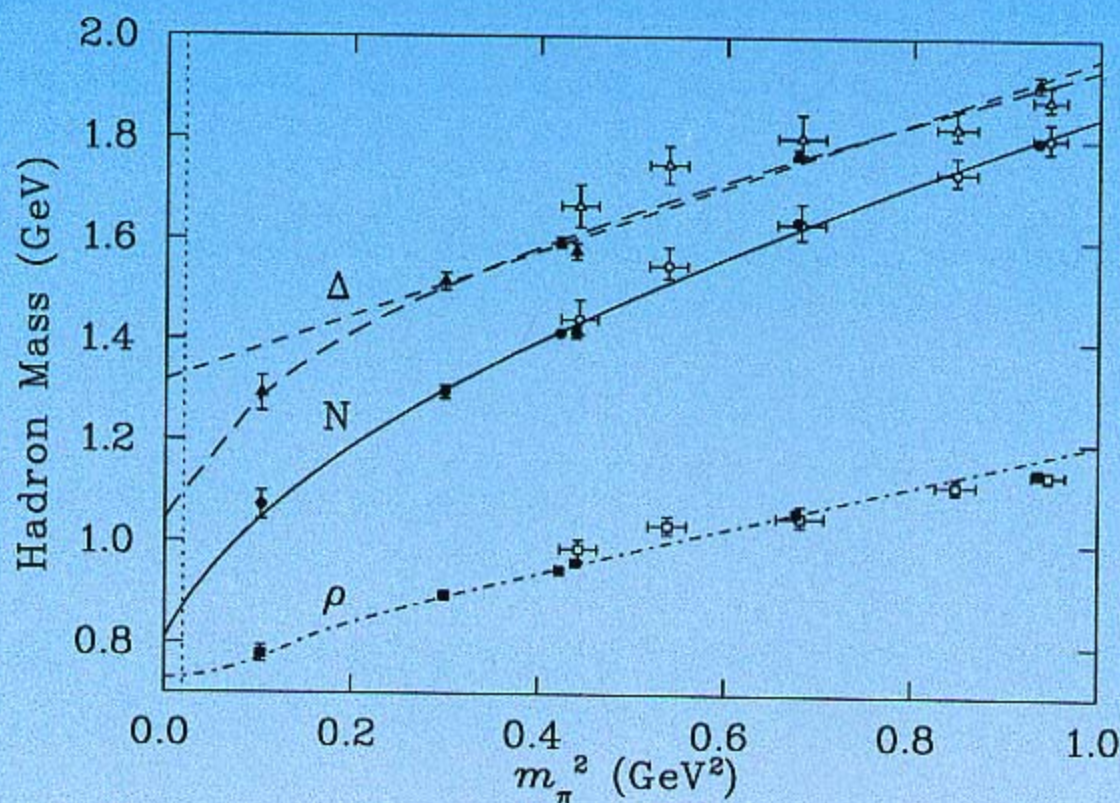
- Need **100's Tflops** to compute hadron properties at  $m_q = 5 \text{ MeV}$
- Will NOT happen for at least **10 years**

**BUT**

- Several groups will have **10's Tflops** within **2-3 years**
- Advances in **controlled chiral extrapolation** should be sufficient for accurate **physical** hadron properties!

## Overview of Hadron Masses

- Behave like constituent quark model for  $m_\pi$  above 400–500 MeV:



Leinweber, Wright.....



## OVERVIEW

### ● Lattice data $\bar{m} > 60$ MeV:

- (i.e.  $m_\pi \geq 400$ – $500$  MeV)
- Hadron properties **smooth**, slowly varying functions of the **constituent quark** mass ( $M \sim M_0 + c\bar{m}$ ):
  - $M_{N,\Delta} \sim 3M$  ( $\sim a + bm_\pi^2$ )
  - $M_{\rho,\omega} \sim 2M$  ( $\sim a' + \frac{2}{3}bm_\pi^2$ )
  - $\mu_H \sim 1/M$

### ● BUT at small $\bar{m} (< 60$ MeV):

- (i.e.  $m_\pi \leq 400$ – $500$  MeV)
- Chiral Symmetry  $\Rightarrow$
- **rapid**, non-analytic variation with  $\bar{m}$ :
  - $\delta M_H \sim \bar{m}^{3/2}$
  - $\delta \mu_H \sim \bar{m}^{1/2}$
  - $\delta \langle r^2 \rangle_{\text{ch}} \sim \ln \bar{m}$

● Models (like CBM) yield a natural explanation of transition from **LNA** to **smooth** behaviour

## Non-Analytic Behaviour (cont.)

- $$\sigma_{NN} = -\frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$
$$= -5.6 m_\pi^3 \text{ GeV}$$
$$= -17 \text{ MeV at } m_\pi^{\text{phys}} = 140 \text{ MeV}$$

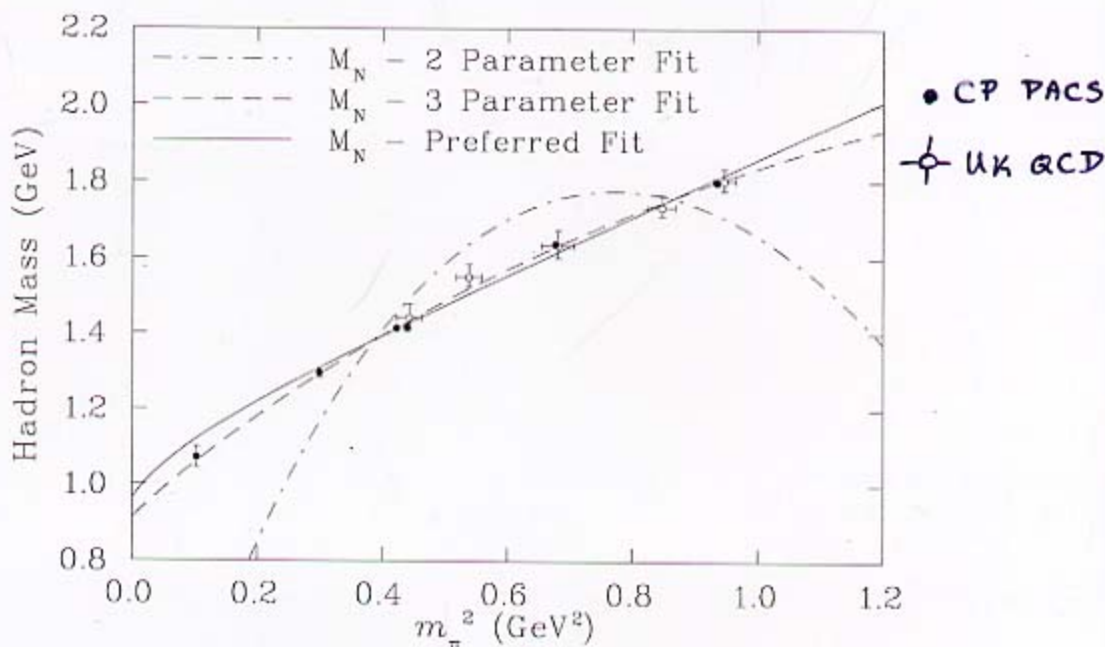
BUT

$$= -460 \text{ MeV at } m_\pi = 420 \text{ MeV}$$

- where lowest lattice data exists!
  - Thus non-analytic behaviour is
  - huge problem for chiral extrapolation
- OR
- Major opportunity
- analogous to study versus  $N_c$
- to learn from study of QCD
- as a function of  $m_q$

## ● COMMON LATTICE METHOD :

- Formula :  $M_B = \alpha + \beta m_\pi^2 + \gamma m_\pi^3$



- Tiny coefficient of  $m_\pi^3$  - incompatible with  $\chi$ PT

$$\gamma = -0.761 \quad \text{c.f.} \quad \gamma = -5.60 \quad \text{for } \chi\text{PT}$$

## Non-Analytic Behaviour

● Dynamical symmetry breaking  
 $\Rightarrow$  dependence of **all** hadron properties on  $m_q$  is not analytic

● e.g. Nucleon Mass

$N \rightarrow N\pi \rightarrow N$ :

$$\delta M_N = -\frac{3g_A^2}{16\pi^2 f_\pi^2} \int_0^\infty dk \frac{k^4 u^2(k)}{k^2 + m_\pi^2}$$

$$= m_0 + c_2 m_\pi^2 + c_3^{\text{LNA}} m_\pi^3 + c_4 m_\pi^4 + \dots$$

(chiral perturbation theory)

●  $c_3^{\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2}$  is model independent  
 - coming from pion pole

●  $c_3^{\text{LNA}} m_\pi^3 \propto m_q^{\frac{3}{2}}$  !!

●  $\Delta \rightarrow N\pi \rightarrow \Delta$  gives  $m_\pi^4 \ln m_\pi$

## Non-Analytic Behaviour

● Dynamical symmetry breaking  
 $\Rightarrow$  dependence of **all** hadron properties on  $m_q$  is not analytic

● e.g. Nucleon Mass

$N \rightarrow N\pi \rightarrow N$ :

$$\delta M_N = -\frac{3g_A^2}{16\pi^2 f_\pi^2} \int_0^\infty dk \frac{k^4 u^2(k)}{k^2 + m_\pi^2}$$

$$= m_0 + c_2 m_\pi^2 + c_3^{\text{LNA}} m_\pi^3 + c_4 m_\pi^4 + \dots$$

(chiral perturbation theory)

●  $c_3^{\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2}$  is model independent  
 – coming from pion pole

●  $c_3^{\text{LNA}} m_\pi^3 \propto m_q^{\frac{3}{2}}$  !!

●  $\Delta \rightarrow N\pi \rightarrow \Delta$  gives  $m_\pi^4 \ln m_\pi$

## Relevance for Lattice QCD

### ● Time:

Small quark masses increase time for calculations:

$$t \propto 1/m_q^{3.0-3.5}$$

### ● Hence extrapolations:

Therefore in practice compute at large quark mass and extrapolate to physical value

### ● State-of-the-art:

CP-PACS (Tsukuba):

$M_N$ : above 40 MeV

$\mu_{II}$ : above 90 MeV

c.f. 5–8 MeV physical values.....

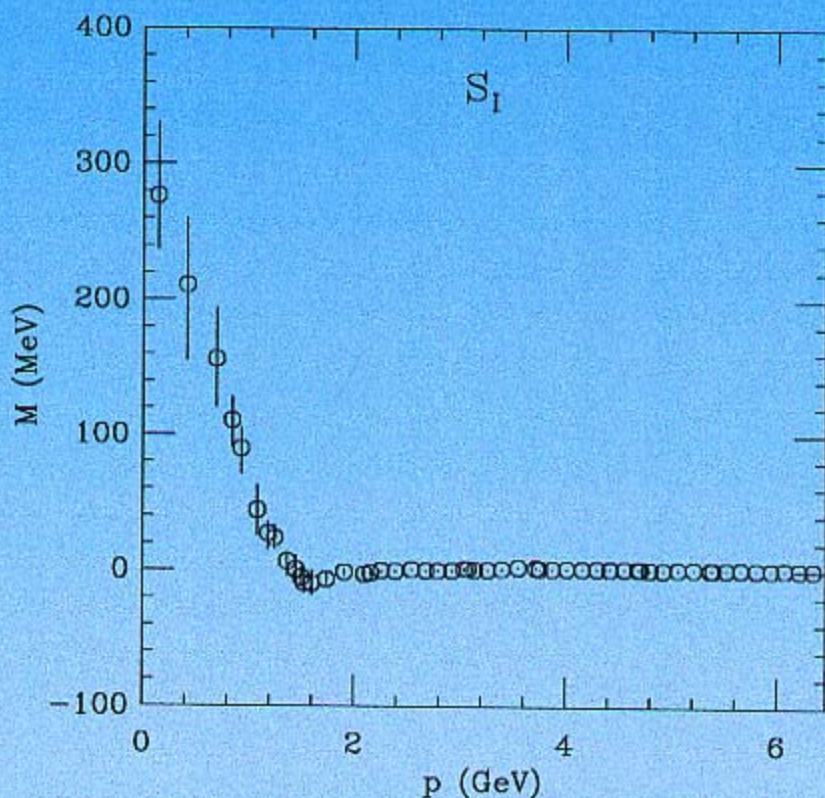
● That is state-of-the-art involves quark masses a factor of 8–20 too large!

## Quark Propagator

- Fix Landau gauge numerically

(Leinweber, Skullerud, Williams... also Aoki et al.)

- $$S_E(p) = \frac{Z(p^2)}{i\gamma_\mu p_\mu + M(p^2)}$$



- Asymptotic form:  $-\frac{4\pi\alpha_s}{3p^2} \langle \bar{q}q \rangle$
- “Constituent” mass  $M(0) \sim 300$  MeV
- very similar to DSE phenomenology

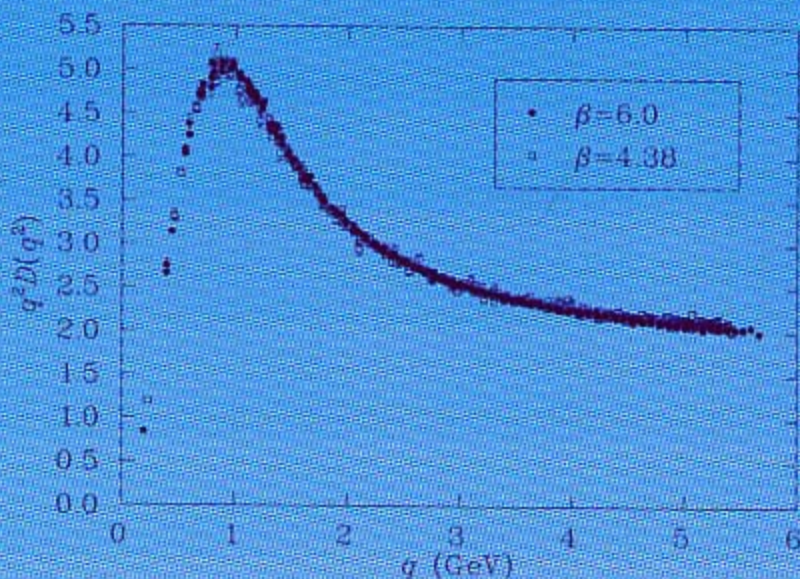
## Gluon Propagator

- Fix Landau gauge numerically

(Leinweber, Williams, Bonnet .....

- pQCD  $\Rightarrow D(q^2) \sim \frac{1}{q^2}$

- DATA  $\Rightarrow$  Non-Perturbative for  $q < 2$



- No infra-red enhancement

- disagrees SDE phenomenology

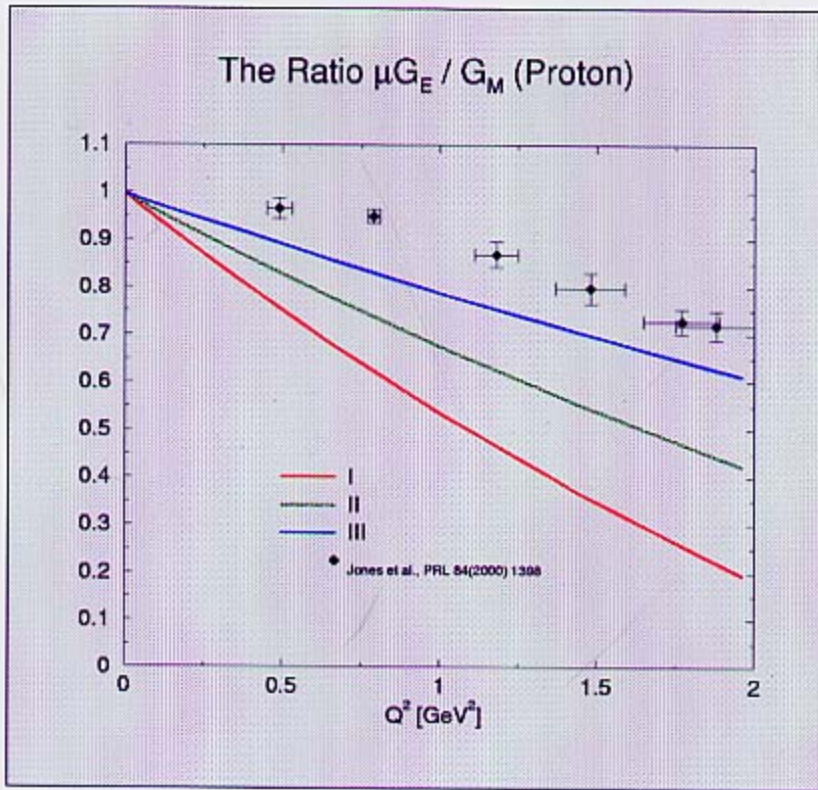
- BUT agrees Alkofer et al.:

ghosts enhance quark-gluon vertex

- to be tested soon .....



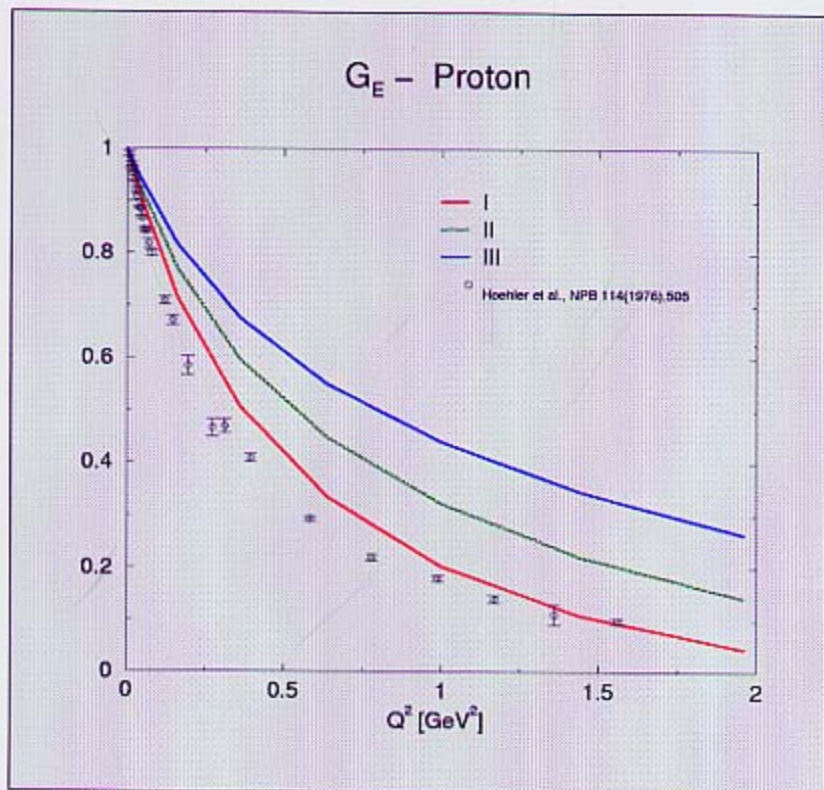
The ratio  $\mu_p G_E / G_M$ :



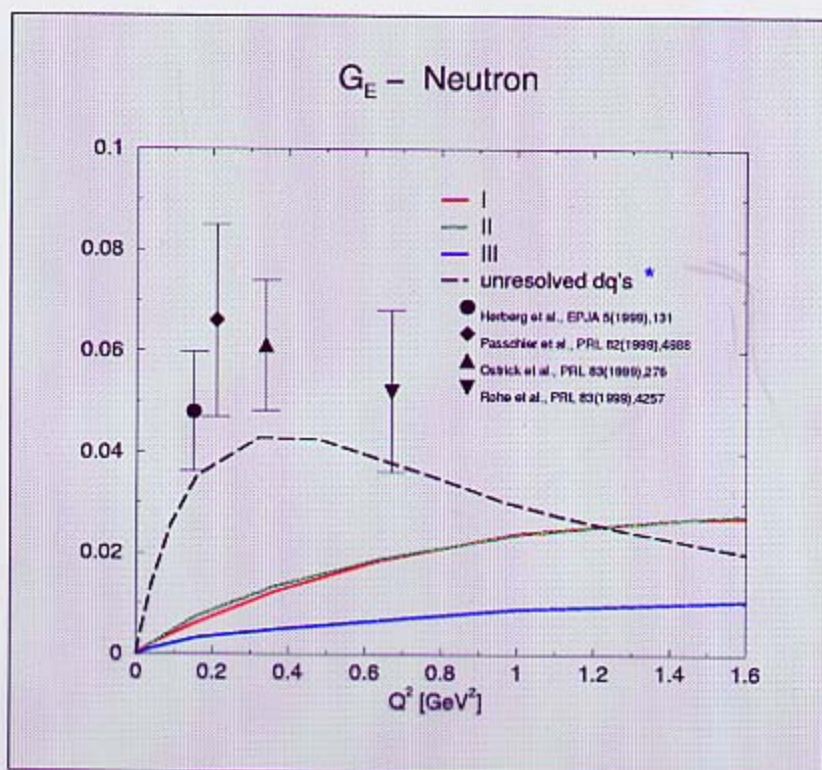
Oettel et al. (2001)

# Nucleon electric form factors:

- compressed  $q - q \rightarrow$  hard form factor
- realistic "diquark" size:  $l \approx 0.3 \text{ fm}$
- $\rho - \omega$ : 20–25% to  $(r_p)_{el}^2$



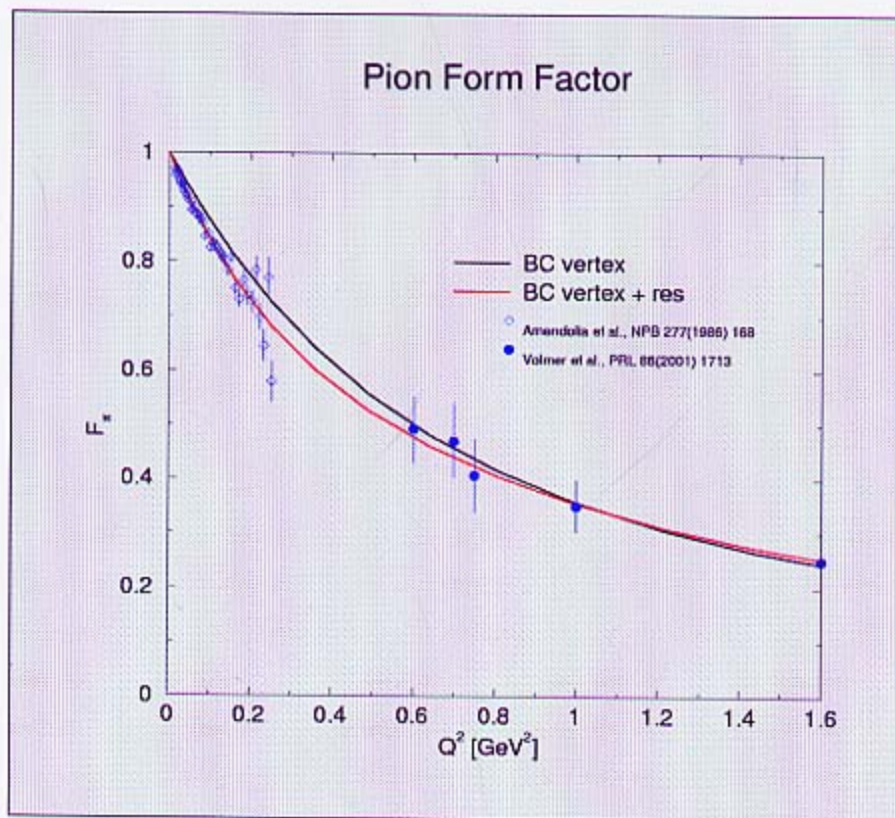
- photon resolves  $q - q \rightarrow$  quenched form factor
- (e.m.'ally) compact dq  $\rightarrow$  mimics  $\pi$  cloud?



\* Oettel, Alkofer, v. Smekal, EPJA8 (2000), 553

# Quark-photon vertex II:

Fit off-shell behaviour to the  $\pi$  form factor:

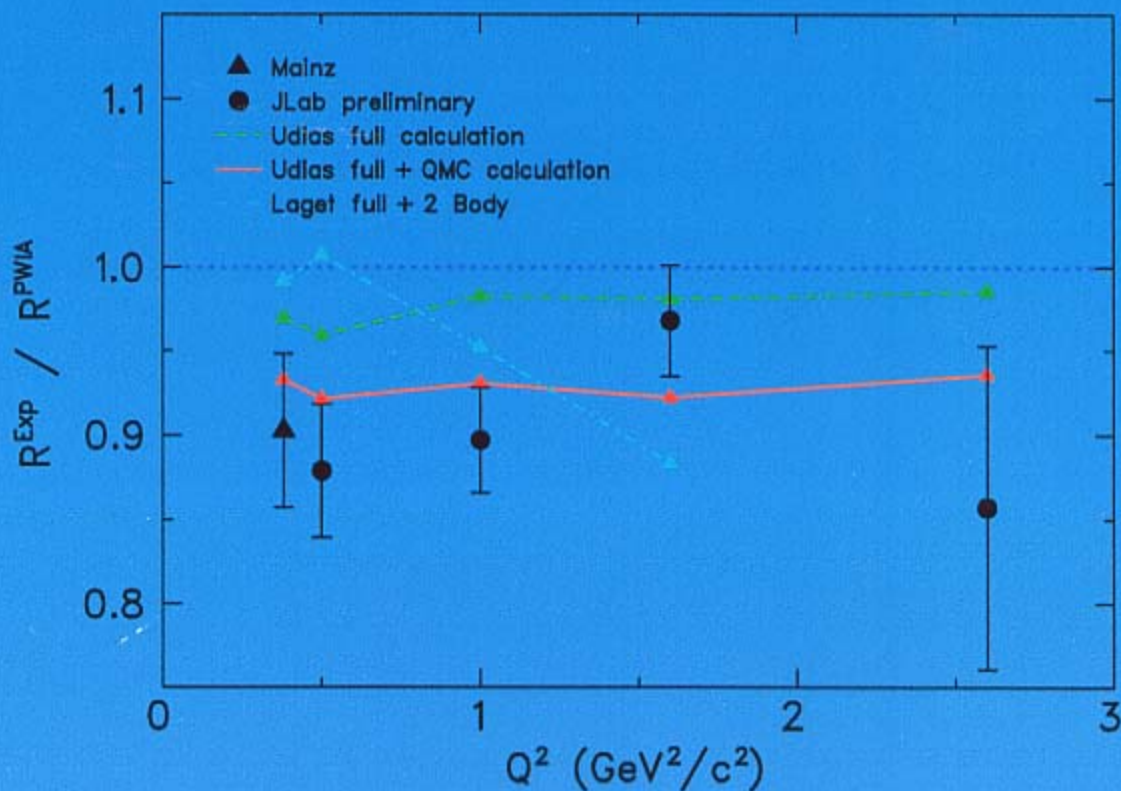


$\rho - \omega$  contributions:

- mainly small  $Q^2$
- 25% to  $r_\pi^2$  (model dependent!)

## Further Tests at Higher- $Q^2$

- More recent measurements **JLAB**  
**E93-049:**



Preliminary results: R. Ransome & C. Glashauser

## Quark Condensate

- Need Hartree + Fock + RPA to understand

Cotanch et al., Krein et al., .....

$$\langle \bar{u}u \rangle \approx \langle \bar{d}d \rangle \approx (-240\text{MeV})^3$$

- Pion is a **Goldstone** excitation:

$$m_\pi^2 \propto \frac{m_u + m_d}{2} = m_q$$

- Dynamical chiral symmetry breaking also implies quark has large effective (**constituent**) mass at low momentum:

$$m(p^2) \rightarrow m_q \quad \text{as } p^2 \rightarrow \infty$$

$$m(p^2) \rightarrow M(0) \quad \text{as } p^2 \rightarrow 0.$$