

# Enhanced Harmonic Up-Conversion Using a Single Laser, Hybrid HGHG-EEHG Scheme

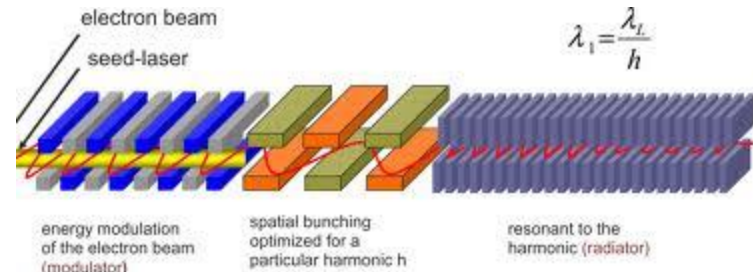
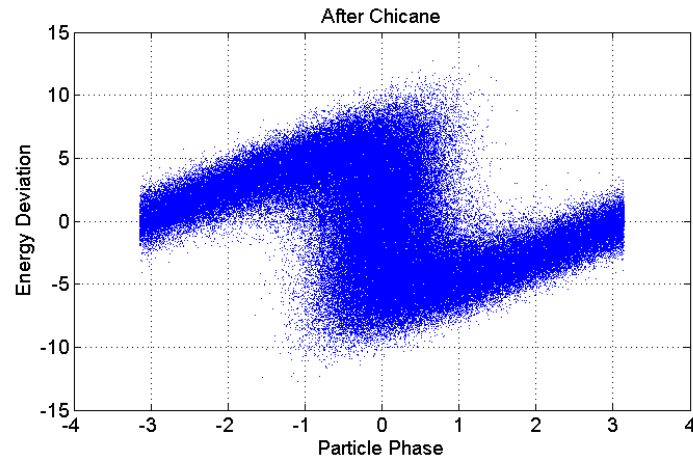
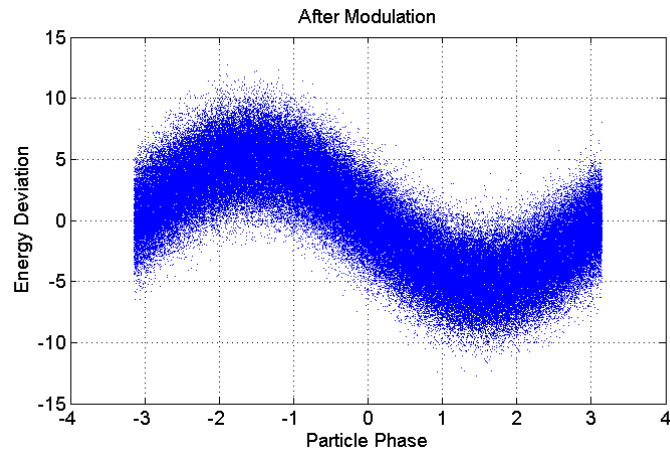
**Quinn Marksteiner**

**Los Alamos National Lab**

Collaborators: Bruce Carlsten, Kip Bishofberger, Leanne Duffy,  
Henry Freund, Nikolai Yampolsky



# High Gain Harmonic Generation



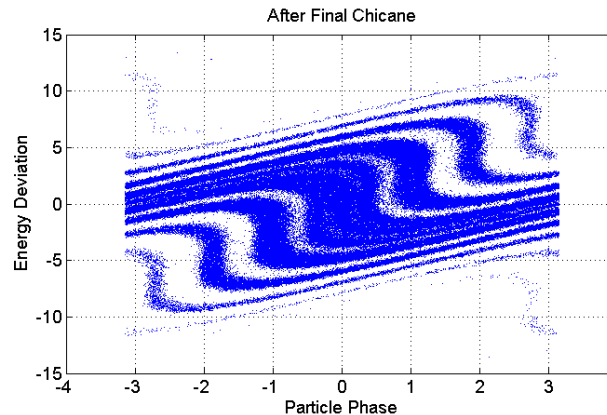
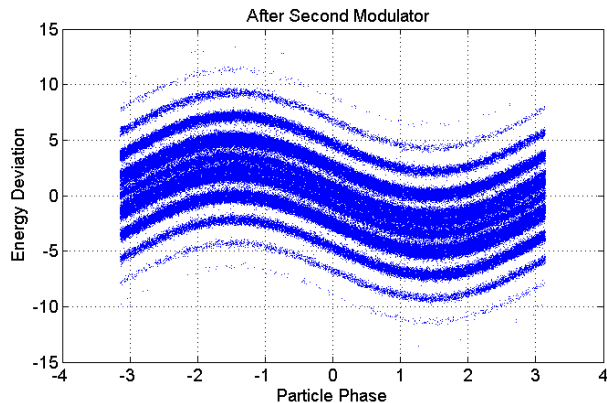
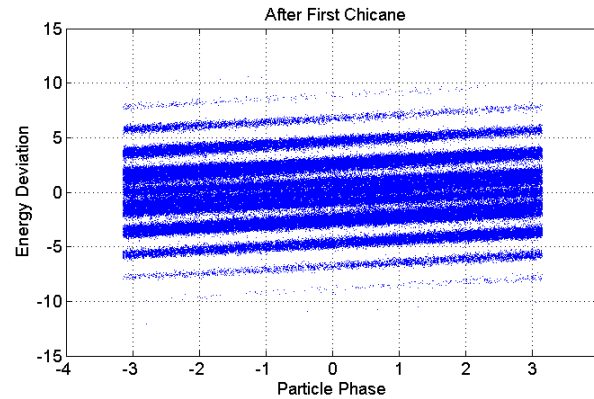
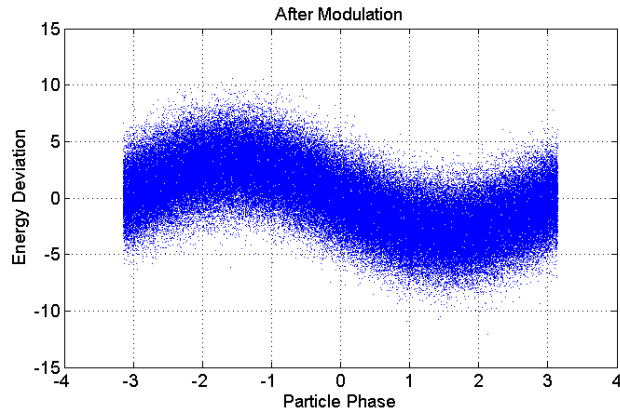
- High Gain Harmonic Generation uses a single modulator and a single chicane to generate harmonics in the electron current profile.
- The bunching factor at higher harmonics is severely limited by the electron beam energy spread<sup>1</sup>:

$$b_n = 2 \exp \left[ -\frac{1}{2} n^2 \left( \frac{\sigma_E}{\Delta E} \right)^2 a^2 \right] J_n(na)$$

[1] L.H. Yu, Phys. Rev. A (1991)



# Echo-Enabled Harmonic Generation



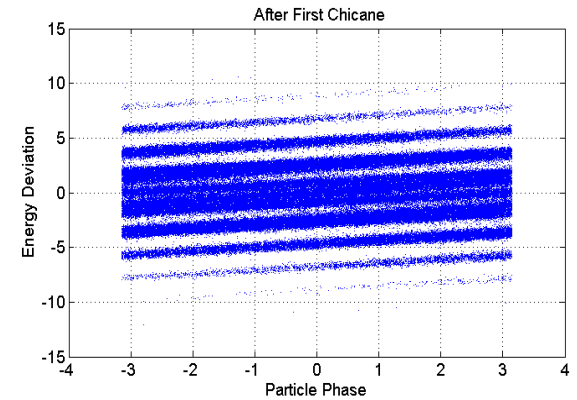
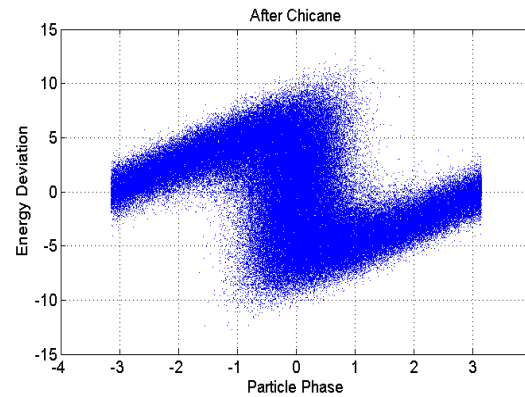
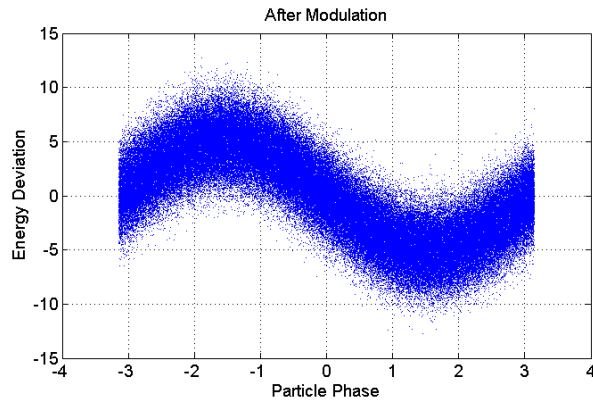
- Echo-Enabled Harmonic Generation<sup>1</sup> uses 2 modulators and 2 chicanes. The 1<sup>st</sup> chicane is large, and breaks the modulated beam into energy bands.
- Theory shows that EEHG has a very favorable scaling with harmonic number<sup>1</sup>:

$$b_n \approx \frac{0.39}{m^{1/3}}$$

[1] Stupakov, PRL (2009)



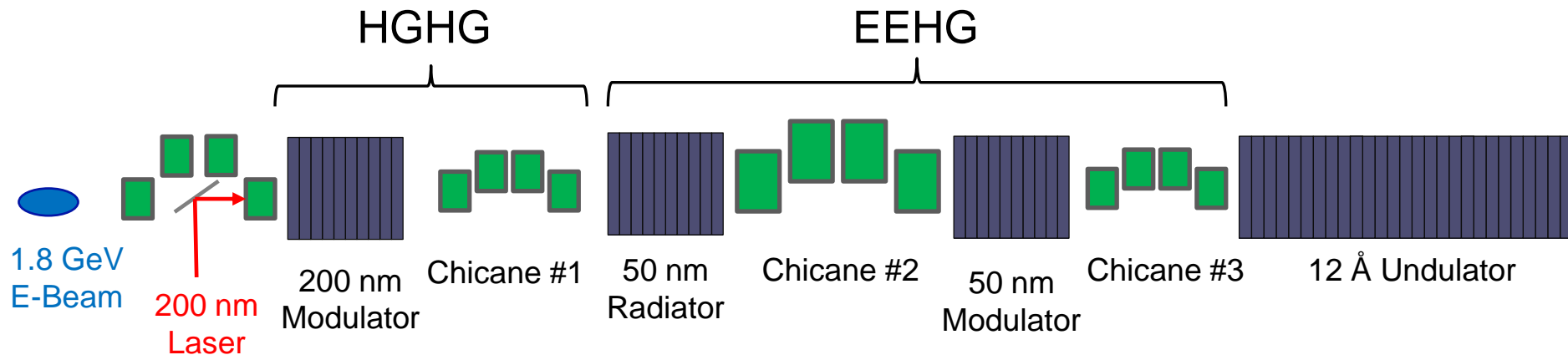
# Before Energy Bands, you get Harmonic Bunching



- With EEHG, you modulate beam, then send through large chicane so that you get energy bands.
- Without any additional modulation, beam passes through a point where there is bunching at higher harmonics.
- Can use beam at this point to get “free” radiation at a higher harmonic, then break beam into energy bands.



## Introduction to Single Laser Hybrid HGHG-EEHG Scheme



- Seed electron beam at 1.8 GeV with 200 nm laser.
- After 200 nm modulator, use chicane #1 to get bunching at 50 nm (HGHG), and then generate 50 nm radiation in radiator.
- Use chicane #2 to create EEHG like energy bands, while delaying and focusing 50 nm radiation with mirrors.
- Chicane #3 produces bunching at large harmonic of laser



## Simple Justification for Idea

- EEHG theory gives:

$$(\kappa m - 1)A_2 B_2 = m + 0.81m^{1/3} \quad \text{and} \quad B_1 = (\kappa m - 1)B_2$$

$$\text{With } A_1 = \Delta E_1 / \sigma_E, \quad B_1 = R_{56}^{(1)} k_1 \sigma_E / E_0, \quad \kappa = k_2 / k_1$$

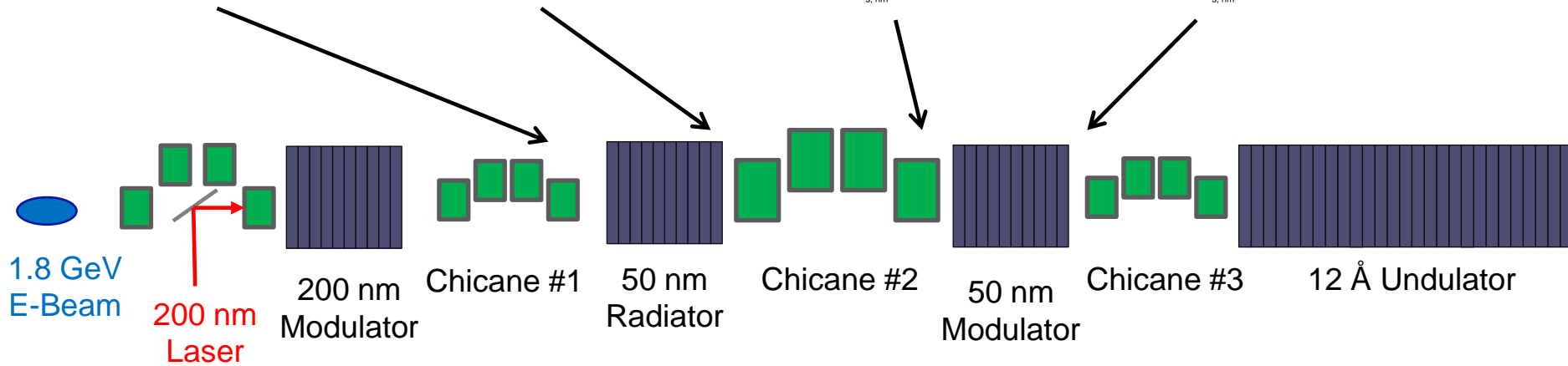
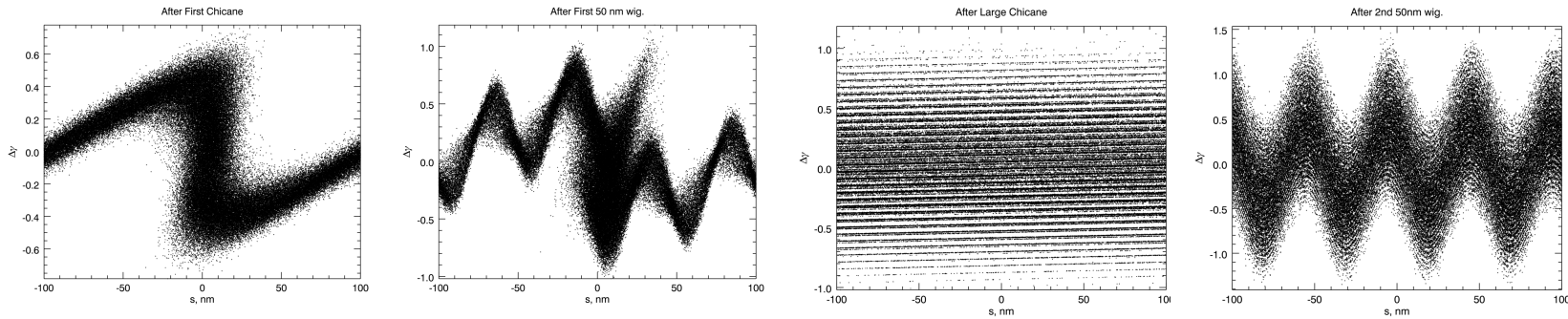
For high harmonic numbers, you get:

$$B_2 \approx \frac{1}{\kappa A_2} \quad \text{and} \quad B_1 \approx \kappa m B_2$$

By producing 4<sup>th</sup> harmonic in the 50 nm, you should get 4 times smaller chicanes! (Ignoring for now the phase space distortion in 50 nm radiator). This means energy bands are 4x farther apart, so that ISR etc. will not be as big of a problem.



# Phase Space of electrons in scheme



- Phase space distorts (compared to EEHG) in 50 nm radiator, changing required  $R_{56}$ s from EEHG theory.



## Parasitic Modulation in 50 nm radiator

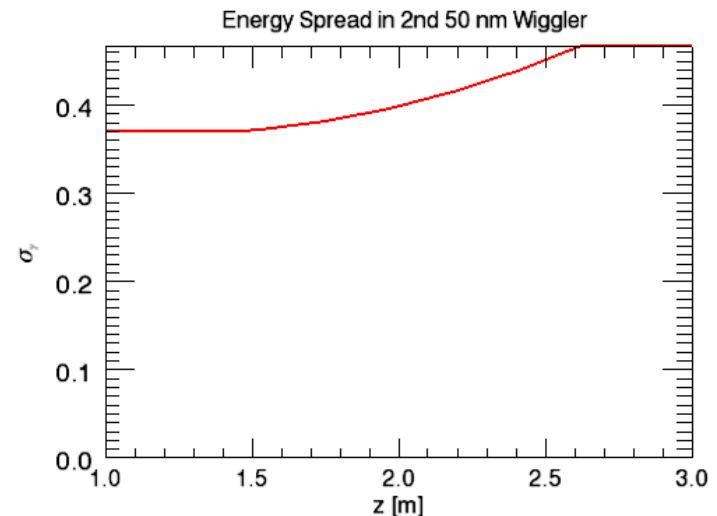
You want to maximize power out, while minimizing increase in energy spread from 50 nm radiator.

Assume 50 nm bunching factor is constant in radiator. Then you get:

$$E_{x0} = -\frac{\mu_0 c \hat{K} z}{4\gamma_r} \tilde{j}_n \quad ; \quad P = \frac{A_b}{\mu_0 c} E^2 \quad ; \quad \frac{d\eta_{\max}}{dz} = \frac{e\mu_0 \tilde{K}^2 \tilde{j}_n}{8mc\gamma_r^3} z$$

Combining these, we get: 
$$\frac{P}{\eta} = \frac{A_b m c^2 \gamma \tilde{j}_n}{e}$$

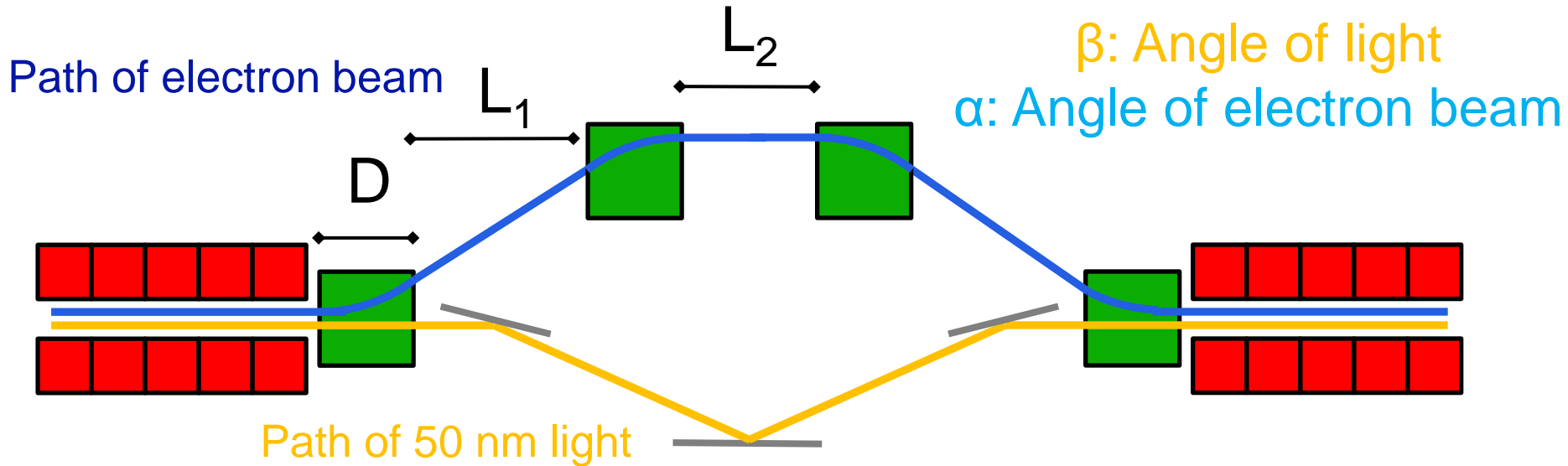
- You need high bunching factor at 4<sup>th</sup> harmonic to get favorable ratio of 50 nm power to induced energy spread. Length of radiator does not effect ratio.





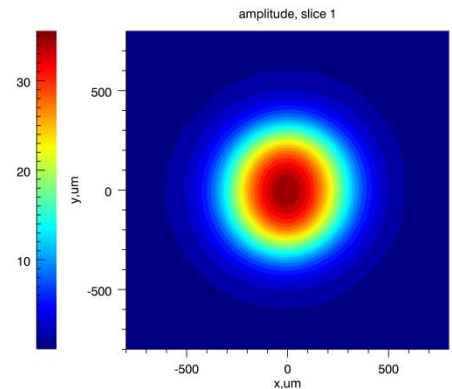


# Mirror Design to Delay 50 nm Radiation



Distance Traveled by Electron Beam:  $4\alpha\rho + \frac{2L_1}{\cos \alpha} + L_2$

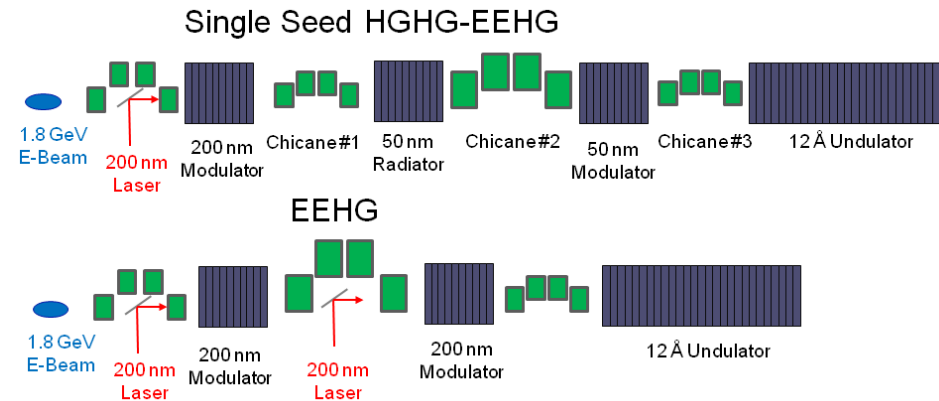
Distance Traveled by Light:  $2D + L_1 + \frac{L_1 + L_2 + 2D}{\cos \beta}$





## Comparison With NGLS EEHG Design

- NGLS has a design to use EEHG to generate coherent 1.2 nm bunching from a 200 nm seed using EEHG<sup>1</sup>.
- EEHG design predicts a theoretical bunching factor of 4.1%, and simulation gives bunching of 2.6%. Requires 2 lasers, one at 31.9 MW and one at 127.1 MW.
- New hybrid design gives a theoretical bunching of 10%, simulations (with ideal  $R_{56}$ s and simple lenses as mirrors) gives 6% bunching at 1.2 nm. Requires only one 16 MW laser, with weaker chicanes than EEHG.
- Hybrid design requires one extra chicane and one extra wiggler.



[1] G. Penn and M. Reinsch, J. Mod. Optics, **58**, 16 (2011) p. 1404



## Description of Parameters in Simulation

Chicane #	$R_{56}$
1	363 $\mu\text{m}$
2	12.1 mm
3	68.3 $\mu\text{m}$

Beam Input	Value
Energy	1.8 GeV
E Spread	50 keV
Current	500 A
$\epsilon_N$	0.6

Wiggler #	$A_w0$	$\lambda_w$	Length
1	4.872	20 cm	1 m
2	4.0	7.29 cm	0.875 m
3	4.0	7.29 cm	0.875 m
4	0.7	2 cm	25 m

Param.	Value
50 nm $b_n$ after chicane #1	13%
50 nm power after wiggler #1	14.1MW



# Harmonic Generation with a Phase Chirp<sup>1</sup>

A Gaussian laser beam with a frequency chirp is given by:

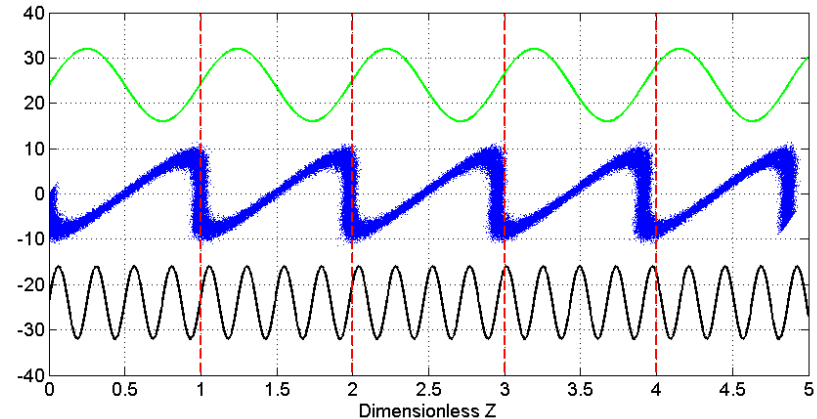
$$E_{in}(\zeta) = E_0 \exp \left[ i \left( \zeta + \frac{\alpha \zeta^2}{2\sigma_L^2} \right) - \frac{\zeta^2}{2\sigma_L^2} \right]$$

Departure from FT limited pulse:

$$\Delta\omega \Delta\tau = 4 \ln 2 M^2 = 4 \ln 2 \sqrt{1 + \alpha^2}$$

For Ti:Sa laser,  $M^2-1 = \sim 10^{-2}$ , ie. almost FT limited. After HG, you get:

$$E_{out}(\zeta) = E_0 \exp \left[ i \left( N\zeta + \frac{N\alpha \zeta^2}{2\sigma_L^2} \right) - \frac{\zeta^2}{2\sigma_L^2} \right]$$



This has a departure from FT limited pulse given by:

$$\Delta\omega \Delta\tau = 4 \ln 2 M^2 = 4 \ln 2 \sqrt{1 + N^2 \alpha^2}$$

For NGLS,  $N \sim 667$ , which will be 94 more broadband than FT limit.

[1] G. Geloni, V. Kocharyan, and E. Saldin, "Analytical studies of constrains on the performance for EEHG FEL seed lasers"



## Phase Chirp with Short Electron Beam

If you are in a regime where the electron beam is shorter than the laser pulse,  $\sigma_e \ll \sigma_L$ , then:

$$E_{out}(\zeta) = E_0 \exp \left[ i \left( N\zeta + \frac{N\alpha\zeta^2}{2\sigma_L^2} \right) - \frac{\zeta^2}{2\sigma_e^2} \right]$$

Let  $s$  denote the factor by which the laser is longer than the electron beam. Then you can substitute  $\sigma_L = s \sigma_e$  into above to get:

$$E_{out}(\zeta) = E_0 \exp \left[ i \left( N\zeta + \frac{N\alpha\zeta^2}{2s^2\sigma_e^2} \right) - \frac{\zeta^2}{2\sigma_e^2} \right]$$

Deviation from FFT limit is now:

$$M_N^2 = \sqrt{1 + N^2\alpha^2 / s^4}$$

For a high quality beam, initial  $M_0^2$  is:

$$M_0^2 - 1 \approx \alpha^2 / 2$$

Then we have a formula for the necessary quality of a long laser, so that the final harmonic is FT limited:

$$\frac{N\sqrt{2(M_0^2 - 1)}}{s^2} \leq 1$$

For  $M_0^2 - 1 = 1\%$ , need a laser that is 11 times longer than electron beam to achieve FT limit.

With less power demands, can you get a longer electron pulse with the same  $M_0^2$ ?



## Introduction to Our Scheme

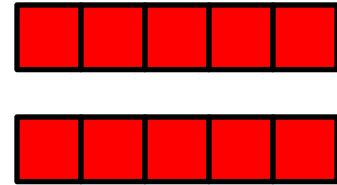
Seed 1 GeV  
electron  
beam with  
200 nm laser

Using 2 Emittance  
Exchangers, bring  
wavelength down  
by ~100 by  
compressing  
electron beam



Accelerate beam to  
12 GeV while  
preserving bunching

Use EEHG or  
EEHG+HGHG  
to bring  
wavelength  
down another  
60x, to 0.29 Å

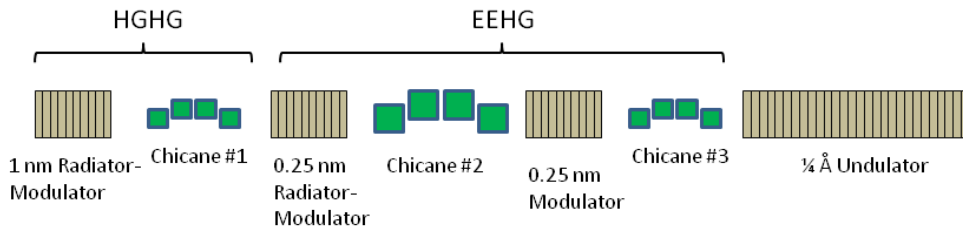


Undulator  
generates 0.29 Å  
x-rays.

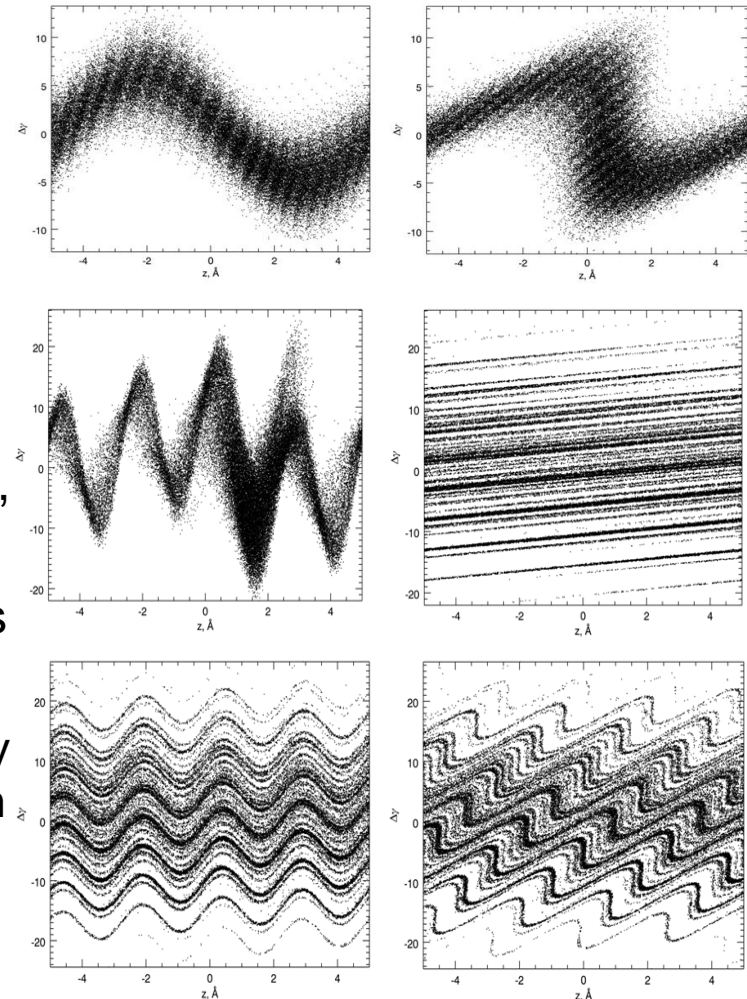
- Seed electron beam at 1 GeV with 200 nm laser.
- Use double emittance exchanger to compress electron beam by ~100, while also compressing bunching down to 2 nm. At higher electron beam energies, bunching would wash out from ISR.
- Accelerate beam to 12 GeV, while preserving the 2 nm bunching.
- Finally, use hybrid EEHG-HGHG scheme seeded with beam bunching to bring wavelength down another 60x.



# Harmonic Generation with Hard X-rays



- We developed an HGHG-EEHG scheme for stepping from 1 nm to 0.25 Å at 20 GeV (old MaRIE parameters).
- Uses bunching in beam, instead of laser seed, to generate harmonics.
- Biggest issue is 10 fs slippage between x-rays and beam in large chicane.
- Input bunching is 10% at 1 nm, with an energy spread of  $5 \times 10^{-5}$ , output is 9.2% at 0.25 Å with an energy spread of  $2.1 \times 10^{-4}$ .





## Conclusions

- New single laser hybrid HGHG-EEHG design only needs a single laser at lower power, and seems to get higher bunching factors at very high harmonic number.
- Requires an extra wiggler and an extra chicane, in addition to grazing incidence mirrors that will delay and focus the HGHG generated light.
- HGHG radiator distorts phase space, but extra energy spread is small, and kinetic code predicts that high bunching can still be achieved.
- It may be possible to mitigate phase chirp problem by using longer bunches (as long as  $M^2$  stays the same).

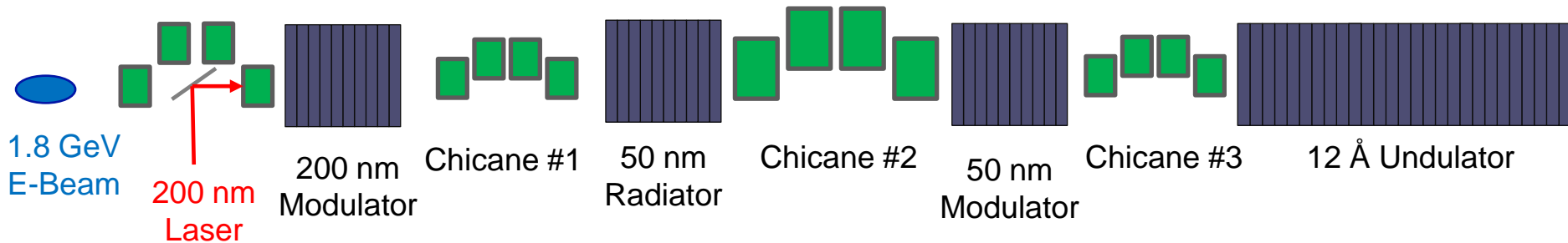




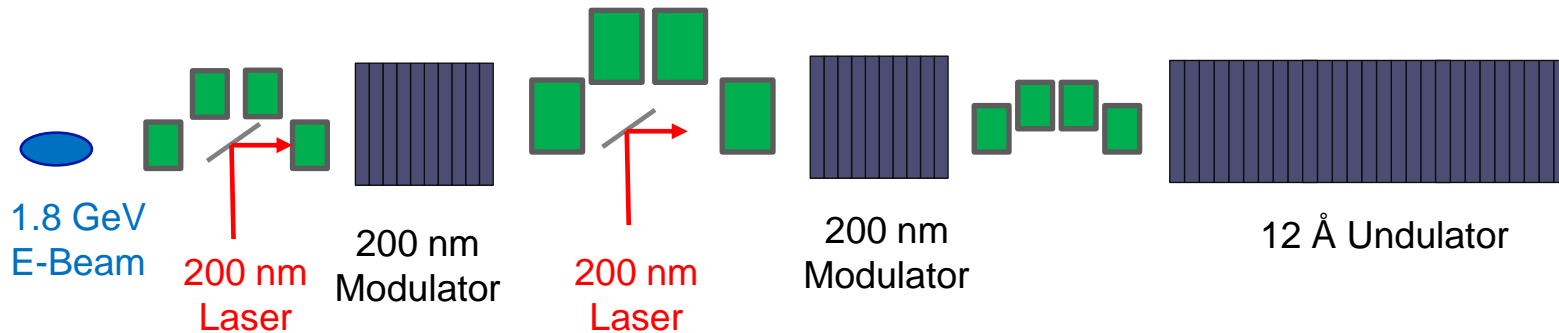
# Extra Slides



# Single Seed HGHG-EEHG



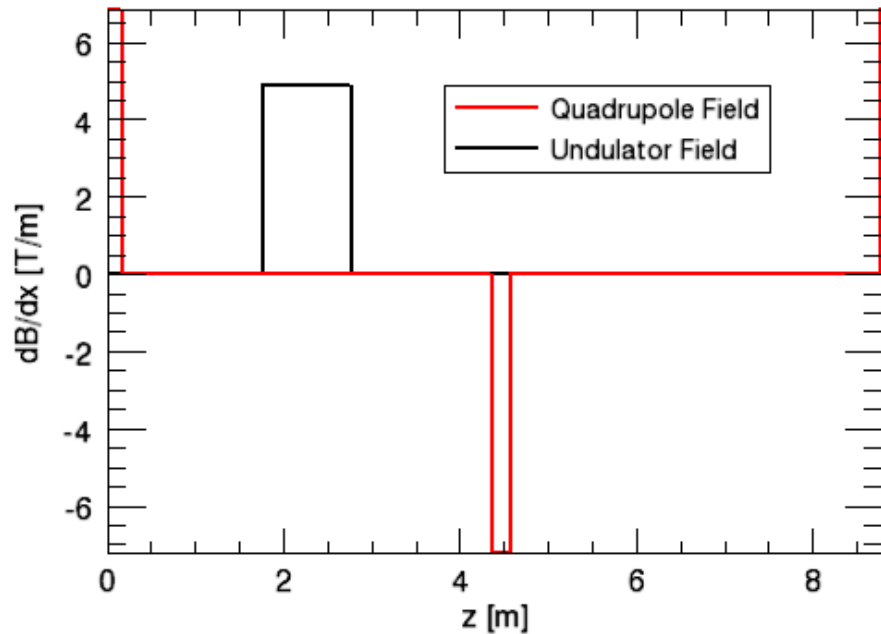
# EEHG



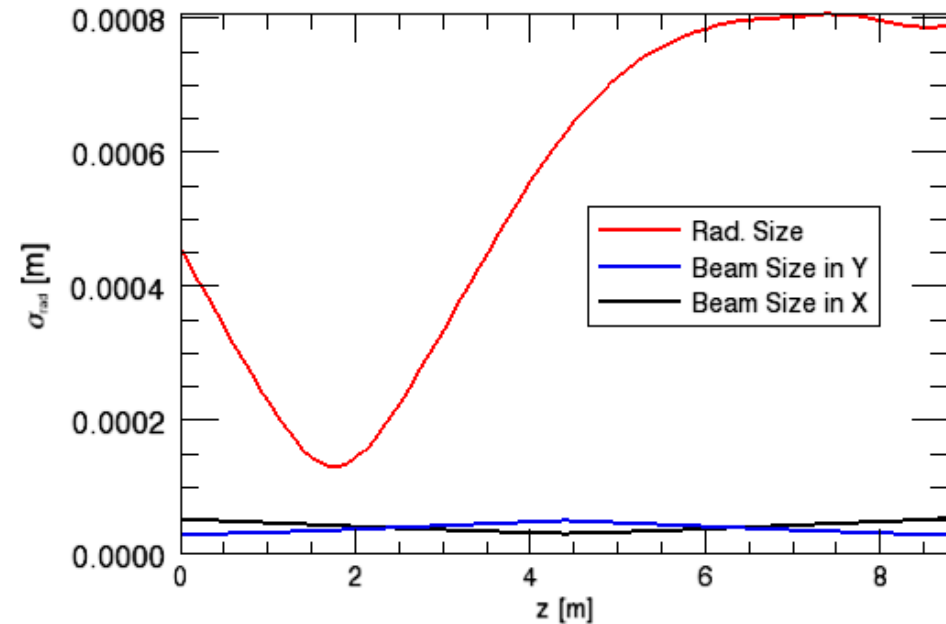
# 200 nm modulator



FODO Lattice for 200 nm Wiggler



Beam Sizes in 200 nm Wiggler

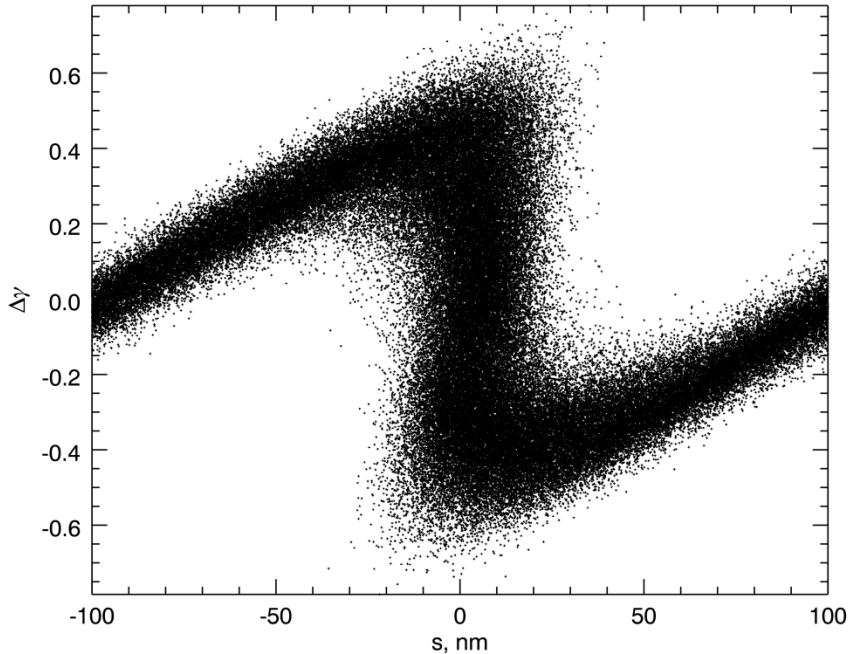


- 200 nm modulator is same design as NGLS – 1 meter long,  $Aw_0 = 4.872$  and wiggler period = 20 cm.
- Light can be focused down a lot, so that power requirement is low.

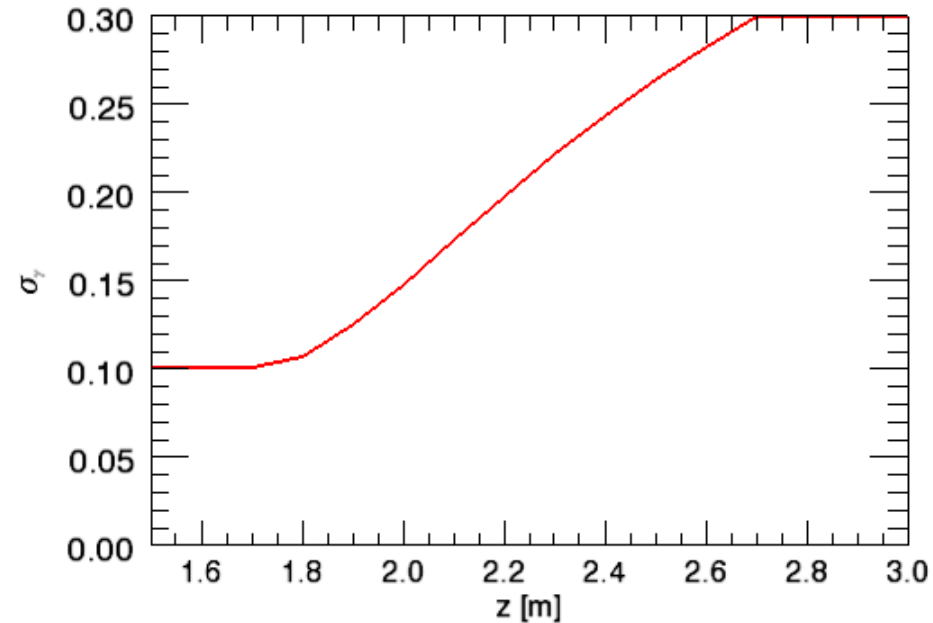
# 200 nm modulator



After First Chicane



Energy Spread Growth in 200 nm Wiggler

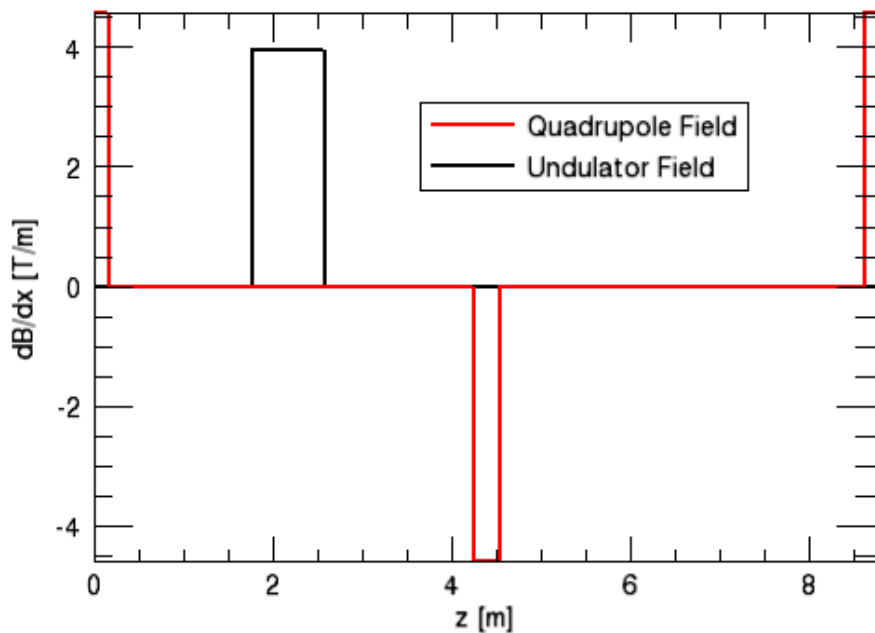


- Right now I apply an ideal R56, but let the particles drift an appropriate amount to approximate the emittance nonlinearity.
  - I include ISR in wigglers, but not chicanes (need Elegant).
  - Bunching at 4<sup>th</sup> harmonic is 13%.

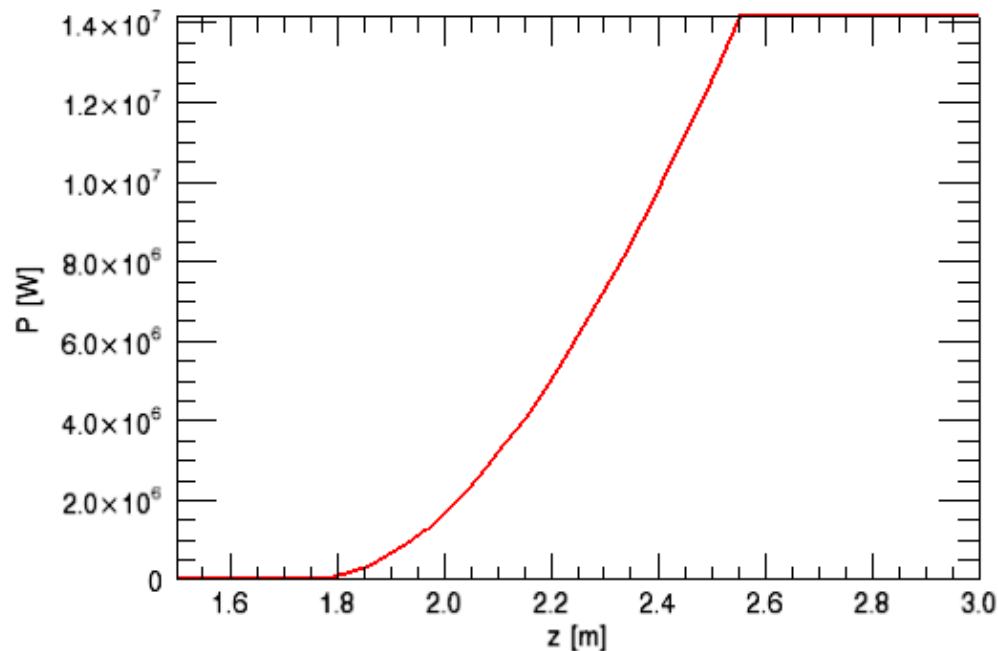
# 50 nm modulator



FODO Lattice for 50 nm Wiggler

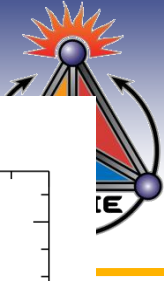


50 nm Power in 1st 50 nm Wiggler

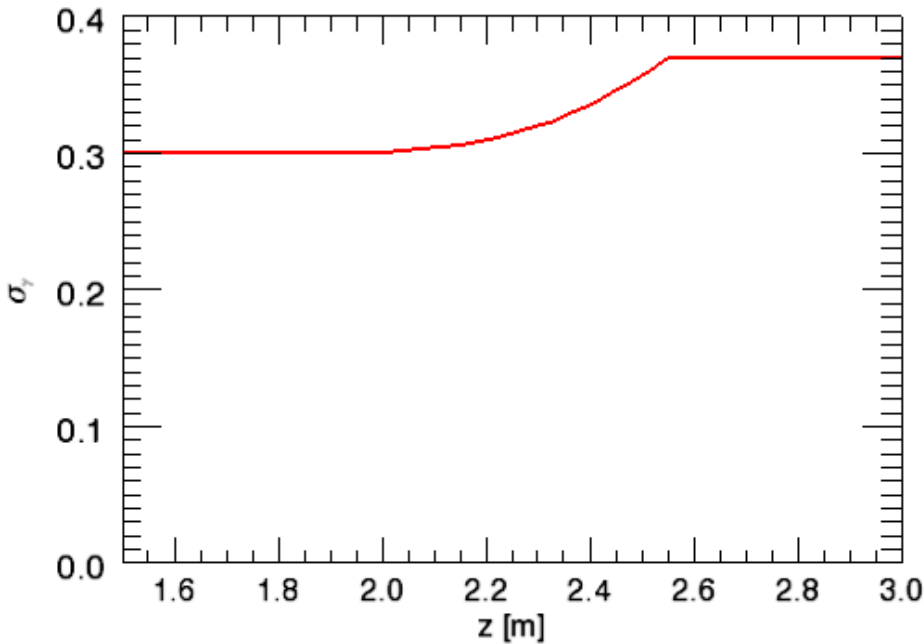


- 50 nm wigglers are  $Aw_0 = 4.0$ , wiggler period 7.29 cm. This has not been optimized.
- The power I get is less than what NGLS design uses in final modulator, but still seems adequate.

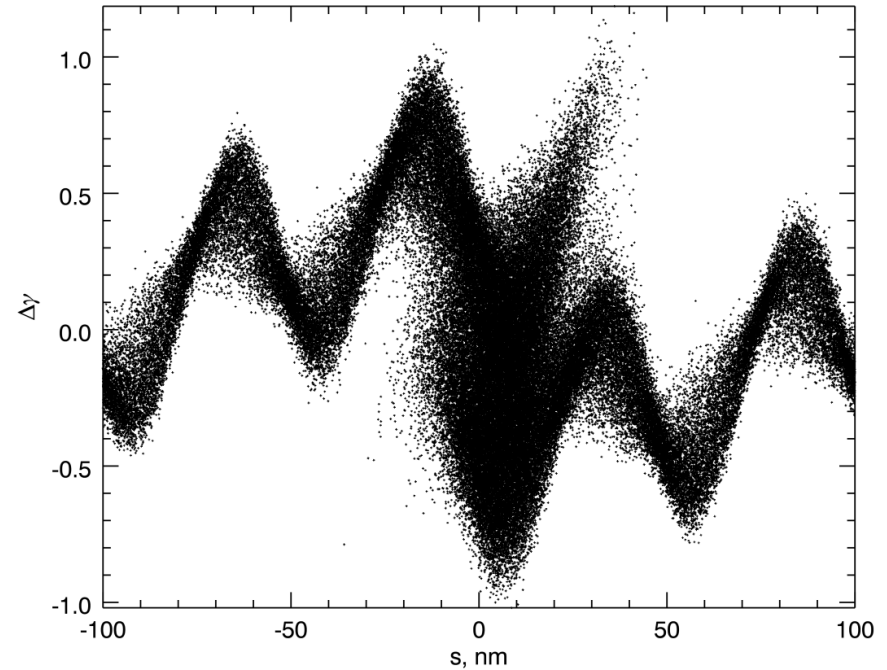
# 50 nm modulator



Energy Spread Growth in 1st 50 nm Wiggler



After First 50 nm wig.



• It is impossible to generate 50 nm radiation without also modulating the beam. Ratio of power to modulation is:

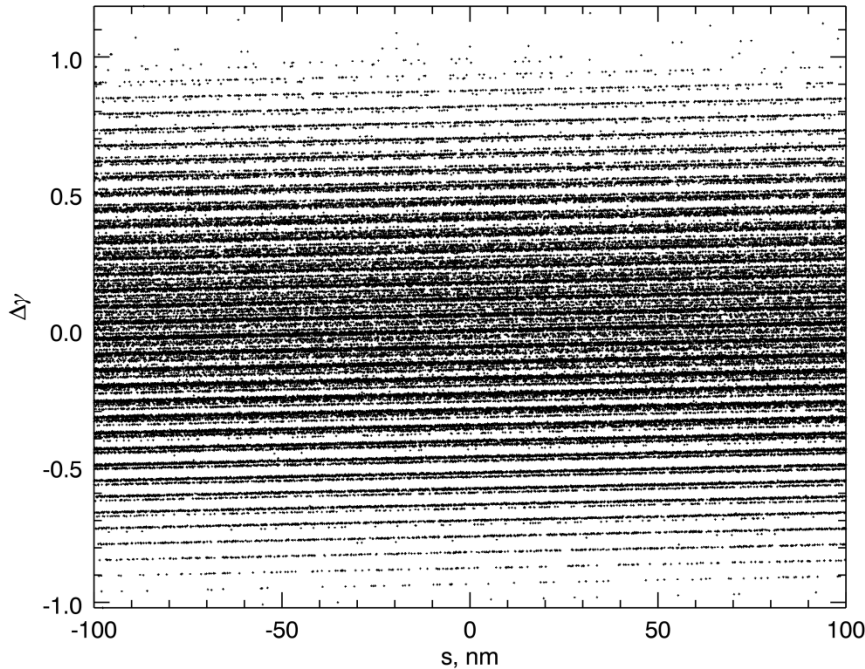
$$\frac{P}{\eta} = \frac{A_b m c^2 \gamma \tilde{j}}{e}$$

• Increase in energy spread is small, but phase space is distorted over traditional EEHG. Kinetic code gives 10% bunching at 12 Å, but large chicane is larger than pure EEHG, and some harmonics may be suppressed.

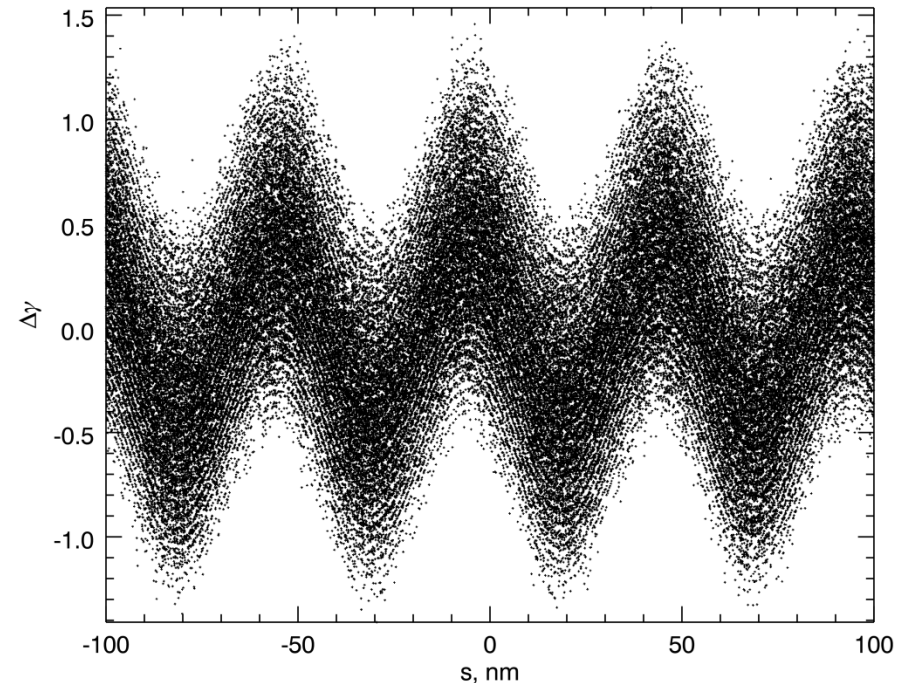
# Second 50 nm modulator



After Large Chicane



After 2nd 50nm wig.

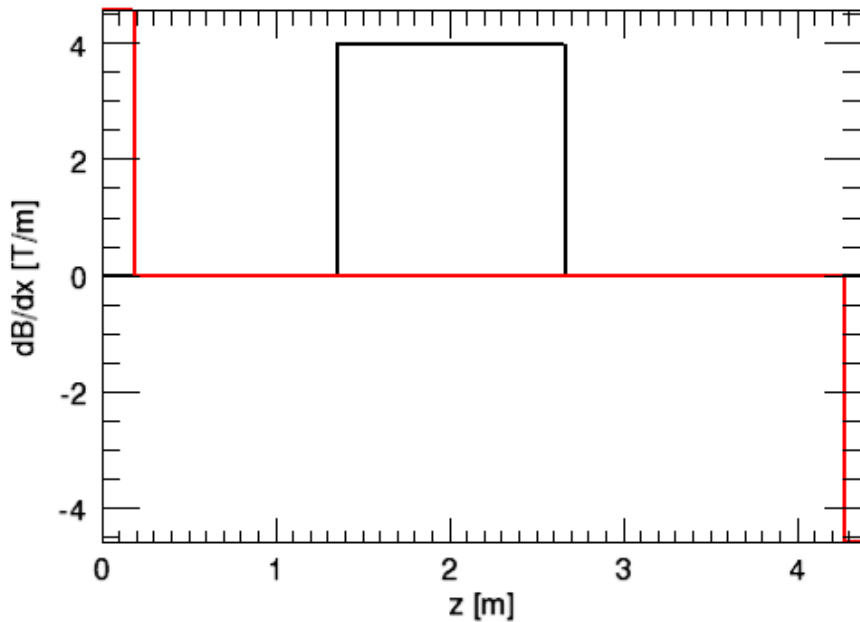


- ISR, nonlinearity, and finite transverse radiation size reduce energy bands in wiggler.
- Still, I am getting 6% bunching at 12 Å, I may be able to improve this by optimizing  $Aw_0$  and transverse electron beam size.

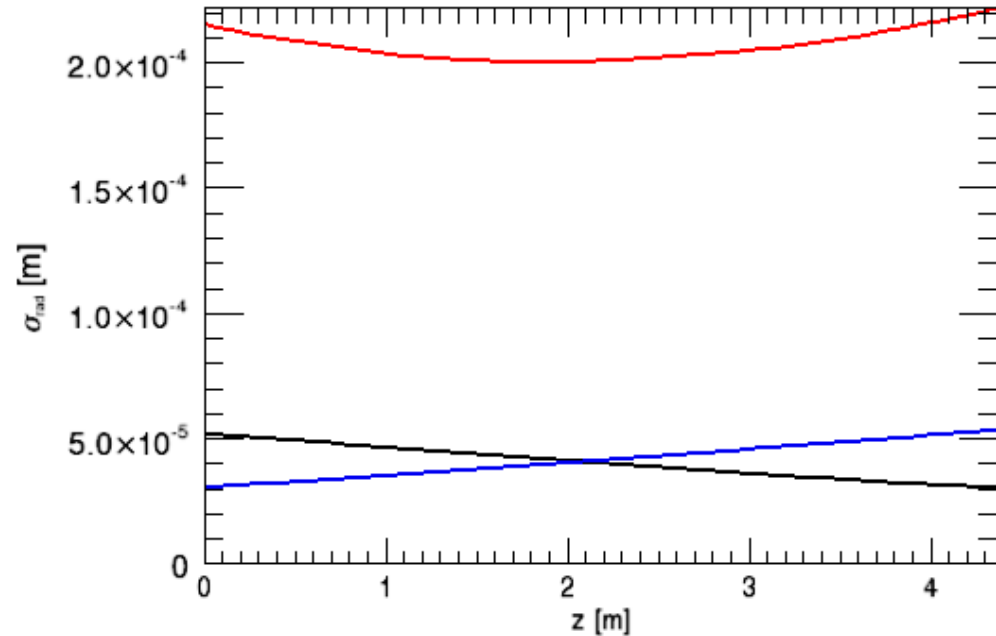
# Second 50 nm modulator



FODO Lattice for 2nd 50 nm Wiggler



Beam Sizes in 2nd 50 nm Wiggler



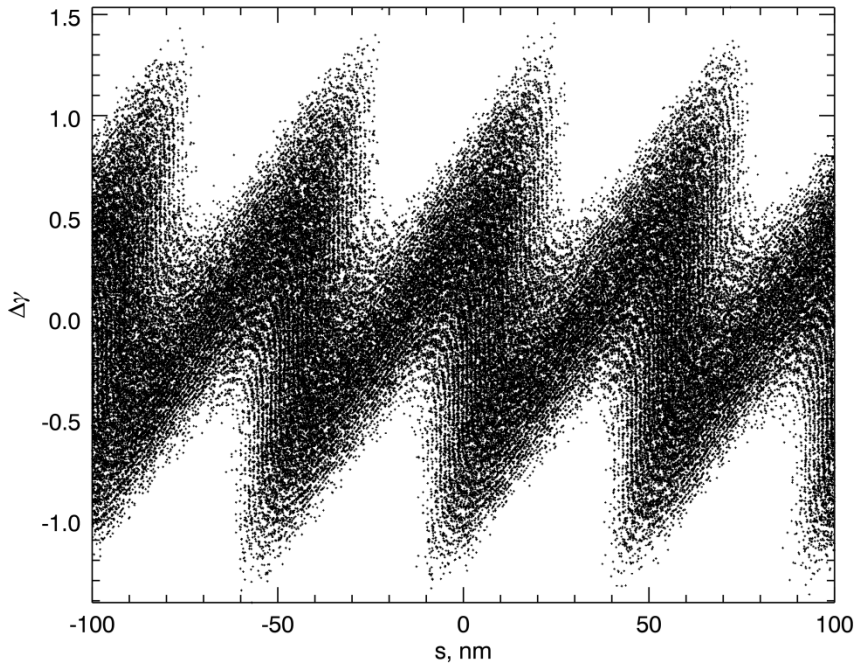
- Size of 50 nm radiation is very important.
- Also need to keep distance between 50 nm and 12 Å wiggler short. Because my final chicane is  $\sim 1/3$  size of current NGLS, we can make this distance  $\sim 1$  m.



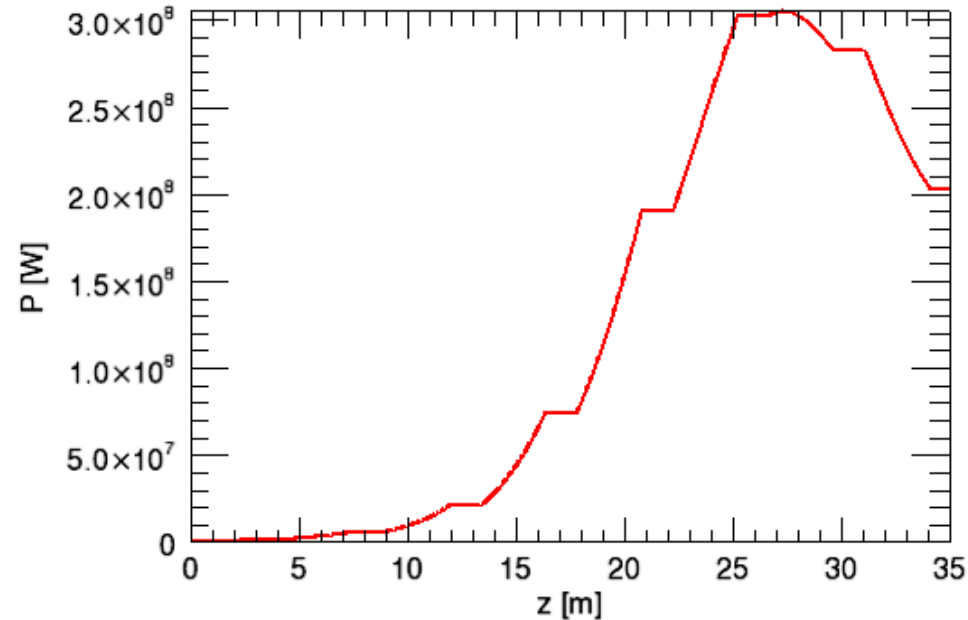
# 12 Å Undulator



After Final Chicane



NGLS Power Growth in 12A Undulators



- Saturation length is 10 m shorter than NGLS design, power about the same.