(Bessel-)weighted asymmetries

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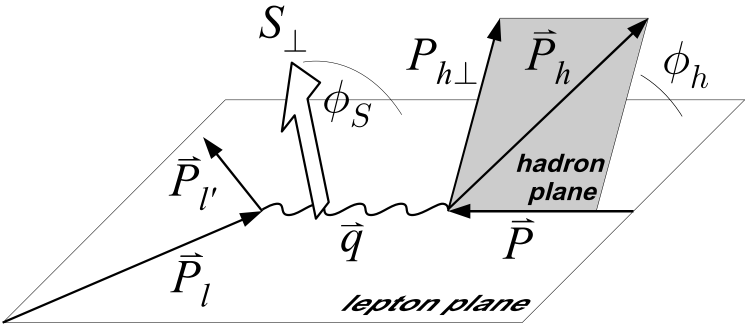
Part I

(Bessel-) Weighted Asymmetries in Semi-Inclusive Deep Inelastic Scattering (SIDIS)

$$\underbrace{l}_{\text{lepton}} + \underbrace{N(P)}_{\text{nucleon}} \longrightarrow l' + \underbrace{h(P_h)}_{\text{hadron}} + \underbrace{X}_{\text{rest}} \tag{1}$$

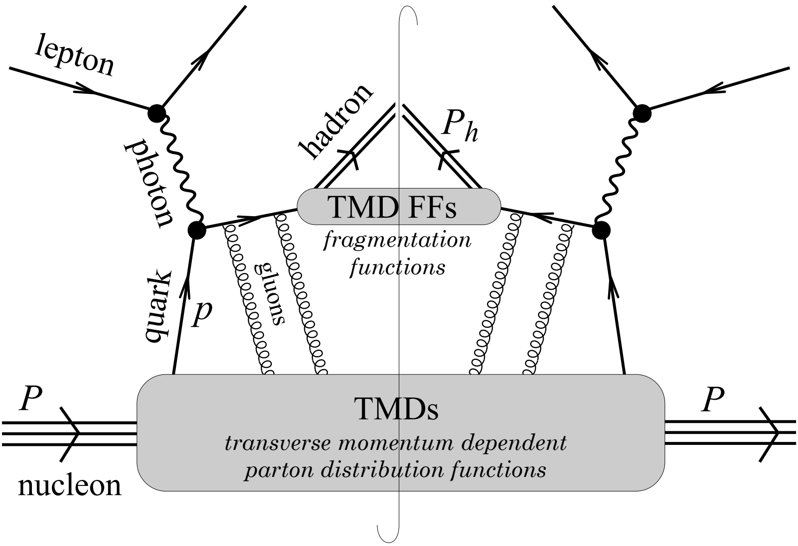


$$x_B = \frac{-q^2}{2P \cdot q}$$

$x \approx x_B$: longitudinal momentum fraction of the struck quark in the nucleon

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$z \approx z_h$: longitudinal momentum fraction deposited in the measured hadron



lepton
photon
hadron
P_h
TMD FFs
fragmentation functions
quark
p
gluons
P
nucleon
TMDs
transverse momentum dependent parton distribution functions
P

$$\frac{d\sigma}{dx_B\, dz_h\, d\phi_S\, d\phi_h\, dP_{h\perp}^2\, dy} \propto \frac{\alpha^2}{x_B\, Q^2} \Big\{ F_{UU,T} + |\boldsymbol{S}_\perp| \sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h-\phi_S)} + 16 \text{ further sturctures} \Big\}$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = x_B\, H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2) \sum_a e_a^2 \times \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{K}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(z\boldsymbol{p}_T + \boldsymbol{K}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}\big) \times \frac{|\boldsymbol{p}_T| \cos(\phi_h - \phi_p)}{M} \underbrace{f_{1T}^{\perp a}(x, \boldsymbol{p}_T^2)}_{\text{TMD}} \underbrace{\mathcal{S}(\boldsymbol{l}_T^2)}_{\text{softf.}} \underbrace{D_1^{\perp a}(z, \boldsymbol{K}_T^2)}_{\text{FF}}$$

3 non-perturbative ingredients:

- TMD: momentum distribution of quarks in nucleon
- TMD FF: fragmentation function
- soft factor $\mathcal{S}$ (soft gluons),
  (absorbed into TMD/FF in [Aybat, Rogers (2011)][Collins tbp])

What's a weighted asymmetry?

weighted asymmetry for general weights $w_{0,1}(\phi_h, |\boldsymbol{P}_{h\perp}|)$

$$A^{w_1} = 2\frac{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, d\phi_S\, w_1(\phi_h, |\boldsymbol{P}_{h\perp}|)\ d\sigma}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, d\phi_S\, w_0(\phi_h, |\boldsymbol{P}_{h\perp}|)\ d\sigma} \quad \begin{array}{l} \leftarrow \text{polarized, } d\sigma^\uparrow - d\sigma^\downarrow \\ \leftarrow \text{unpolarized, } d\sigma^\uparrow + d\sigma^\downarrow \end{array}$$

traditional weighed asymmetry: $w_0 = 1$, $w_1 \propto \sin(n\phi_h + \ldots)|\boldsymbol{P}_{h\perp}|^n$

e.g.,

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{zM}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\ f_{1T}^{\perp(1)a}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\ f_1^{a(0)}(x)\ D_1^{a(0)}(z)}\ ,$$

where

$$f^{(n)}(x) \equiv \int d^2\boldsymbol{p}_T\ \left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x, \boldsymbol{p}_T^2)$$

[Kotzinian, Mulders PLB (1997)] [Boer, Mulders PRD (1998)]

generalized to Bessel weights: $w_{0,1} \propto \sin(n\phi_h + \ldots)\ J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)$

$$A_{UT}^{\frac{2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, \tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}{\sum_a e_a^2\, \tilde{f}_1^{(0)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)},$$

now $\tilde{f}_1$, $\tilde{f}_{1T}^{\perp(1)}$ and $\tilde{D}_1$ are Fourier-transforms of TMDs/FFs (see later).

advantages

- "deconvolution": simple products instead of convolutions of TMDs and FFs
- soft factor $\mathcal{S}$ cancels

Problem: The $\boldsymbol{p}_T$-moments $f_1^{(0)}$, $f_{1T}^{\perp(1)}$, ... are ill-defined.

example: $f_1(x, \boldsymbol{p}_T^2) \sim 1/\boldsymbol{p}_T^2$ for large $\boldsymbol{p}_T^2$ [Bacchetta et al. JHEP (2008)].

$\Rightarrow \quad f_1^{(0)}(x) \equiv \int d^2\boldsymbol{p}_T \; f(x, \boldsymbol{p}_T^2)$ undefined without regularization

Why Bessel-weights?

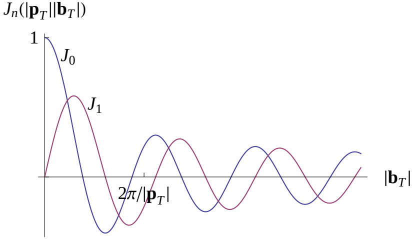
- natural generalization
- naturally more sensitive to low $\boldsymbol{P}_{h\perp}$
- circumvent the problem of ill-defined $\boldsymbol{p}_T$-moments

$$\int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\; e^{-i\boldsymbol{p}_T\cdot\boldsymbol{b}_T}\; f(\boldsymbol{b}_T)$$

$$= \int \frac{d|\boldsymbol{b}_T|}{2\pi}\,|\boldsymbol{b}_T| \int_0^{2\pi} \frac{d\phi_b}{2\pi}\; e^{-i|\boldsymbol{p}_T||\boldsymbol{b}_T|\cos(\phi_p-\phi_b)} \sum_{n=-\infty}^{\infty} e^{in\phi_b}\, f_n(|\boldsymbol{b}_T|)$$

$$= \sum_{n=-\infty}^{\infty} e^{in\phi_p} \int \frac{d|\boldsymbol{b}_T|}{2\pi}\,|\boldsymbol{b}_T|\;(-i)^n J_n(|\boldsymbol{p}_T||\boldsymbol{b}_T|)\; f_n(|\boldsymbol{b}_T|)$$

$J_n(x)$: Bessel function



definition

$$\tilde{f}(x, \boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \; f(x, \boldsymbol{p}_T^2)$$

$$= 2\pi \int_0^\infty d|\boldsymbol{p}_T| \; |\boldsymbol{p}_T| \; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f(x, \boldsymbol{p}_T^2)$$

$$\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) \equiv n! \left(-\frac{2}{M^2} \partial_{\boldsymbol{b}_T^2} \right)^n \; \tilde{f}(x, \boldsymbol{b}_T^2)$$

$$= \frac{2\pi n!}{(M^2)^n} \int d|\boldsymbol{p}_T| \; |\boldsymbol{p}_T| \; \left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|} \right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f(x, \boldsymbol{p}_T^2)$$

connection to $\boldsymbol{p}_T$-moments

$$\tilde{f}^{(n)}(x, 0) = \int d^2\boldsymbol{p}_T \; \left(\frac{\boldsymbol{p}_T^2}{2M^2} \right)^n f(x, \boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

At $|\boldsymbol{b}_T| = 0$, the Fourier-transformed TMDs $\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2)$ are equivalent to $\boldsymbol{p}_T$-moments of TMDs, and can be UV divergent [Ji,Ma,Yuan PRD (2005)].

$$\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} \propto \frac{\alpha^2}{x_B Q^2} \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\; \tilde{\mathcal{S}}(\boldsymbol{b}_T^2) \Big\{$$

$$+ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, H_{UU,T}(Q^2)\, \mathcal{P}[\tilde{f}_1^{(0)}\, \tilde{D}_1^{(0)}]$$

$$+ |\boldsymbol{S}_T| \sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, H_{UT,T}^{\sin(\phi_h - \phi_S)}(Q^2)\, \mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\, \tilde{D}_1^{(0)}]$$

$$+ \epsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, H_{UU}^{\cos(2\phi_h)}(Q^2)\, \mathcal{P}[\tilde{h}_1^{\perp(1)}\, \tilde{H}_1^{\perp(1)}]$$

$$+ \langle \dots\ \text{15 further structures}\ \dots \rangle + \underbrace{\tilde{Y}}_{\text{assume small}} \Big\},$$

where

$$\mathcal{P}[\tilde{f}^{(n)}\, \tilde{D}^{(m)}] \equiv x_B (zM|\boldsymbol{b}_T|)^n\, (zM_h|\boldsymbol{b}_T|)^m$$
$$\times \sum_a e_a^2\; \tilde{f}^{a(n)}(x, z^2\boldsymbol{b}_T^2)\; \tilde{D}^{a(m)}(z, \boldsymbol{b}_T^2)\,.$$

similar as in [Idilbi, Ji, Ma, Yuan PRD (2004)]

$$A^{w_1} = 2\frac{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, d\phi_S\, w_1(\phi_h, |\boldsymbol{P}_{h\perp}|)\; d\sigma}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, d\phi_S\, w_0(\phi_h, |\boldsymbol{P}_{h\perp}|)\; d\sigma}$$

Choose weights that project onto the desired Fourier-mode, e.g.,

$$w_0 = J_0(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\,, \qquad w_1 = \frac{J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T} 2\sin(\phi_h - \phi_S)$$

| use $\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\, J_n(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)\, J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T) = \delta(|\boldsymbol{b}_T| - \mathcal{B}_T)/\mathcal{B}_T$

$$A_{UT}^{\frac{2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T}\sin(\phi_h-\phi_s)}$$

$$= -2\frac{\cancel{\tilde{S}(\mathcal{B}_T^2)}\, H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\, \sum_a e_a^2\, \tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}{\cancel{\tilde{S}(\mathcal{B}_T^2)}\, H_{UU,T}\,(Q^2) \sum_a e_a^2\, \tilde{f}_1^{(0)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}$$

- accessible window $[\mathcal{B}_T^{\min}, \mathcal{B}_T^{\max}] \sim [\,|\boldsymbol{P}_{h\perp}|_{\max}^{-1}, |\boldsymbol{P}_{h\perp}|_{\text{resolution}}^{-1}\,]$
- traditional weighted asymmeties at $\mathcal{B}_T = 0 \Rightarrow$ UV divergences.

$$A^{w_1} = 2\frac{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, d\phi_S\, w_1(\phi_h, |\boldsymbol{P}_{h\perp}|)\; d\sigma}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, d\phi_S\, w_0(\phi_h, |\boldsymbol{P}_{h\perp}|)\; d\sigma}$$

Choose weights that project onto the desired Fourier-mode, e.g.,

$$w_0 = J_0(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\,, \qquad w_1 = \frac{J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T} 2\sin(\phi_h - \phi_S)$$

$$w_0 \xrightarrow{\mathcal{B}_T\to 0} 1 \qquad w_1 \xrightarrow{\mathcal{B}_T\to 0} |\boldsymbol{P}_{h\perp}|/(zM)\sin(\phi_h - \phi_S)$$

use $\int d|\boldsymbol{P}_{h\perp}|\, |\boldsymbol{P}_{h\perp}|\, J_n(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)\, J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T) = \delta(|\boldsymbol{b}_T| - \mathcal{B}_T)/\mathcal{B}_T$

$$A_{UT}^{\frac{2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T}\sin(\phi_h-\phi_s)}$$

$$= \; -2\frac{\cancel{\tilde{S}(\mathcal{B}_T^2)}\, H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\,\sum_a e_a^2\, \tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}{\cancel{\tilde{S}(\mathcal{B}_T^2)}\, H_{UU,T}\,(Q^2)\sum_a e_a^2\, \tilde{f}_1^{(0)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}$$

- accessible window $[\mathcal{B}_T^{\min}, \mathcal{B}_T^{\max}] \sim [\,|\boldsymbol{P}_{h\perp}|_{\max}^{-1}, |\boldsymbol{P}_{h\perp}|_{\text{resolution}}^{-1}\,]$
- traditional weighted asymmeties at $\mathcal{B}_T = 0 \Rightarrow$ UV divergences.

Part II

Fourier-transformed TMDs at the level of matrix elements

lightcone coordinates

$w^\pm = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$, so $w = w^+ \hat{n}_+ + w^- \hat{n}_- + w_T$; P^+ large, $P_T = 0$

Several factorization frameworks have been proposed.
Here we choose [Ji,Ma,Yuan PRD (2005)] as an example:

$$\Phi^{[\Gamma]} \equiv \frac{1}{2} \int \frac{d^4 b}{(2\pi)^4} e^{ip\cdot b} \frac{\overbrace{\langle P,S|\ \bar{q}(0)\,\Gamma\,\mathcal{U}[0,\infty v,\infty v+b,b]\,q(b)\ |P,S\rangle}^{\equiv \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,v,\mu)}}{\underbrace{\langle 0|\ \mathcal{U}[0,\infty v,\infty v+b_T,b_T,b_T-\infty\tilde{v},-\infty\tilde{v},0]\,|0\rangle}_{\equiv \tilde{\mathcal{S}}(b_T^2,\rho,\mu)}}$$

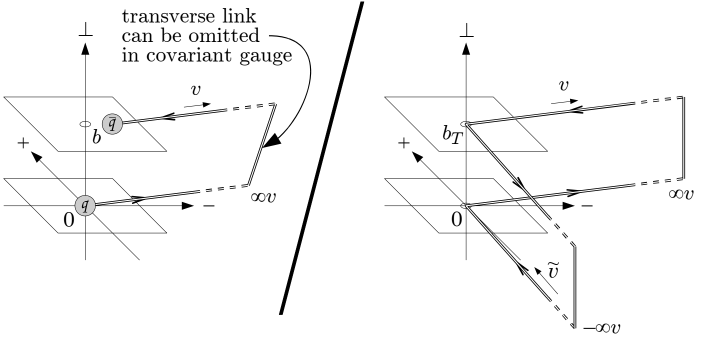
parametrization in terms of TMDs, example $\Gamma = \gamma^+$

$$\int dp^-\ \Phi^{[\gamma^+]}\Big|_{p^+=xP^+} = f_1(x,\boldsymbol{p}_T^2;\hat{\zeta},\rho,\mu) - \frac{\boldsymbol{\epsilon}_{ij}\boldsymbol{p}_i\boldsymbol{S}_j}{m_N} f_{1T}^\perp(x,\boldsymbol{p}_T^2;\hat{\zeta},\rho,\mu)$$

[Ralston, Soper NPB 1979], [Mulders, Tangerman NPB 1996], [Goeke, Metz, Schlegel PLB 2005]

where $\hat{\zeta}^2 \equiv \dfrac{(P\cdot v)^2}{|P^2||v^2|}$, $\rho \mathrel{\hat{=}} \dfrac{(v\cdot\tilde{v})^2}{|v^2||\tilde{v}^2|}$

$$\mathcal{U}[a,b,c,\ldots] \;\equiv\; \mathcal{P}\exp\left(-ig\int_a^b d\xi^\mu A_\mu(\xi) - ig\int_b^c d\xi^\mu A_\mu(\xi) + \ldots\right)$$



$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,v,\mu) \qquad\qquad \tilde{\mathcal{S}}(b_T^2,\rho,\mu)$$

$\hat{\zeta}^2 \equiv \dfrac{(P\cdot v)^2}{|P^2||v^2|}$, $\rho \,\hat{=}\, \dfrac{(v\cdot\tilde{v})^2}{v^2\tilde{v}^2}$. $\quad v,\tilde{v}$ lightlike for $\hat{\zeta},\rho\to\infty$

$\zeta = 4M^2\hat{\zeta}^2$: "Collins-Soper evolution param." [CS NPB (1981)]

evolution eqns. for large $\hat{\zeta},\rho$ [Idilbi,Ji,Ma,Yuan PRD (2004)] as in [Goeke,Metz,Schlegel PLB (2005)] but in Fourier-space

$$\frac{1}{2}\widetilde{\Phi}^{[\gamma^\mu]}_{\text{unsubtr.}}(b,P,S,v,\mu) = \langle P,S|\ \bar{q}(0)\,\gamma^\mu\,\mathcal{U}[0,\infty v,\infty v+b,b]\,q(b)\ |P,S\rangle$$
$$= P^\mu\,\widetilde{A}_2 - iM^2 b^\mu\,\widetilde{A}_3 - iM\epsilon^{\mu\nu\alpha\beta}P_\nu b_\alpha S_\beta\,\widetilde{A}_{12}$$
$$+\frac{M^2}{(v{\cdot}P)}v^\mu\,\widetilde{B}_1 + \frac{M}{v{\cdot}P}\epsilon^{\mu\nu\alpha\beta}P_\nu v_\alpha S_\beta\,\widetilde{B}_7 - \frac{iM^3}{v{\cdot}P}\epsilon^{\mu\nu\alpha\beta}b_\nu v_\alpha S_\beta\,\widetilde{B}_8$$
$$-\frac{M^3}{v{\cdot}P}(b{\cdot}S)\epsilon^{\mu\nu\alpha\beta}P_\nu b_\alpha v_\beta\,\widetilde{B}_9 - \frac{iM^3}{(v{\cdot}P)^2}(v{\cdot}S)\epsilon^{\mu\nu\alpha\beta}P_\nu b_\alpha v_\beta\,\widetilde{B}_{10}$$

in total 32 amplitudes $\widetilde{A}_i^{(\pm)}(b^2, b{\cdot}P, b{\cdot}v/(v{\cdot}P), \hat{\zeta})$, $\widetilde{B}_i^{(\pm)}(\ldots)$,

denoted $^{(+)}$ for $v{\cdot}P > 0$ (SIDIS), $^{(-)}$ for $v{\cdot}P < 0$ (Drell-Yan)

"time-reversal even" : $\widetilde{A}_i^{(+)} = \widetilde{A}_i^{(-)}$

"time-reversal odd" : $\widetilde{A}_i^{(+)} = -\widetilde{A}_i^{(-)}$

as in [Goeke,Metz,Schlegel PLB (2005)] but in Fourier-space

$$\frac{1}{2}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}}(b,P,S,v,\mu) = \langle P,S|\ \bar{q}(0)\,\gamma^+\,\mathcal{U}[0,\infty v,\infty v+b,b]\,q(b)\ |P,S\rangle$$

$$= P^+\underbrace{\left(\widetilde{A}_2 + R(\hat{\zeta})\widetilde{B}_1\right)}_{\widetilde{A}_{2B}} + iMP^+\boldsymbol{\epsilon}_{ij}\boldsymbol{b}_i\boldsymbol{S}_j\underbrace{\left(\widetilde{A}_{12} - R(\hat{\zeta})\widetilde{B}_8\right)}_{\widetilde{A}_{12B}}$$

where $R(\hat{\zeta}) \equiv 1 - \sqrt{1+\hat{\zeta}^{-2}}$, note that $\lim_{\hat{\zeta}\to\infty} R(\hat{\zeta}) = 0$

$\tilde{f}_1(x,\boldsymbol{b}_T^2;\zeta,\rho,\mu)$

$$= \frac{2}{\widetilde{\mathcal{S}}(-\boldsymbol{b}_T^2,\mu^2,\rho)}\int\frac{d(b\cdot P)}{(2\pi)}\,e^{ix(b\cdot P)}\ \widetilde{A}_{2B}\left(-\boldsymbol{b}_T^2, b\cdot P, \frac{(b\cdot P)R(\hat{\zeta})}{M^2}, \hat{\zeta}, \mu\right)$$

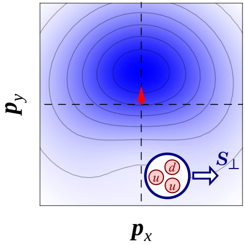
$\tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2;\zeta,\rho,\mu)$

$$= \frac{2}{\widetilde{\mathcal{S}}(-\boldsymbol{b}_T^2,\mu^2,\rho)}\int\frac{d(b\cdot P)}{(2\pi)}\,e^{ix(b\cdot P)}\ \widetilde{A}_{12B}\left(-\boldsymbol{b}_T^2, b\cdot P, \frac{(b\cdot P)R(\hat{\zeta})}{M^2}, \hat{\zeta}, \mu\right)$$

unpolarized quark density in a transversely polarized nucleon

$$\rho_{TU}(x, \boldsymbol{p}_T, \boldsymbol{S}_T) = f_1(x, \boldsymbol{p}_T^2) - \frac{\epsilon_{ij} \boldsymbol{p}_i \boldsymbol{S}_j}{M} f_{1T}^{\perp}(x, \boldsymbol{p}_T^2) = \int dp^- \; \Phi^{[\gamma^+]}$$

$$\langle \boldsymbol{p}_y \rangle_{TU} \equiv \frac{\int dx \int d^2\boldsymbol{p}_T \; \boldsymbol{p}_y \; \rho_{TU}(x, \boldsymbol{p}_T, \boldsymbol{S}_T = (1,0))}{\int dx \int d^2\boldsymbol{p}_T \; \rho_{TU}(x, \boldsymbol{p}_T, \boldsymbol{S}_T = (1,0))} = M \frac{\int dx \, f_{1T}^{\perp(1)}(x)}{\int dx \, f_1^{(0)}(x)}$$



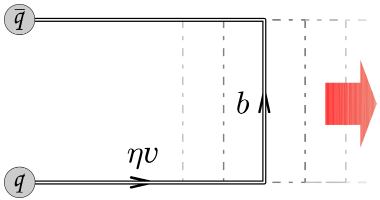
$\langle \boldsymbol{p}_y \rangle_{TU}$:= average quark momentum in transverse y-direction measured in a proton polarized in transverse x-direction.

"dipole moment", "shift"

attention divergences from high-$\boldsymbol{p}_T$-tails!

⇒ "generalized" average transverse momentum shift

$$\langle \boldsymbol{p}_y \rangle_{TU}(\mathcal{B}_T) \equiv M \frac{\int dx \, \tilde{f}_{1T}^{\perp(1)}(x, \mathcal{B}_T^2)}{\int dx \, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\cancel{\widetilde{\mathcal{S}}(-\mathcal{B}_T^2, \ldots)} \; \widetilde{A}_{12B}(-\mathcal{B}_T^2, 0, 0, \hat{\zeta}, \mu)}{\cancel{\widetilde{\mathcal{S}}(-\mathcal{B}_T^2, \ldots)} \; \widetilde{A}_{2B}(-\mathcal{B}_T^2, 0, 0, \hat{\zeta}, \mu)}$$



lattice limitations

- link spacelike, finite length

$\Rightarrow$ look for plateau at large η

- $\boxed{\hat{\zeta}_{\max} = \frac{|\boldsymbol{P}_{\mathrm{lat}}|}{M}}$
- $\mathcal{B}_T \gtrsim a$ (at least a few lattice spacings a)

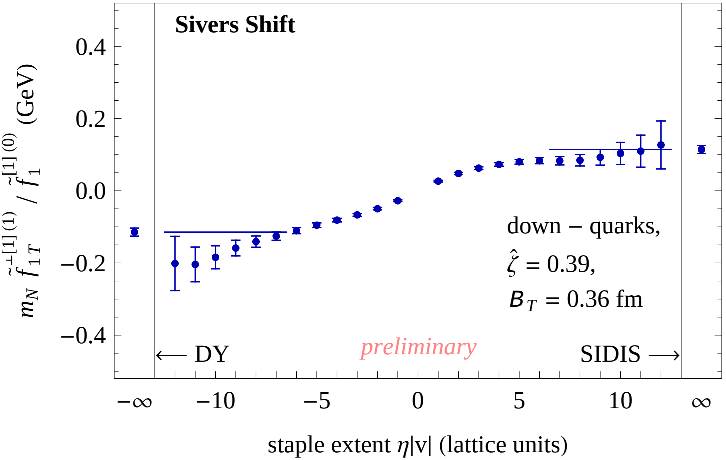
$+$ typical lattice limitations

$$\langle \boldsymbol{p}_y \rangle_{TU}(\mathcal{B}_T;\zeta) \equiv M\,\frac{\int dx\, \tilde{f}_{1T}^{\perp(1)}(x,\mathcal{B}_T^2)}{\int dx\, \tilde{f}_1^{(0)}(x,\mathcal{B}_T^2)}$$

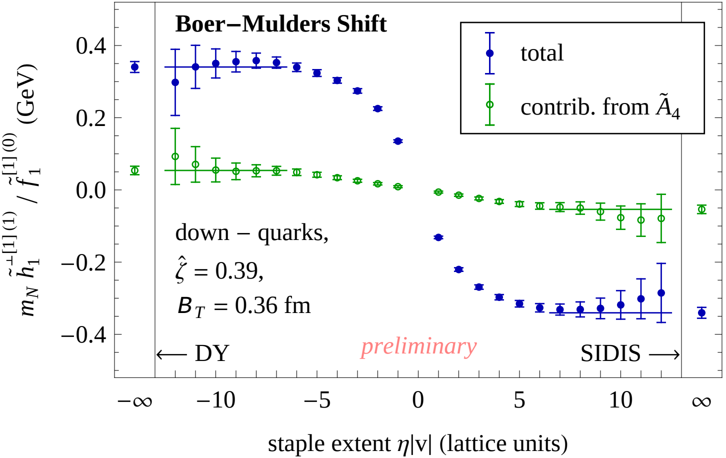
$$= \lim_{\eta\to\infty} \frac{\widetilde{A}_{12}^{\mathrm{lat}}(-\mathcal{B}_T^2,0,0,\hat{\zeta},\mu,\eta) - R(\hat{\zeta})\ \widetilde{B}_8^{\mathrm{lat}}(-\mathcal{B}_T^2,0,0,\hat{\zeta},\mu,\eta)}{\widetilde{A}_2^{\mathrm{lat}}(-\mathcal{B}_T^2,0,0,\hat{\zeta},\mu,\eta) + R(\hat{\zeta})\ \widetilde{B}_1^{\mathrm{lat}}(-\mathcal{B}_T^2,0,0,\hat{\zeta},\mu,\eta)}$$

preliminary lattice results

- MILC lattices (staggered)
- LHPC propagators (domain wall)
- pion mass $m_\pi \approx 500$ MeV
- box 20^3, spacing $a \approx 0.12$ fm

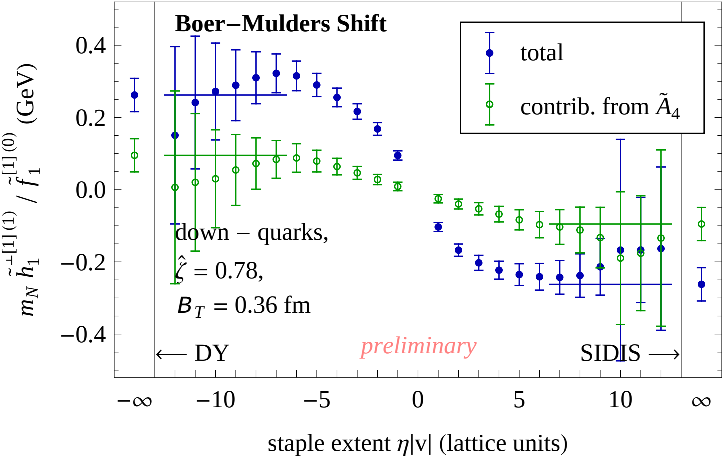


$$M \frac{\int_{-1}^{1} dx\, \tilde{f}_{1T}^{\perp(1)}(x, \mathcal{B}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_{1}^{(0)}(x, \mathcal{B}_T^2)} = \frac{\widetilde{A}_{12} - R(\hat{\zeta})\, \widetilde{B}_8}{\widetilde{A}_2 + R(\hat{\zeta})\, \widetilde{B}_1}$$



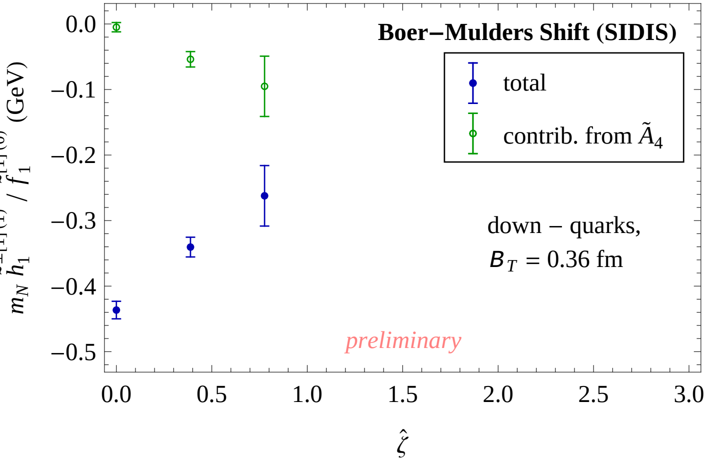
$$M\,\frac{\int_{-1}^{1} dx\, \tilde{h}_1^{\perp(1)}(x,\mathcal{B}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_1^{(0)}(x,\mathcal{B}_T^2)} = \frac{\widetilde{A}_4 - R(\hat{\zeta})\,\widetilde{B}_3}{\widetilde{A}_2 + R(\hat{\zeta})\,\widetilde{B}_1}$$

$R(\hat{\zeta}) \xrightarrow{\hat{\zeta}\to\infty} 0$ $\Rightarrow$ expect numerator dominated by $\widetilde{A}_4$ close to lighcone.
(We are still far from that region.)



$$M\,\frac{\int_{-1}^{1} dx\, \tilde{h}_1^{\perp(1)}(x, \mathcal{B}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\widetilde{A}_4 - R(\hat{\zeta})\, \widetilde{B}_3}{\widetilde{A}_2 + R(\hat{\zeta})\, \widetilde{B}_1}$$

$R(\hat{\zeta}) \xrightarrow{\hat{\zeta}\to\infty} 0$ $\Rightarrow$ expect numerator dominated by $\widetilde{A}_4$ close to lighcone.
(We are still far from that region.)



$$M \frac{\int_{-1}^{1} dx\, \tilde{h}_1^{\perp(1)}(x, \mathcal{B}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\widetilde{A}_4 - R(\hat{\zeta})\, \widetilde{B}_3}{\widetilde{A}_2 + R(\hat{\zeta})\, \widetilde{B}_1}$$

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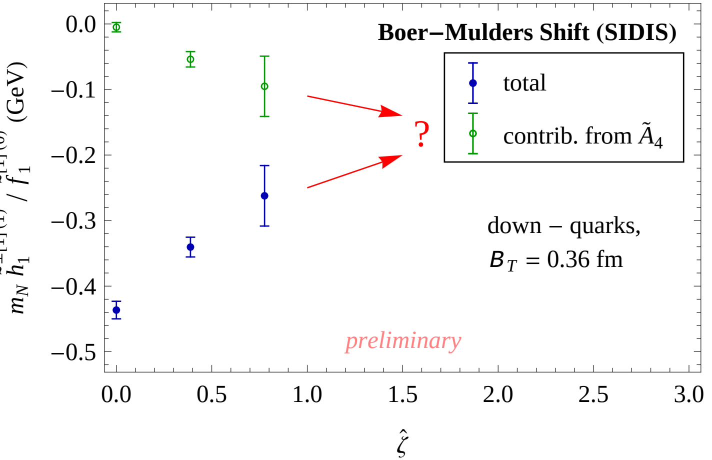

$$M \frac{\int_{-1}^{1} dx\, \tilde{h}_1^{\perp(1)}(x, \mathcal{B}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\widetilde{A}_4 - R(\hat{\zeta})\, \widetilde{B}_3}{\widetilde{A}_2 + R(\hat{\zeta})\, \widetilde{B}_1}$$

$R(\hat{\zeta}) \xrightarrow{\hat{\zeta}\to\infty} 0 \;\Rightarrow$ expect numerator dominated by $\widetilde{A}_4$ close to lighcone.

- Soft factors cancel in weighted asymmetries.
- generalization to Bessel-weights
  $\Rightarrow$ avoid divergences from high $\boldsymbol{p}_T$-tails

- similar advantages found in ratios of Fourier-transformed TMDs
- first lattice calculations for the Sivers- and the Boer-Mulders shift albeit at a relatively low Collins-Soper parameter ζ

$\longrightarrow$ towards observables with minimal model-dependence.

Thanks to the MILC and LHP lattice collaborations for providing gauge configurations and propagators.