Argonne NATIONAL LABORATORY

# π & K Distribution Amplitudes & Distribution Functions

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Nuclear Theory

Physics Division

Office of Nuclear Physics

Exploring Nuclear Matter - Quarks to Stars

U.S. DEPARTMENT OF ENERGY

*Students, Postdocs*
Asst. Profs.

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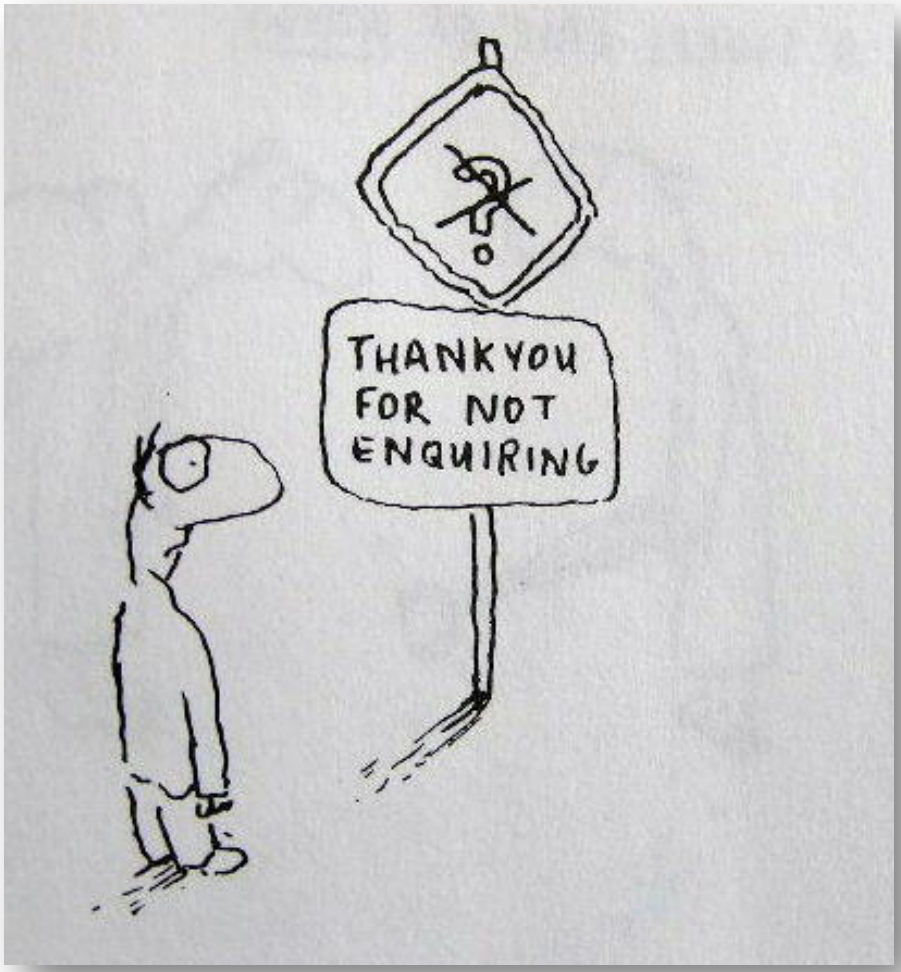
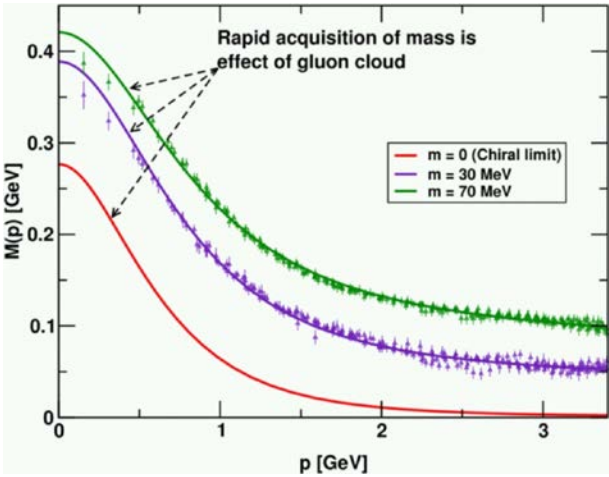



# Enigma of Mass

# Dynamical Chiral Symmetry Breaking




- *DCSB is a fact in QCD*
  - ***Dynamical***, not spontaneous
    - Add nothing to *QCD , no Higgs field, nothing!* Effect achieved purely through quark+gluon dynamics.
  - It's the most important mass generating mechanism for visible matter in the Universe.
    - Responsible for ≈98% of the proton's mass.
    - Higgs mechanism is (*almost*) irrelevant to light-quarks.
  - Just like gluons and quarks, and for the same reasons, condensates are confined within hadrons.
    - There are *no* vacuum condensates.

*Confinement contains condensates,* S.J. Brodsky, C.D. Roberts, R. Shrock and P.C. Tandy, arXiv:1202.2376 [nucl-th], Phys. Rev. C**85** (2012) 065202

*Maris, Roberts and Tandy*
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

# Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

***Pseudovector components necessarily nonzero. Cannot be ignored!***

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

***Owing to DCSB & Exact in Chiral QCD***

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

# Enigma of mass

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

$$f_\pi E_\pi(p^2) = B(p^2)$$

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

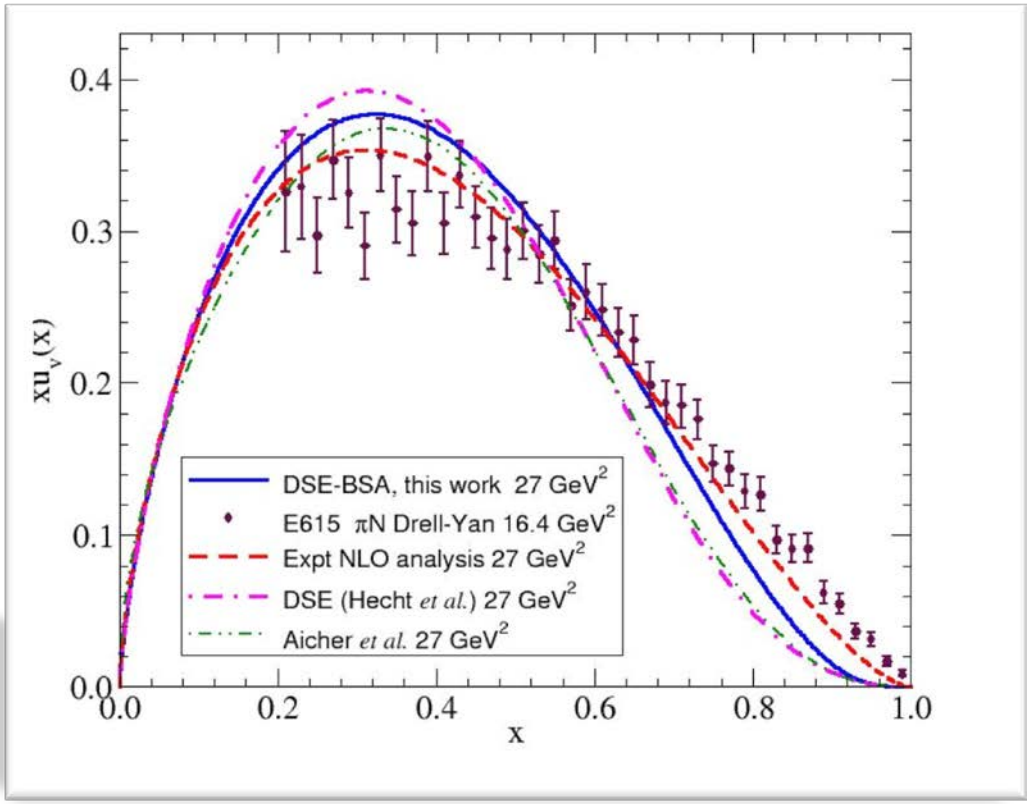
# Parton Structure of Hadrons

- Valence-quark structure of hadrons
  - Definitive of a hadron.
    After all, it's how we distinguish a proton from a neutron
  - Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers
  - Via evolution, determines background at LHC
- Foreseeable future will bring precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*

*Valence quark distributions in the pion, M.B. Hecht, Craig D. Roberts, S.M. Schmidt*, nucl-th/0008049, Phys.Rev. C63 (2001) 025213 .

# Parton Structure of Hadrons

- Need for calculation is emphasised by Saga of pion's valence-quark distribution:
  - 1989: $u_v^\pi \sim (1-x)^1$ – inferred from LO-Drell-Yan & disagrees with QCD;
  - 2001: DSE- QCD predicts $u_v^\pi \sim (1-x)^2$ argues that distribution inferred from data can't be correct;



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# Parton Structure of Hadrons

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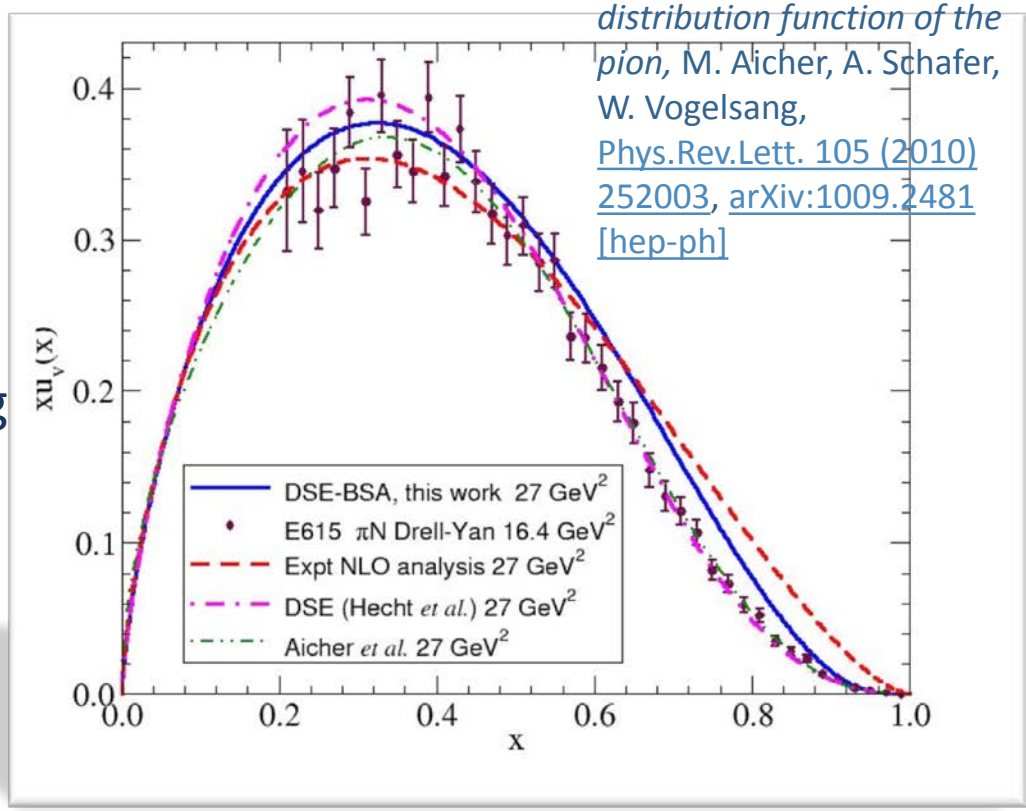
  - 2001: DSE- QCD predicts $u_v^\pi \sim (1-x)^2$ argues that distribution inferred from data can't be correct;
  - 2010: NLO reanalysis including soft-gluon resummation, inferred distribution agrees with DSE and QCD

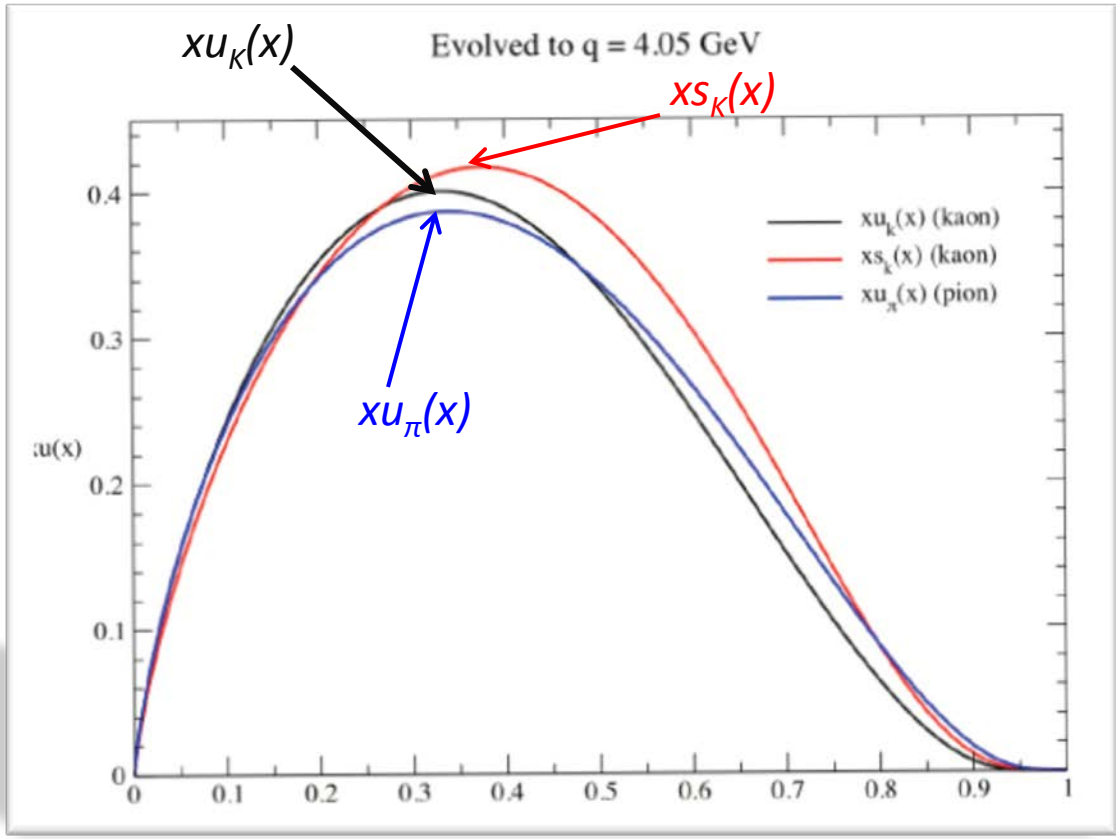
*Soft-gluon resummation and the valence parton distribution function of the pion,* M. Aicher, A. Schafer, W. Vogelsang, Phys.Rev.Lett. 105 (2010) 252003, arXiv:1009.2481 [hep-ph]



*Trang, Bashir, Roberts & Tandy,* "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

# $q_v^{\pi}(x)$ & $q_v^{K}(x)$

- $m_s \approx 24\, m_u$ & $M_s \approx 1.25\, M_u$
  Expect the *s*-quark to carry more of the kaon's momentum than the *u*-quark, so that $xs_K(x)$ peaks at larger value of $x$ than $xu_K(x)$
- Expectation confirmed in computations, with *s*-quark distribution peaking at 15% larger value of $x$
- *Even though deep inelastic scattering is a high-$Q^2$ process, constituent-like mass-scale explains the shift*



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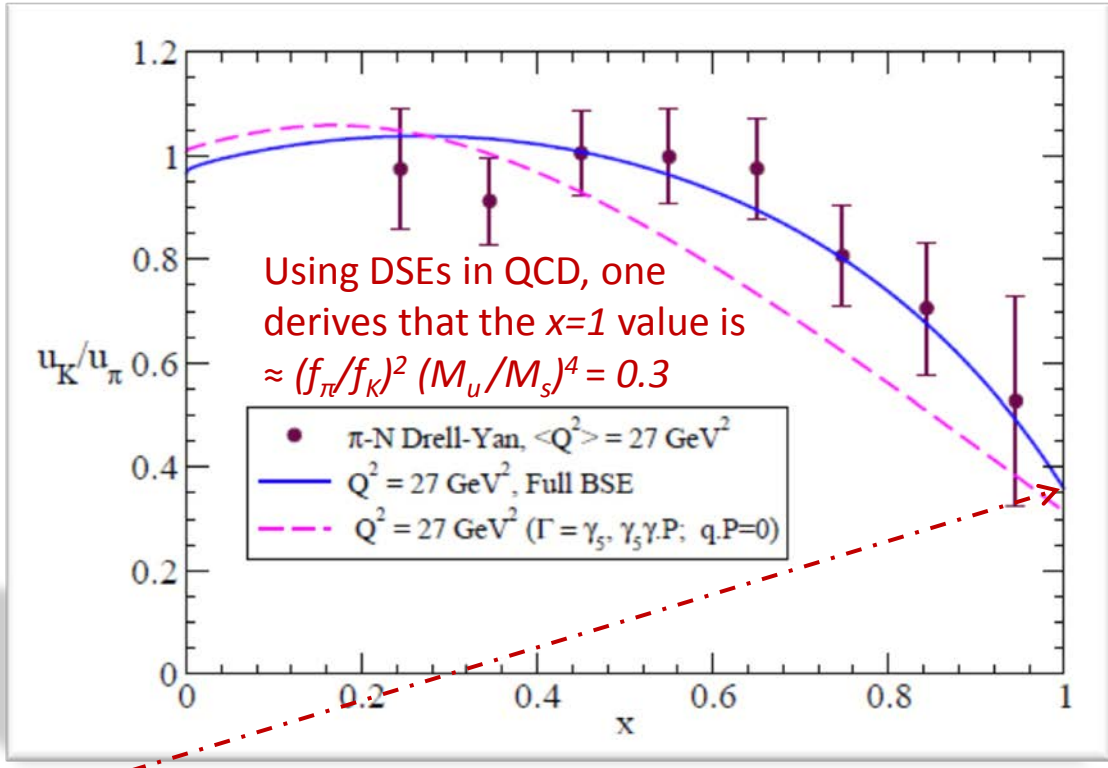
# $u_K(x)/u_\pi(x)$

- Drell-Yan experiments at CERN (1980 & 1983) provide the only extant measurement of this ratio
- No-parameter DSE prediction in complete accord with the measurement
- New experiments at modern facilities are capable of validating this comparison
- Necessary … so that complete understanding can be claimed

*Value of ratio at x=0 will approach "1" under evolution to higher resolving scales. This is a feature of perturbative dynamics*




Using DSEs in QCD, one derives that the *x=1* value is $\approx (f_\pi/f_K)^2 (M_u/M_s)^4 = 0.3$

*Value of ratio at x=1 is a fixed point of the evolution equations*
*Hence, x≈1 domain is always a direct measure of nonperturbative dynamics*

*Imaging dynamical chiral symmetry breaking: pion wave function on the light front,* Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

# Pion's valence-quark Distribution Amplitude

- Developed DSE methods to compute $\varphi_\pi(x)$ = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.

$$x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5$$

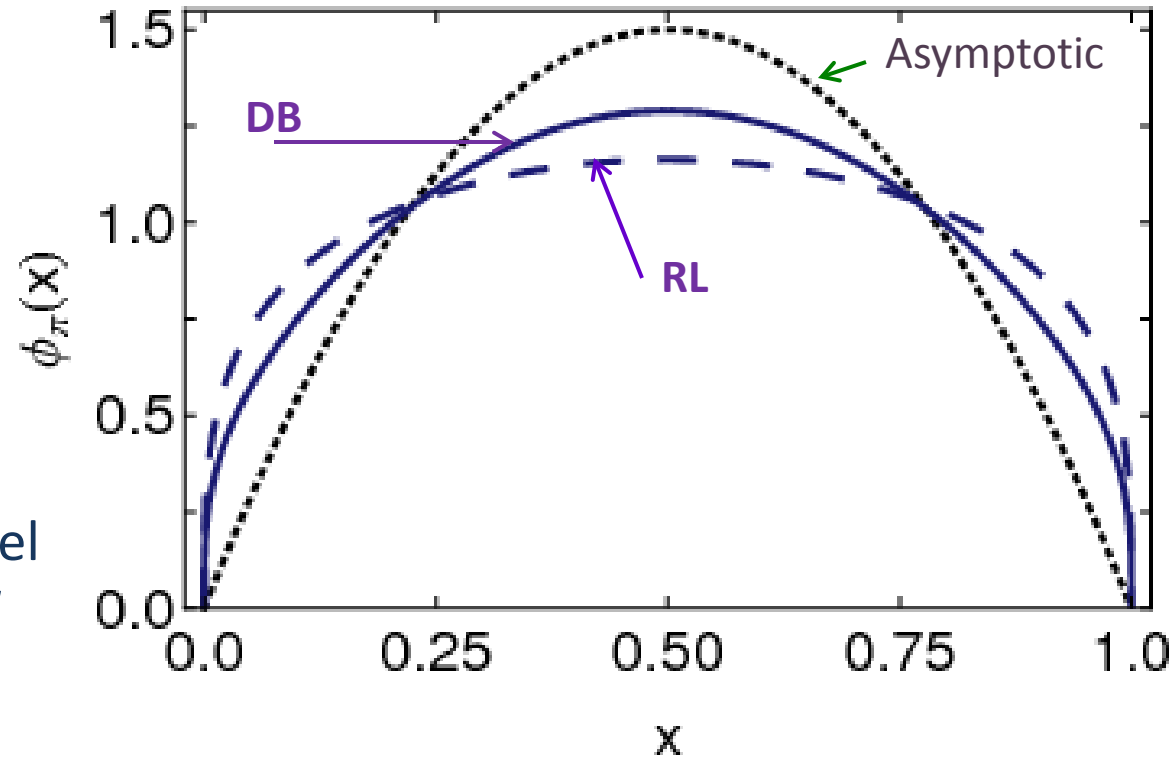
*Imaging dynamical chiral symmetry breaking: pion wave function on the light front,* Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

# Pion's valence-quark Distribution Amplitude

- Both kernels agree: marked broadening of $\varphi_\pi(x)$, which owes to DCSB

- This may be claimed because PDA is computed at a low renormalisation scale in the chiral limit, whereat the quark mass function owes entirely to DCSB.
- Difference between RL and DB results is readily understood: $B(p^2)$ is more slowly varying with DB kernel and hence a more *balanced* result

*PDAs of ground-state mesons are concave functions at all scales*



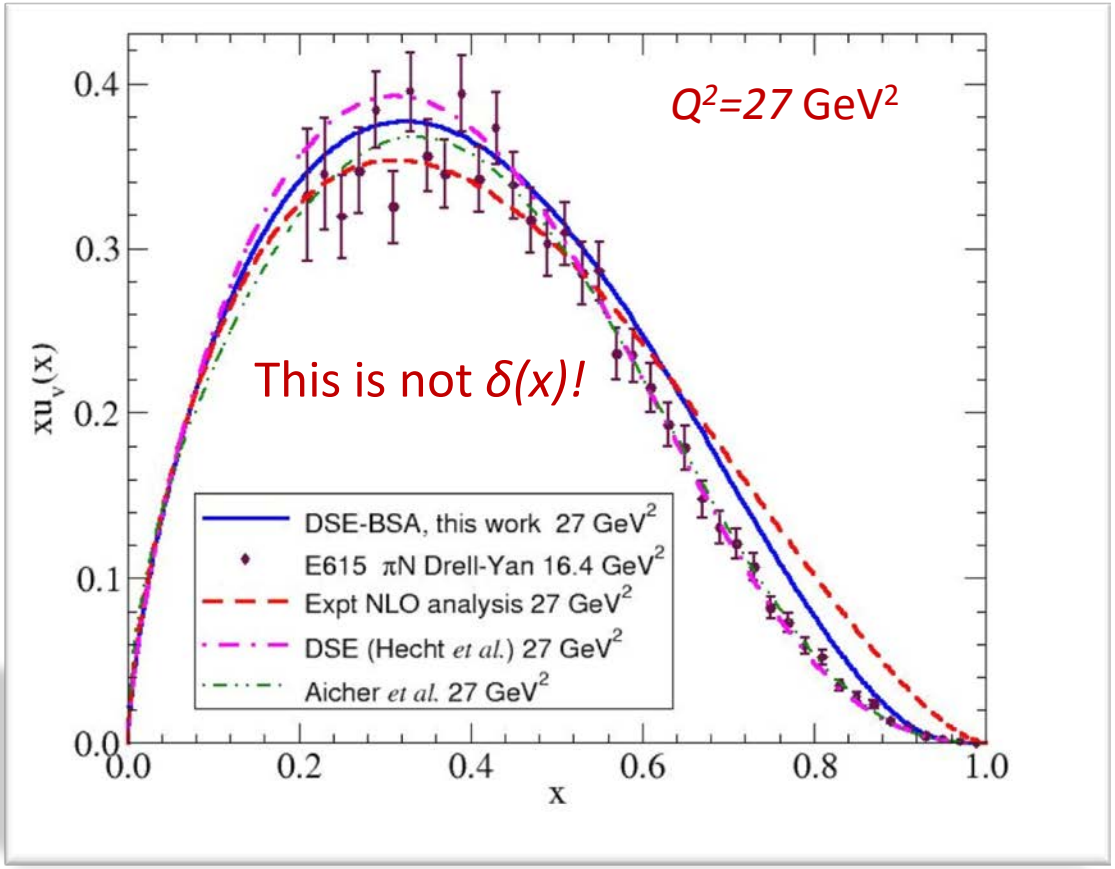
# When is asymptotic PDA valid?

- PDA is a wave function
  ∴ not directly observable
  *But … PDF is.*
- $\varphi_\pi^{asy}(x)$ can only be a good approximation to the pion's PDA when it is accurate to write

$$u_v^\pi(x) \approx \delta(x)$$

for the pion's valence-quark distribution function.

- This is far from valid at currently accessible scales



# When is asymptotic PDA valid?

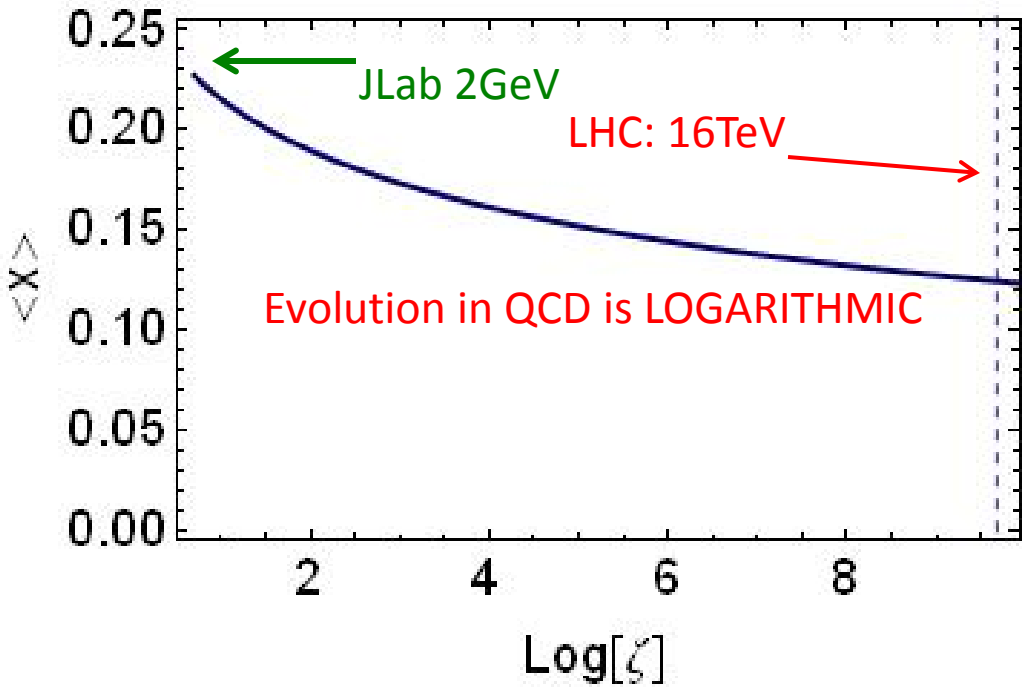
- When is asymptopia reached?
- If $u_v^\pi(x) \approx \delta(x)$, then

  $$\langle x \rangle = \int_0^1 dx\, x\, u_v^\pi(x) = 0;$$

  i.e., the light-front momentum fraction carried by valence-quarks is ZERO

  ∴ Asymptopia is reached when $\langle x \rangle$ is "small"
- As usual, the computed valence-quark distribution produces ($\pi = u + d_{bar}$)

  $$2\langle x \rangle_{2\mathrm{GeV}} = 44\%$$
- When is $\langle x \rangle$ small?




- NLO evolution of PDF, computation of $\langle x \rangle$.
- Even at LHC energies, light-front fraction of the $\pi$ momentum:

  $$\langle x \rangle_{\text{dressed valence-quarks}} = 25\%$$

  $$\langle x \rangle_{\text{glue}} = 54\%,\ \langle x \rangle_{\text{sea-quarks}} = 21\%$$

# When is asymptotic PDA valid?

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- If $u_v^\pi(x) \approx \delta(x)$, then

  $\langle x \rangle = \int_0^1 dx\, x\, u_v^\pi(x) = 0$:

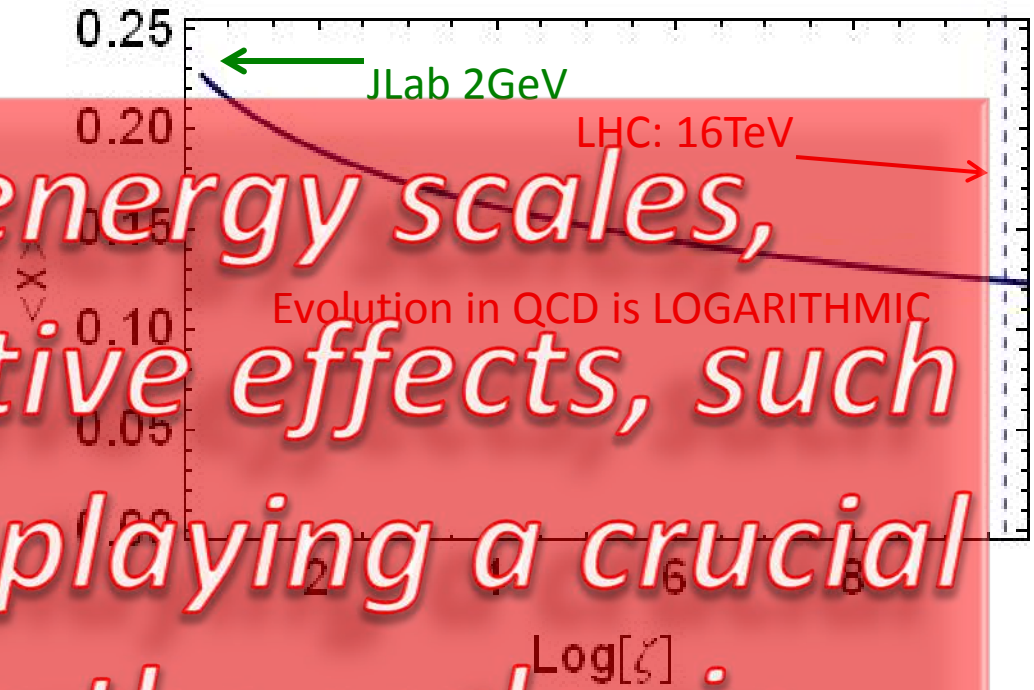
  i.e., the light-front momentum fraction carried by valence quarks is ZERO

  ∴ Asymptopia is reached when $\langle x \rangle$ is small

- As usual, the computed valence quark distribution produces ($\pi = u + d_{bar}$)

  $\langle x \rangle_{2GeV} = 44\%$

- When is $\langle x \rangle$ small?




- NLO evolution of PDF, computation of $\langle x \rangle$.
- Even at LHC energies, light-front fraction of the π momentum:

  $\langle x \rangle_{\text{dressed valence-quarks}} = 25\%$

  $\langle x \rangle_{\text{glue}} = 54\%$, $\langle x \rangle_{\text{sea-quarks}} = 21\%$

**Even at LHC energy scales, nonperturbative effects, such as DCSB, are playing a crucial role in setting the scales in PDAs and PDFs.**

*Distribution amplitudes of light-quark mesons from lattice QCD,* Jorge Segovia, *et al.* arXiv:1311.1390 [nucl-th]

# Distribution Amplitudes of light-quark mesons from lattice-QCD

- PDA Moments from lattice-QCD
- Best available results show no distinction between pseudoscalar and vector, nor between light-front parallel and perpendicular polarisations
- DSE studies underway, reveal there are certainly differences:

$\varphi_{\|}^{V}$ narrower-than $\varphi_{\perp}^{V}$ narrower-than $\varphi^{P}$

R. Arthur *et al.*, Phys.Rev. D83 (2011) 07450

Table 1: Meson PDA moments obtained using numerical simulations of lattice-regularised QCD with $N_f = 2+1$ domain-wall fermions and nonperturbative renormalisation of lattice operators [17]: linear extrapolation to physical pion mass, $\overline{\text{MS}}$-scheme at $\zeta = 2$ GeV, two lattice volumes. The first error is statistical, the second represents an estimate of systematic errors, including those from the $s$-quark mass, discretisation and renormalisation.

| meson | $\langle(2x-1)^n\rangle$ | $16^3 \times 32$ | $24^3 \times 64$ |
|---|---|---|---|
| $\pi$ | n=2 | 0.25(1)(2) | 0.28(1)(2) |
| $\rho_{\|}$ | n=2 | 0.25(2)(2) | 0.27(1)(2) |
| $\phi$ | n=2 | 0.25(2)(2) | 0.25(2)(1) |
| $K$ | n=1 | 0.035(2)(2) | 0.036(1)(2) |
| $K^*_{\|}$ | n=1 | 0.037(1)(2) | 0.043(2)(3) |
| $K$ | n=2 | 0.25(1)(2) | 0.26(1)(2) |
| $K^*_{\|}$ | n=2 | 0.25(1)(2) | 0.25(2)(2) |

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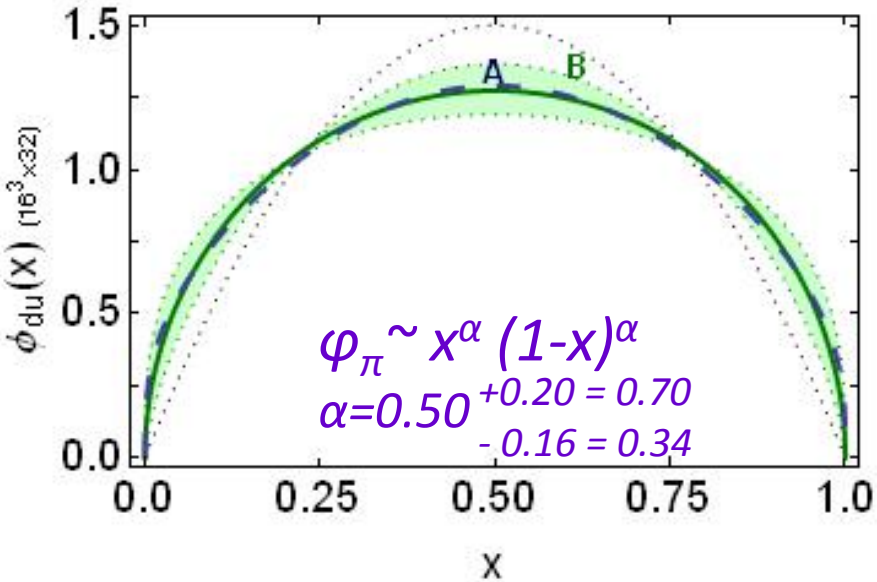
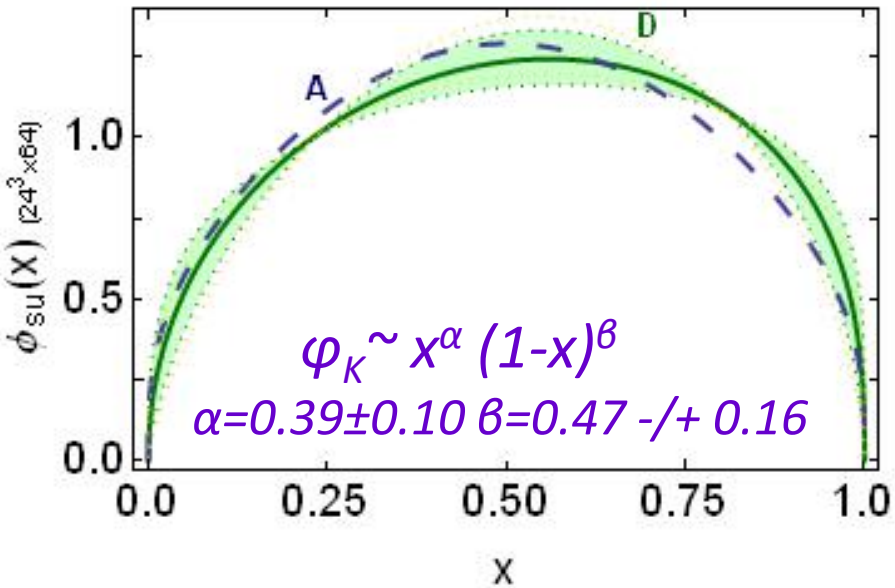



- Reconstruct PDAs using generalisation of method described elsewhere (I. C. Cloët *et al.*, arXiv:1306.2645 [nucl-th], Phys. Rev. Lett. **111** (2013) 092001 [5 pages] )
- SU(3) breaking is measured by shift in peak: ratio =1.10

  $x_{max} = 0.55$ for $\varphi_K(x_{max})$ cf. $x_{max} = ½$ for $\varphi_\pi(x)$ ***DCSB-driven***

  cf. ratio of 1.15 for peak of $[x\, s_v^K(x_{max})]/[x\, u_v^K(x_{max})]$
- Lattice-based estimate DSE ratio=*1.13*

| $Q^2 = 4\,\mathrm{GeV}^2$ | $16^3 \times 32$ | $24^3 \times 64$ |
|---|---|---|
| $F_K(\zeta_2^2)/F_\pi(\zeta_2^2)$ | $1.21^{+1.22}_{-0.62}$ | $0.74^{+1.20}_{-0.51}$ |

*Appearance of precision in lattice moments is misleading*
*Large-lattice result for pion is unreliable – systematic error*

*Light front distribution of the chiral condensate,* Lei Chang, Craig D. Roberts and Sebastian M. Schmidt, arXiv:1308.4708 [nucl-th], Phys. Lett. B **727** (2013) pp. 255–259

# pion PDAs
# Twist-3 & 2-particle ($\gamma_5$)

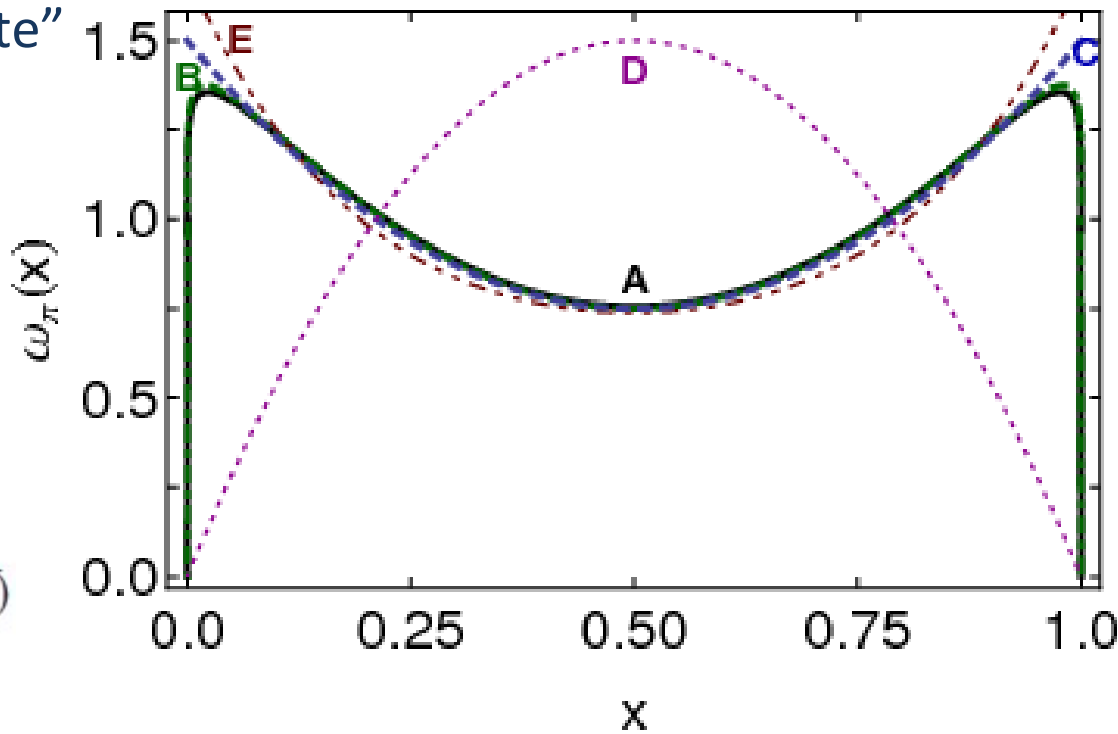
- Pseudoscalar projection of pion's Bethe-Salpeter amplitude onto the light-front

$$\rho_\pi^\zeta\,\omega_\pi(x) = \mathrm{tr}_{\mathrm{CD}} Z_4 \int_{dk}^{\Lambda} \delta(n\cdot k_\eta - x\, n\cdot K)\,\gamma_5\,\chi_\pi(k;K)$$

$\rho_\pi^\zeta$ is the "in-pion condensate"

- (A) DSE result at $\zeta$= *2*GeV
  (B) DSE result at $\zeta$=*19*GeV
  (C) conformal QCD

$$\omega_\pi^{\mathrm{asy}}(x) = {}_1\omega_\pi(x) = 1 + \frac{1}{2} C_2^{(1/2)}(x-\bar{x})$$



Chen Chen, Lei Chang, C.D. Roberts and S.M. Schmidt, S-l Wang, in progress.

# pion PDAs
# Twist-3, 2-particle ($\sigma_{\mu\nu}$)

- Pseudtensor projection of pion's Bethe-Salpeter amplitude onto the light-front

$$\frac{1}{4}\rho_\pi^\zeta\,(n\cdot P)\,\tau_\pi(x) = \mathrm{tr}_{\mathrm{CD}}\int_{dq}^{\Lambda}\delta(n\cdot q_\eta/n\cdot P - x)\,\gamma_5\sigma_{\mu\nu}n_\mu P_\nu\chi_\pi(q;P)$$

$\rho_\pi^\zeta$ is the "in-pion condensate"

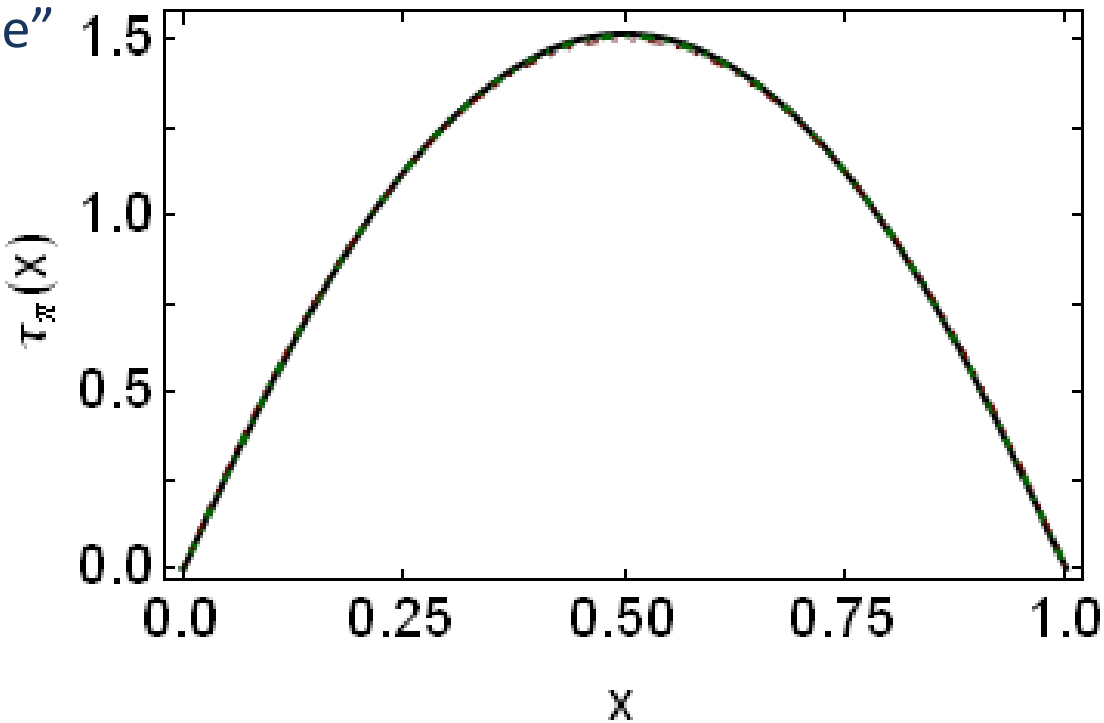
- DSE result at $\zeta$= 2GeV

  DSE result at $\zeta$=19GeV

  conformal QCD

  $\tau_\pi^{asy}(x) = 6\,x\,(1-x) = \varphi_\pi^{asy}(x)$

- This PDA is slightly narrower at accessible scales



*Nucleon spin structure at very high-x*
Craig D. Roberts, Roy J. Holt and Sebastian M. Schmidt
arXiv:1308.1236 [nucl-th], Phys. Lett. B **727** (2013) 249–254

# Far valence domain x≃1

- Endpoint of the far valence domain: x ≃ 1, is especially significant
  - All familiar PDFs vanish at x=1; but ratios of any two need not
  - Under DGLAP evolution, the value of such a ratio is invariant.
- Thus, e.g.,
  - $\lim_{x\to 1} d_v(x)/u_v(x)$
    is unambiguous, scale invariant, nonperturbative feature of QCD.

  ∴ keen discriminator between frameworks
  that claim to explain nucleon structure.
- Furthermore, Bjorken-*x=1* corresponds strictly to the situation in which the invariant mass of the hadronic final state is precisely that of the target; viz., elastic scattering.

  ∴ Structure functions inferred experimentally on x≃1
  are determined theoretically by target's elastic form factors.

I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) 1-36

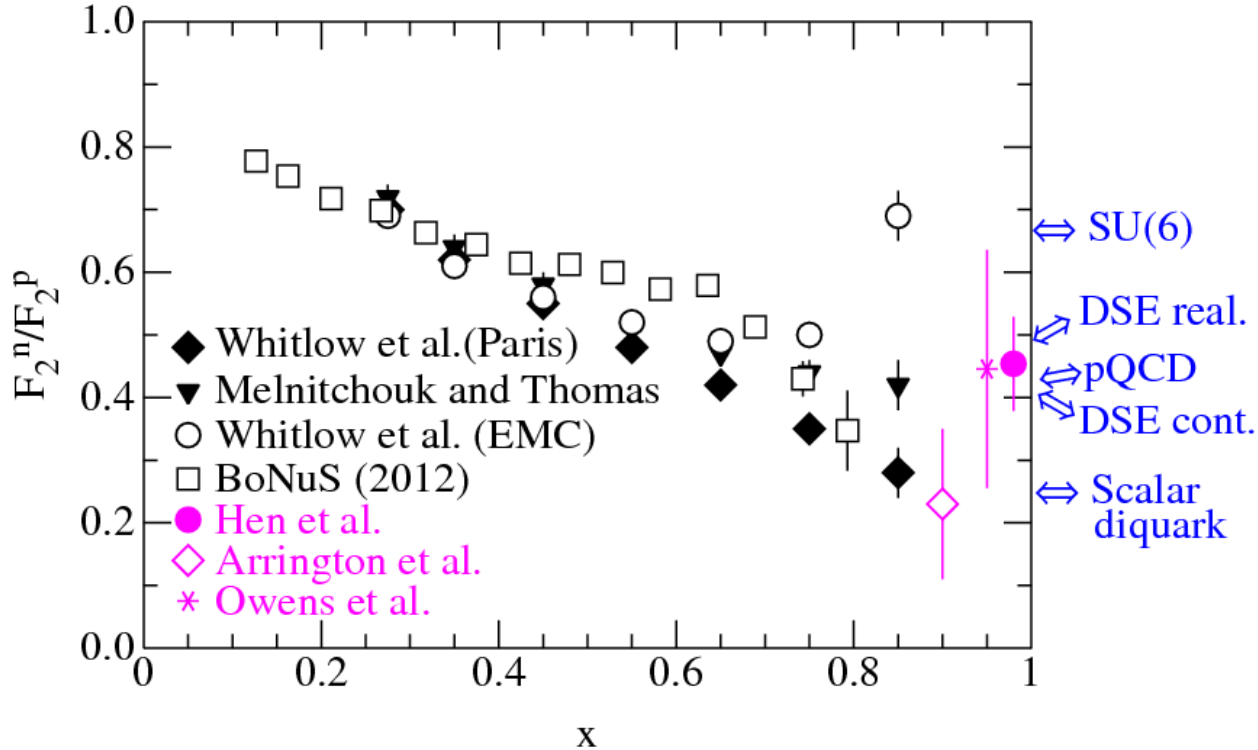
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

# Neutron Structure Function at high-*x*

*Nucleon spin structure at very high-x*
Craig D. Roberts, Roy J. Holt and Sebastian M. Schmidt
arXiv:1308.1236 [nucl-th], Phys. Lett. B **727** (2013) pp. 249–254

- Faddeev equation analyses of nucleon structure show that this and similar ratios measure the relative strength of scalar- and axial-vector diquark correlations.

$$\left.\frac{d_v(x)}{u_v(x)}\right|_{x\to 1} = \frac{P_1^{p,d}}{P_1^{p,u}} = \frac{\frac{2}{3}P_1^{p,a} + \frac{1}{3}P_1^{p,m}}{P_1^{p,s} + \frac{1}{3}P_1^{p,a} + \frac{2}{3}P_1^{p,m}}$$



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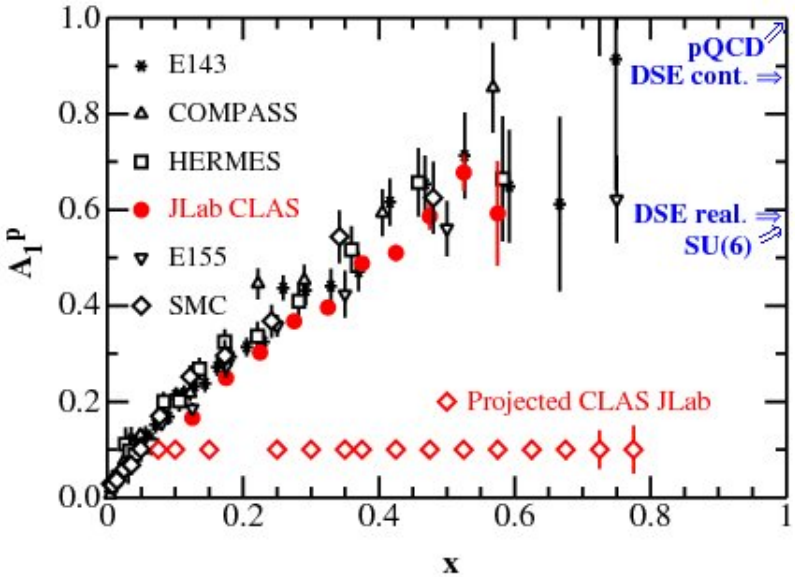
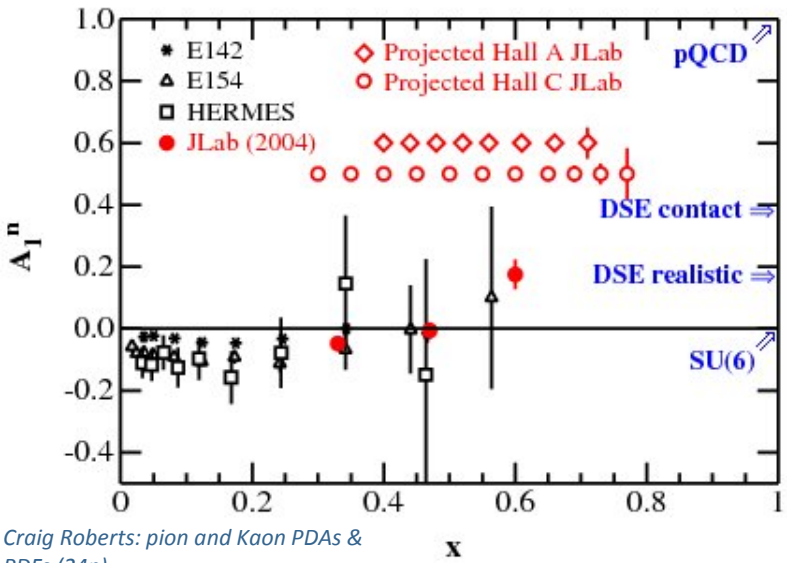
# Nucleon spin structure at very high *x*




- Similar formulae for nucleon longitudinal structure functions.
- Plainly, existing data cannot distinguish between modern pictures of nucleon structure
- Empirical results for nucleon longitudinal spin asymmetries on $x \simeq 1$ promise to add greatly to our capacity for discriminating between contemporary pictures of nucleon structure.

NB. pQCD is actually model-dependent: assumes *SU(6)* spin-flavour wave function for the proton's valence-quarks and the corollary that a hard photon may interact only with a quark that possesses the same helicity as the target.

*Craig Roberts: pion and Kaon PDAs & PDFs (24p)*

# Epilogue

- DCSB →equivalence between 1-body and $0^+$ 2-body problem
  - Complete understanding of pion … (almost) no model dependence
- Prediction of valence-quark PDF in pion is confirmed
  - Predictions for kaon are verifiable in foreseeable future
- Leading-twist PDAs of ground-state mesons are concave
  - They are broader than asymptotic distribution owing to DCSB
  - I.C. Cloët … this is measurable in pion's electromagnetic form factor
- Leading-twist *asymptotic distribution,* $\varphi^{asy}$, is poor approximation to $\varphi_{\pi,K}$ at all mass-scales accessible terrestrially.
- Using Bayesian analysis, it's possible to infer pointwise form of meson PDAs from lattice-QCD
  - SU(3) breaking in meson PDAs is a measure of DCSB
- Predictions for twist-3 pion PDAs
  - Model-independent results
  - They are almost indistinguishable from respective asymptotic forms
- Combination of unpolarised and polarised structure function data will discriminate between pictures of nucleon structure


# This is not the end

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