Constrained γZ interference corrections to PVES

Peter G. Blunden
University of Manitoba

Adelaide-JLab-Manitoba (AJM) Collaboration

Nathan Hall, Wally Melnitchouk, Tony Thomas, Ross Young

Outline

- Calculation of γZ boxes by forward angle dispersion relations
    - vector (V) hadron coupling: γγ → γZ model dependencies
    - constraints from PDF region
    - constraints from PV inelastic data
- Qweak (1.165 GeV)
- Moller (11 GeV)

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\,p \rightarrow e\,p$ scattering

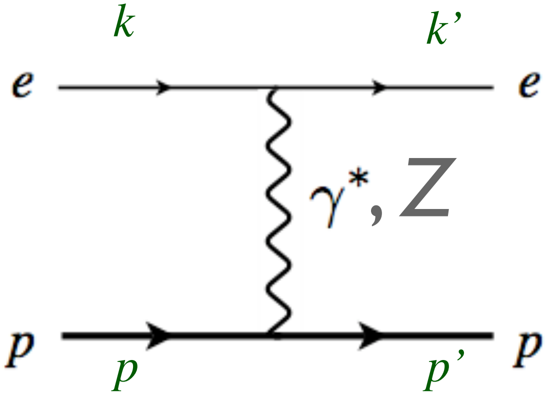
$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \longrightarrow \frac{G_F\,Q_W^p}{4\sqrt{2}\pi\alpha}\,t$$

forward limit

$$t = (k - k')^2 \rightarrow 0$$

$$s = (k + p)^2$$

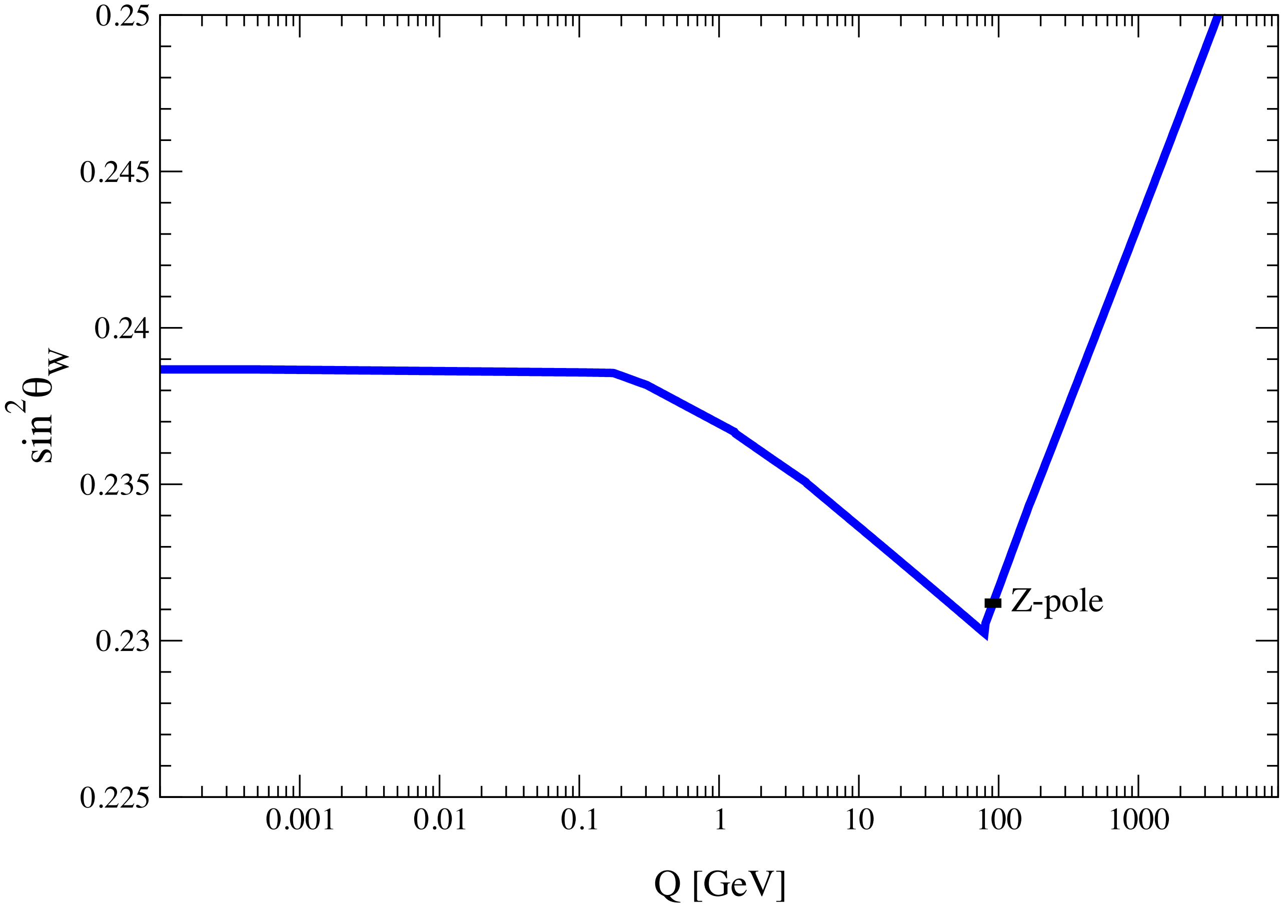
$$= M(M + 2E)$$



→ in forward limit, gives proton weak charge

$$Q_W^p = 1 - 4\sin^2\theta_W$$

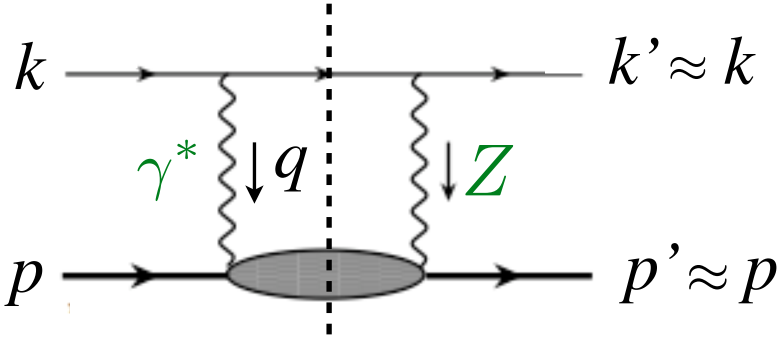
(tree level only)



Forward angle dispersion method

Gorchtein, Horowitz, PRL ***102*** *(2009) 091806*

→ repeat calculation using forward dispersion relations with realistic (structure function) input



$$S = 1 + i\mathcal{M}$$

$$S^\dagger = 1 - i\mathcal{M}^\dagger$$

$$SS^\dagger = 1$$

Unitarity → $-i\left(\mathcal{M} - \mathcal{M}^\dagger\right) = 2\Im m\, \mathcal{M} = \mathcal{M}^\dagger\mathcal{M}$

$$\Im m\, \langle f|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho\, \sum_n \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle$$

Forward scattering amplitude

$$\Im m\langle i|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho \sum_n |\langle n|\mathcal{M}|i\rangle|^2 \sim \int d^3k_1 \;\frac{L_{\mu\nu}W^{\mu\nu}}{q^2(q^2 - M_Z^2)}$$

vector *h*

axial *h*

hadronic tensor: $MW^{\mu\nu}_{\gamma Z} = -g^{\mu\nu}F_1^{\gamma Z} + \frac{p^\mu p^\nu}{p\cdot q}F_2^{\gamma Z} - i\varepsilon^{\mu\nu\lambda\rho}\frac{p_\lambda q_\rho}{2p\cdot q}F_3^{\gamma Z}$

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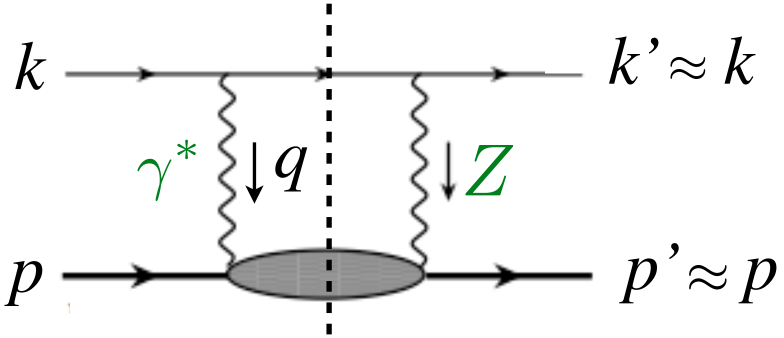

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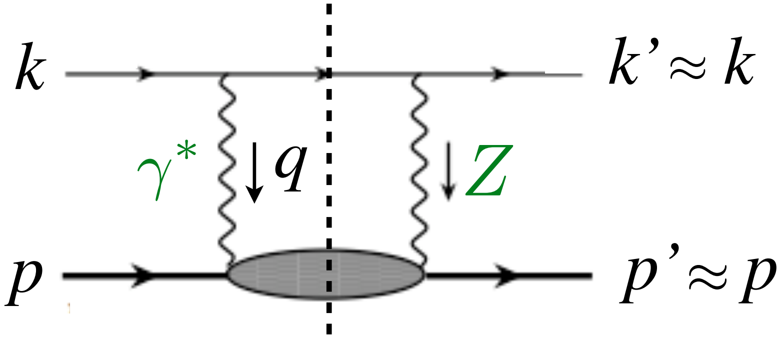

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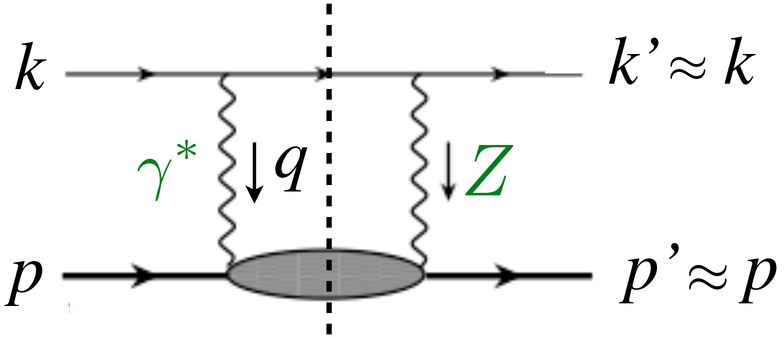

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Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1\ \mathrm{GeV}$ – what is energy dependence?

→ forward dispersion relation

- ★ $\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$
- ★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

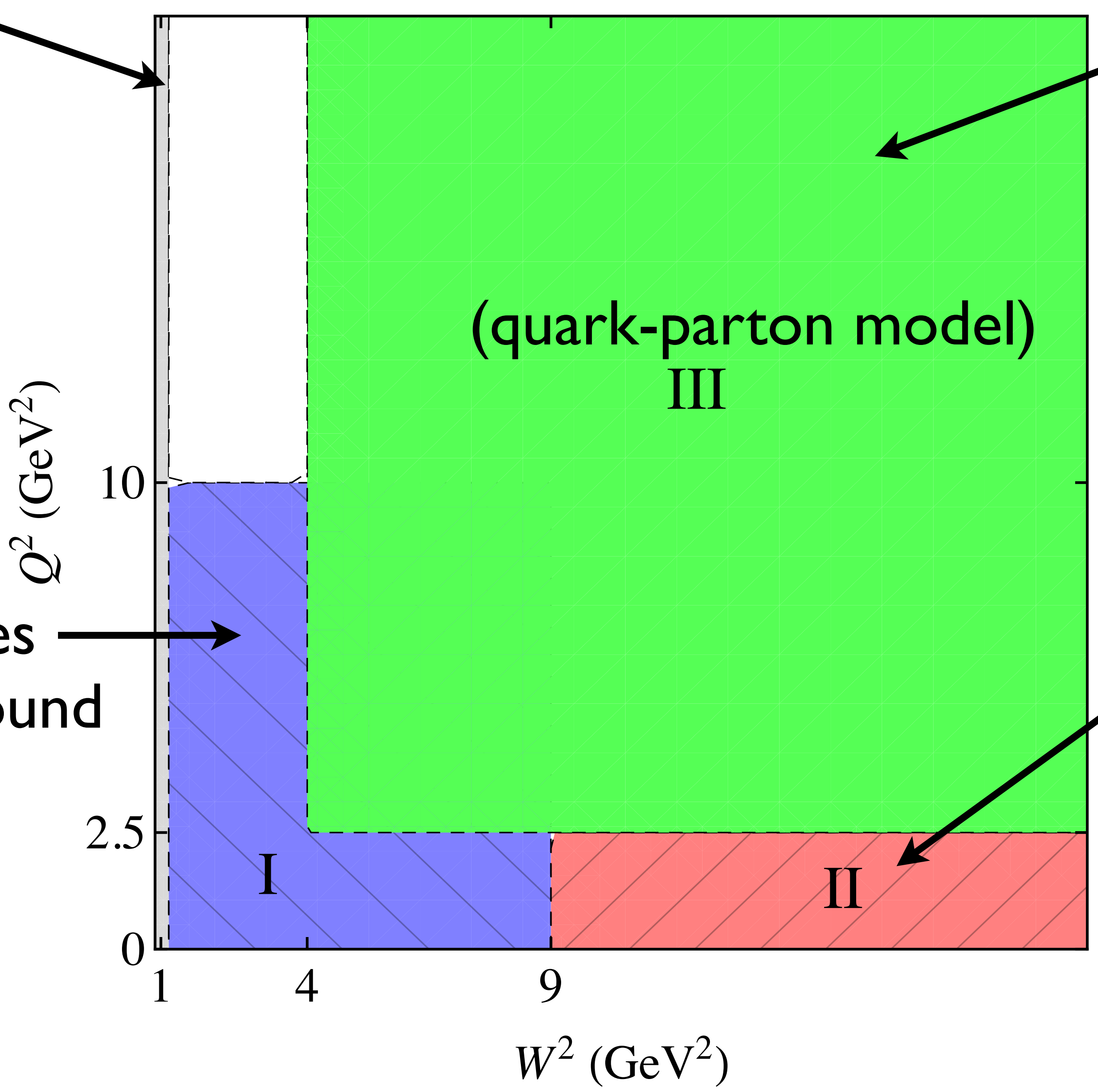
$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

- 3 groups doing independent analyses
- At Qweak energy *E* = 1.165 GeV:

$$\begin{aligned} \text{GH (2009)} & \\ \text{SBMT (2010)} & \ (4.7^{+1.1}_{-0.4}) \times 10^{-3} \\ \text{GHRM (2011)} & \ (5.4 \pm 2.0) \times 10^{-3} \\ \text{RC (2011)} & \ (5.7 \pm 0.9) \times 10^{-3} \\ \text{AJM (2013)} & \ (5.6 \pm 0.4) \times 10^{-3} \end{aligned}$$

- Mainly different treatments of low Q^2, low *W* region background contributions
- Agree on overall magnitude, but disagree on errors

Integration region



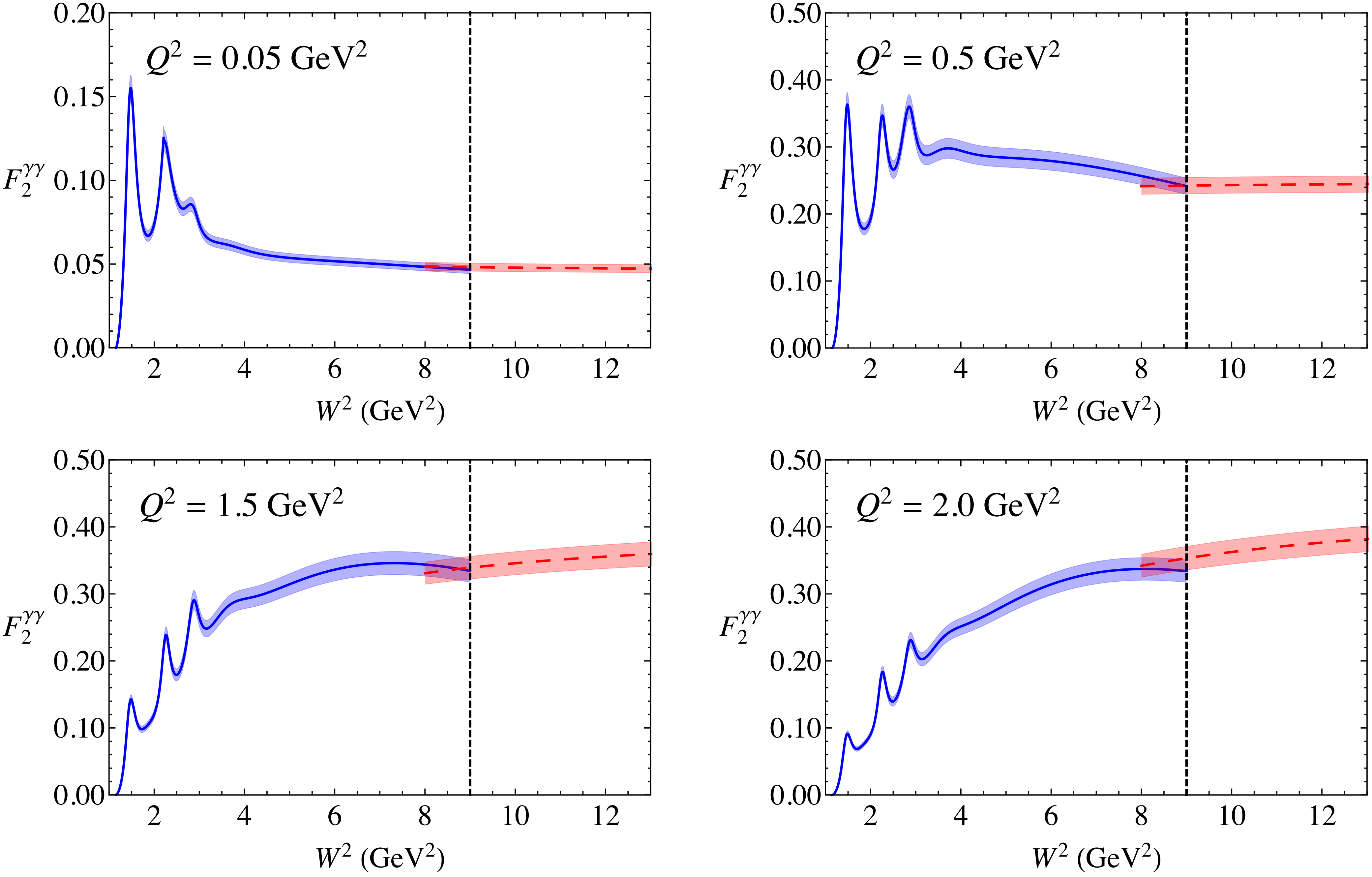
elastic

Deep Inelastic Scattering (PDF's)

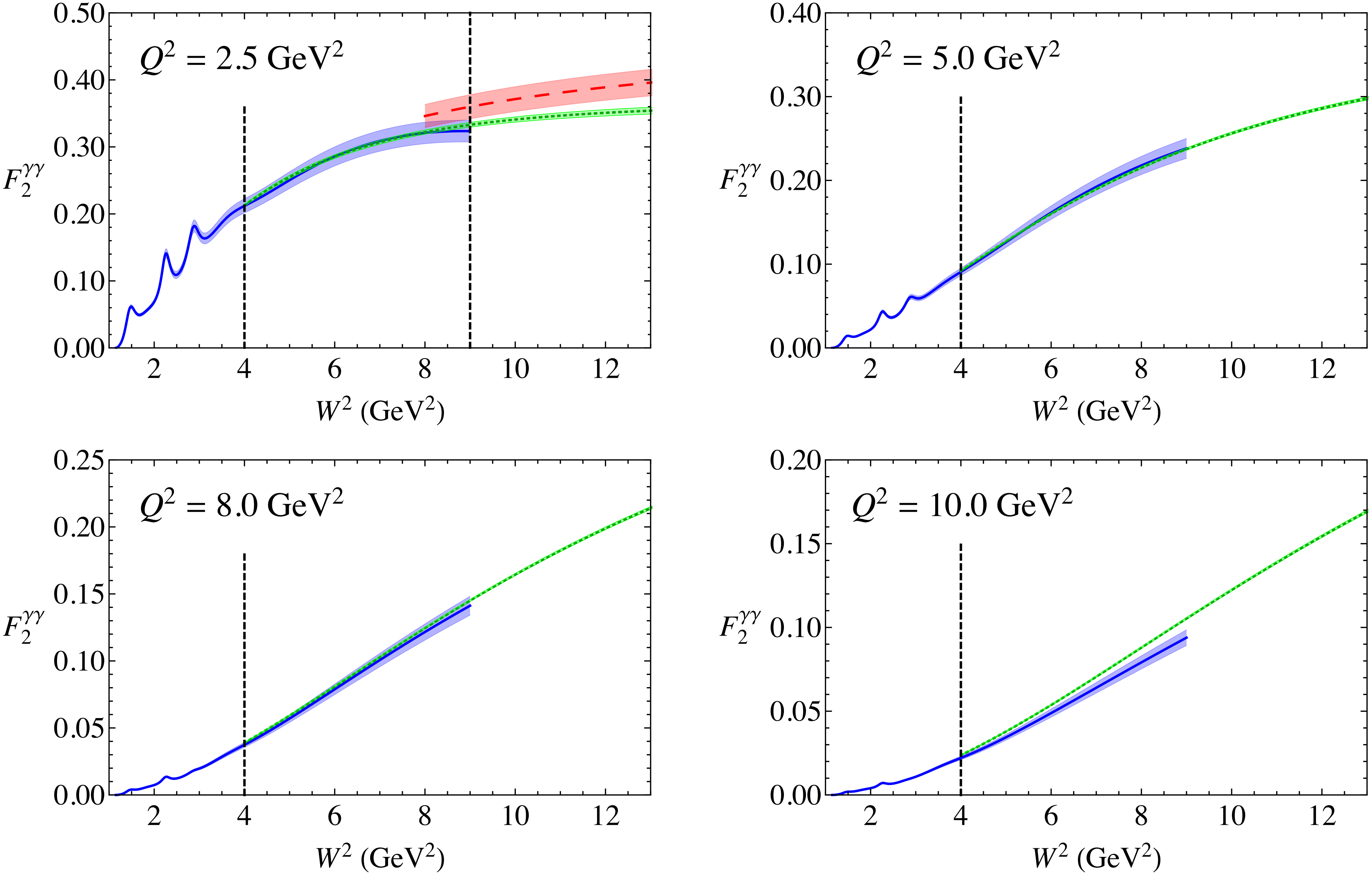
resonances + background (BC fit)

background + low Q^2 fit (VMD + Regge)

Region I and II matching



Region I and III matching



Basic issue: how to relate $F_{1,2}^{\gamma Z}$ to $F_{1,2}^{\gamma}$?

Scaling region III

$$F_2^{\gamma} = \sum_q e_q^2 x(q+\bar{q})$$

$$F_2^{\gamma Z} = \sum_q 2 e_q g_V^q x(q+\bar{q})$$

Resonance region I

largest contribution, unlike $F_3^{\gamma Z}$

Christy-Bosted (CB) fit to *ep*

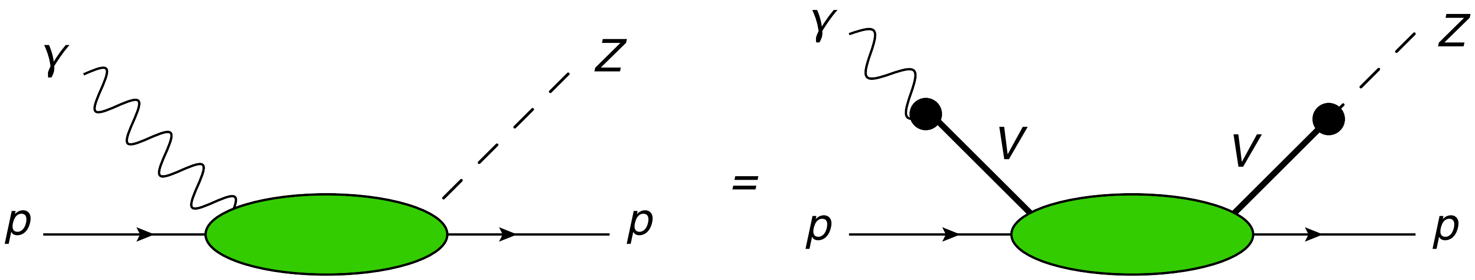
$$\sigma_{T,L} = \sigma_{T,L}(\mathrm{res}) + \sigma_{T,L}(\mathrm{bg})$$

$\sigma_{T,L}(\mathrm{res})$ 7 resonances from 1232 to 1934 MeV. Modify fit by ratio of e.m. to weak transition amplitudes.

Background

$\sigma_{T,L}(\mathrm{bg})$

- GHRM use Vector Meson Dominance (VMD) models fit to high energy data, plus isospin rotations
- Assign 100% uncertainty on continuum contribution (dominates errors)



V = ρ, ω, φ + continuum

- AJM model: constrain continuum (higher Q^2) contribution by matching with PDF ratios (γZ to γγ) across boundaries of Regions I, II and III.

AJM γZ model

- $F_{1,2}^{\gamma Z}$ structure functions
  - for background at low Q^2, weak isospin rotation uses VMD

$$\sigma_V^{\gamma Z} = \kappa_V \, \sigma_V^{\gamma\gamma}$$

$$\kappa_\rho = 2 - 4\sin^2\theta_W\,, \quad \kappa_\omega = -4\sin^2\theta_W\,, \quad \kappa_\phi = 3 - 4\sin^2\theta_W$$

$$\frac{\sigma^{\gamma Z}}{\sigma^{\gamma\gamma}} = \frac{\kappa_\rho + \kappa_\omega R_\omega + \kappa_\phi R_\phi + \kappa_C R_C}{1 + R_\omega + R_\phi + R_C}$$

$$R_V = \frac{\sigma^{\gamma^* p \to V p}}{\sigma^{\gamma^* p \to \rho p}}$$

production cross section ratio for vector meson V to ρ meson

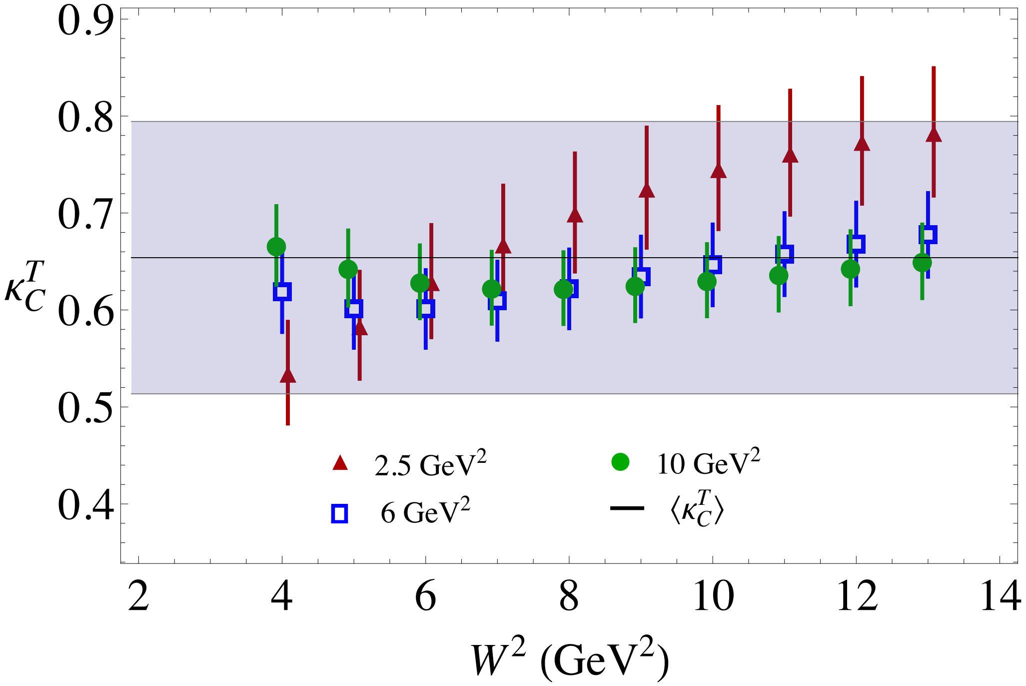
→ continuum parameter κ_C not constrained in VMD

→ GHRM assume $\kappa_C = \boxed{1 \pm 1}$ ← largest source of error!

AJM γZ model

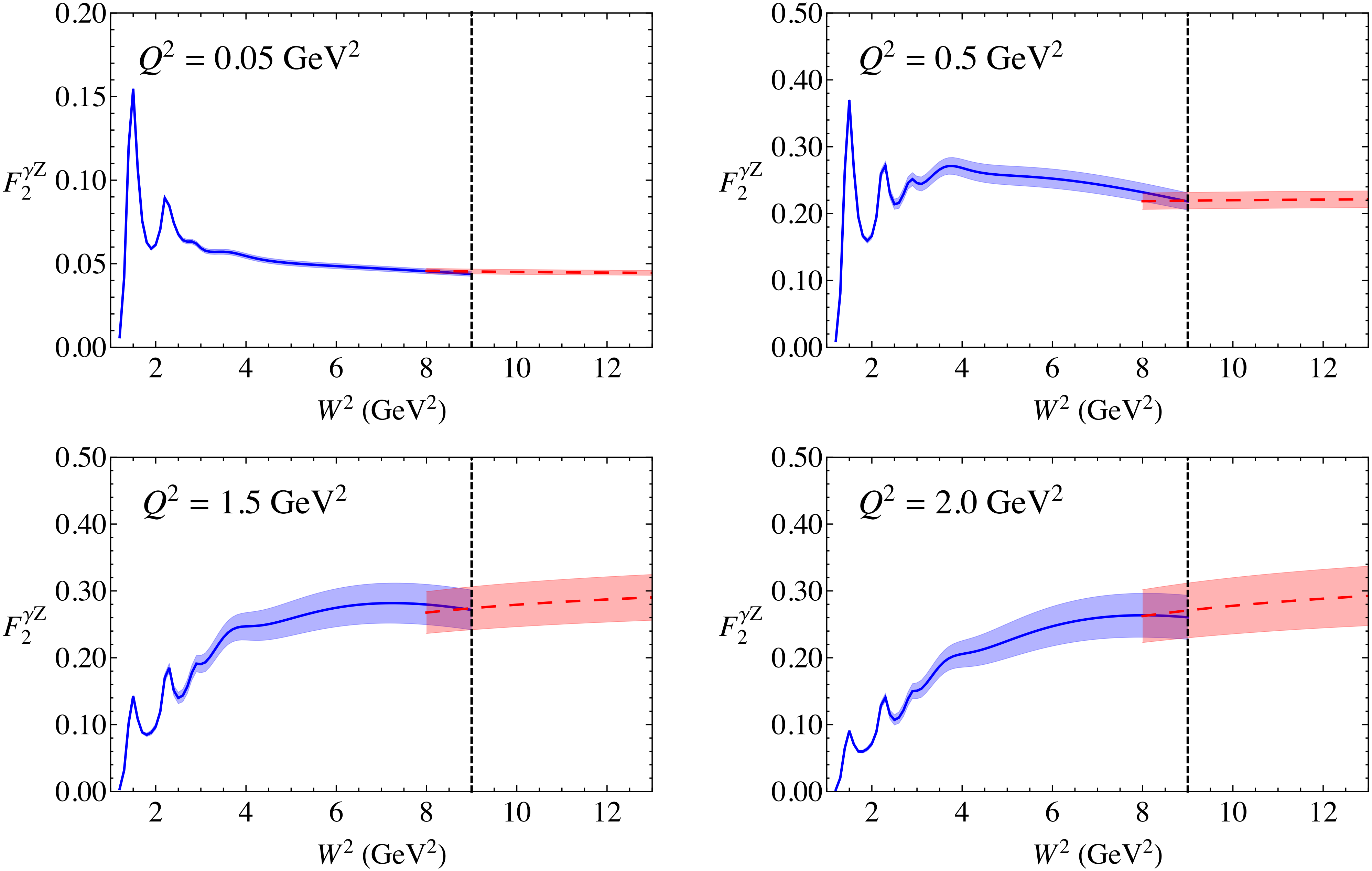
- Region where continuum contributions are relevant overlaps with typical reach of global PDF fits

→ constrain κ_C using PDF parametrizations by requiring matching of $F_{1,2}^{\gamma Z}$ to DIS structure functions



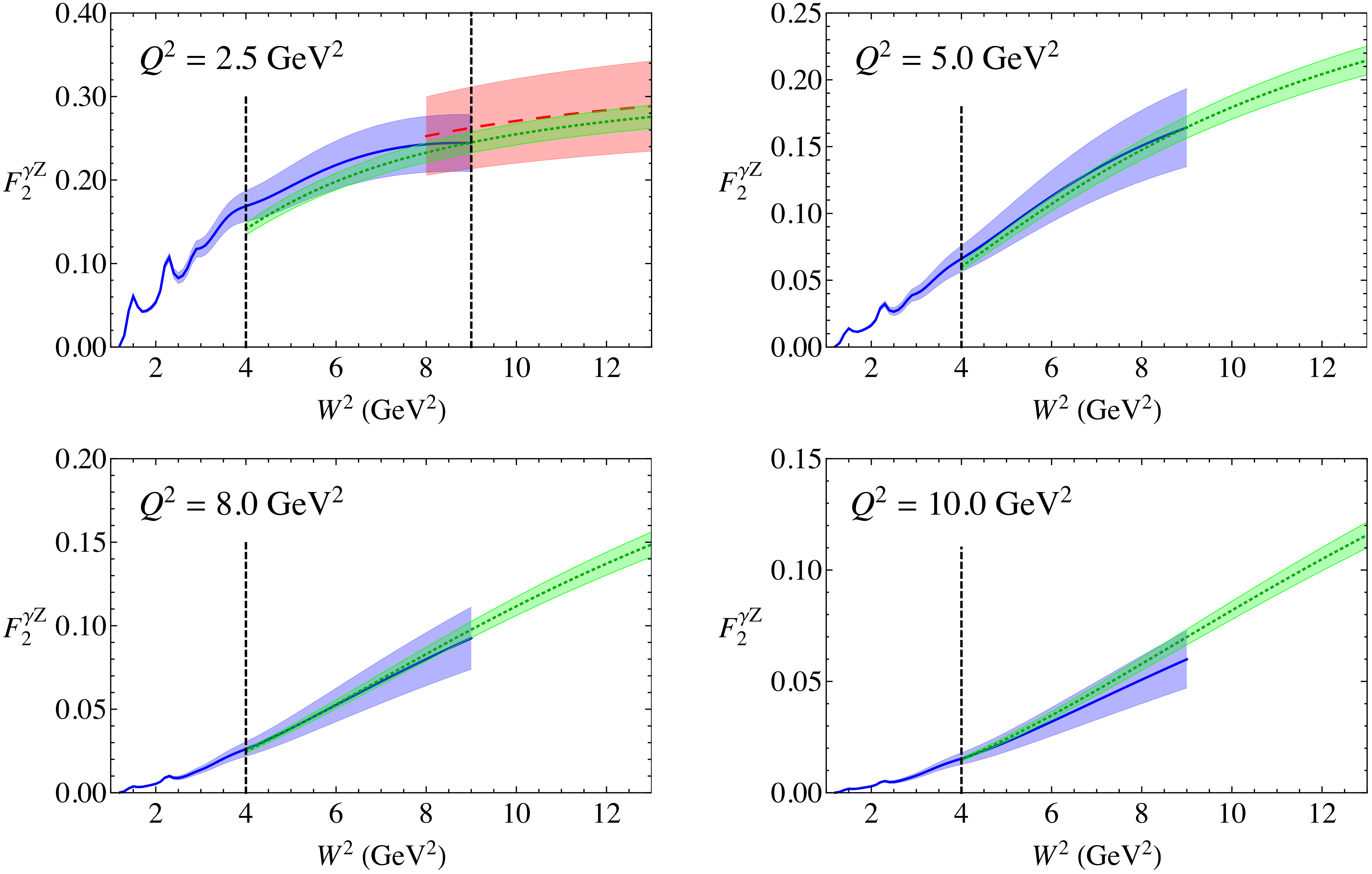
→ $$\boxed{\kappa_C^T = 0.65 \pm 0.14, \quad \kappa_C^L = -1.3 \pm 1.7}$$

(small contribution to asymmetry)



Region I and II matching

Hall et al. (2013)



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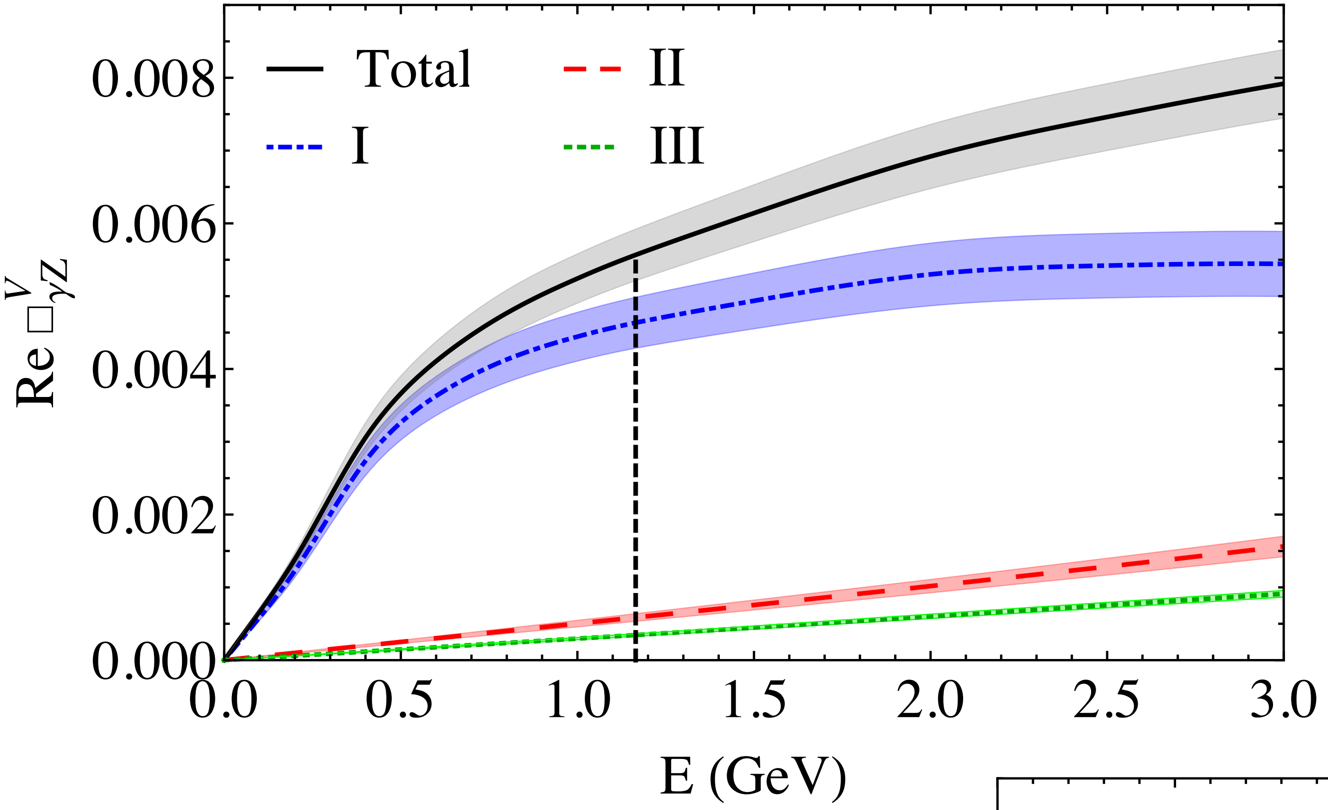
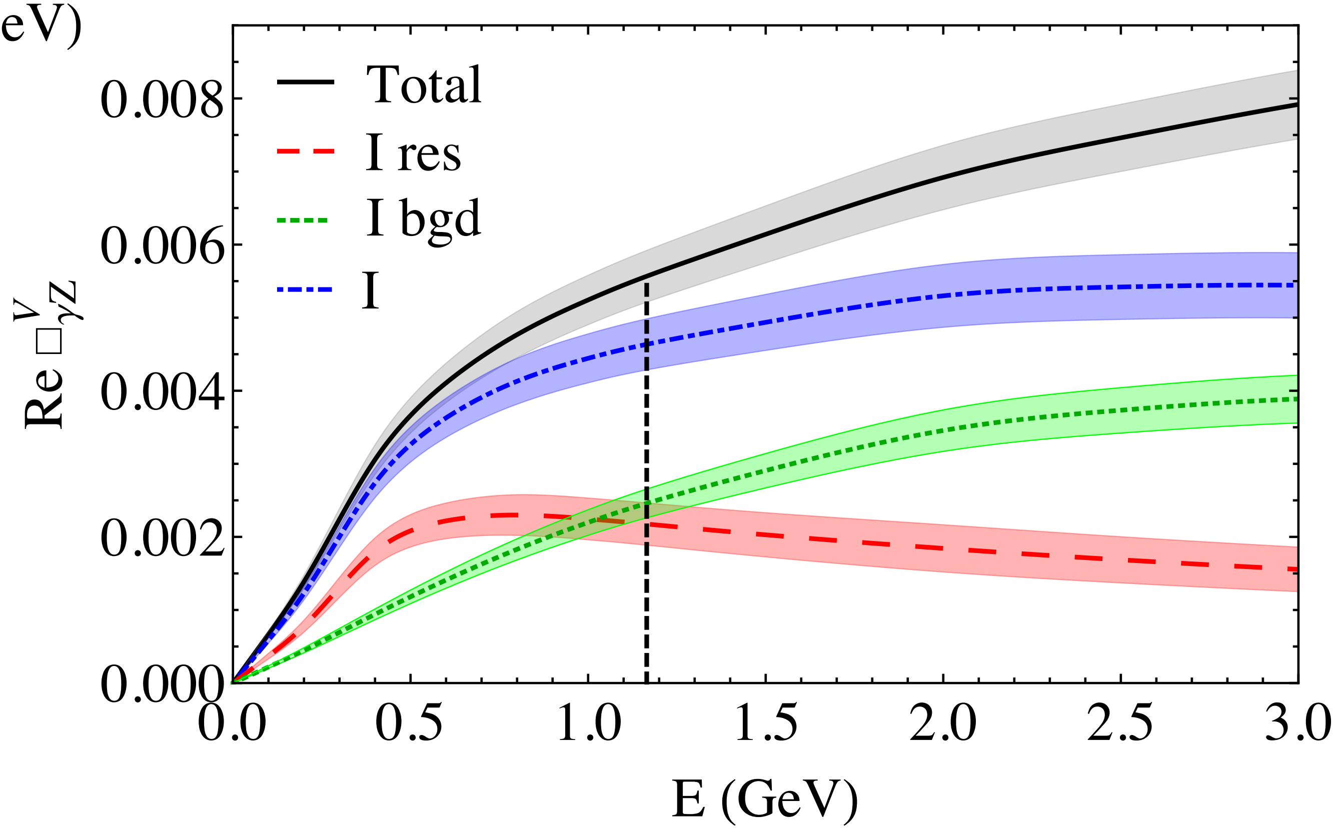
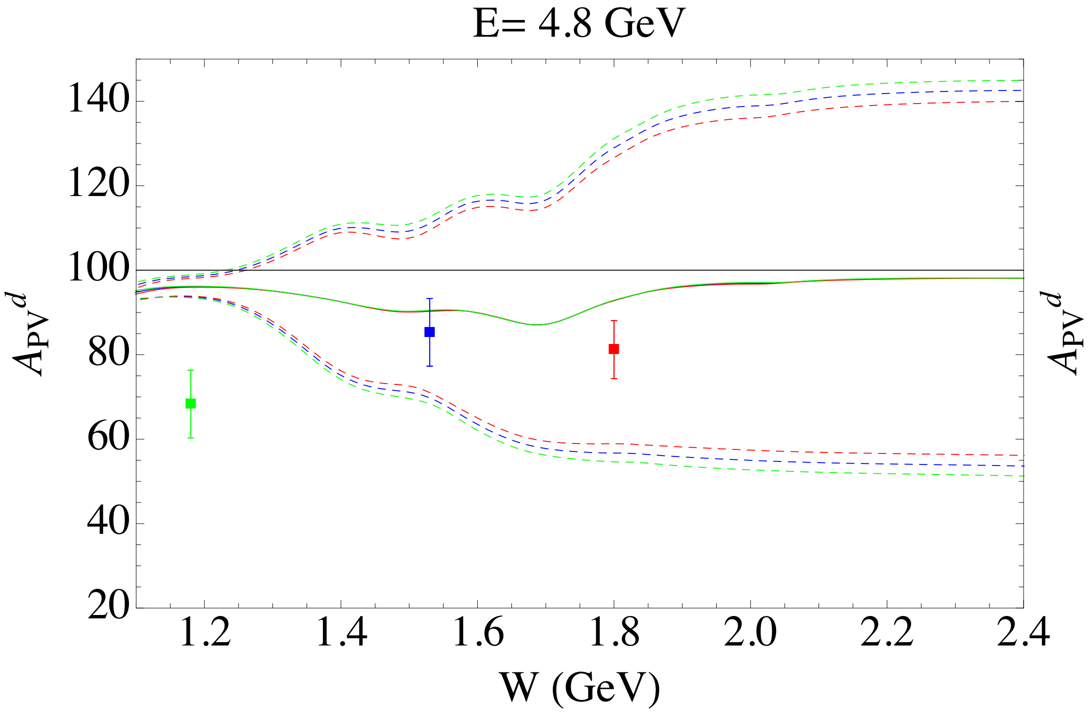
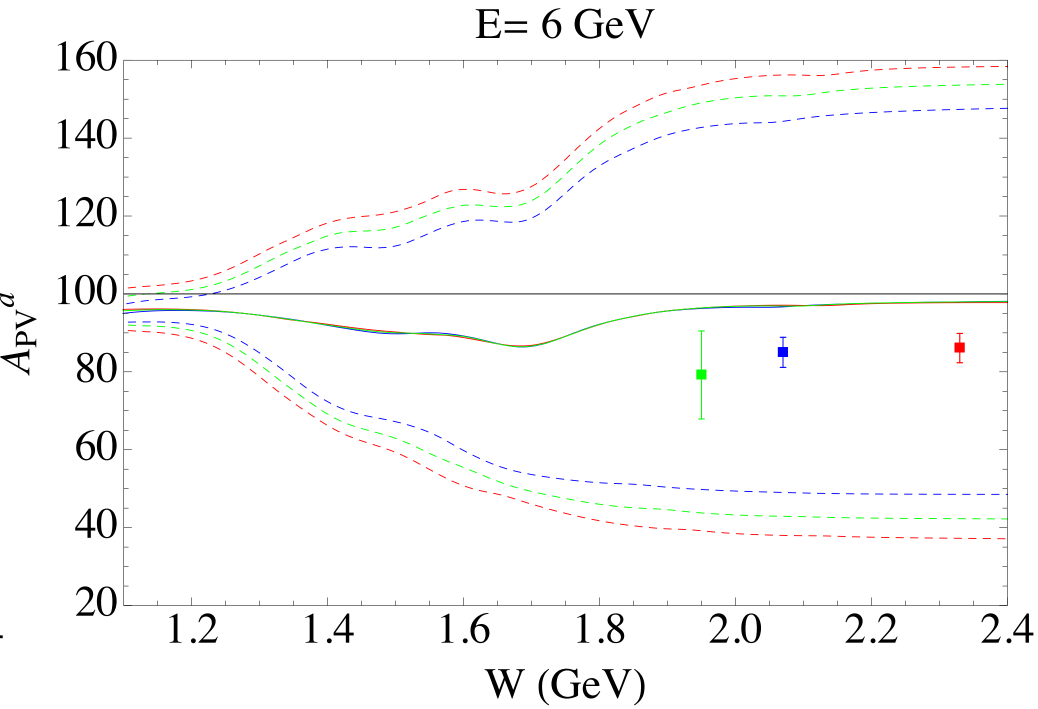


Contribution from different regions

Breakdown of region I resonance + background

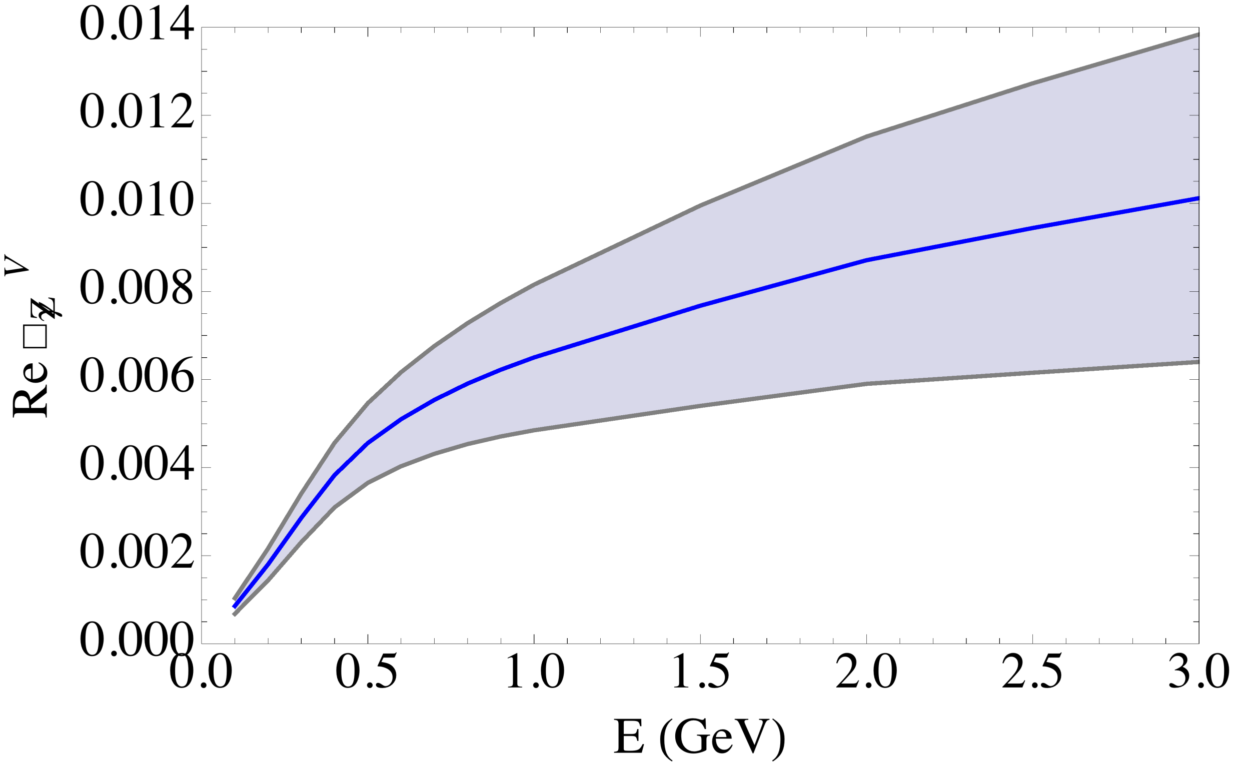


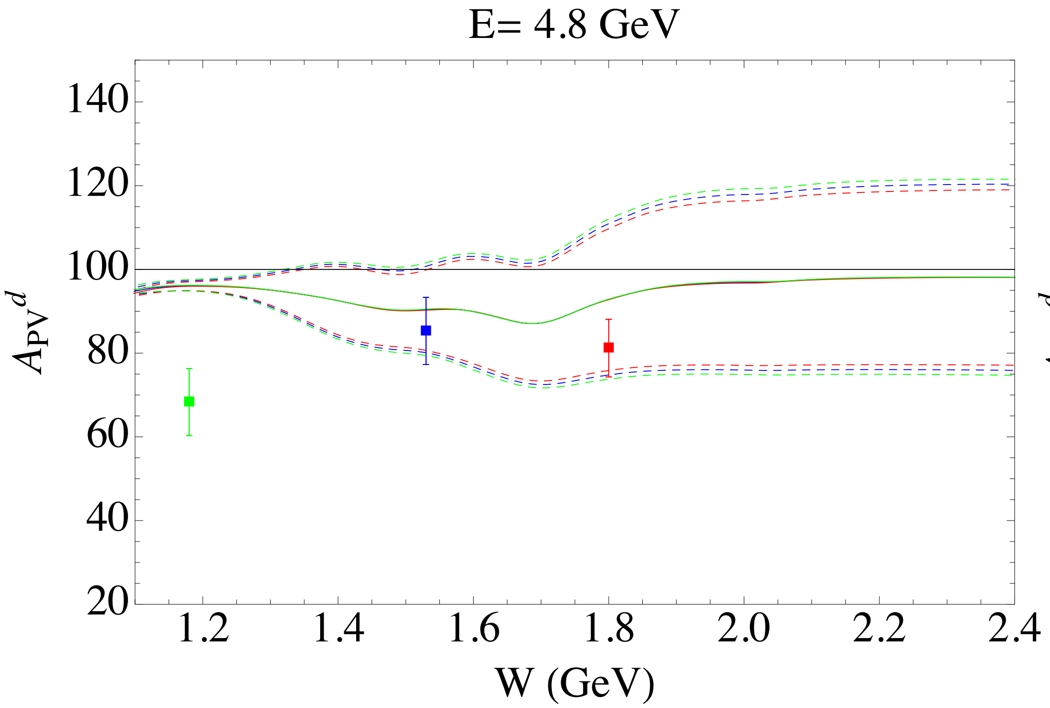


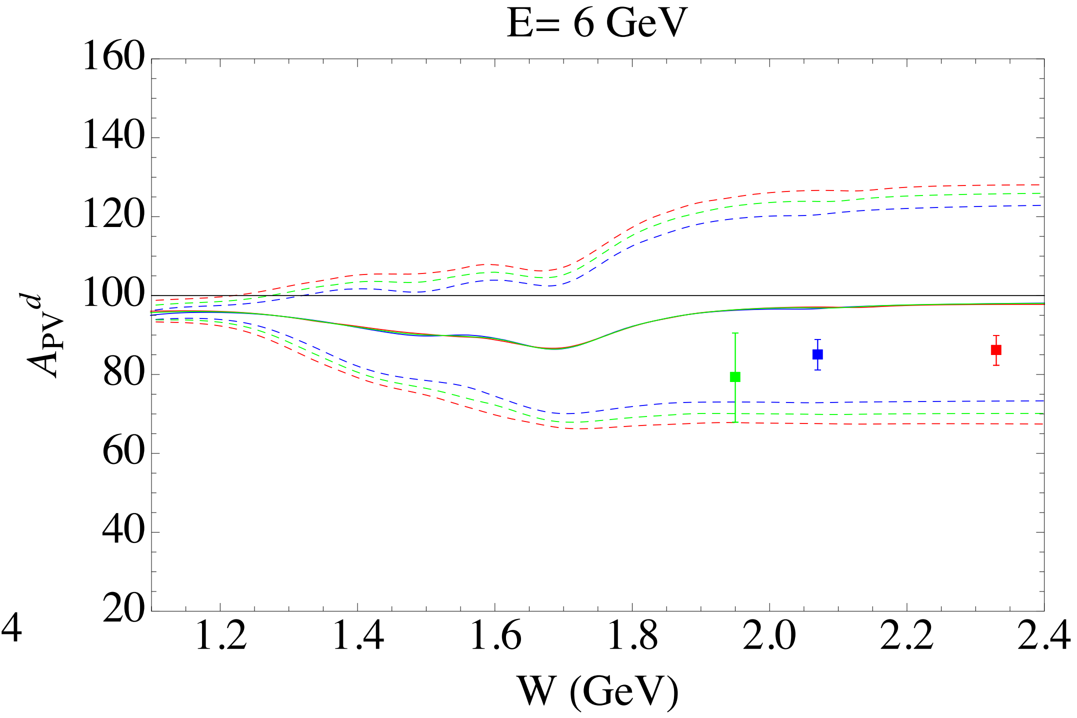


Potential impact of constraints from deuteron PV inelastic asymmetries

100% uncertainty on continuum background

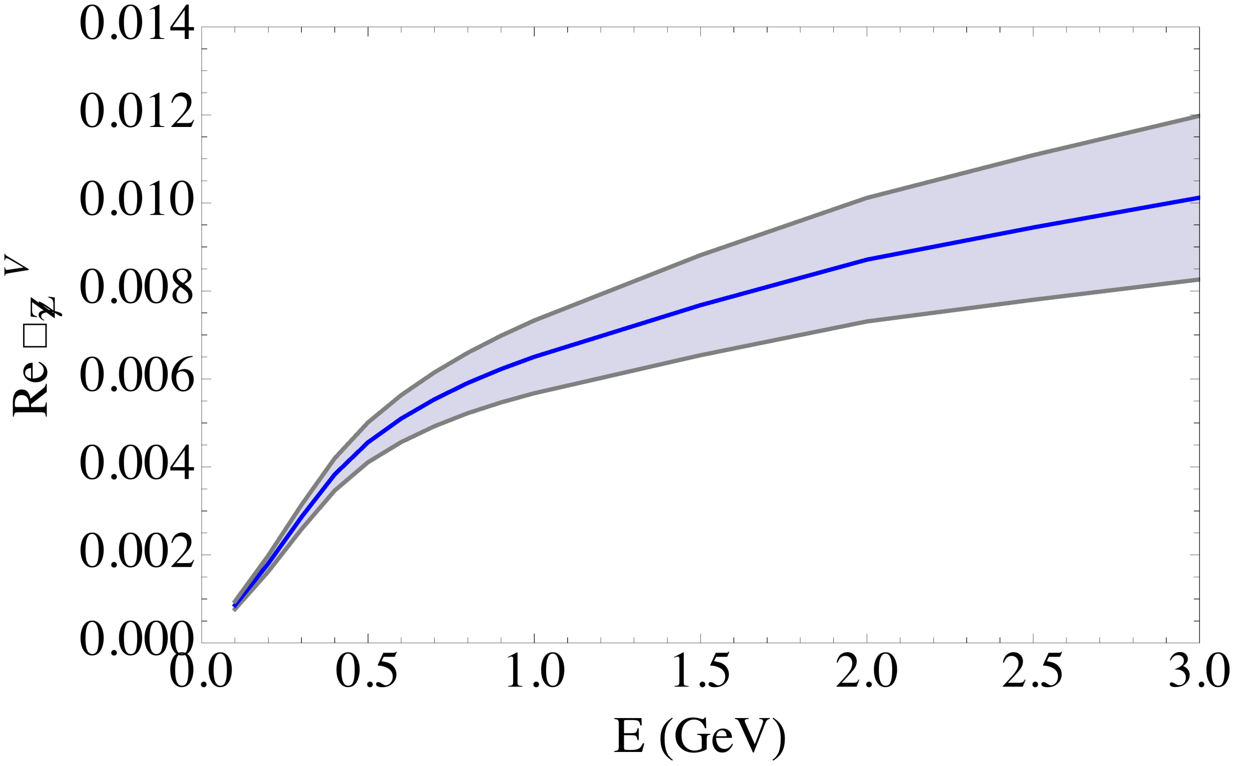


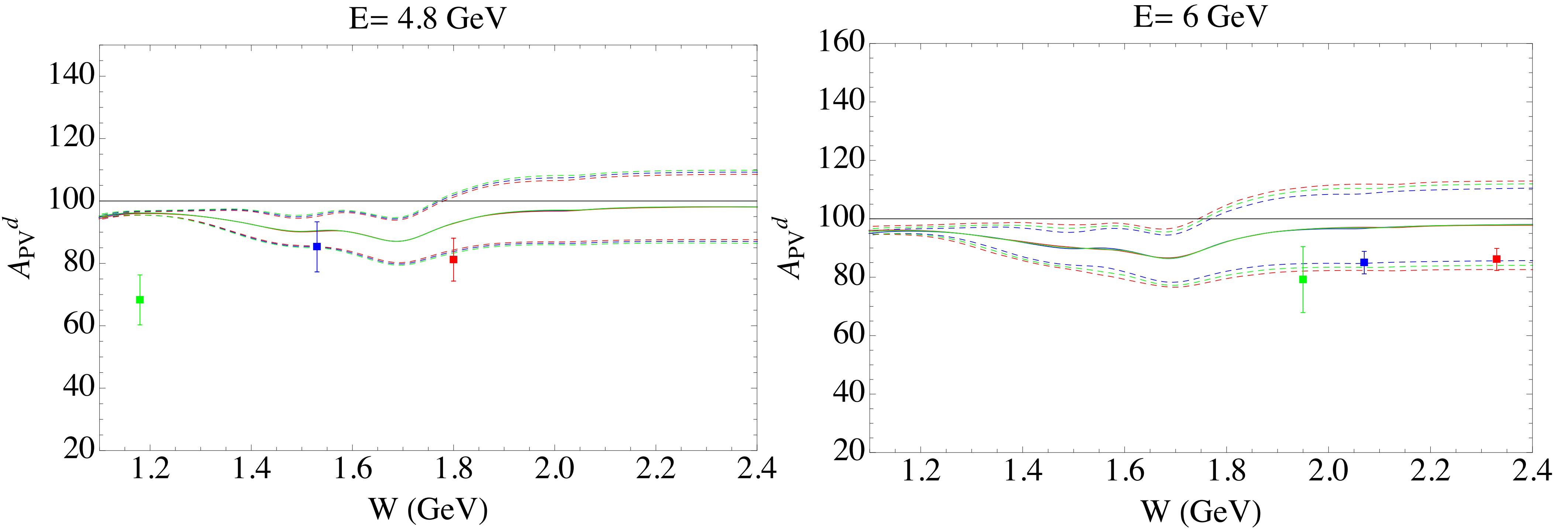




Potential impact of constraints from deuteron PV inelastic asymmetries

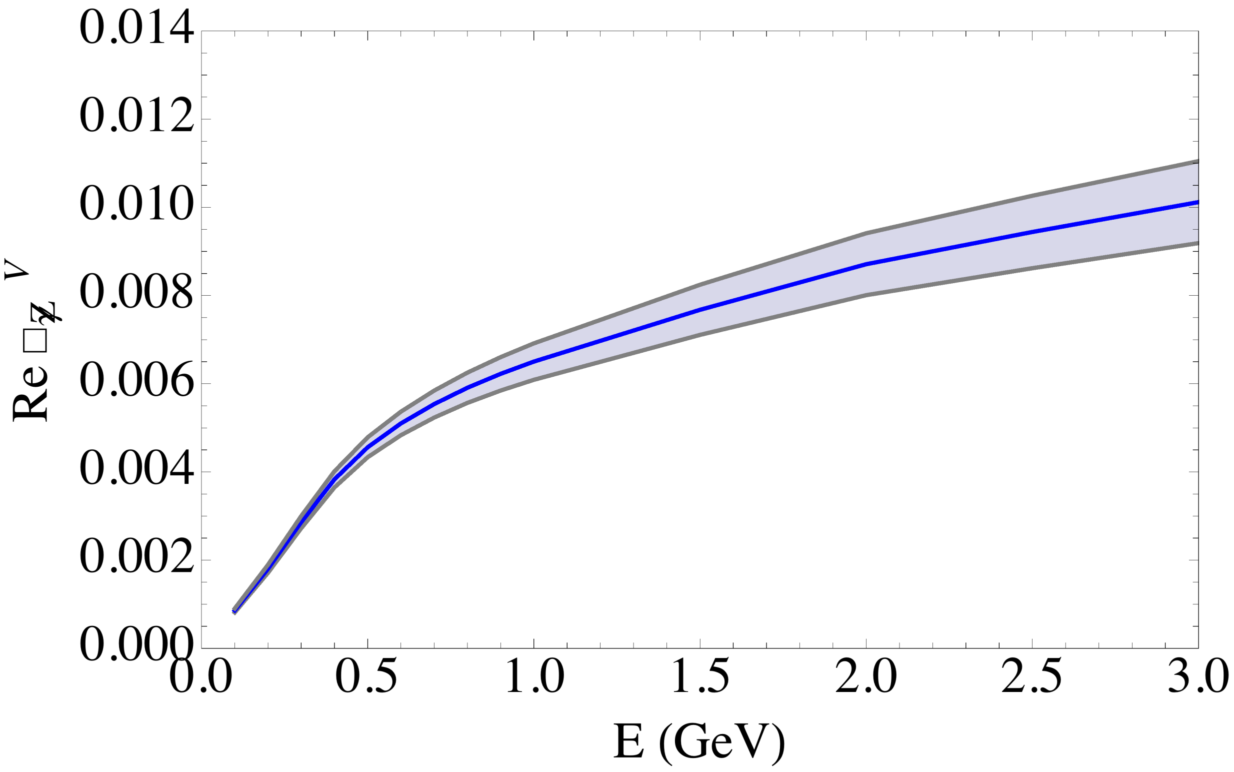
50% uncertainty on continuum background

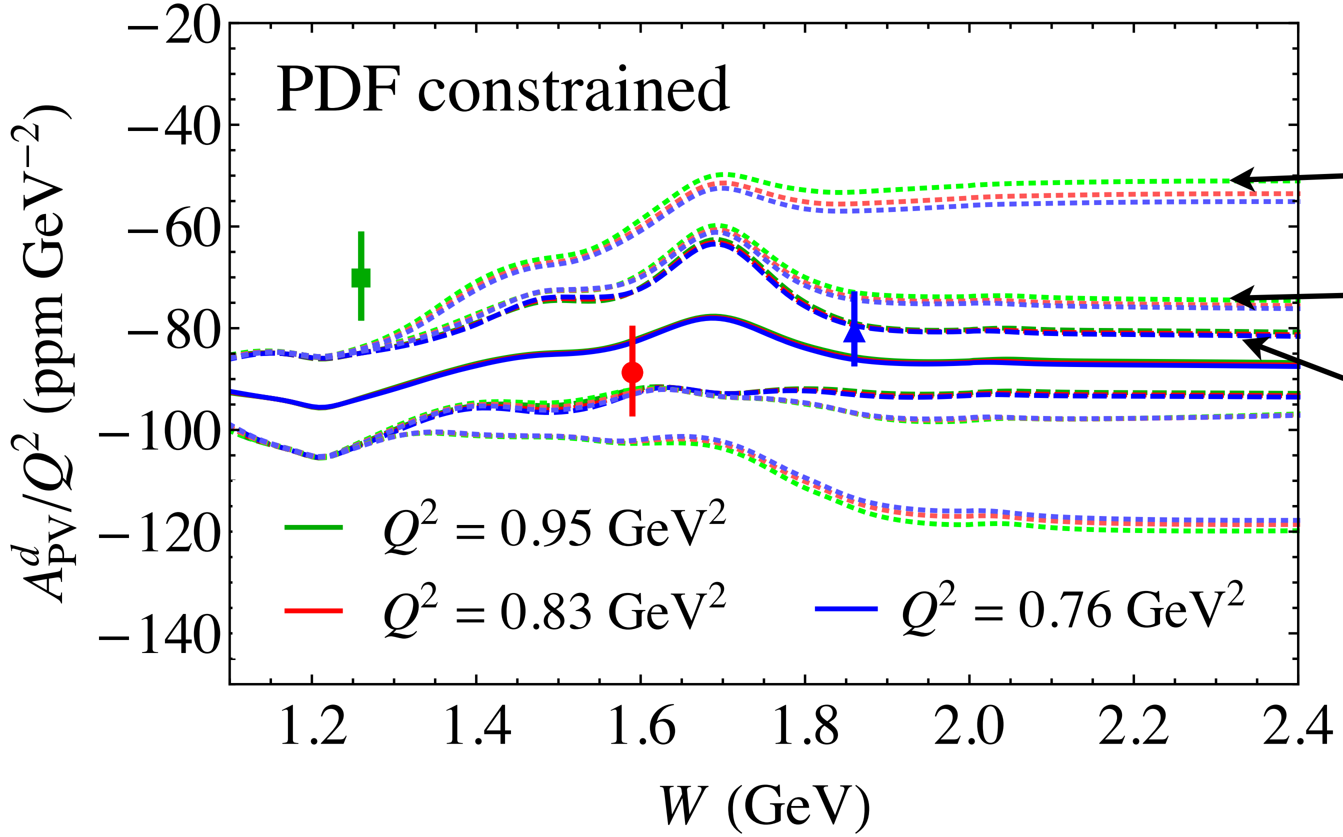




Potential impact of constraints from deuteron PV inelastic asymmetries

25% uncertainty on continuum background



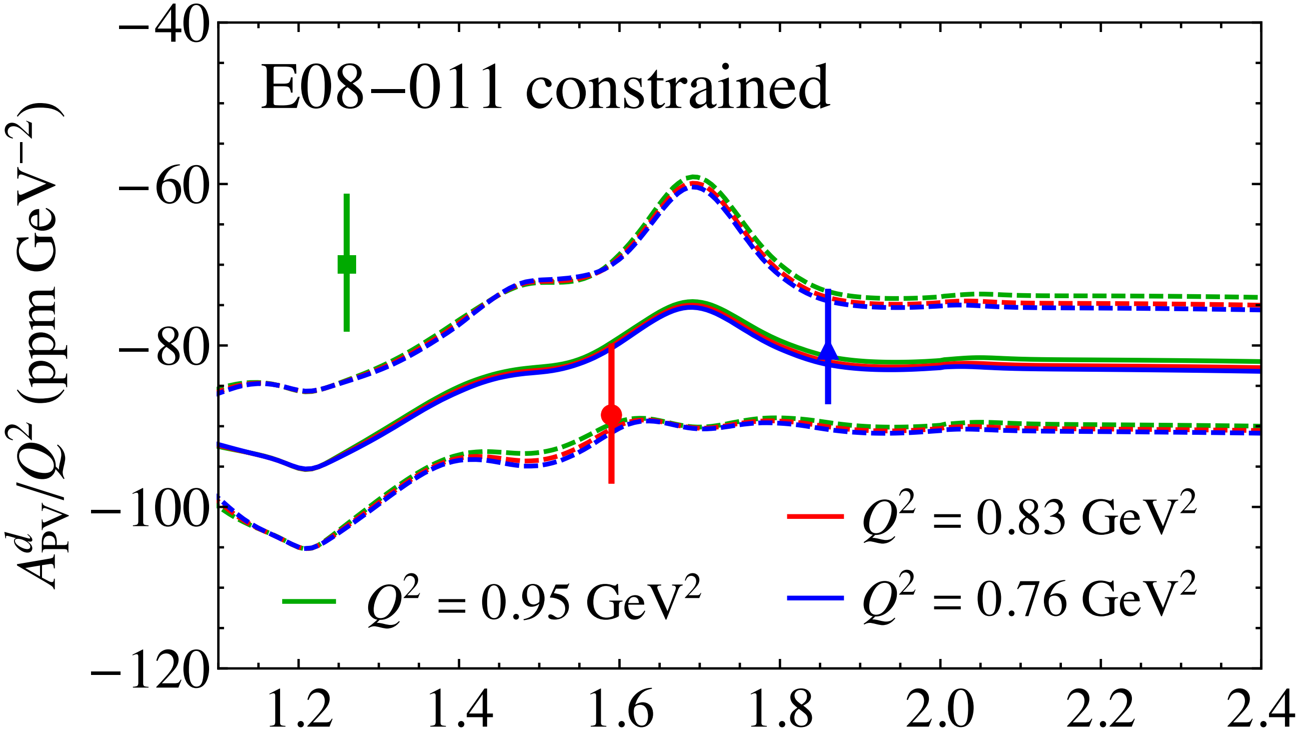


100% bg uncertainty

25% bg uncertainty

AJM bg uncertainty

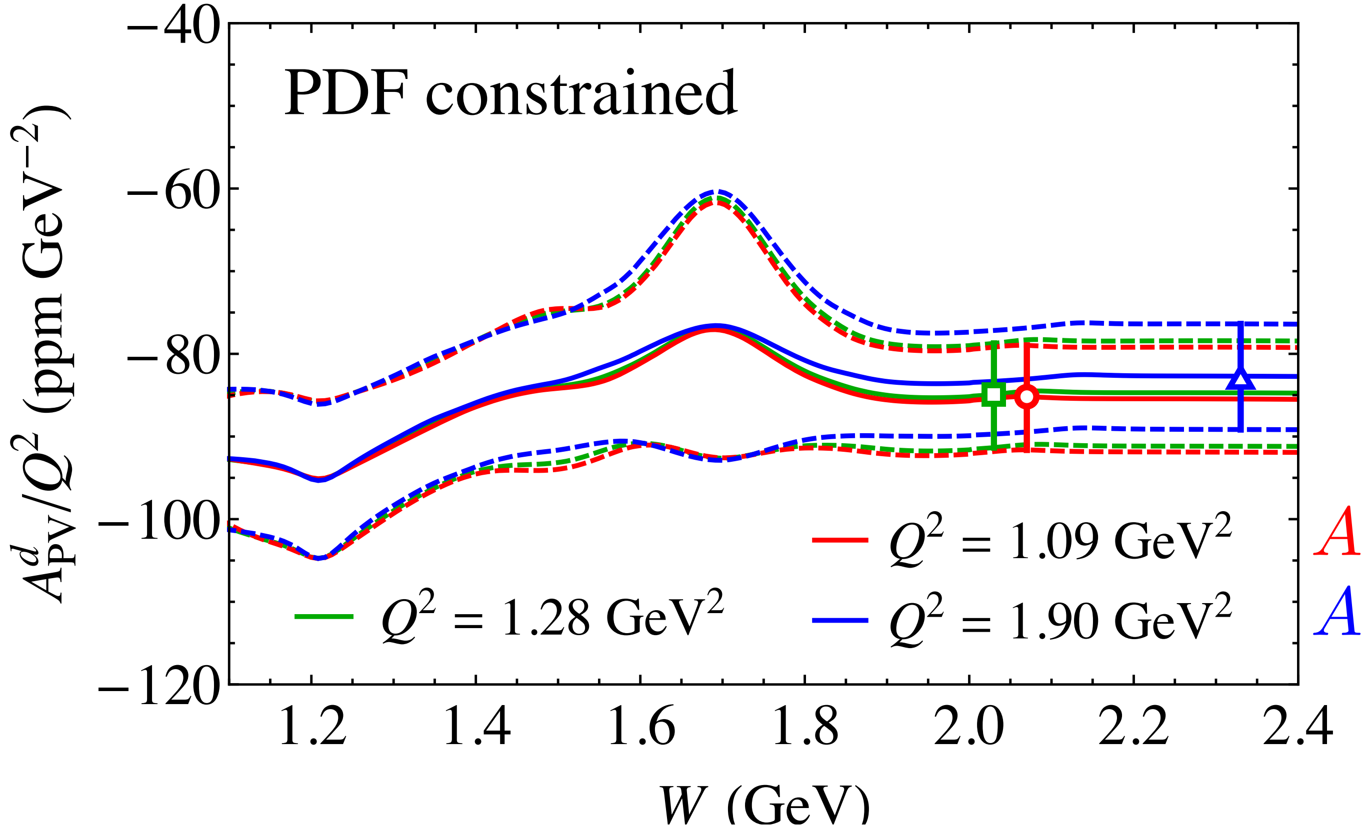
Constraints from PV inelastic asymmetries



AJM model asymmetries and uncertainties for PV deuteron asymmetry constrained by fit to E08-011 data

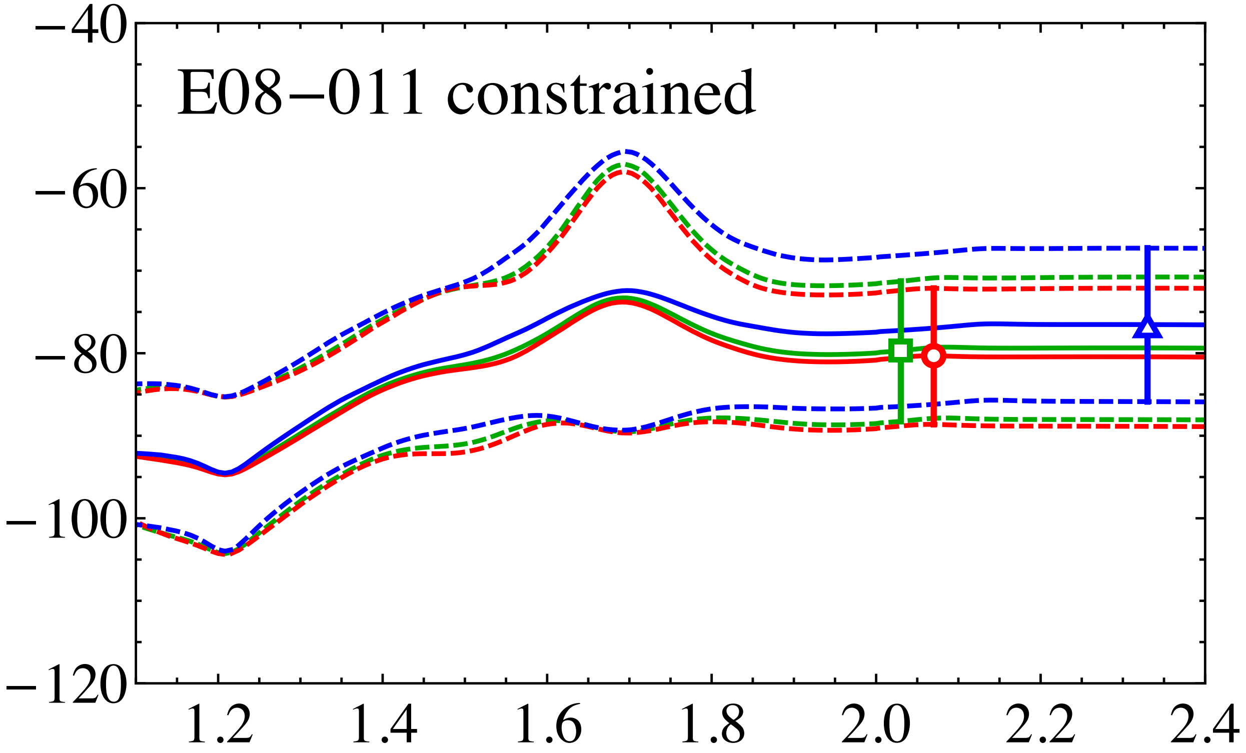
Wang et al. PRL ***111****, 082501 (2013)*

Predictions for PV deuteron asymmetry in DIS kinematics



$A_{\rm PV} = -92.4 \pm 6.8$ ppm

$A_{\rm PV} = -157.2 \pm 12.2$ ppm



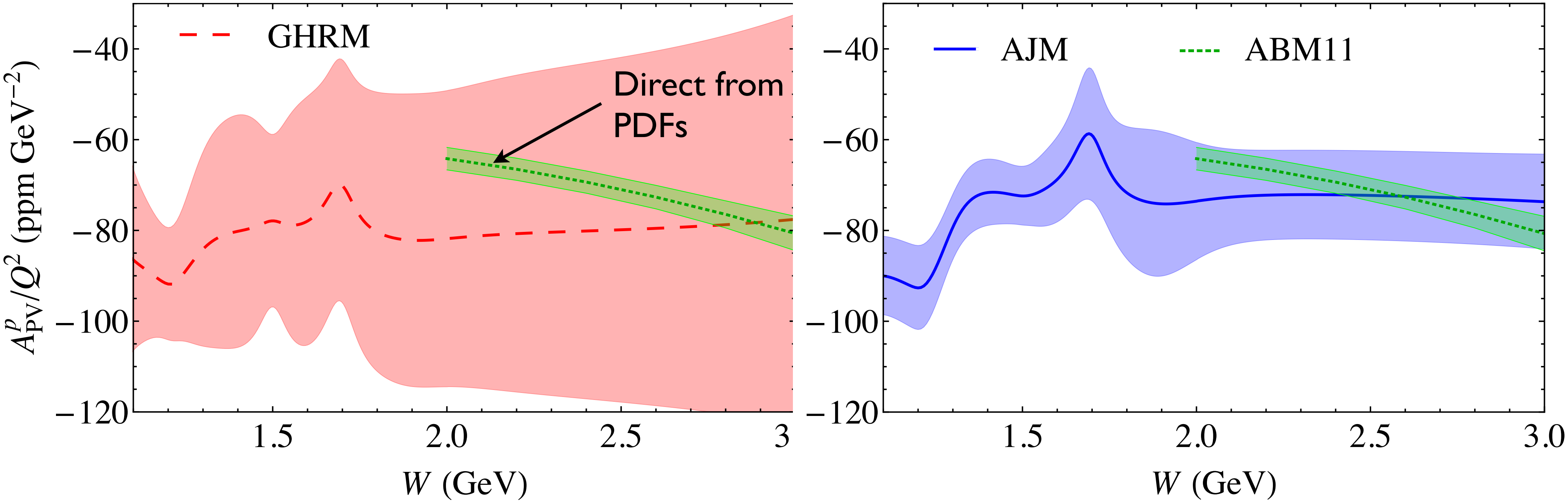
See expt. result in talk by R. Michaels Tuesday morning

Hall et al. (2013)

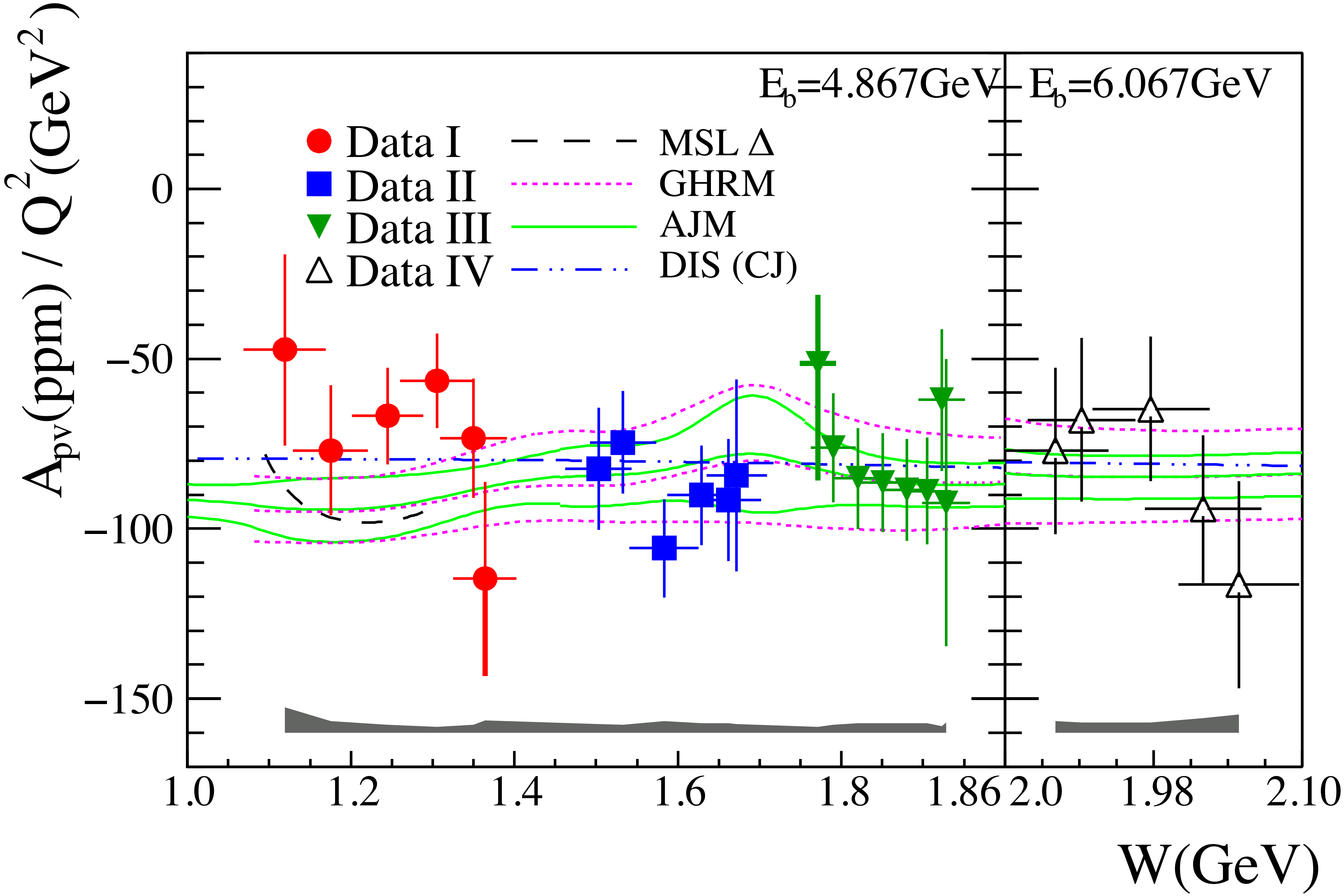
AJM γZ model

- PVDIS asymmetry

$$A_{\rm PV} = g_A^e \left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}(y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

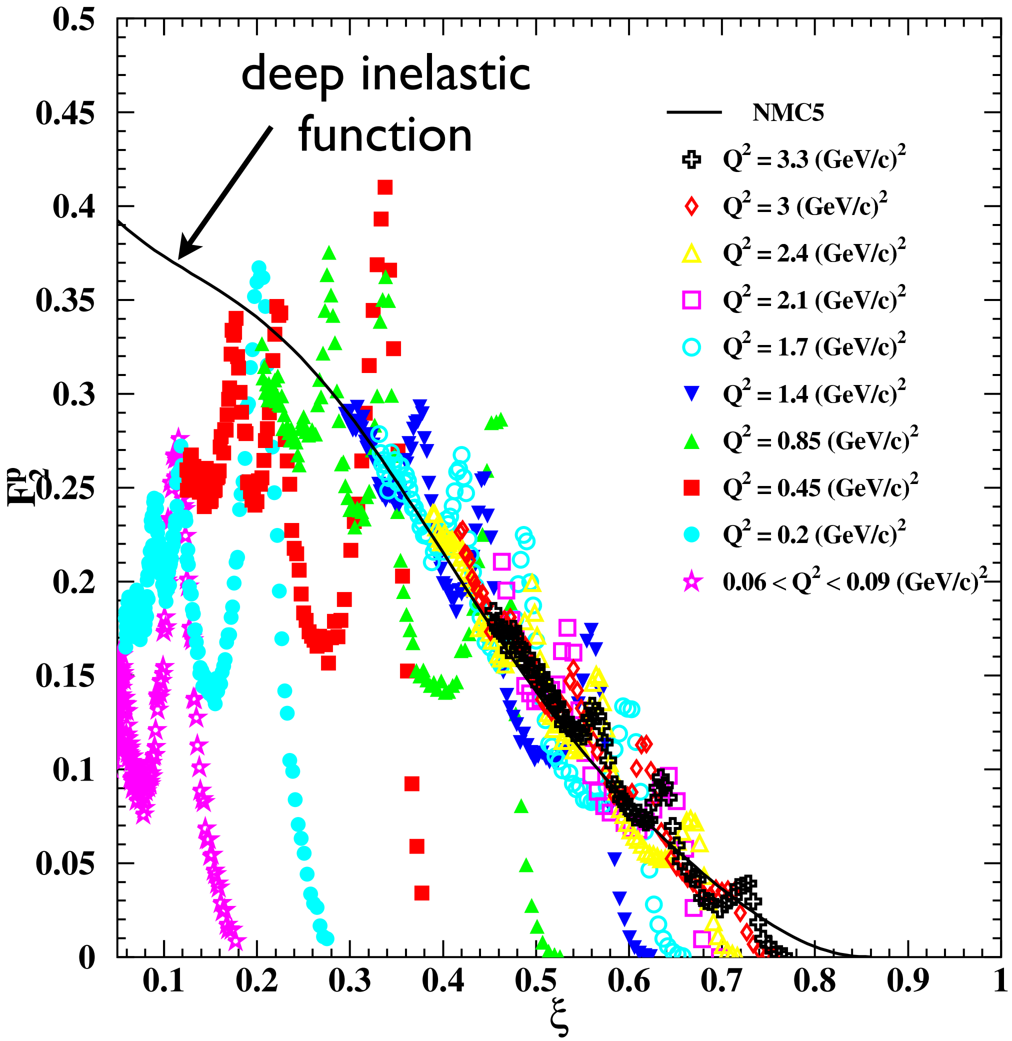


→ significantly smaller uncertainties (at typical JLab kinematics) for constrained model



Wang et al. PRL ***111****, 082501 (2013)*

Duality in electron-nucleon scattering



average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

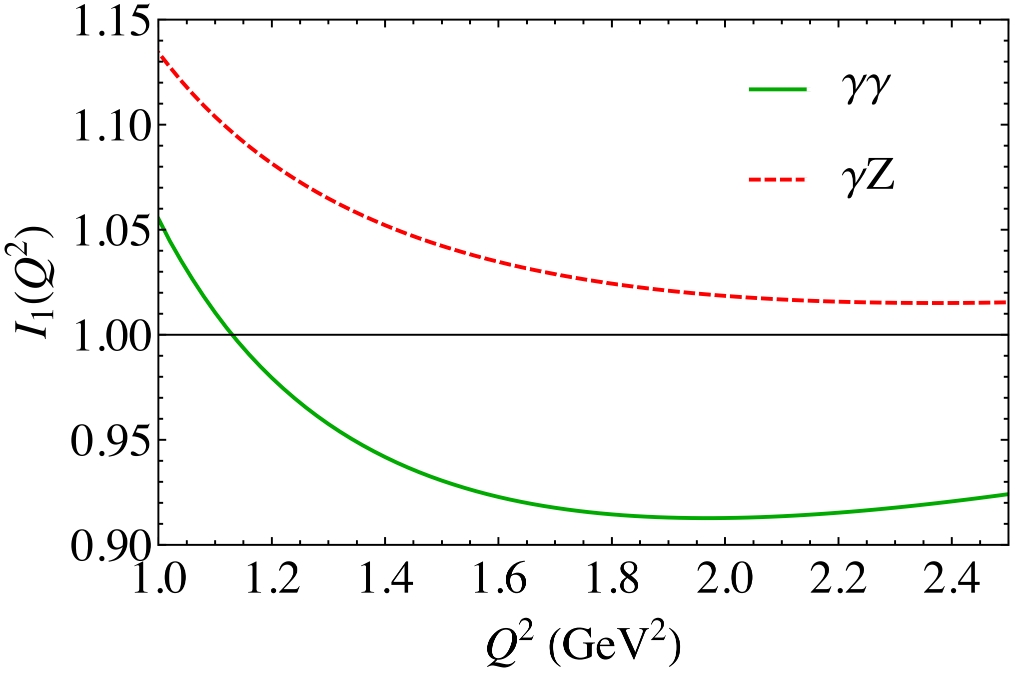
$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

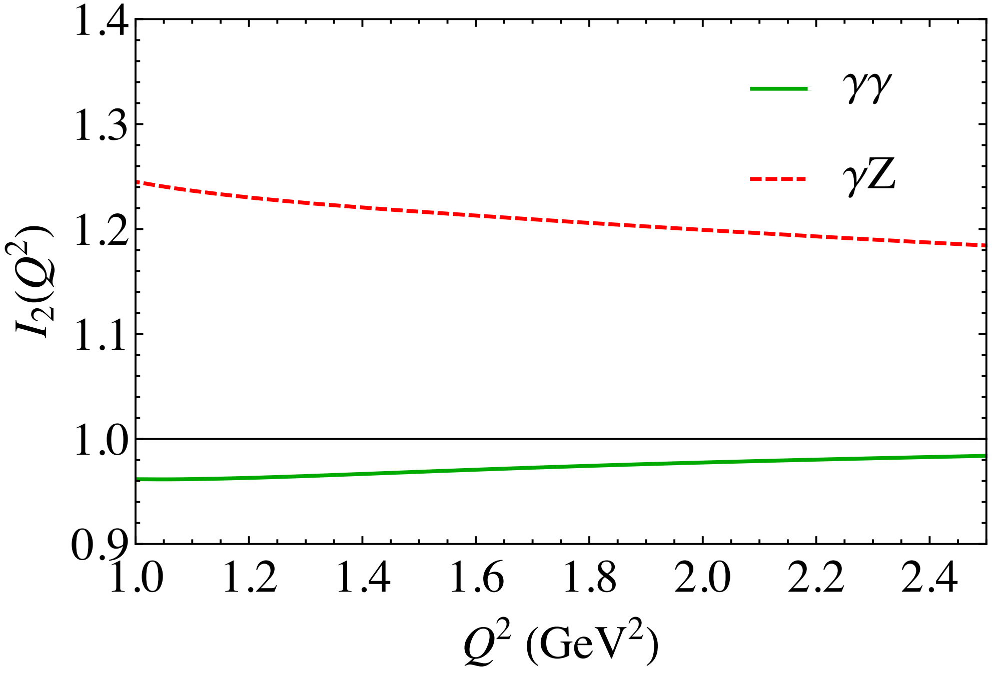
Niculescu et al., PRL ***85****, 1182 (2000)*
Melnitchouk, Ent, Keppel, PRep. ***406****, 127 (2005)*

Duality in γZ vs. γγ

Ratio of model/PDF integrated over resonance region

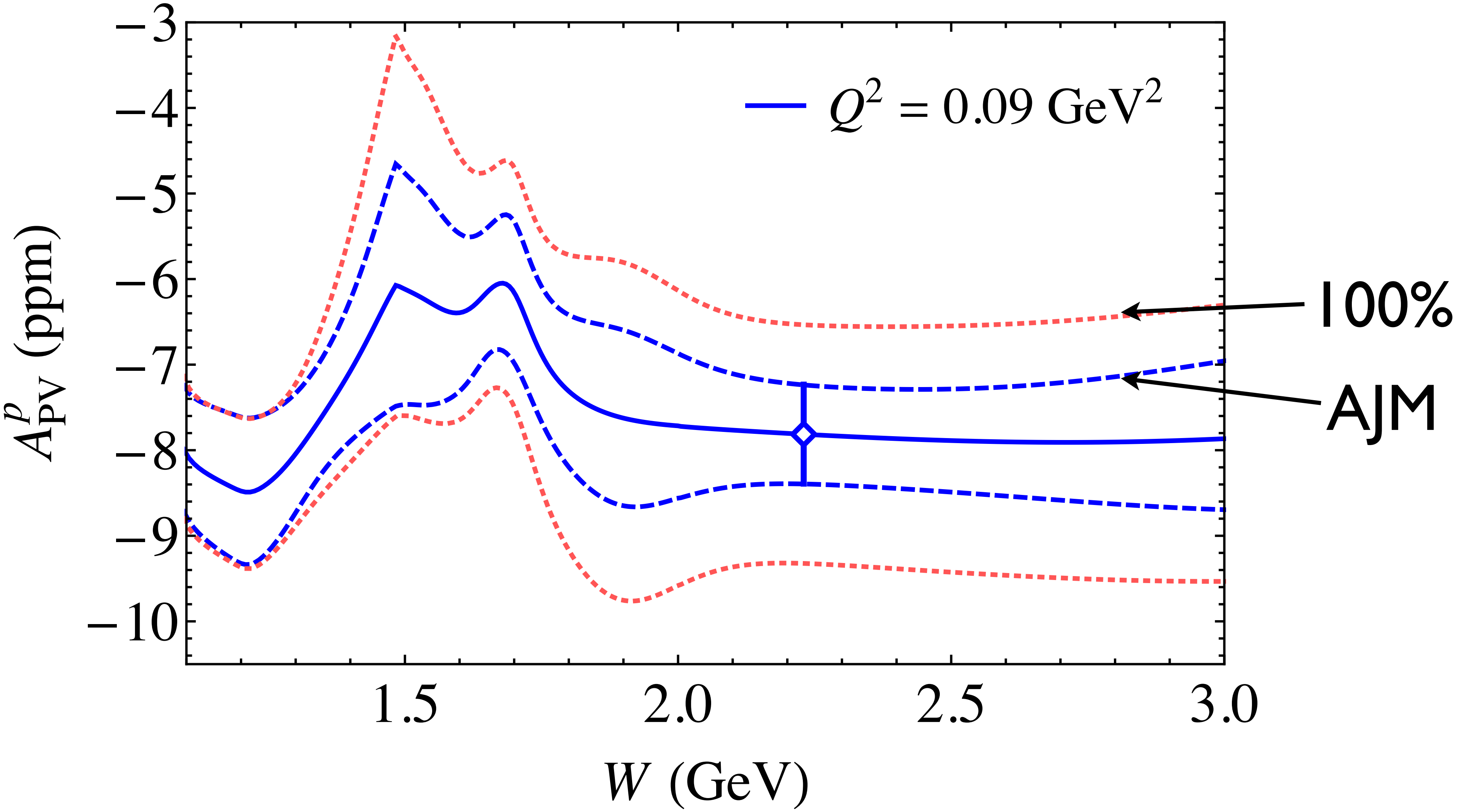
$$\frac{\int_{\rm res} dW\, F_1^{\gamma Z}(Q^2,W)_{\rm AJM}}{\int_{\rm res} dW\, F_1^{\gamma Z}(Q^2,W)_{\rm PDF}} \qquad \frac{\int_{\rm res} dW\, F_2^{\gamma Z}(Q^2,W)_{\rm AJM}}{\int_{\rm res} dW\, F_2^{\gamma Z}(Q^2,W)_{\rm PDF}}$$





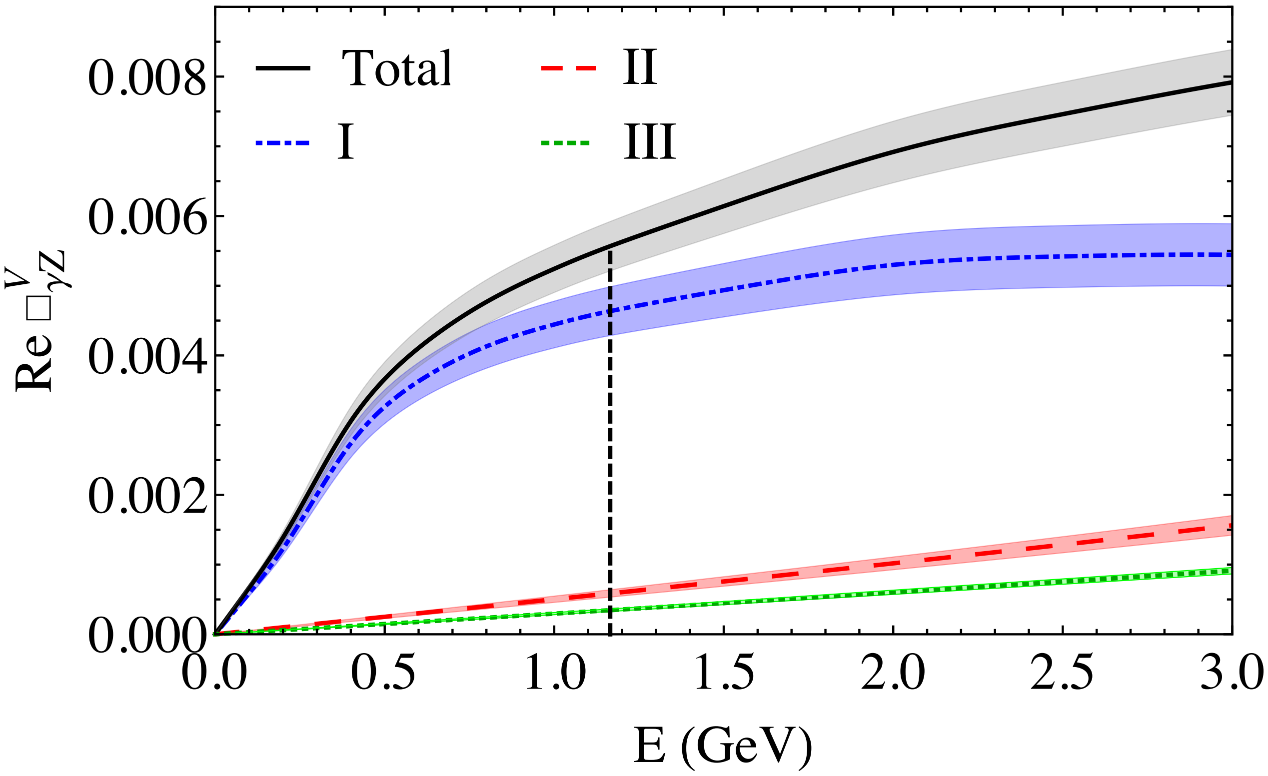
Parity-violating inelastic asymmetries

- Expected inelastic asymmetry data from Qweak



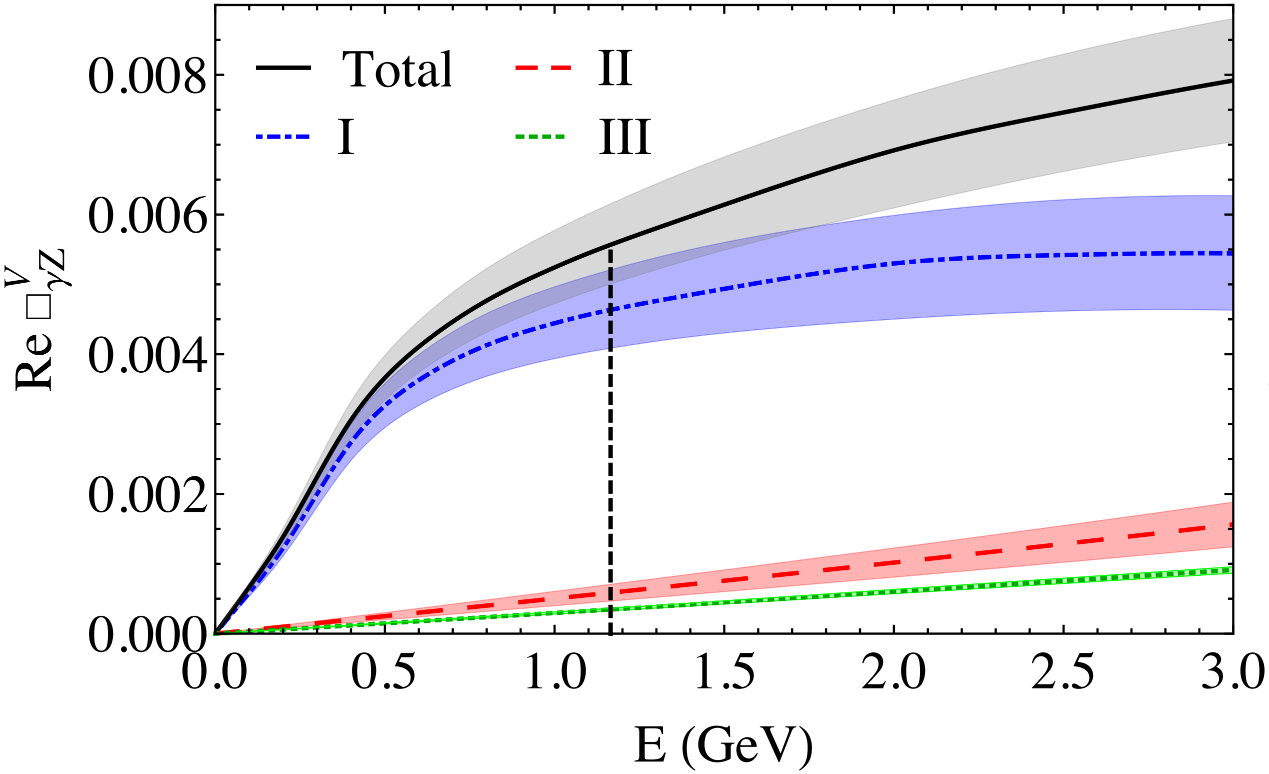
→ AJM model uncertainties compared with 100% on continuum contribution

Hall et al. (2013)



κ_C^T constrained at Q^2=2.5 GeV^2

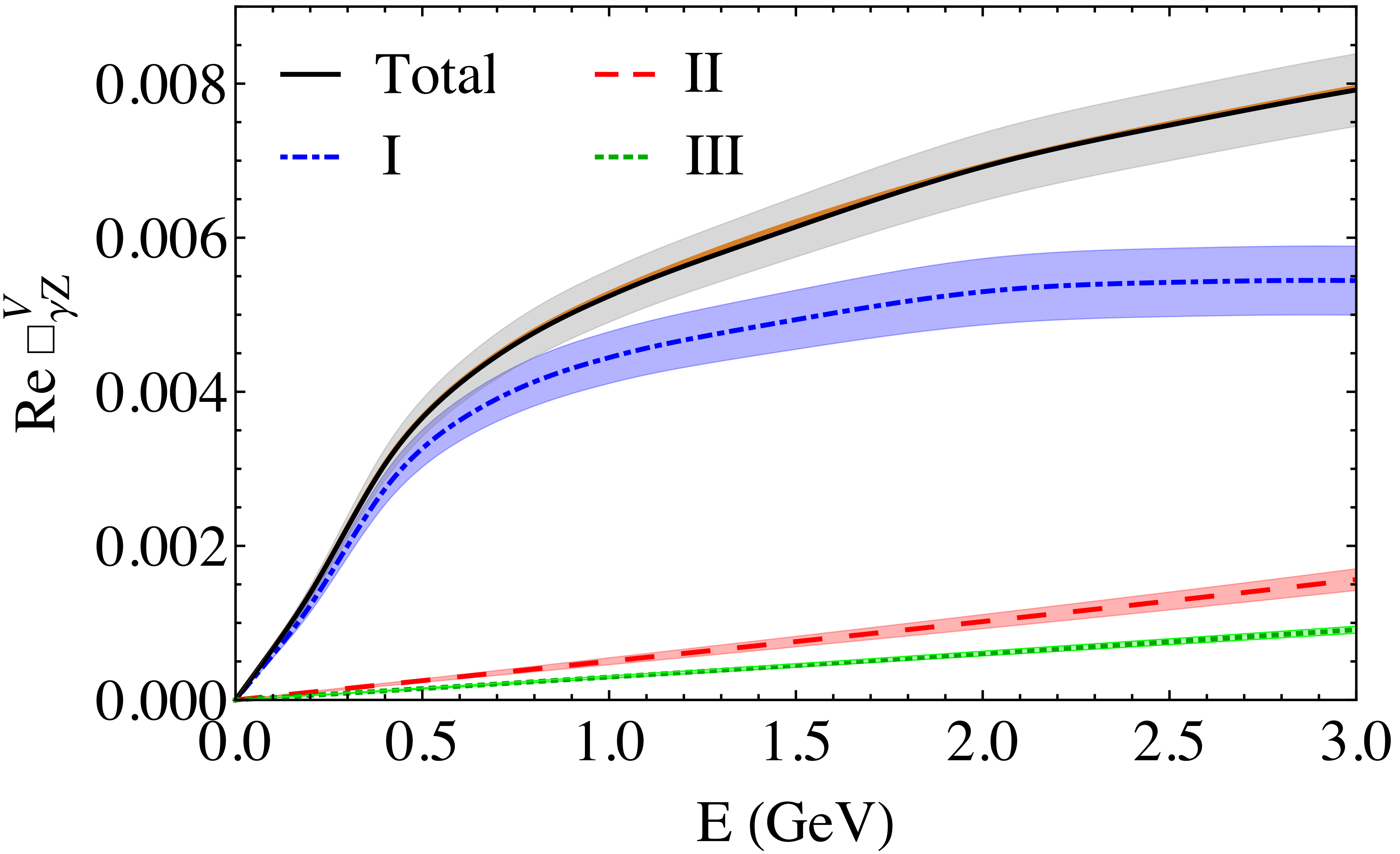
$$(5.57 \pm 0.36) \times 10^{-3}$$



κ_C^T constrained at Q^2=2.5 GeV^2 and 100% uncertain at Q^2=0, with a linear interpolation in Q^2

$$(5.57 \pm 0.59) \times 10^{-3}$$

Using MSTW PDFs down to Q²=1 GeV² (orange curve)



$(5.62 \pm 0.36) \times 10^{-3}$ at 1.165 GeV (previous is 5.57)

Summary

- PDF region provides constraints on model-dependence

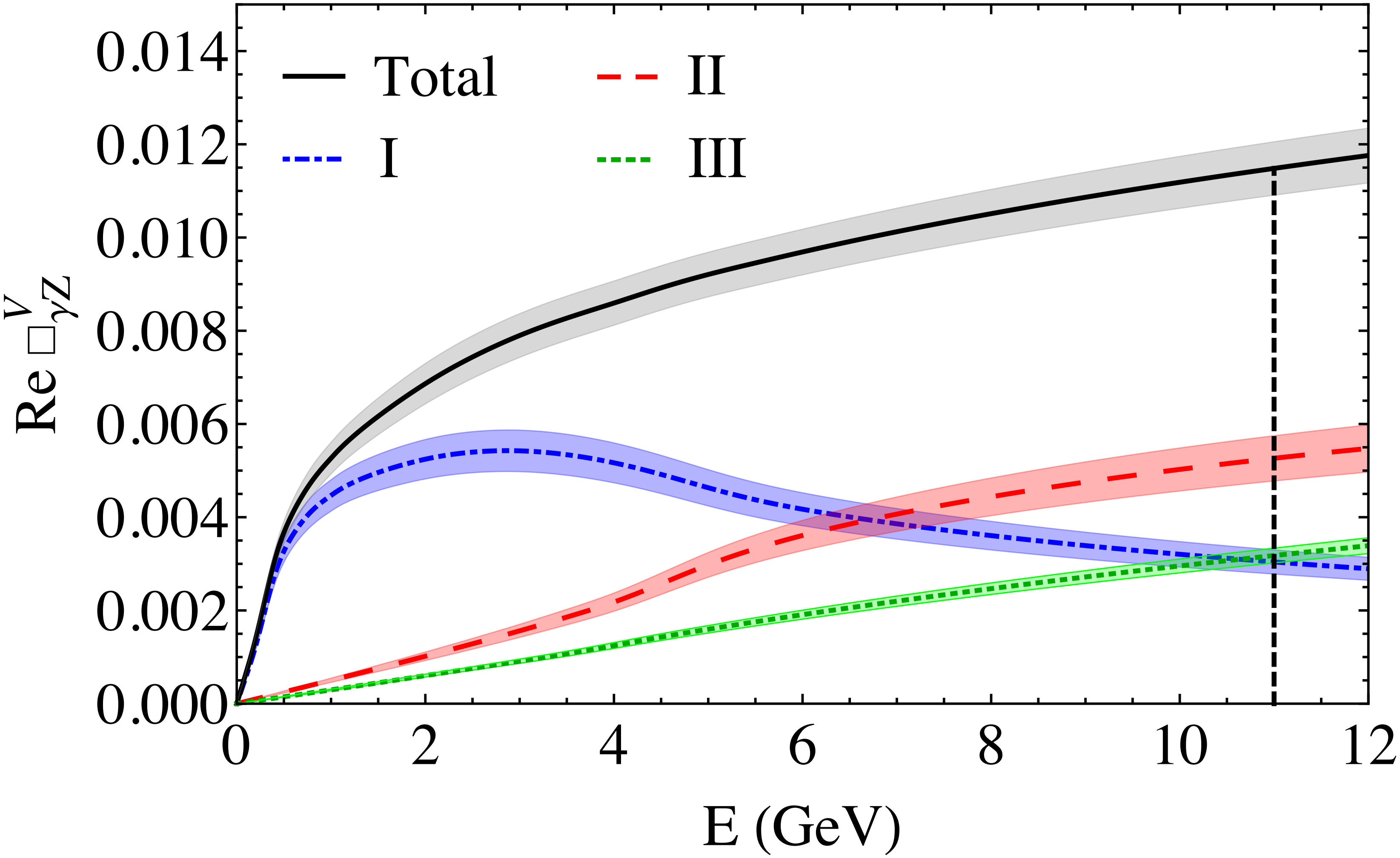
- We’ve taken great care with error, including crosschecks:
  - Linear increase in error on κ_C^T from $Q^2=2.5$ GeV^2 to 0
  - Low Q^2 MSTW calculation
  - Direct comparison with PV inelastic data in resonance and DIS regions

Model passes every check

- E08-011 PVDIS asymmetry should be able to strongly constrain the error

- Checking Δ region would be good for Mainz

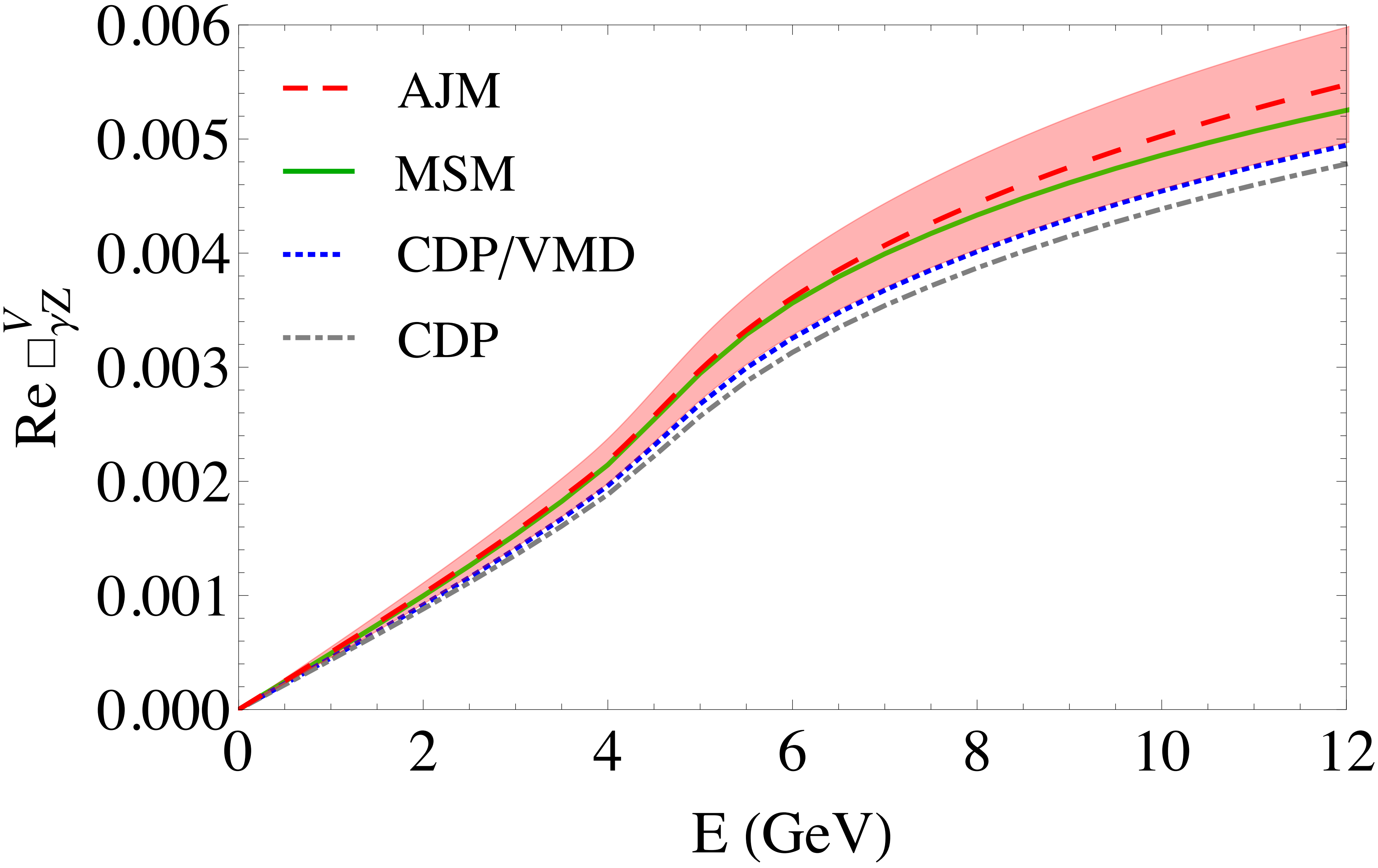
Moller scattering (background)

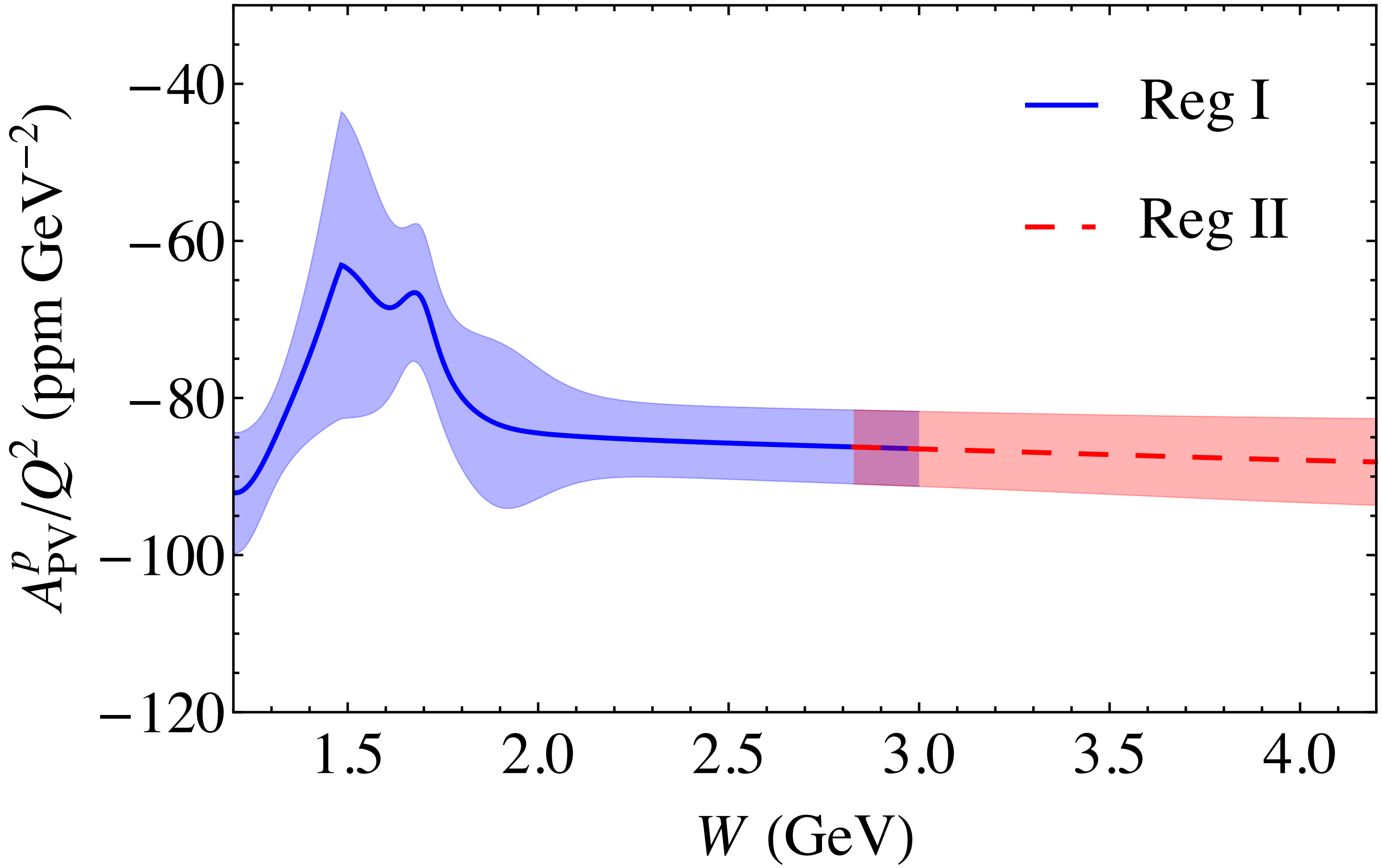


Region II (Regge) relatively more important

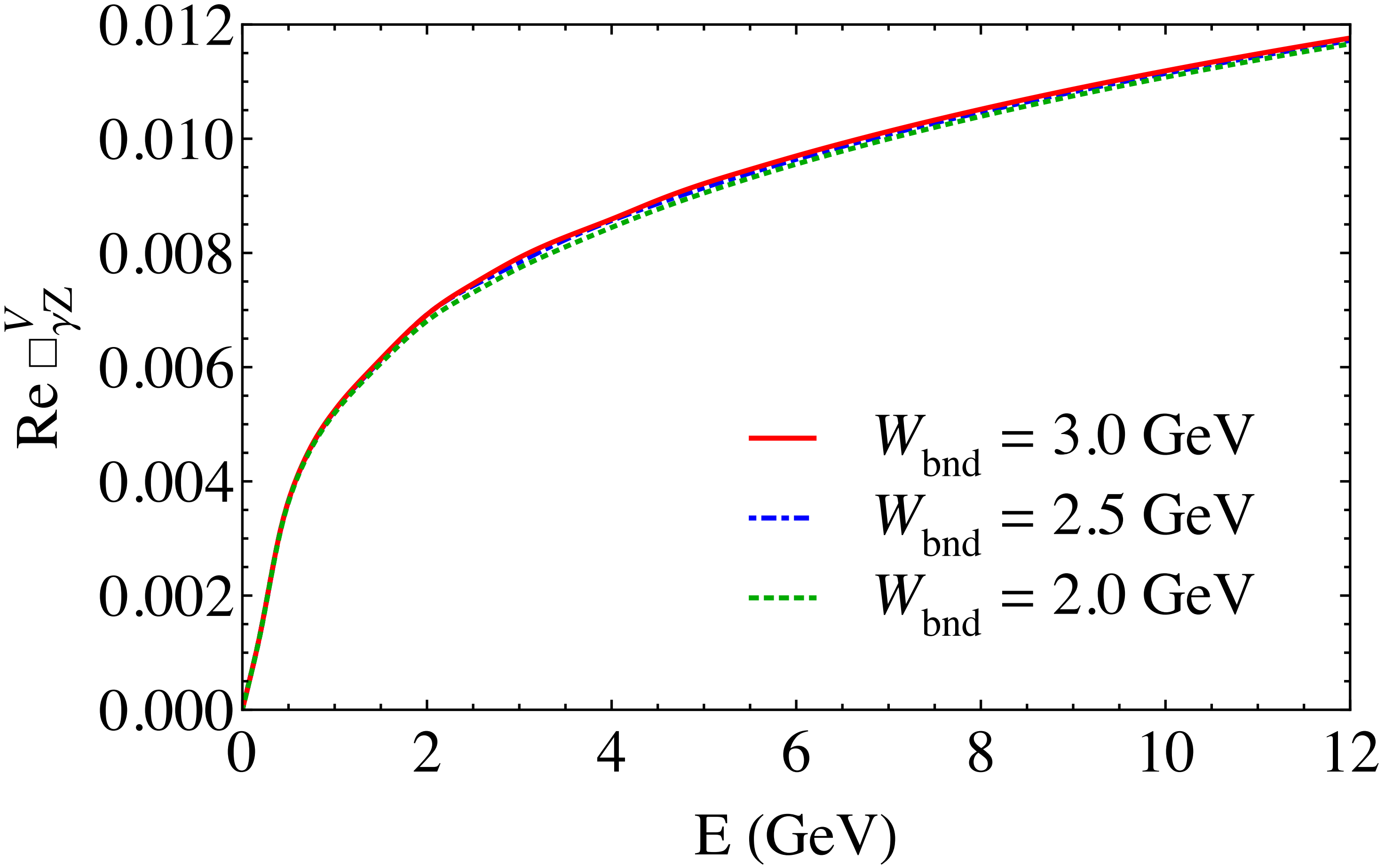
$$(11.5 \pm 0.8) \times 10^{-3} \text{ at } 11 \text{ GeV}$$

Various models for Regge contribution

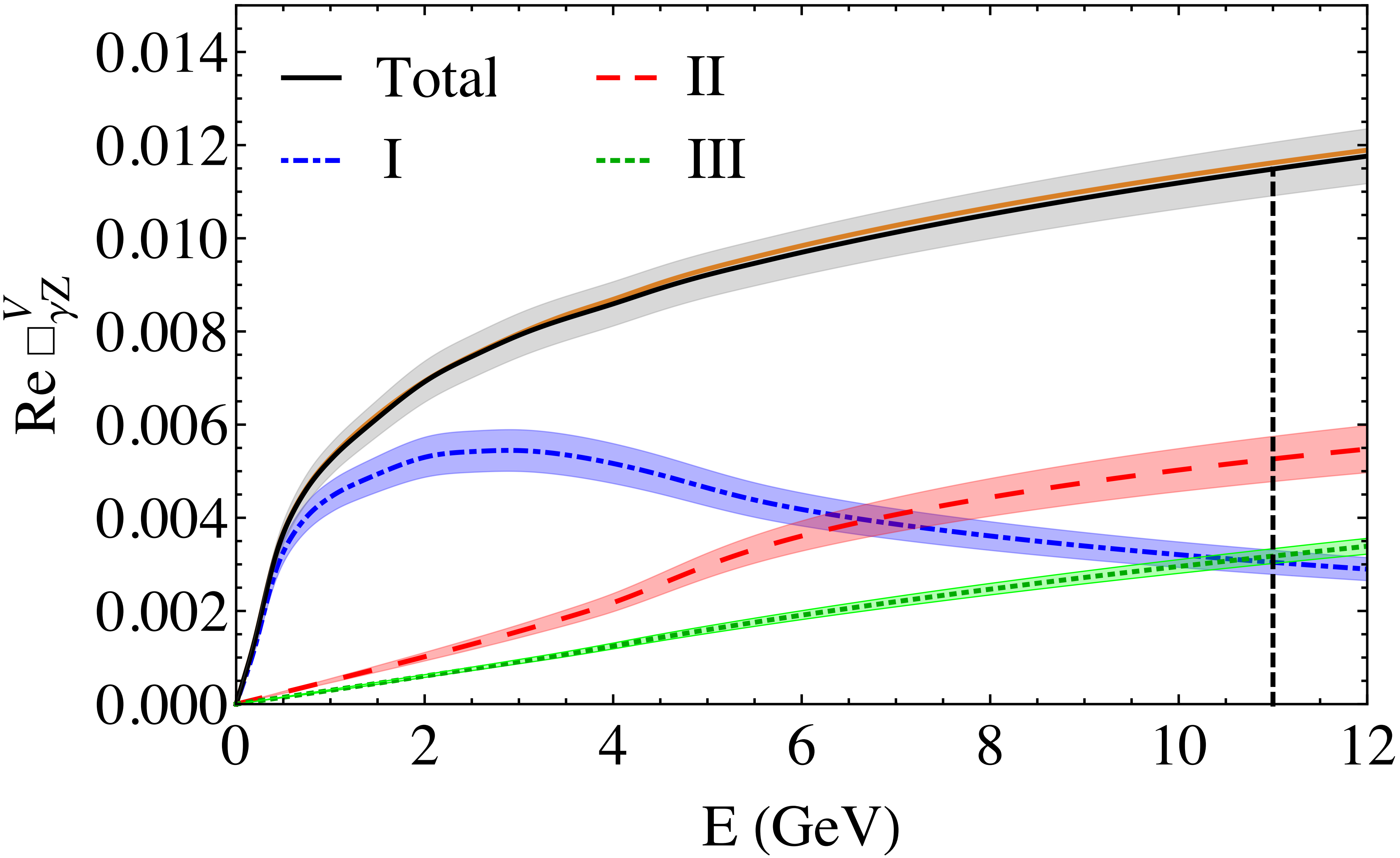



Reg I
Reg II
A^p_{PV}/Q^2 (ppm GeV^{-2})
−40
−60
−80
−100
−120
1.5
2.0
2.5
3.0
3.5
4.0
W (GeV)

Check that varying the matching point from *W* = 3 GeV doesn't affect the calculations.



Using MSTW PDFs down to $Q^2=1$ GeV^2 (orange curve)



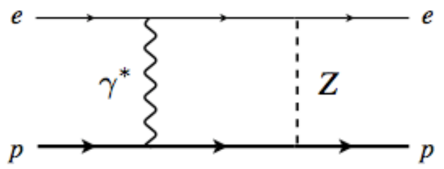
$(11.6 \pm 0.8) \times 10^{-3}$ at 11 GeV (previous is 11.5)

Backup slides

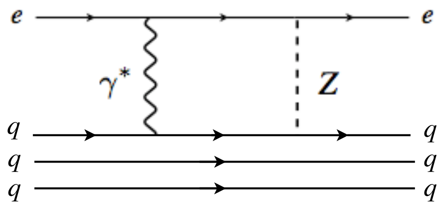
Axial h correction

- axial h correction $\Box^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin as sum of two parts:



- low-energy part approximated by *Born* contribution (elastic intermediate state)



- high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box^{A}_{\gamma Z} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

$$\approx 0.0052 \pm 0.0005$$

short–distance

long–distance: $\frac{3}{2} \pm 1$

Marciano, Sirlin, PRD ***29*** *(1984) 75; Erler et al., PRD* ***68*** *(2003) 016006*

Axial h correction

- Axial $V_e \times A_h$ correction $\Box^A_{\gamma Z}$

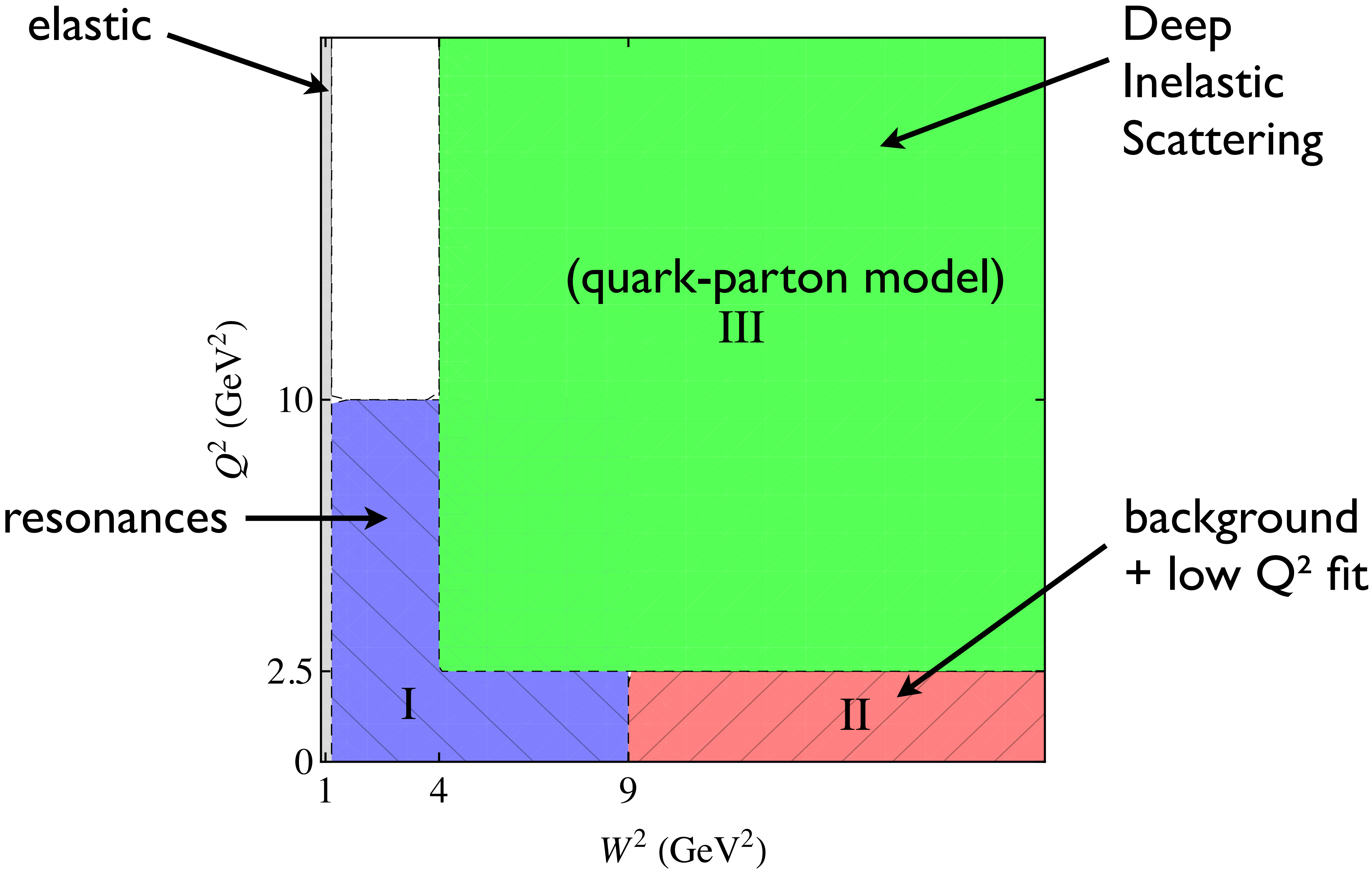
$$\Re e\, \Box^A_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im m\, \Box^A_{\gamma Z}(E')$$

→ imaginary part given by $F_3^{\gamma Z}$ structure function

$$\Im m\, \Box^A_{\gamma Z}(E) = \frac{1}{(2ME)^2} \int_{M^2}^{s} dW^2 \int_0^{Q^2_{\rm max}} dQ^2\, \frac{v_e(Q^2)\, \alpha(Q^2)}{1 + Q^2/M_Z^2} \times \left(\frac{2ME}{W^2 - M^2 + Q^2} - \frac{1}{2} \right) F_3^{\gamma Z}$$

with $v_e(Q^2) = 1 - 4\kappa(Q^2) \sin^2\theta_W(Q^2)$

Integration region



elastic

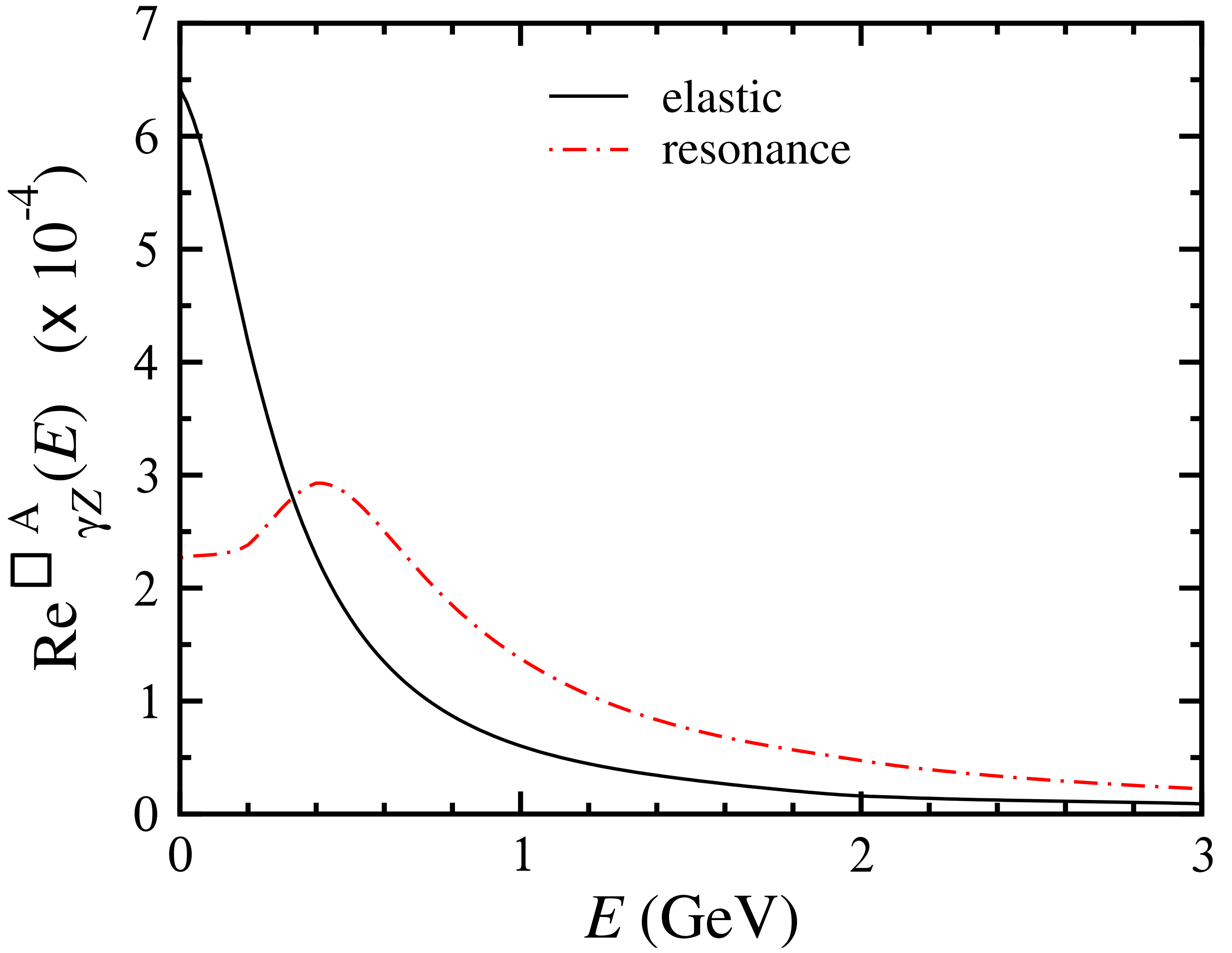
Deep
Inelastic
Scattering

resonances

background
+ low Q^2 fit

Axial h elastic + resonance correction

- elastic part: $F_3^{\gamma Z(\mathrm{el})}(Q^2) = -Q^2 G_M^p(Q^2) G_A^Z(Q^2) \delta(W^2 - M^2)$
- resonance part from parametrization of ν scattering data; includes lowest 4 spin1/2 and 3/2 states (Lalakulich-Paschos)



Axial h correction DIS part

- change integration variable $W^2 \to x$ and switch order of integration. Energy integral can be done analytically.

- DIS part dominated by leading twist PDFs at small x

(MSTW, CTEQ, Alekhin)

$$F_3^{\gamma Z(\mathrm{DIS})}(x,Q^2) = \sum_q 2\, e_q\, g_A^q\ \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$$

→ in DIS region ($Q^2 \gtrsim 1\ \mathrm{GeV}^2$), expand integrand in powers of *x*

$$\Re e\Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi}\int_{Q_0^2}^{\infty} dQ^2\, \frac{v_e(Q^2)\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)}$$

$$\times\left[M_3^{(1)}(Q^2) + \frac{2M^2}{9Q^4}(5E^2-3Q^2)M_3^{(3)}(Q^2) + \ldots\right]$$

with moments $$M_3^{\gamma Z(n)} = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$$

Axial h correction

- structure function moments

$n = 1$ $$M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

→ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\,\Box_{\gamma Z}^{A(\mathrm{DIS})} \approx (1 - 4\hat{s}^2)\,\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

→ precisely result from Marciano & Sirlin! $\sim \log\frac{M_Z^2}{Q_0^2}$
(works because result depends on lowest moment of *valence* PDF, with model-independent normalization!)

$n = 3$ $$M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$$

→ related to x^2 -weighted moment of valence quarks

Parity-violating e scattering

- ★ "DIS" region at $Q^2 < 1\ \mathrm{GeV}^2$ does not afford PDF description

  ⟶ in absence of data, consider models with general constraints

  - ★ $F_3^{\gamma Z}(x_{\max}, Q^2)$ should not diverge in limit $Q^2 \to 0$
  - ★ $F_3^{\gamma Z}(x, Q^2)$ should match PDF description at $Q^2 \sim 1\ \mathrm{GeV}^2$

Model 1 $$F_3^{\gamma Z}(x, Q^2) = \left(\frac{1+\Lambda^2/Q_0^2}{1+\Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)$$

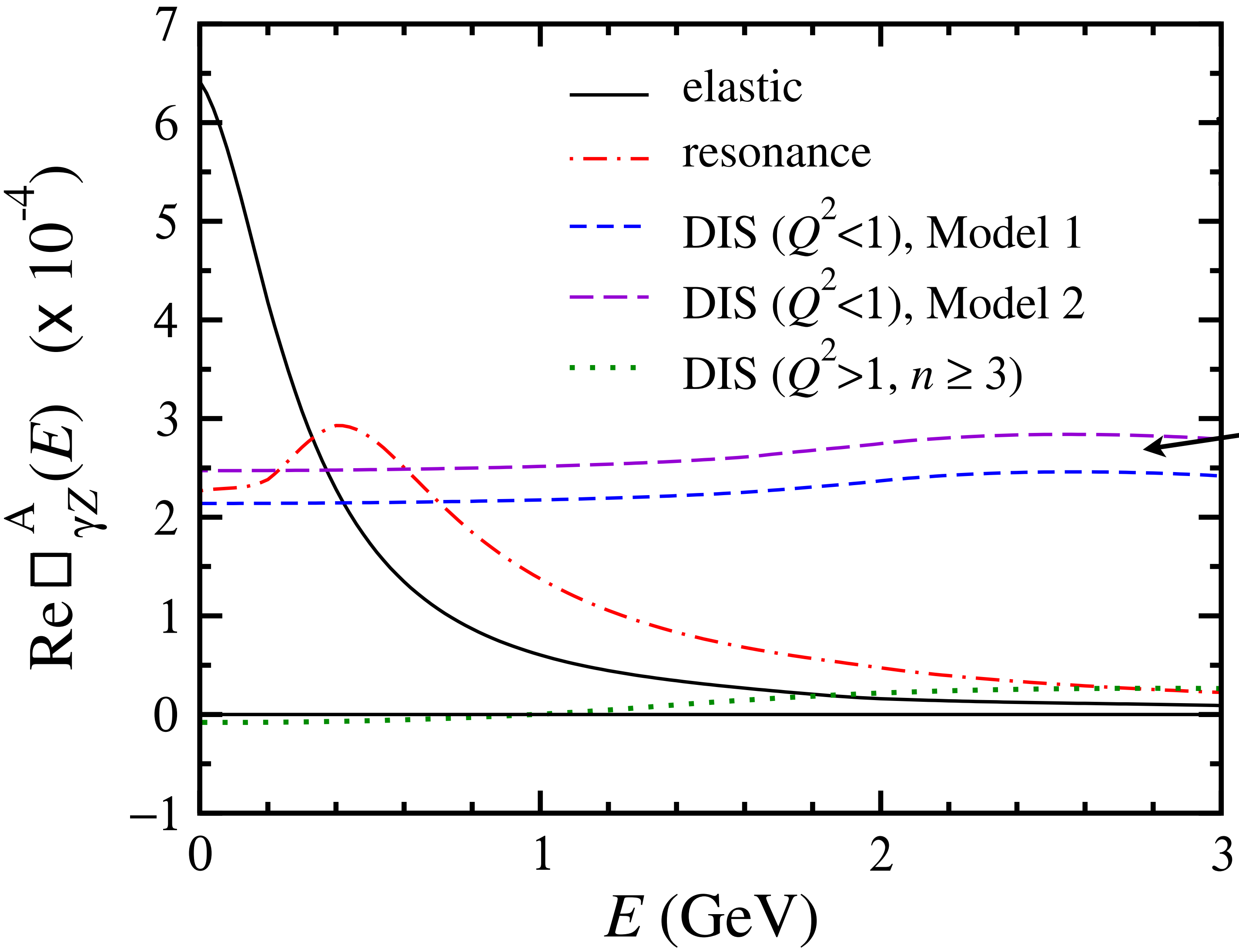
$$F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \to 0$$

Model 2 $F_3^{\gamma Z}$ frozen at $Q^2 = 1$ value for all W^2

$$F_3^{\gamma Z} \text{ finite as } Q^2 \to 0$$

- ★ Result should be independent of matching point
  (e.g. setting Q_0^2 = 2 GeV^2 gives almost identical results)

Parity-violating e scattering



Take this as the uncertainty

Blunden et al.,
PRL ***107****, 081801 (2011)*

→ dominated by $n = 1$ DIS moment: $\underline{32.8 \times 10^{-4}}$
(no E dependence)

Parity-violating e scattering

- correction at $\underline{E = 0}$

$$\Re e\,\Box^{A}_{\gamma Z} = \underset{\underset{\boxed{\text{elastic}}}{\uparrow}}{0.00064} + \underset{\underset{\boxed{\text{resonance}}}{\uparrow}}{0.00023} + \underset{\underset{\boxed{\text{DIS}}}{\uparrow}}{0.00350} \to \underline{0.0044(4)}$$

- correction at $\underline{E = 1.165 \text{ GeV}}$ (Qweak)

$$\Re e\,\Box^{A}_{\gamma Z} = 0.00005 + 0.00011 + 0.00352 = \underline{0.0037(4)}$$

cf. MS* value: $\underline{0.0052(5)}$ (~1% shift in Q^p_W)

* *Marciano, Sirlin, PRD* ***29****, 75 (1984)*

- shifts Q^p_W from $\underline{0.0716(8)} \longrightarrow \underline{0.0708(8)}$

Update of Atomic Parity Violation calculation

	MS	free nucleons	bound nucleons
B^p	11.8(1.0)	9.95(40)	9.36(40)
B^n	11.5(1.0)	9.82(40)	9.32(40)
Q_W^p	0.0716(8)	0.0708(6)	0.0705(6)
Q_W^n	-0.9882(4)	-0.9888(2)	-0.9890(2)
$Q_W(\mathrm{Cs})$	-73.14(6)	-73.23(4)	-73.26(4)
$Q_W(\mathrm{Ra})$	-117.22(10)	-117.37(7)	-117.42(7)

Blunden et al., *PRL* ***109***, *262301 (2012)*

↑

Includes Pauli-blocking of elastic contribution (*cf.* Coulomb sum rule)

$$Q_W(Cs) = 55Q_W^p + 78Q_W^n$$

$$\Box_{\gamma Z}^N = \frac{\hat{\alpha}}{2\pi}(1 - 4\hat{s}^2) \times \begin{cases} 5B^p, & p, \\ 4B^n, & n. \end{cases}$$

$$\text{MS: } B^p = \ln \frac{M_Z^2}{m^2} + \frac{3}{2}; \quad m = 0.77 \text{ GeV}$$

- Find $Q_W(\mathrm{Cs}) = -73.26 \pm 0.04$ Expt: $Q_W(\mathrm{Cs}) = -73.20 \pm 0.35$
- Errors: $\kappa(0)\hat{s}^2$ (± 0.033), WW boxes (± 0.006), ρ (± 0.016), γZ (± 0.021).