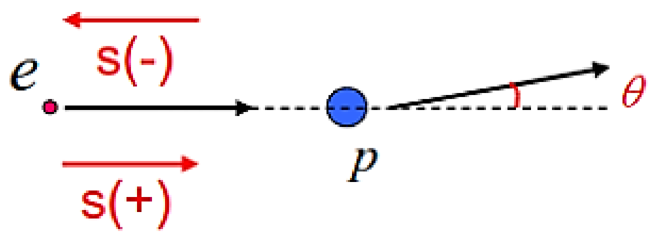
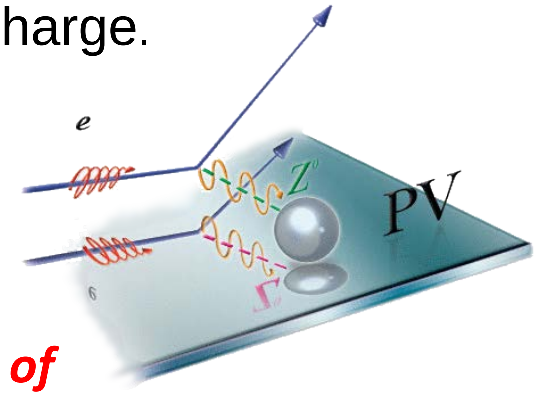
The Q-weak Experiment

A search for parity violating new physics at the TeV scale
by measurement of the Proton's weak charge.



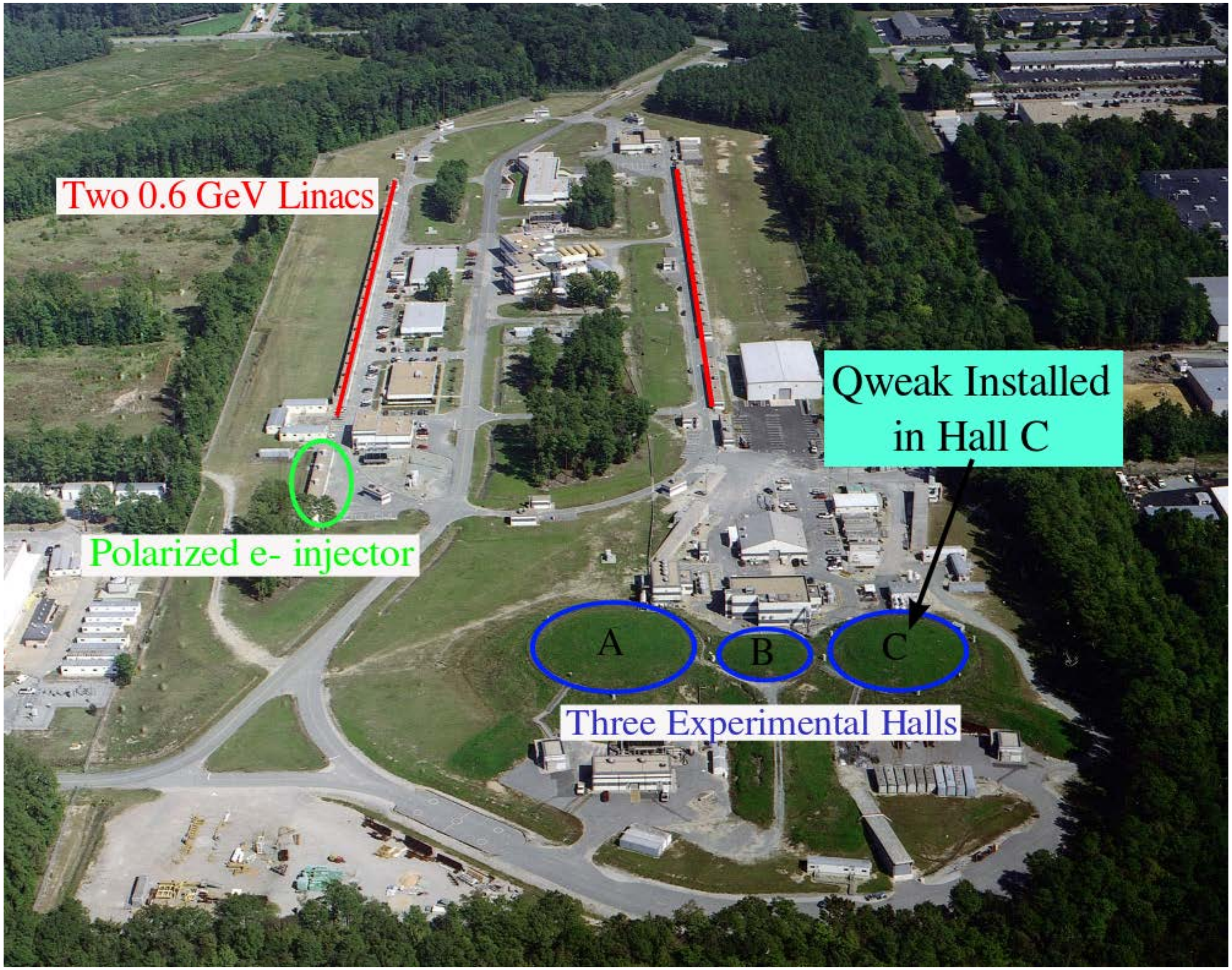
Roger D. Carlini
Jefferson Laboratory



(Content of this talk includes the work of students, postdocs and collaborators)

- Scatter longitudinally polarized electrons from liquid hydrogen
- Flip the electron spin and see how much the scattered fraction changes
- The difference is proportional to the weak charge of the proton
- Hadronic structure effects determined from global PVES measurements.

Jefferson Lab Site



Qweak Installation:
May 2010-May 2012

~1 year of beam in
3 running periods:

Run 0
Jan – Feb 2011

Run 1
Feb – May 2011

Run 2
Nov 2011 – May 2012

Precision Tests of the Standard Model

- Standard Model is known to be the effective low-energy theory of a more fundamental underlying structure. (Meaning its not complete!)

- Finding new physics beyond the SM: Two complementary approaches:
  - Energy Frontier (direct) : *eg*. Tevatron (deceased), LHC (dry well so far)
  - Precision Frontier (indirect) : *Often at modest or low energy…*
    - μ(g-2) , EDM, $\beta\beta$ decay, $\mu \rightarrow e\,\gamma$, $\mu A \rightarrow eA$, $K^+ \rightarrow \pi^+ \nu\nu$, *etc.*
    - ν *- oscillations*
    - *Atomic Parity violation*
    - *Parity-violating electron scattering*

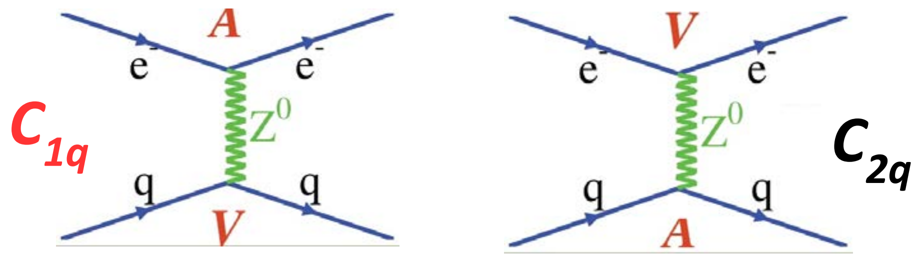
Hallmark of the Precision Frontier: Choose observables that are "*precisely predicted*" or "*suppressed*" in Standard Model.

If new physics is "eventually" found in direct measurements, precision measurements also useful to determine e.g. couplings…

The Weak Charges

Electron-quark scattering, general four-fermion contact interaction:

$$\mathcal{L}_{eq}^{PV} = -\frac{G_F}{\sqrt{2}} \sum_i \left[C_{1i} \bar{e}\gamma_\mu\gamma_5 e \bar{q}\gamma^\mu q + C_{2q} \bar{e}\gamma_\mu e \bar{q}\gamma^\mu\gamma^5 q \right] + \mathcal{L}_{new}^{PV}$$

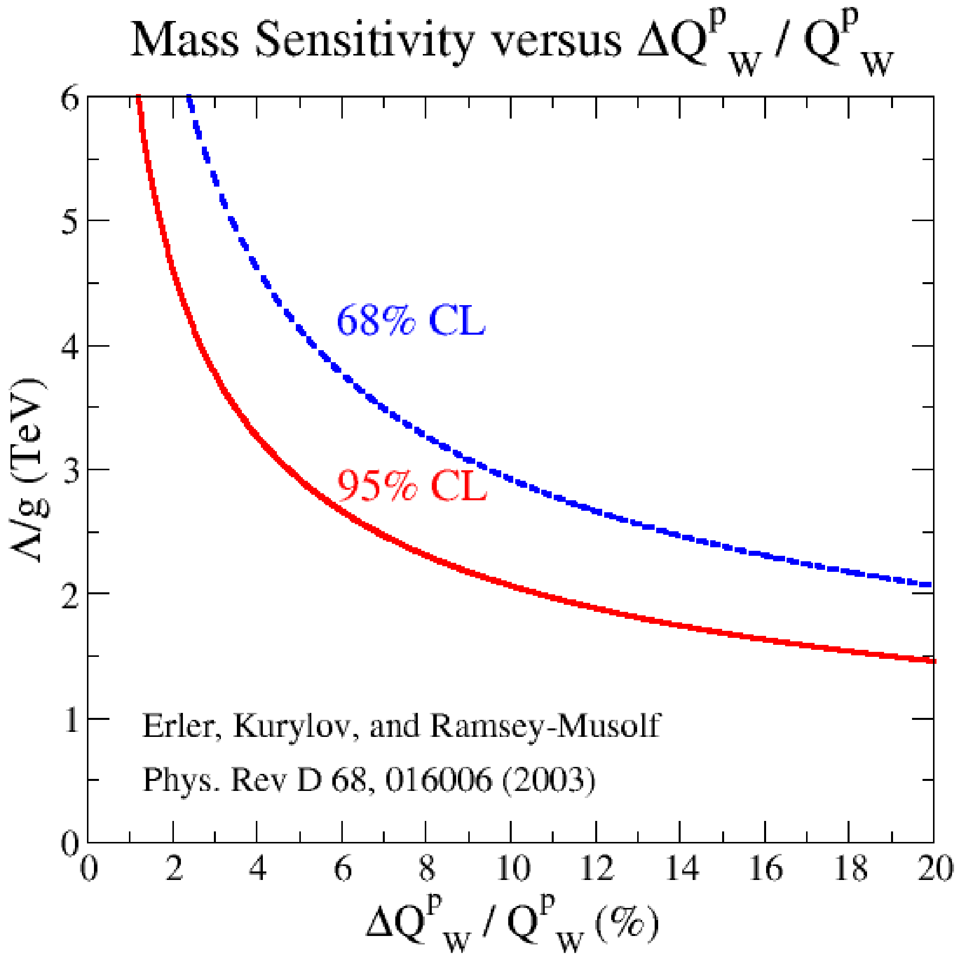


Note “accidental” suppression of Q^p_W → ***sensitivity to new physics***

Particle	Electric charge	Weak vector charge ($\sin^2\theta_W \approx \frac{1}{4}$)
e	-1	$Q_W^e = -1 + 4\sin^2\theta_W \approx 0$
u	$+\frac{2}{3}$	$-2C_{1u} = +1 - \frac{8}{3}\sin^2\theta_W \approx +\frac{1}{3}$
d	$-\frac{1}{3}$	$-2C_{1d} = -1 + \frac{4}{3}\sin^2\theta_W \approx -\frac{2}{3}$
p(uud)	$+1$	$Q_W^p = 1 - 4\sin^2\theta_W \approx 0.07$
n(udd)	0	$Q_W^n = -1$

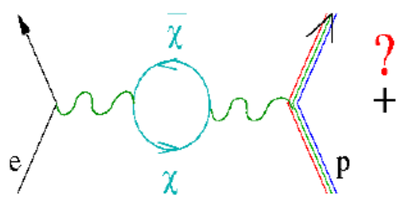
Q^p_W has a definite prediction in the electroweak Standard Model

Sensitivity to New Physics

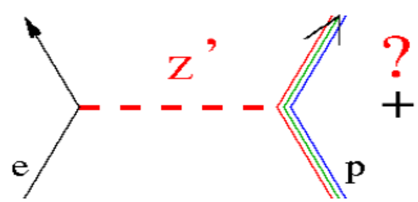


The vertical axis is Λ/g

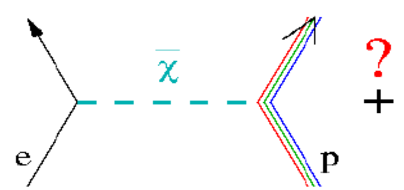
Depending on how you construct the PV "new physics" Lagrangian and select a model dependent "g" the mass reach can become much greater.



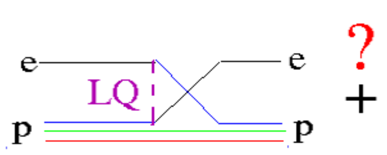
RPC SUSY



Generic Z'



RPV SUSY



Leptoquarks

Qweak Experiment Objectives

10 years of development + 2 years on floor (~1 year beam on target)
International Collaboration: **23 institutions, 95 Collaborators**
(23 grad students,10 postdocs)

- Measured parity-violating e-p analyzing power with high precision at Q^2 ~ 0.025 $(GeV/c)^2$. Determine: $\mathbf{Q^p_W, Q^n_W, \Lambda/g_{e\text{-}p}, C_{1u}, C_{1d}, \sin^2\theta_W}$

Ancillary / Calibration Measurements: (Will be published as standalone results.)

- Parity-violating and conserving e-C and e-Al analyzing powers.
- Parity-allowed analyzing power with transverse-polarized beam on H and Al.
- Parity-violating and allowed analyzing powers on H in the $N \rightarrow \Delta(1232)$ region.
- PV asymmetries in pion photo-production.
- Transverse asymmetries in pion photo-production.
- Non-resonant inelastic measurement at 3.3 GeV to help constrain γ-Z Box uncertainty.
- Transverse asymmetry in the PV inelastic scattering region (3.3 GeV).

Published 10/2/2013: PRL **111,**141803 (2013)

Current Experiment Status

- The Qweak experiment finished successfully
  - Precise measurement of $\vec{e}$-p analyzing power at low Q^2
  - 2 years in situ, ~1 year of beam
  - Commissioning run (a.k.a. Run 0 results) published in: PRL 111,141803 (2013)

> - ~ 4% of total data collected
> - 1st "clean" determination of Q^p_W, C_{1u}, C_{1d}, & Q^n_W

- Remainder of experiment still being analyzed
  - Expect final results by late 2014

PVES and Hadronic Structure Effects

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{\pi\alpha\sqrt{2}}\right] \frac{\varepsilon G_E^{p\gamma} G_E^{pZ} + \tau G_M^{p\gamma} G_M^{pZ} - \frac{1}{2}(1 - 4\sin^2\theta_W)\varepsilon' G_M^{p\gamma} \tilde{G}_A^p}{\varepsilon (G_E^{p\gamma})^2 + \tau (G_M^{p\gamma})^2}$$

Neutral-weak form factors

Axial form factor

assume charge symmetry:

$$4G_{E,M}^{pZ} = (1 - 4\sin^2\theta_W) G_{E,M}^{p\gamma} - G_{E,M}^{n\gamma} - G_{E,M}^{s}$$

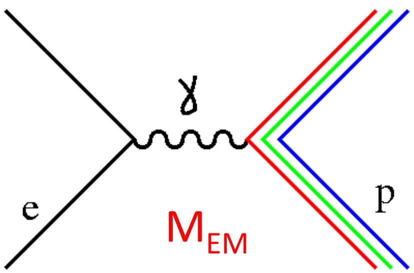
Proton weak charge (tree level)

Strangeness

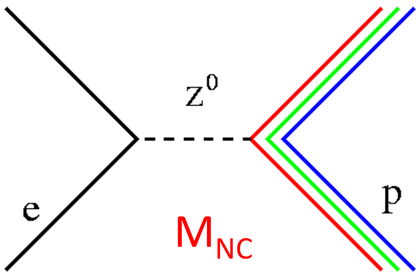
(Now measured to be relatively small!)

Note: Parity-violating asymmetry is sensitive to both weak charges *and* to hadron structure.

Q^p_{Weak} : Extract from Parity-Violating Electron Scattering



As $Q^2 \to 0$



measures Q^p – proton's electric charge

measures Q^p_{Weak} – proton's weak charge

$$A = \frac{2M_{NC}}{M_{EM}} = \left[\frac{-G_F}{4\pi\alpha\sqrt{2}}\right]\left[Q^2\, Q^p_{weak} + F^p(Q^2,\theta)\right]$$

$$\xrightarrow[\theta\to 0]{Q^2\to 0} \left[\frac{-G_F}{4\pi\alpha\sqrt{2}}\right]\left[Q^2\, Q^p_{weak} + Q^4 B(Q^2)\right]$$

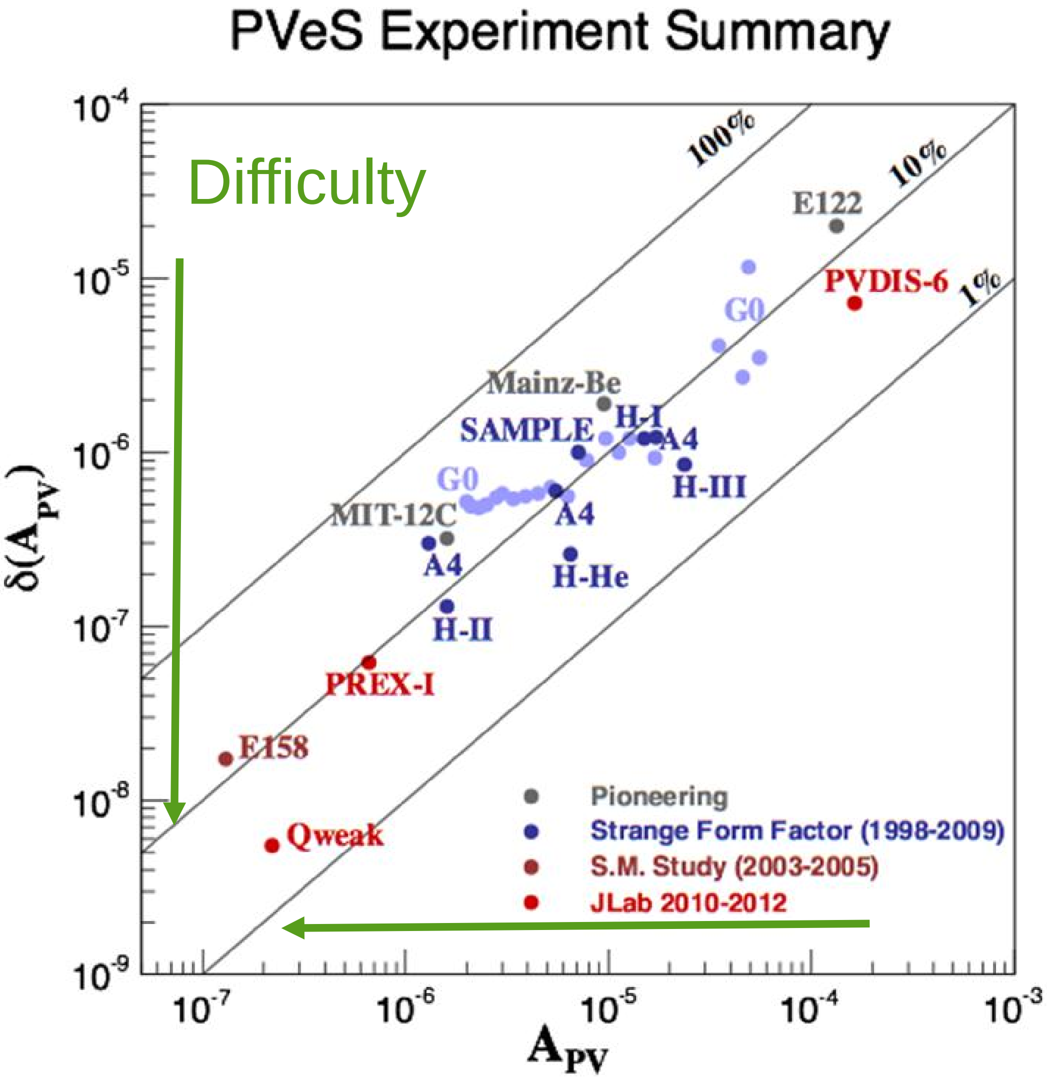
$$Q^p_{weak} = 1 - 4\sin^2\theta_W \sim 0.072$$ (at tree level)

contains $G^{\gamma}_{E,M}$ and $G^{Z}_{E,M}$

Correction involving hadronic form factors.
Exp determined using global analysis of recently completed PVES experiments.

The **lower** the momentum transfer, Q, the more the proton looks like a point and the less important are the form factor corrections.

PV Measurements Relative *"difficulty factor"*



Technical challenges:

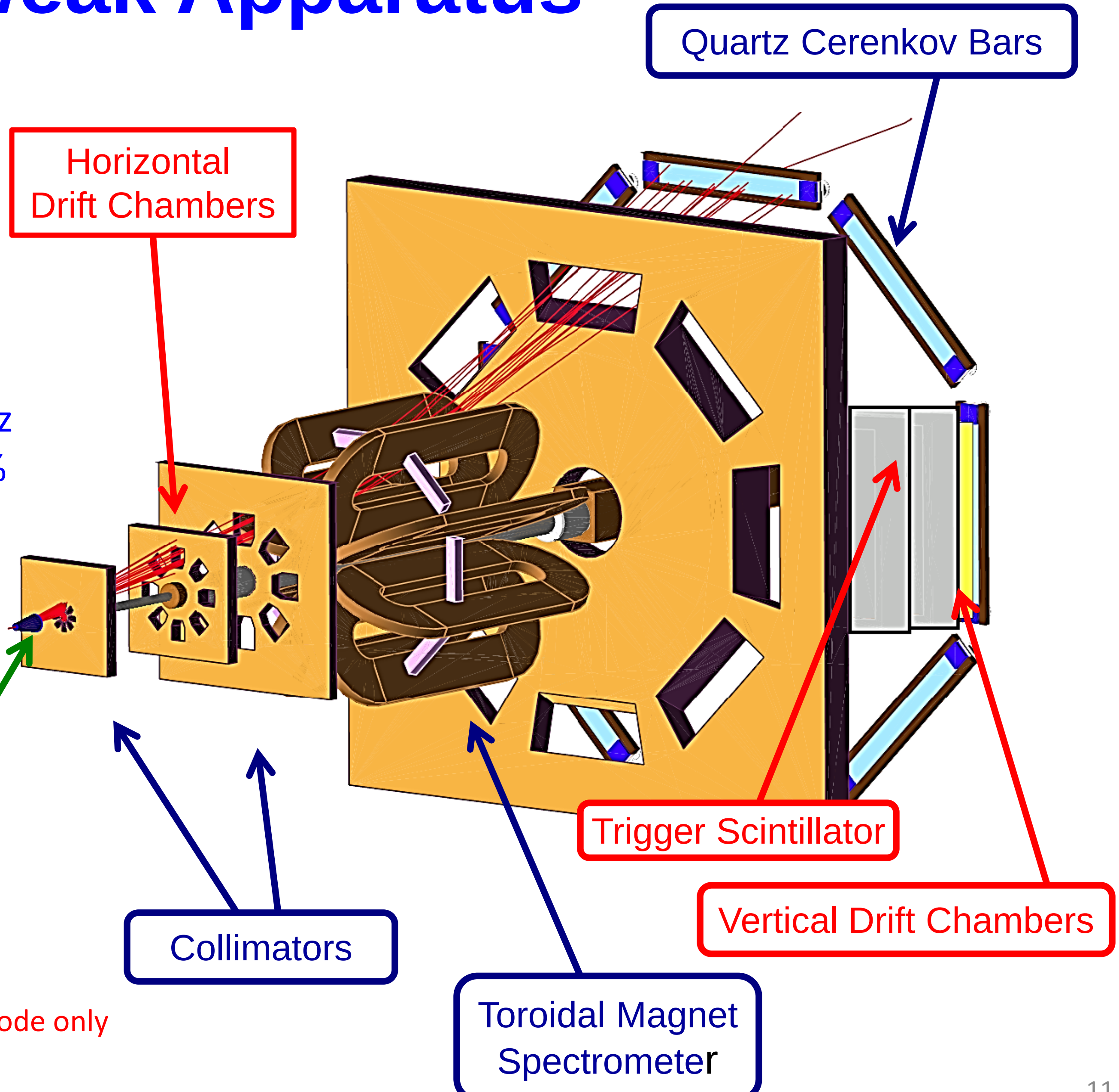
- Counting Statistics
  - High rate, beam polarization, beam current, high-power target, large acceptance detectors
- Low noise
  - Electronics, target density fluctuations, detector resolution
- Systematics
  - Helicity-correlated beam asymmetry, backgrounds, precision beam polarimetry, precise Q^2 determination

Q-weak goal: ~5 ppb on A_{ep}

Qweak Apparatus

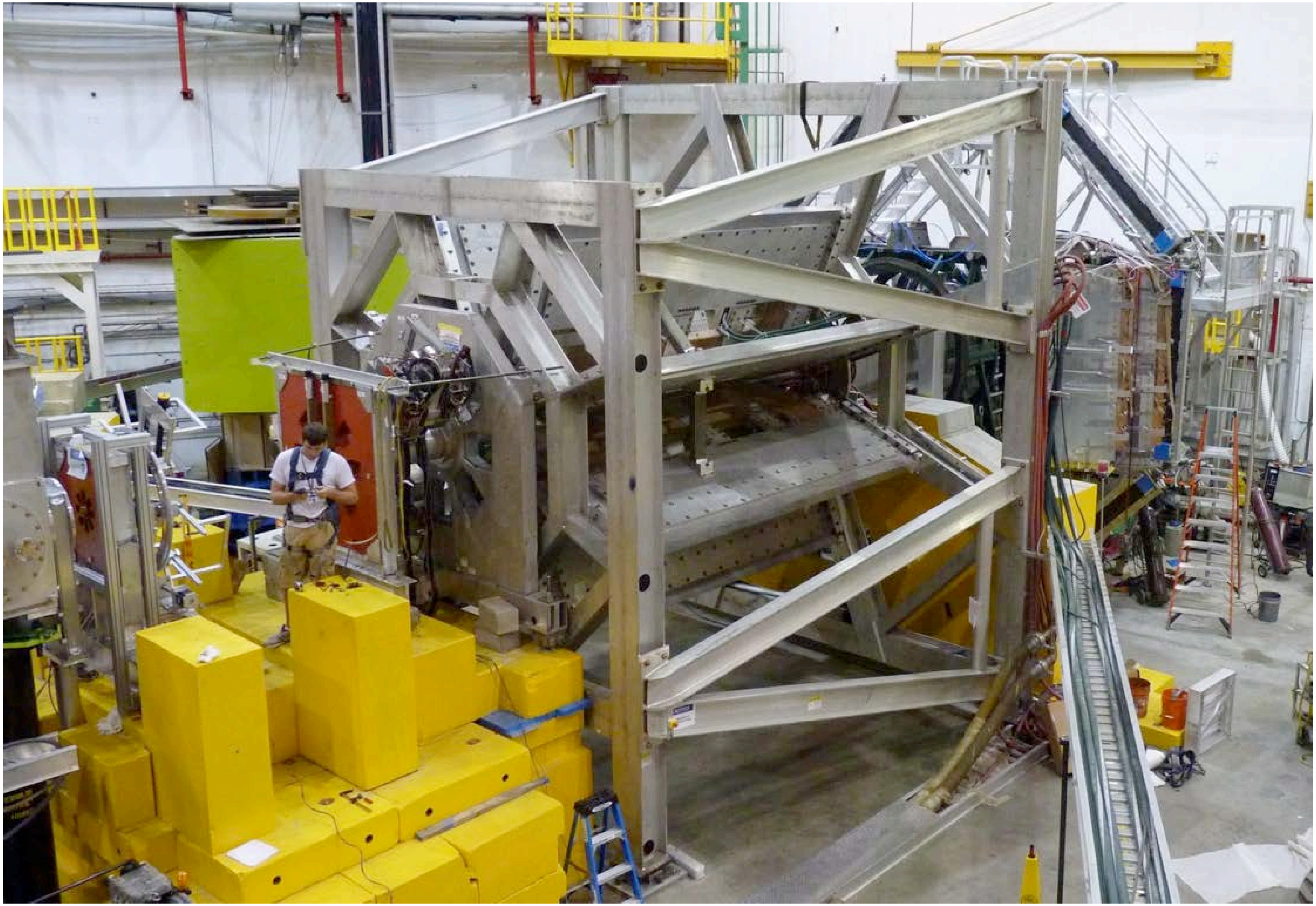
Parameters:

E_{beam}= 1.165 GeV
$<Q^2>$ = 0.025 GeV^2
$<\theta>$ = 7.9° $\pm$ 3°
ϕ coverage = 50% of 2π
I_{beam} = 180 μA
Integrated rate = 6.4 GHz
Beam Polarization = 88%
Target = 35 cm LH_2
Cryopower = 3 kW

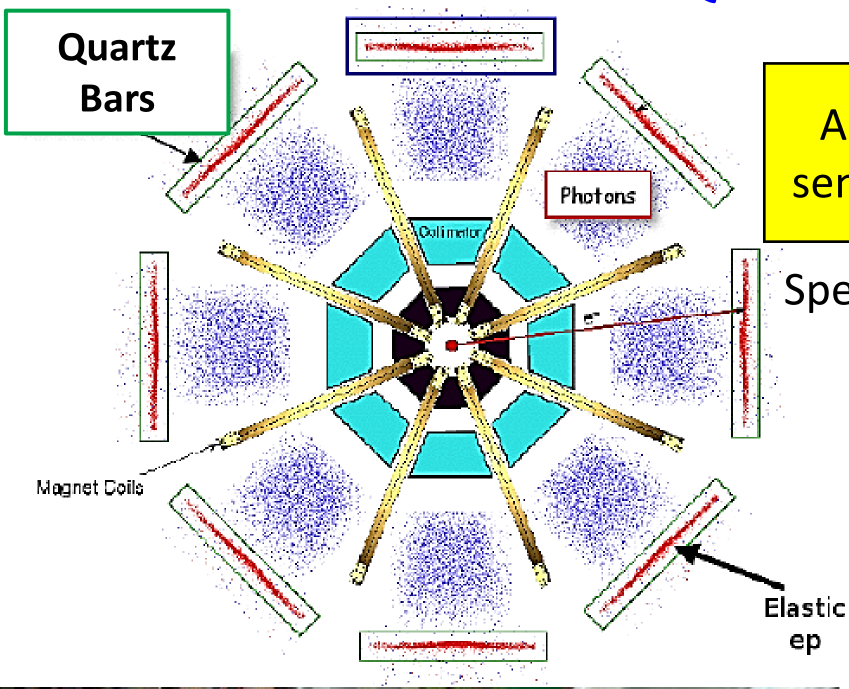


Red = low-current tracking mode only

The Apparatus (before shielding)



Quartz Cerenkov Detectors

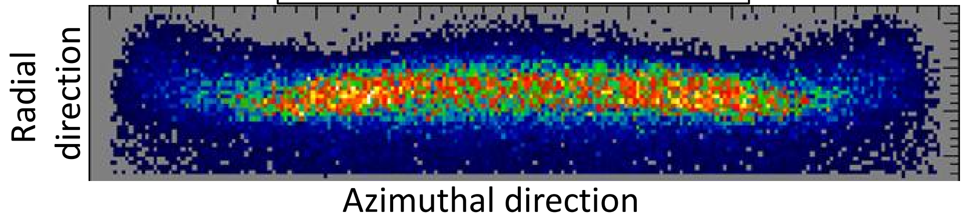


Azimuthal symmetry maximizes rate and decreases sensitivity to HC beam motion, transverse asymmetry.

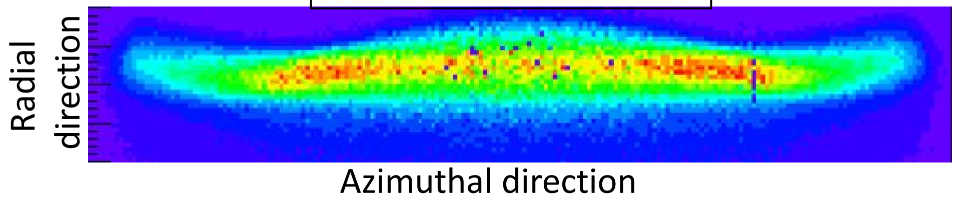
Spectrosil 2000: Eight bars, each 2 m long, 1.25 cm thick

- Rad-hard
- Non-scintillating, low-luminescence

Simulation of MD face:



Measured

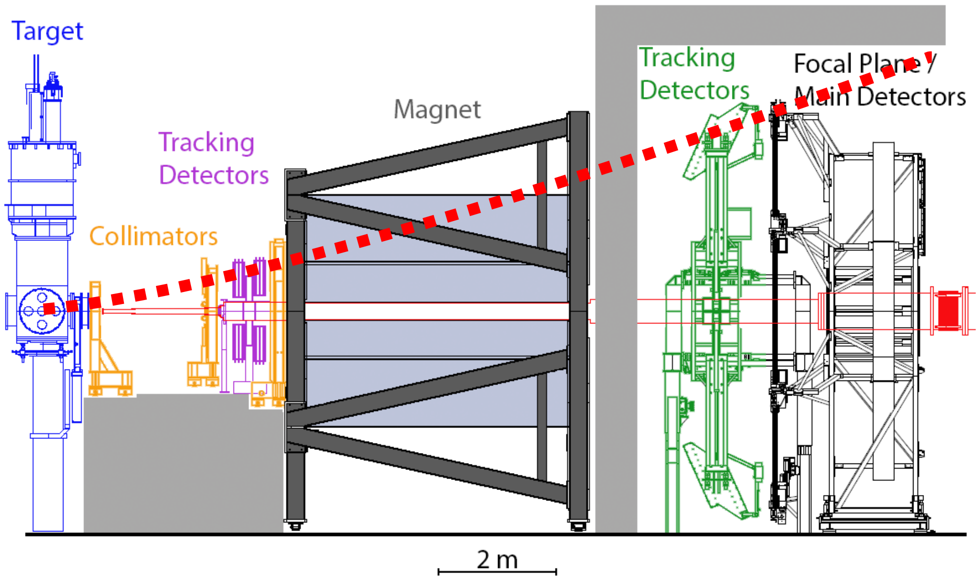


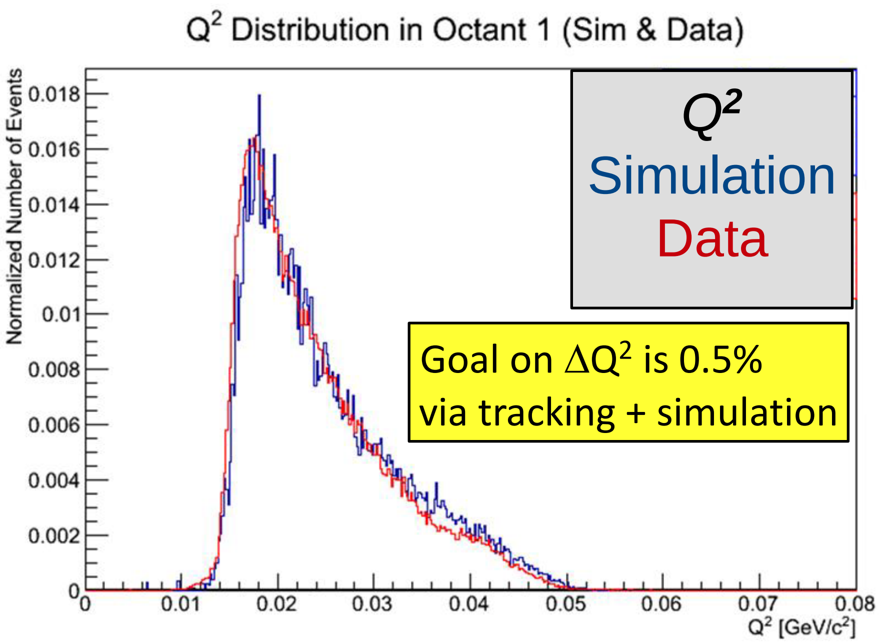
Yield 100 pe's/track with 2cm Pb pre-radiators
Resolution limited by shower fluctuations.

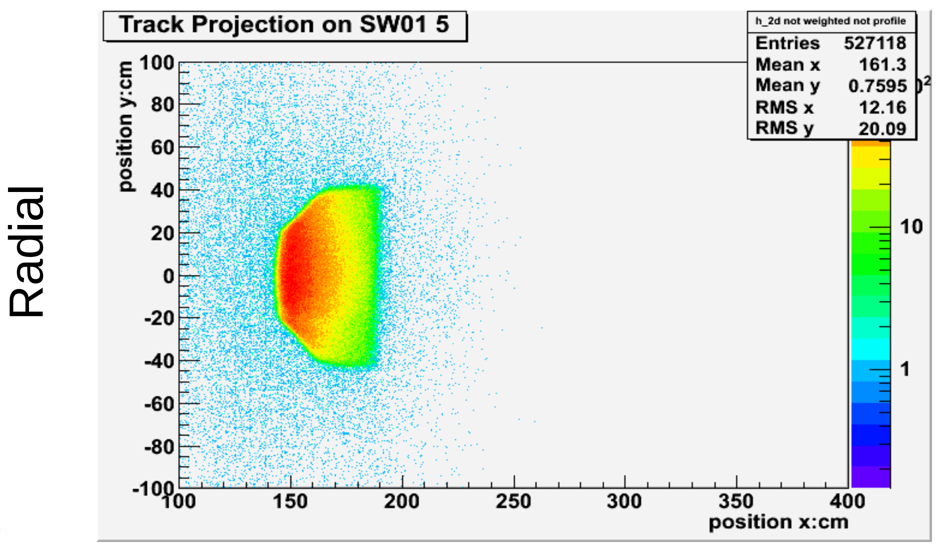
Kinematics Determination

$$A_{PV} = -\frac{Q^2 G_F}{4\sqrt{2}\pi\alpha}[Q_W^p + F(\theta, Q^2)]$$

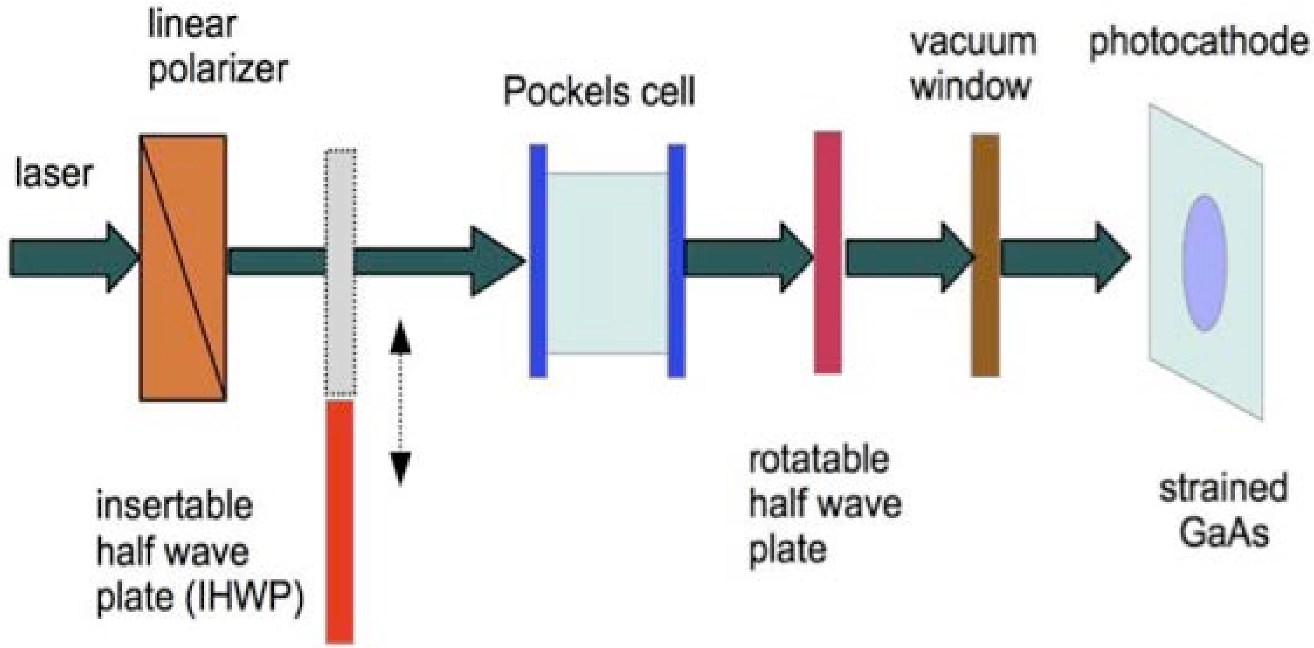
- Drift chambers before and after magnetic field
- Low current, reconstruct individual events
- Systematic studies





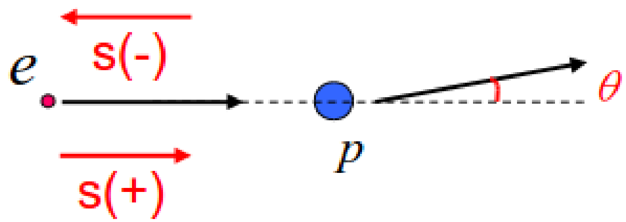


Polarized Injector

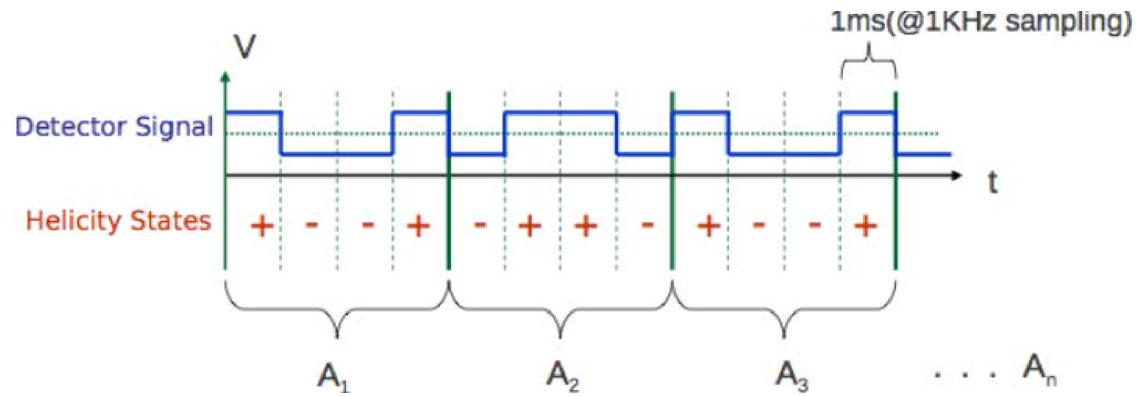


- Pockels cell for fast helicity reversal
- Helicity reversal frequency: 960 Hz (to "freeze" bubble motion in the target)
- Helicity pattern: pseudo-random "quartets" (+--+ or -++-, asymmetry calculated for each quartet)
- Insertable Half-Wave Plate: for "slow reversal" of helicity to check systematic effects and cancel certain false asymmetries. Less frequently, by Wien filter.

"Phase Locked" Detection Methodology



Helicity of electron beam flipped at 960 Hz, delayed helicity reporting to prevent direct electrial pick up of reversal signal by ADC's



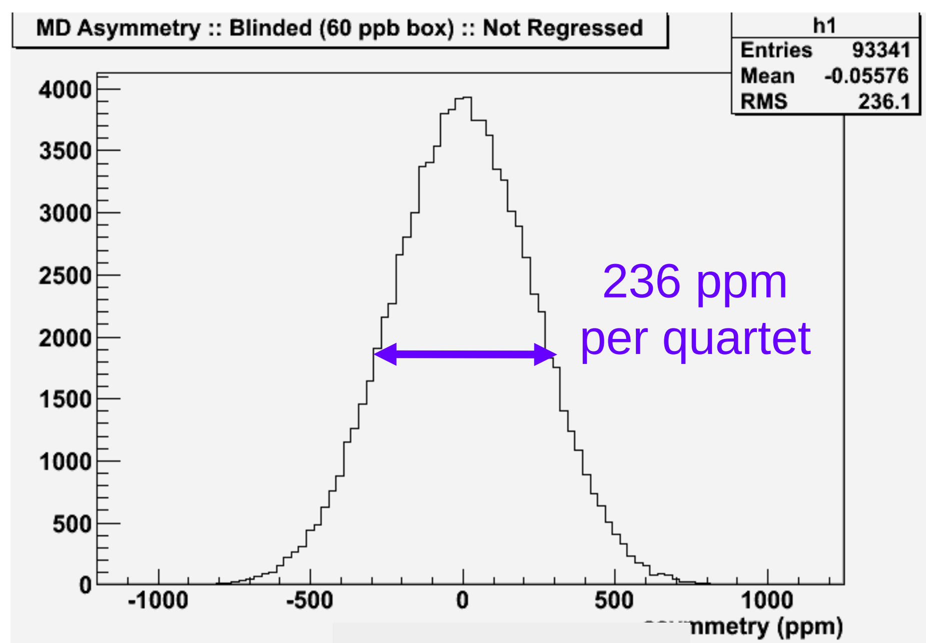
Detector signal integrated
For each helicity window

Asymmetry formed by quartet (4 ms)

Statistical power is

$$\Delta A = s_{width}/\sqrt{N_{quartets}}$$

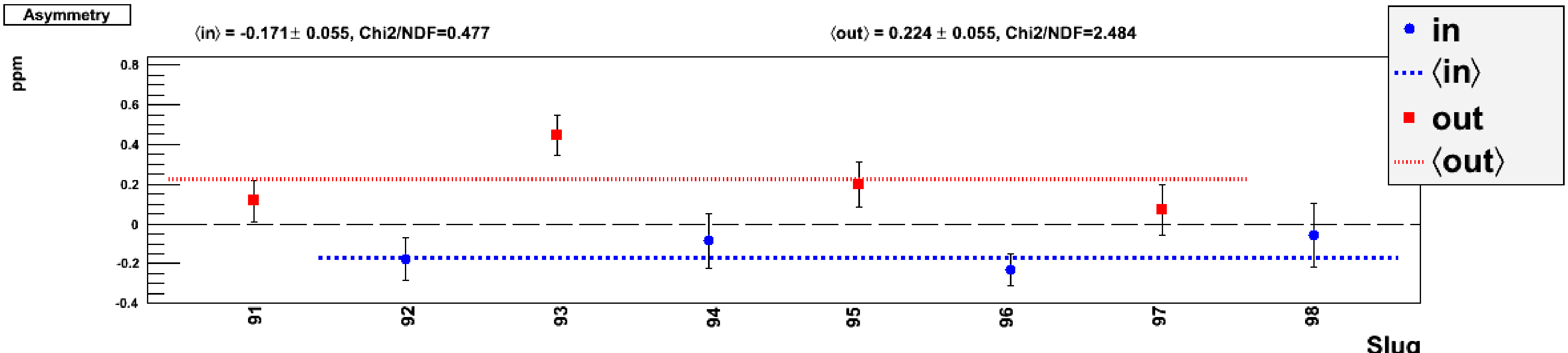
Measured asymmetry has unknown additive "blinding factor" for analysis (± 60 ppb blinding box)



LH_2 Asymmetry

Constructing the Asymmetry

- Asymmetry measured in blocks of runs “a.k.a. Slugs” of data – IHWP in/out
- Blinded Asymmetry: example Slug plot for “run 0” showing reversal



False Asymmetry:

$$A_{msr} = A_{raw} + A_T + A_L + A_{reg}$$

A_{raw} = $(Y^+-Y^-)/(Y^++Y^-)$
charge normalized yields
A_T = remnant transverse asymmetry
A_L = potential non-linearity in PMT
A_{reg} = helicity-correlated false asymmetry

Backgrounds:

$$A_{ep} = R_{tot} \frac{A_{msr}/P - \sum_{i=1}^{4} f_i A_i}{1 - f_{tot}}$$

R_{tot} = includes radiative correction and correction for light-variation
f = background fraction
A = background asymmetry
(backgrounds: Al windows, beamline, neutral backgrounds, N→Δ)

Helicity-Correlated Corrections

(Example: Commissioning Result)

$$A_{corr} = \sum_{i=1}^{5} \left(\frac{\partial A}{\partial x_i} \right) \Delta x_i$$

$$(x, x', y, y', E)$$

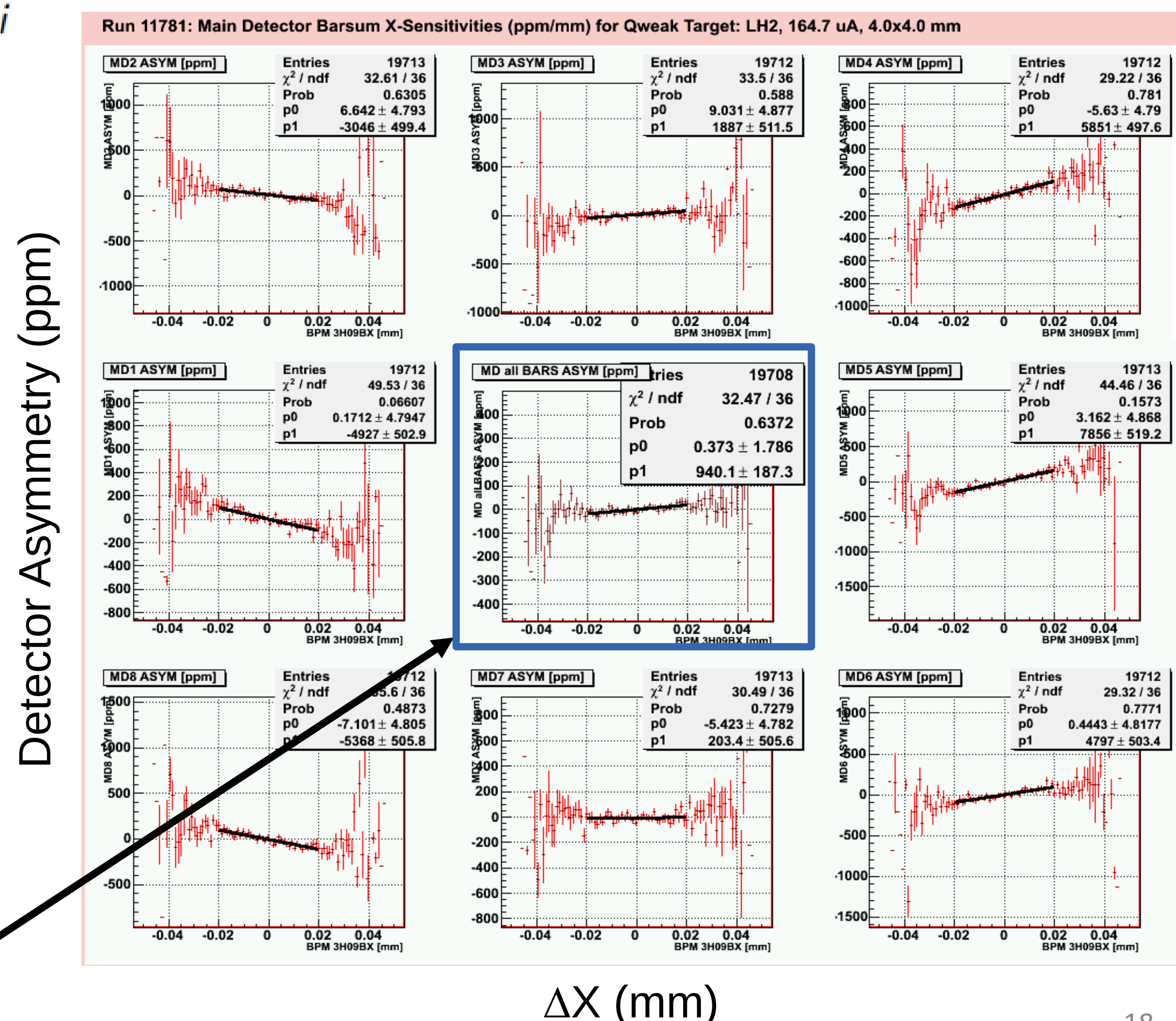
Regression Correction:
A_{reg} = -31 ± 11 ppb

Detector sensitivity to helicity-correlated beam parameters and broken symmetry can cause false asymmetry.

Sensitivity "slopes" determined from linear regression with natural beam jitter.

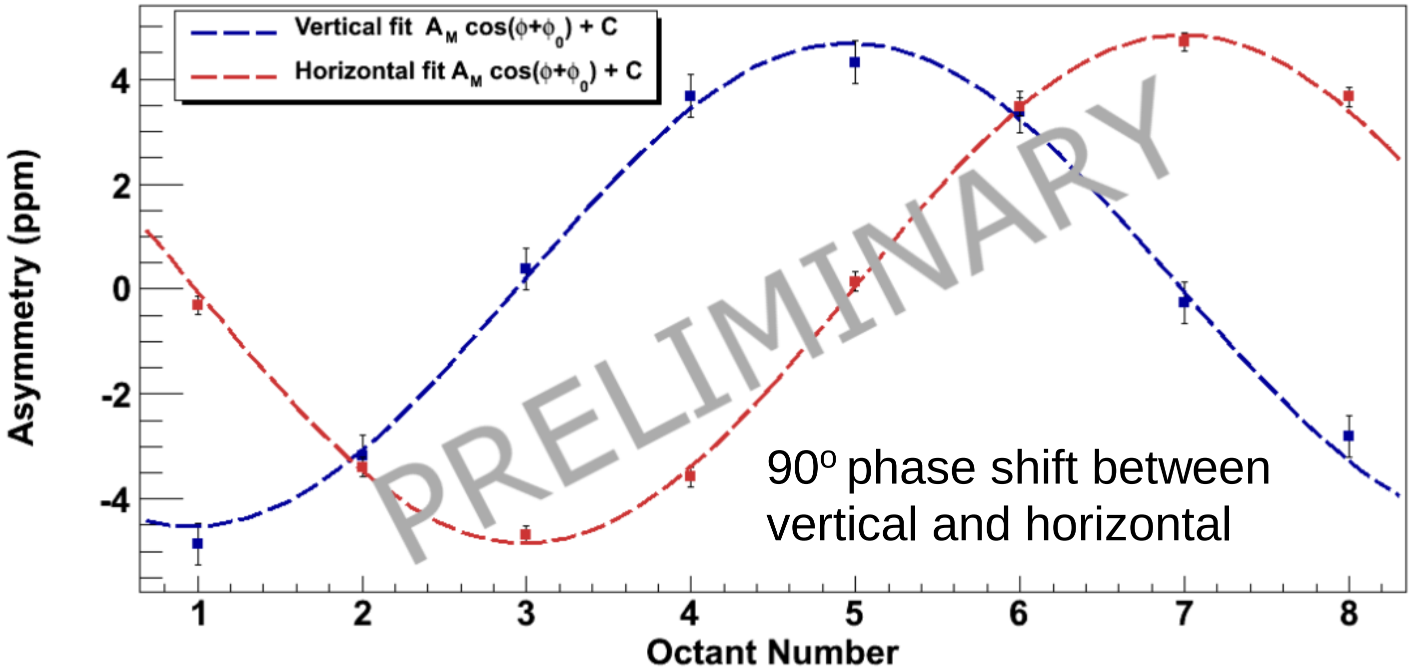
Order of magnitude suppression in sum

Example: Detector Sensitivity to X Beam Jitter



Residual Transverse Asymmetry

- Dedicated measurement with fully transverse beam
  - Constrains false asymmetry for A_{ep} result



- Good cancellation (symmetry factor)

- Small residual P_T when running

- Correction < 4 ppb

- Transverse result: nucleon structure and 2γ exchange
  - Comparison to theory models

Aluminum Window Background

Large *A* (asymmetry) & *f* (fraction) make this our largest correction. Determined from explicit measurements using Al dummy targets & empty H_2 cell.

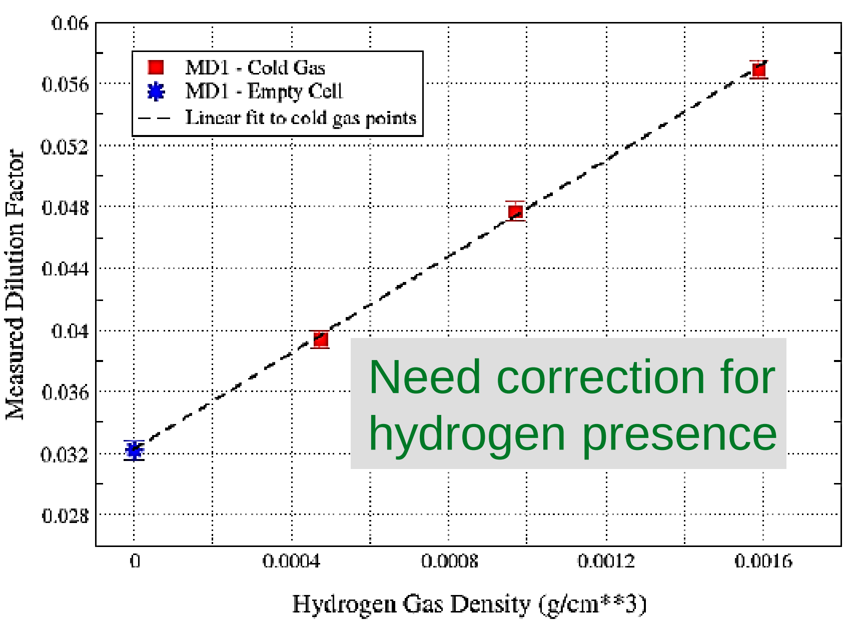
$$C_{\rm Al} = -64 \pm 10 \text{ ppb}$$
$$A_{\rm Al} = 1.76 \pm 0.26 \text{ ppm}$$

$$f_{\rm Al} = 3.23 \pm 0.24 \ \%$$

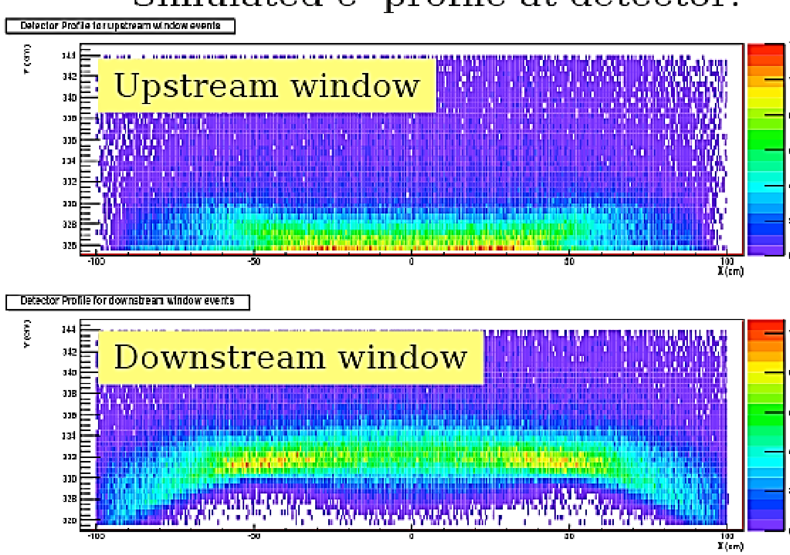
- Dilution from windows measured with empty target (actual target cell windows).
- Corrected for effect of H_2 using simulation and data driven models of elastic and quasi-elastic scattering.

- Asymmetry measured from thick Al targets
- Measured asymmetry agrees with expectations from scaling.

$$A_{PV}(^N_Z X) = -\frac{Q^2 G_F}{4\pi\alpha\sqrt{2}}\left[Q_W^p + \left(\frac{N}{Z}\right) Q_W^n\right]$$



Simulated e- profile at detector:

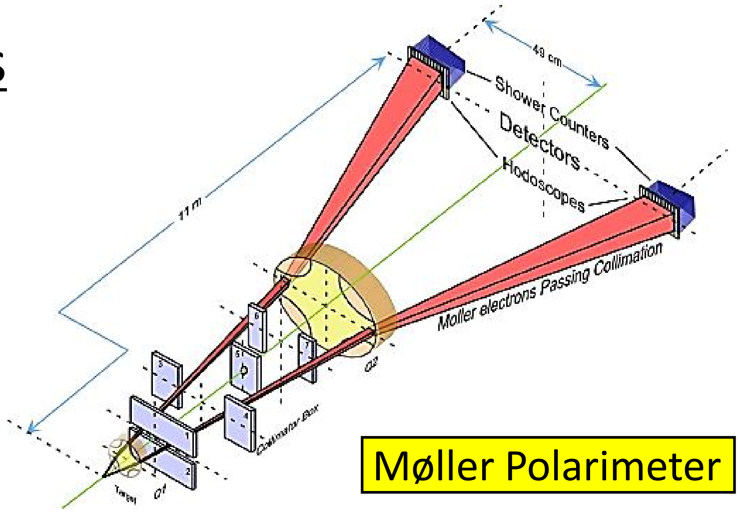


Precision Polarimetry

Qweak requires ΔP/P ≤ 1% (Expect final uncertainty ~0.8%)

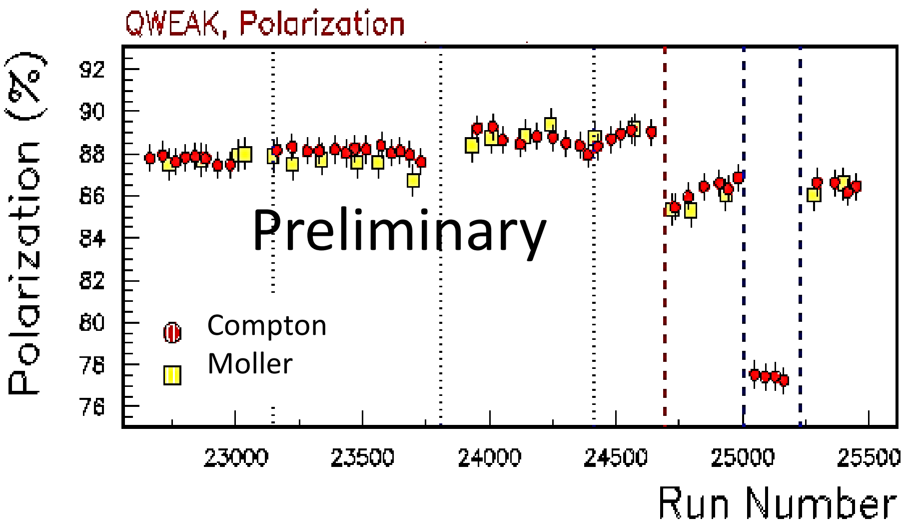
Strategy: use 2 independent polarimeters

- Use existing <1% Hall C Møller polarimeter:
  - Low beam currents, invasive
  - Known analyzing power provided by polarized "saturated" Fe foil in a 3.5 T field.

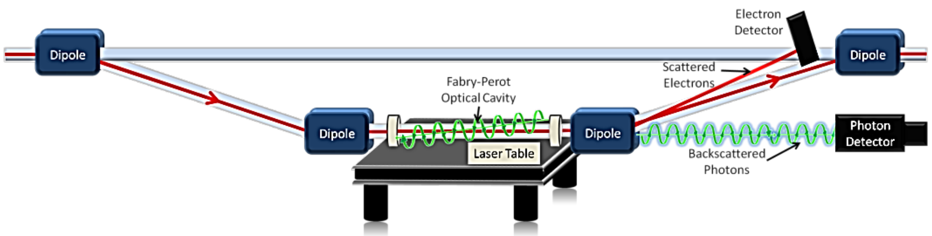


Møller Polarimeter

- Compton (photon & electron) polarimeter (1%/h)
  - Continuous, non-invasive
  - Known analyzing power provided by circularly-polarized laser

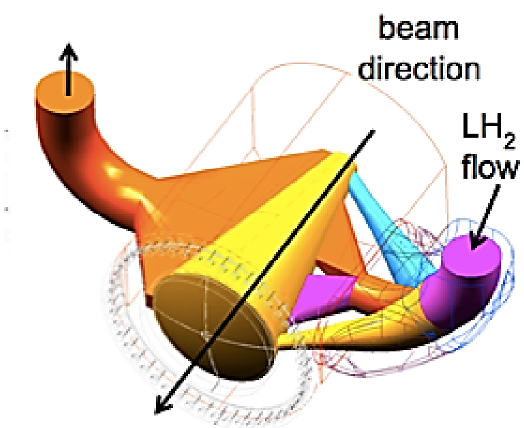


Compton Polarimeter



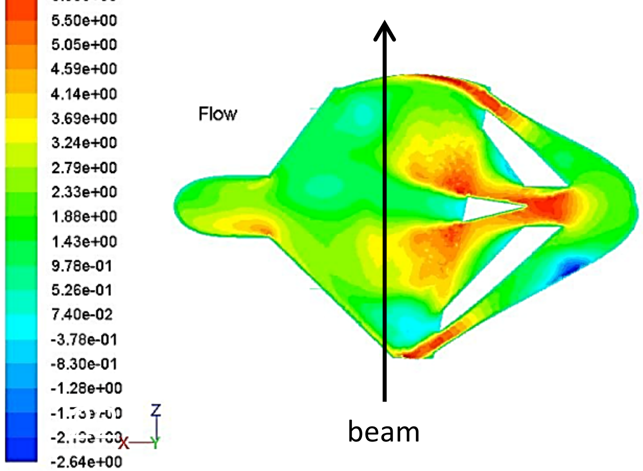
LH_2 Target Design

- World's highest power cryogenic target ~3 kW
- Designed with computational fluid dynamics (CFD) to reduce density fluctuations



I_{Beam} = 180 uA
L = 35 cm (4% X_0)
P_{beam} = 2.2 kW
A_{spot} = 4x4 mm^2
V = 57 liters
T = 20.00 K
P ~ 220 kPa

Fluid velocity



ANSYS

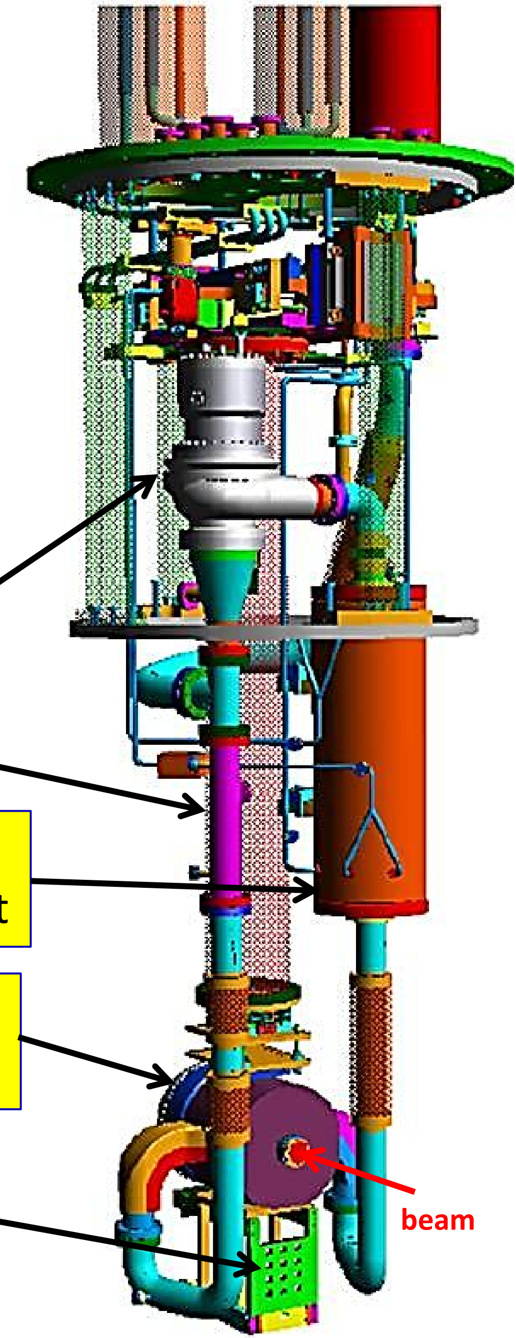
Centrifugal pump (15 l/s, 7.6 kPa)

3 kW Heater

3 kW HX utilizing 4K & 14K He coolant

35 cm cell (beam interaction volume)

Solid Tgts

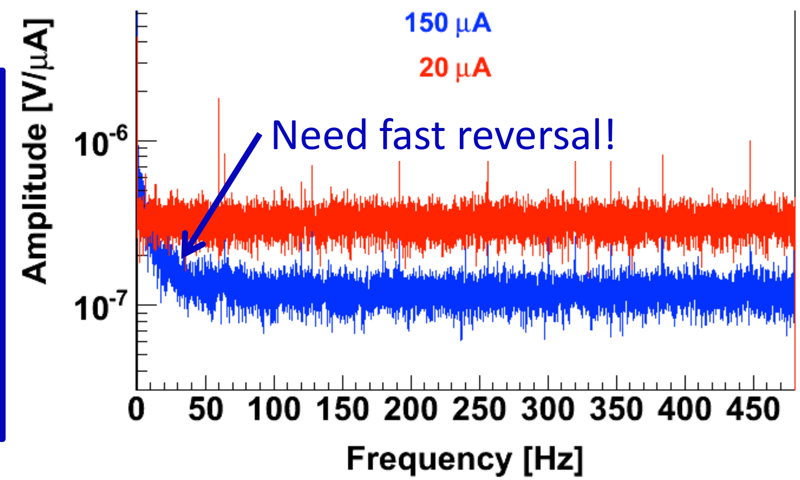


Target Performance

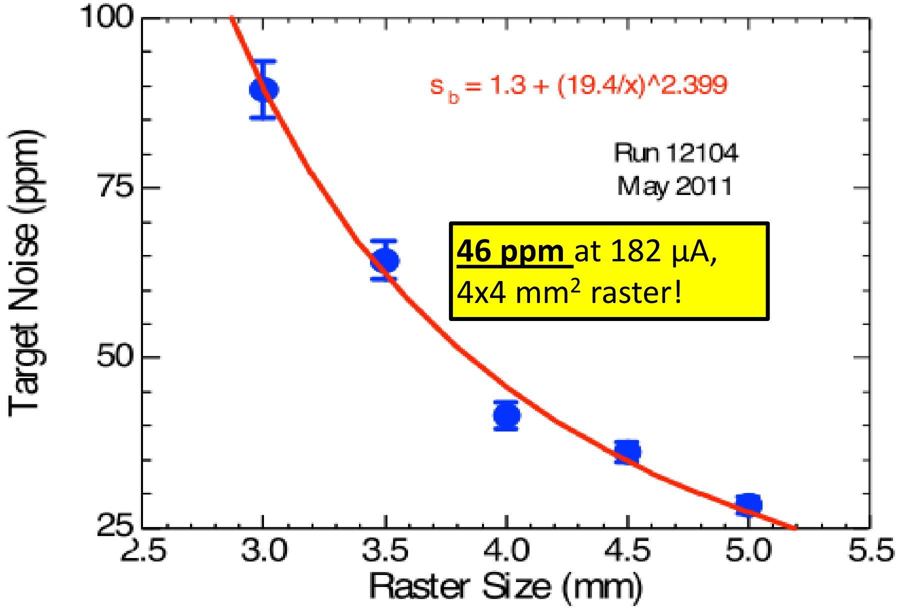
Measured helicity correlated noise.

At **960 Hz reversal rate**, the target noise (< 50 ppm) is very small compared to our helicity quartet (±∓∓±) asymmetry width (~230 ppm).
(statistical power ~$\Delta A_{quartet}$ / □$N_{quartets}$)

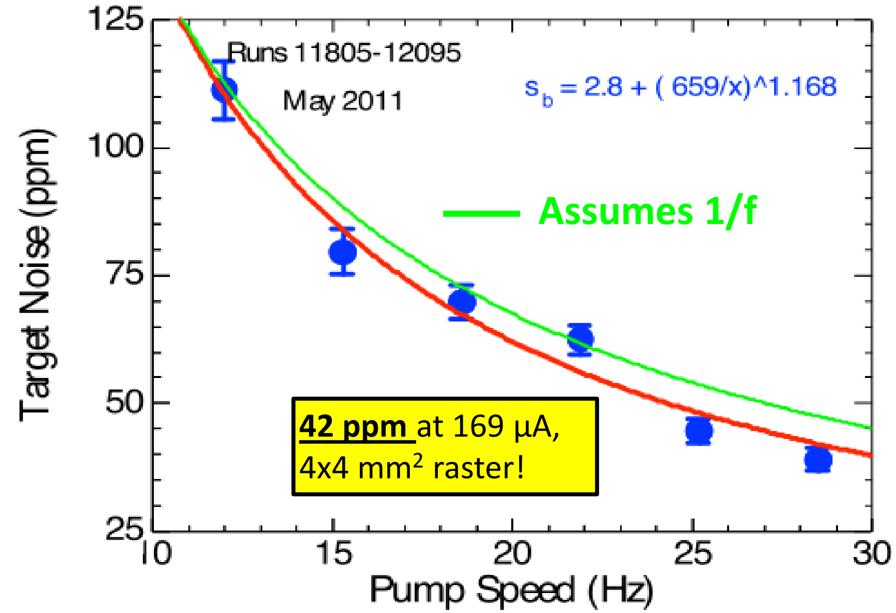
FFT of noise spectrum



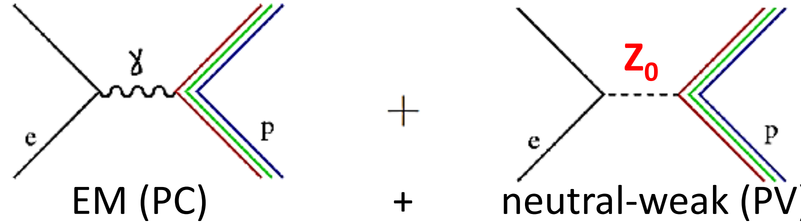
Raster Scan @ 182 µA



Pump Scan @ 169 µA, 4x4 mm Raster



Determining Q^p_{Weak}



- $A_{ep} = \left[\frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}\right] \sim \frac{|M^{PV}_{weak}|}{|M_{EM}|}$ where $\sigma^{\pm}$ is $\vec{e}p$ x-sec for e's of helicity ±1

- $$A_{ep} = \left[\frac{G_F\, Q^2}{4\pi\alpha\sqrt{2}}\right] \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1-4\sin^2\theta_W)\epsilon' G_M^\gamma G_A^Z}{\varepsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

  – where $\varepsilon = [1 + 2(1+\tau)\tan^2(\theta/2)]^{-1}$, $\varepsilon' = \sqrt{\tau(1+\tau)(1-\varepsilon^2)}$,
  $\tau = Q^2/4M^2$, $G^\gamma_{E,M}$ are EM FFs, $G^Z_{E,M}$ & G^Z_A are strange & axial FFs,
  and $\sin^2\theta_W = 1 - (M_W / M_Z)^2$ = weak mixing angle

- Recast $A_{ep} = \frac{G_F\, Q^2}{4\pi\alpha\sqrt{2}}\left[Q^p_w + Q^2 B(Q^2,\theta)\right]$

  – So in a plot of $A_{ep}/\left[\frac{G_F\, Q^2}{4\pi\alpha\sqrt{2}}\right]$ vs Q^2:
  - **Q^p_w is the *intercept*** (anchored by precise data near Q^2=0) ← **This Experiment**
  - $B(Q^2,\theta)$ is the ***slope*** (determined from higher Q^2 PVES data)

Global PVES Fit Details

(Example: Commisioning Result)

- Effectively 5 free parameters:
  - C_{1u}, C_{1d}, ρ_s, μ_s, & isovector axial FF G_A^Z
  - $G_E^S = \rho_s Q^2 G_D$, $G_M^S = \mu_s G_D$, & G_A^Z use G_D where
    - $G_D = (1 + Q^2/\lambda^2)^{-2}$ with $\lambda = 1$ GeV/c
- Employs all PVES data up to $Q^2 = 0.63$ $(GeV/c)^2$
  - On p, d, & ^4He targets, forward and back-angle data
    - SAMPLE, HAPPEX, G^0, PVA4
- Uses constraints on isoscalar axial FF G_A^Z
  - Zhu, et al., PRD 62, 033008 (2000)
- All ep data corrected for E & Q^2 dependence of □$_{\gamma Z}$ RC
  - Hall et al., arXiv:1304.7877 (2013) & Gorchtein et al., PRC84, 015502 (2011)
- Effects of varying Q^2, θ, & λ studied, found to be small

Mechanics of our γZ Correction

- Get E-dependent correction:
  - From fit to Hall et al. (arxiv.org/pdf/1304.7877v1.pdf) Fig. 14
- Get t dependent correction:
  - Using $\square_{\gamma Z}(E,t) / \square_{\gamma Z}(E,0) = e^{-B|t|/2}/F^{\gamma p}_{1}(t)$, with $B=7\pm1$ GeV^{-2} from Gorchtein et al., PRC84, 015502 (2011), Eq. 60.
- Example (our point at 1.165 GeV, 0.025 (GeV/c)^2):
  - E-dep part = 0.00567 ± 0.0003
  - Q^2-dep factor = 0.9776 ± 0.0120
  - Combined correction = (E-dep part)*(Q^2-dep factor) = 0.00555 ± 0.0003
  - Subtract this (*A_0) from the asymmetry (-278.8 ppb → -266.3 ppb)
    - reduced asymmetry 0.1240 → 0.1185
    - add error in quadrature to the systematic error
- Apply this to all proton data $A_{ep}(E,t)$ used in our fit
  - not d, or He (yet)

Electroweak Corrections

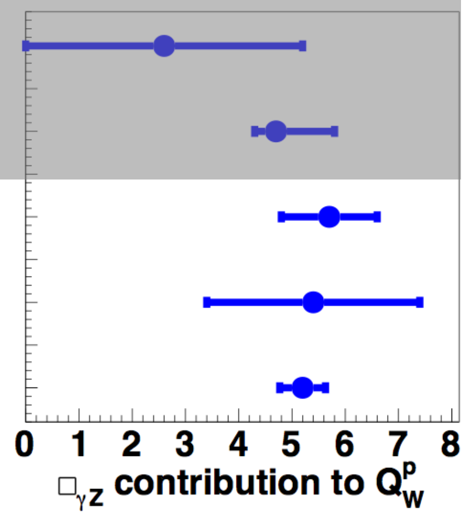
$$Q_W^p = [\rho_{NC} + \Delta_e][1 - 4\sin^2\hat{\theta}_W(0) + \Delta'_e] + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

~7% correction

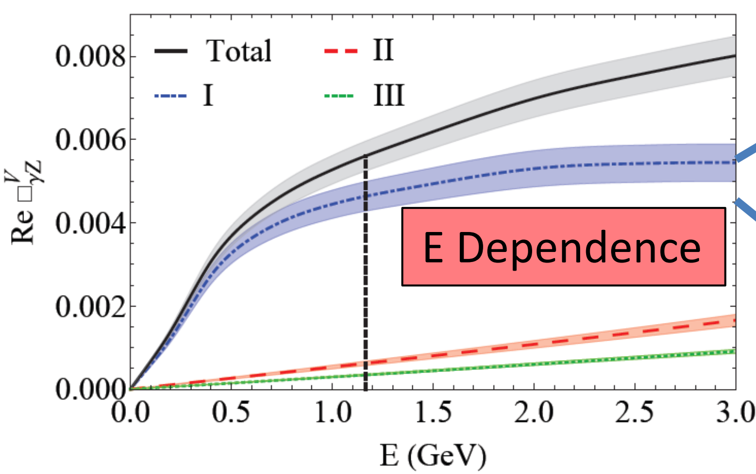
$\Box_{\gamma Z}$ contribution to Q_W^p (Qweak kinematics)

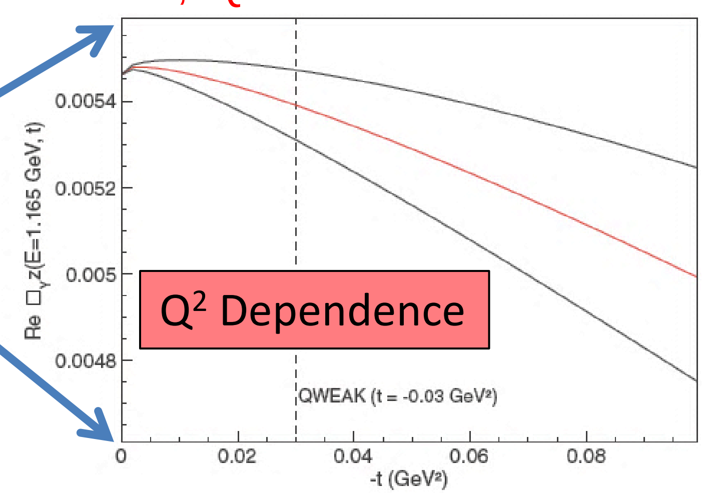
OBSOLUTE CALCULATIONS

Gorchtein & Horowitz *PRL 102, 091806 (2009)*	0.0026 ± 0.0026
Sibirtsev, Blunden & Melnitchouk, Thomas *PRD 82, 013011 (2010)*	$0.0047^{+0.0011}_{-0.0004}$
Rislow & Carlson *PRD 83, 13007 (2011)*	0.0057 ± 0.0009
Gorchtein, Horowitz & Ramsey-Muslof *PRC 84, 015502 (2011)*	0.0054 ± 0.0020
Hall, Blunden, Melnitchouk, Thomas & Young *arXiv:1304:7877 (2013)* (calculation constrained by PVDIS data)	0.0052 ± 0.00043



- Calculations are primarily dispersion theory type
  - error estimates can be firmed up with data!
- Qweak: inelastic asymmetry data taken at W ~ 2.3 GeV, $Q^2 = 0.09$ GeV^2





Electroweak Corrections

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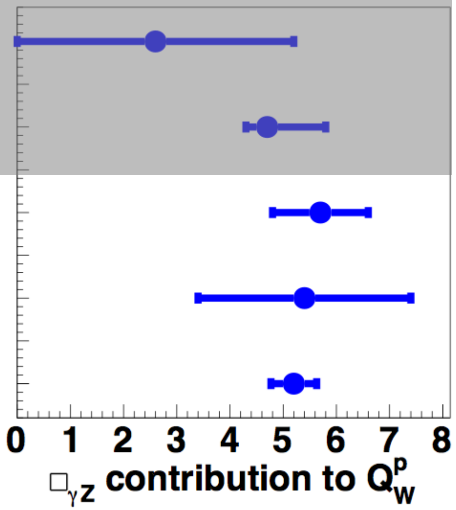


- **The central values of all 3 calculations are essentially in agreement!**
- **However, the errors are significantly different – or so it would seem at first glance.**

– BUT –

- **Whereas errors from Rislow and Hall (~0.5% uncertainty on Q^p_W) are for normal (gaussian) distributions,**
- **Gorchtein shows the extreme limits between two distinct models – (therefore a uniform distribution) - which needs to be converted into an effective "sigma" before folding into our uncertainty (→ becomes a ~1.4% uncertainty on Q^p_W)**

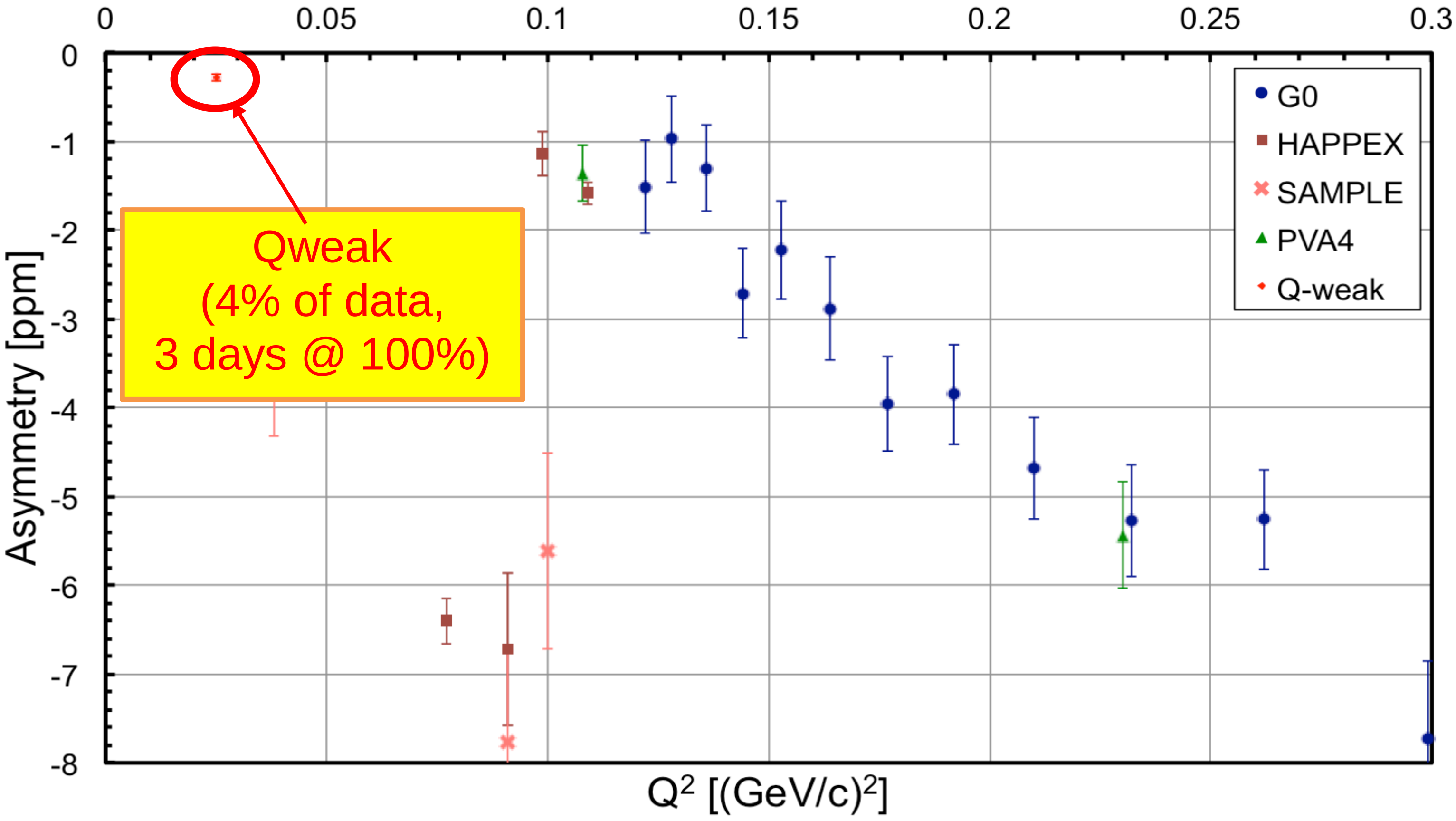
Published 10/2/2013: PRL **111**,141803 (2013)

First Results: Asymmetry

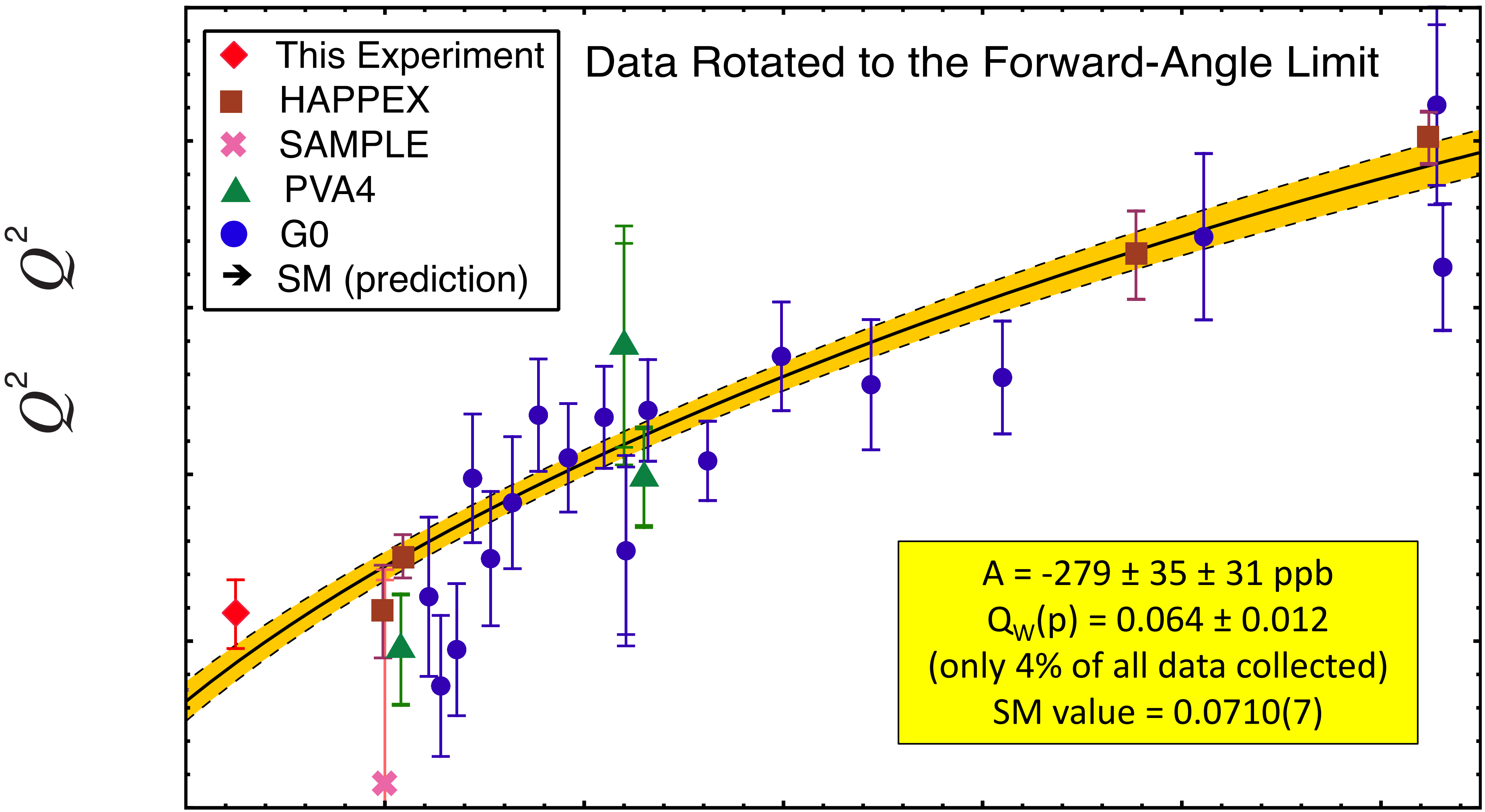
- Run 0 Results
  (1/25th of total data set)

Kinematics: $\langle Q^2 \rangle = 0.0250 \pm 0.0006\ \mathrm{GeV}^2$
$\langle E_{beam} \rangle = 1.155 \pm 0.003\ \mathrm{GeV}$

$$A_{ep} = -279 \pm 35\ (\mathrm{stat})\ \pm 31\ (\mathrm{syst})\ \mathrm{ppb}$$



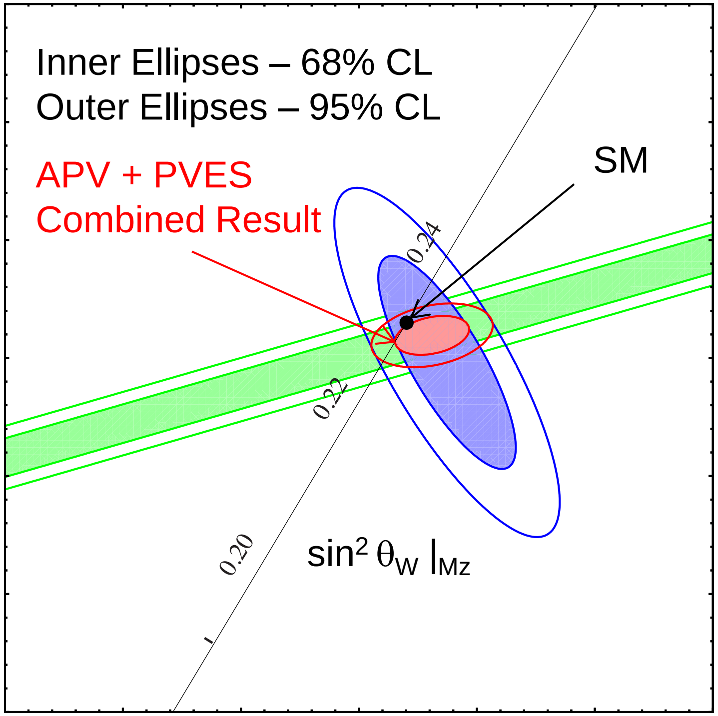
Global Fit of Q^2<0.63 $(GeV/c)^2$ PVES Data



Published 10/2/2013: PRL **111,**141803 (2013)

Combined Analysis

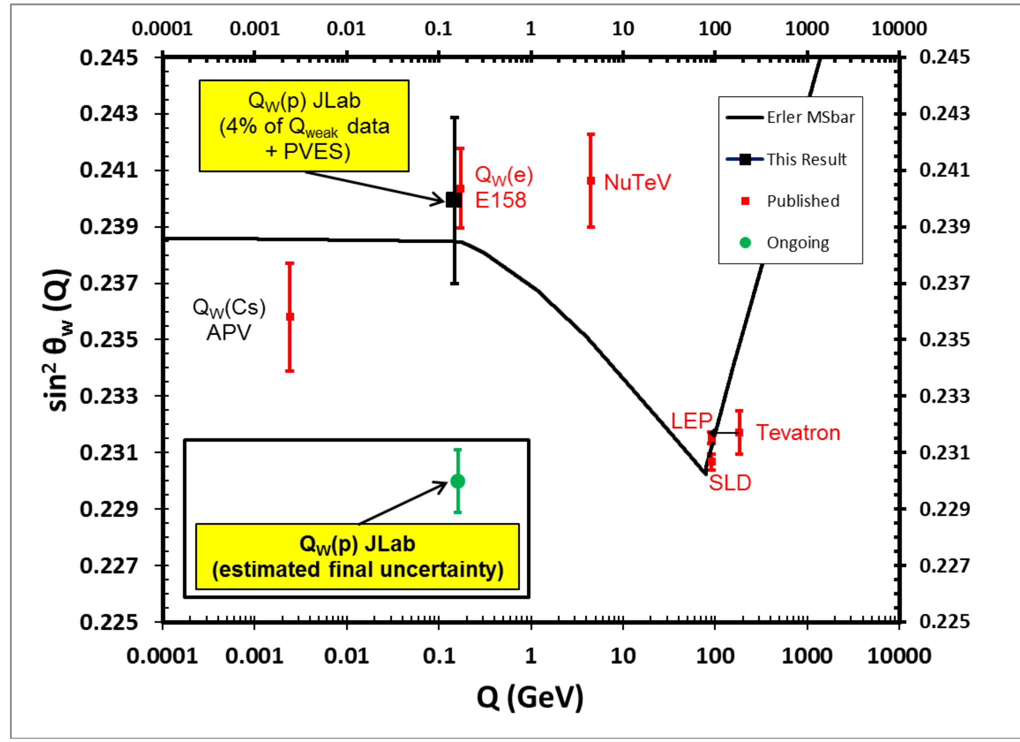
Extract: C_{1u}, C_{1d}, Q^n_W



Qweak + Higher Q² PVES

Extract: Q^p_W, $\sin^2\theta_W$

Weak Mixing Angle: Running of $\sin^2\theta_W$



$Q^n_W = -2\,(C_{1u} + 2\,C_{1d})$
$= -0.975 \pm 0.010$

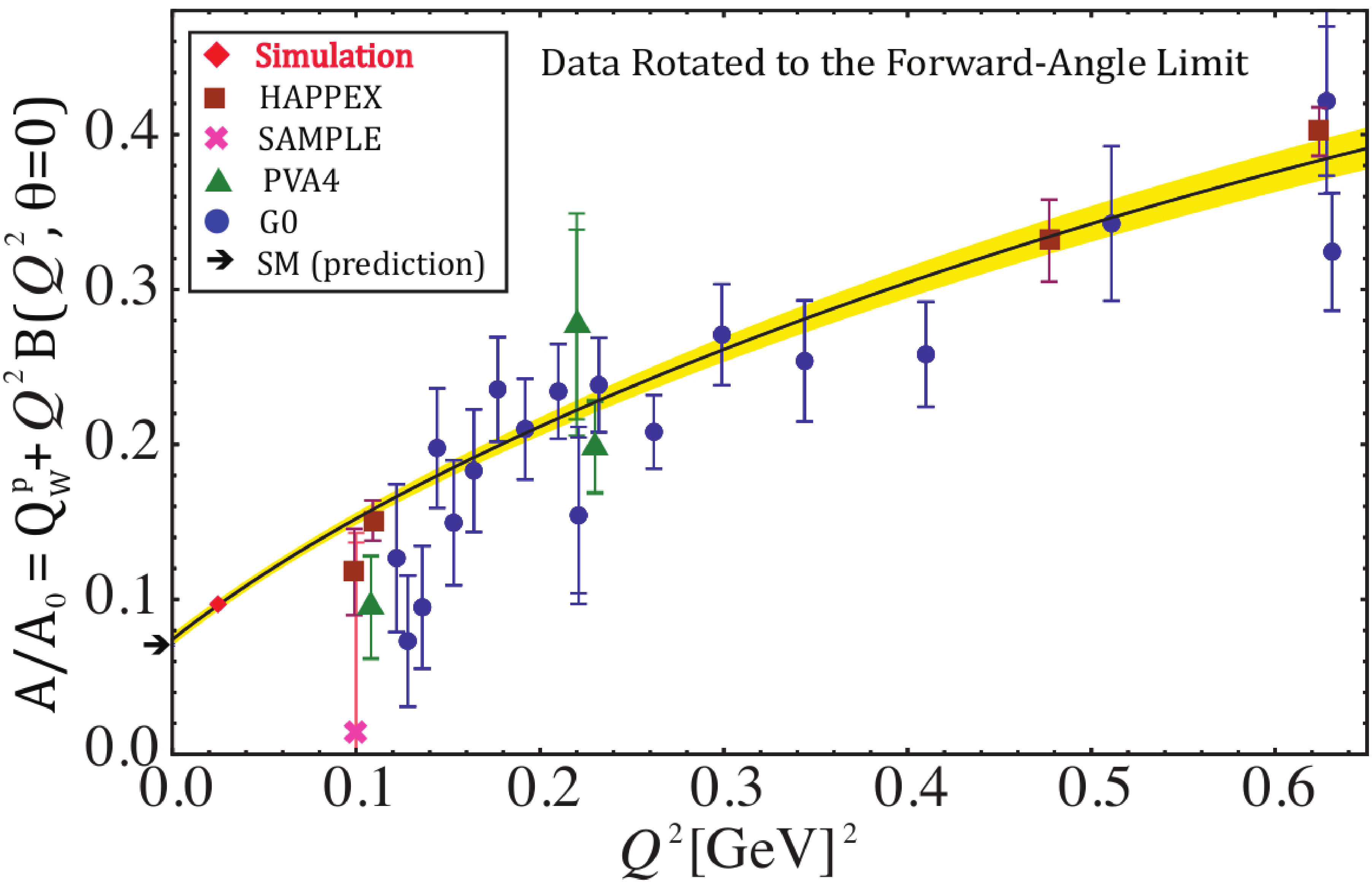
$C_{1u} = -0.184 \pm 0.005$
$C_{1d} = 0.336 \pm 0.005$

$Q^p_W = -2\,(2\,C_{1u} + C_{1d})$
$= 0.064 \pm 0.012$
SM prediction = 0.0710(7)

Remainder of experiment still being analyzed, final result before end of 2014.

Teaser: Simulated Fit !!

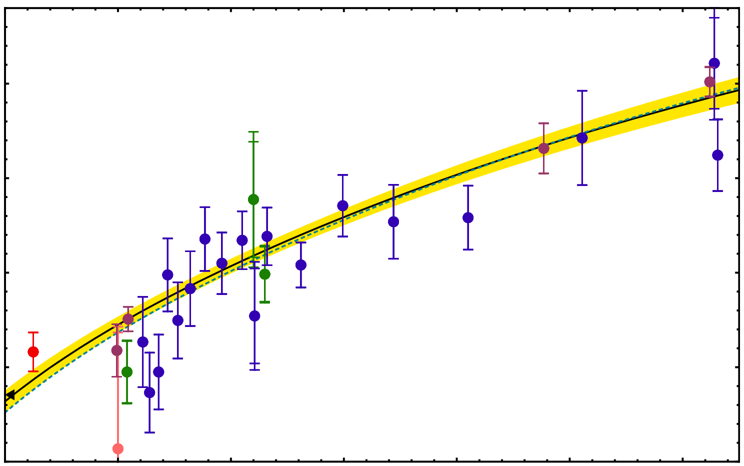
(Assuming anticipated final uncertainties and SM result)

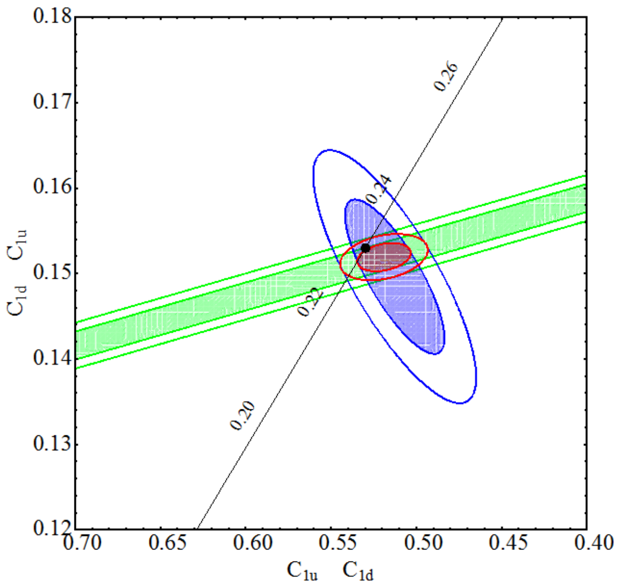


Effect of Applying γ-Z Correction to Higher Q^2 PVES Data – Commissioning Result

With γ-Z correction to all ep data

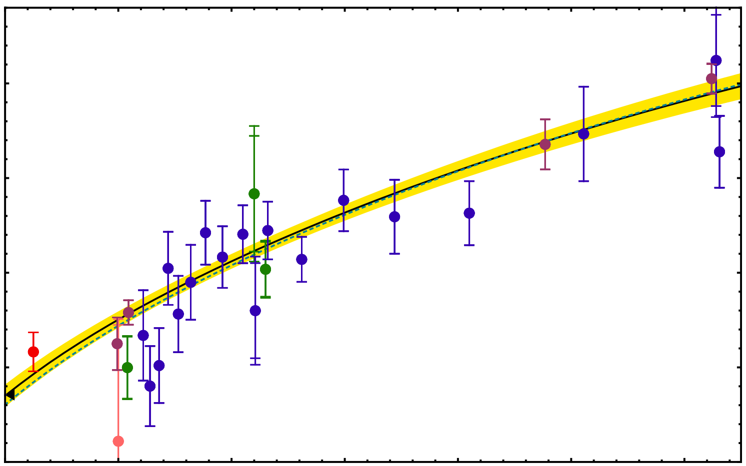
$Q^p_W = 0.064 \pm 0.012$

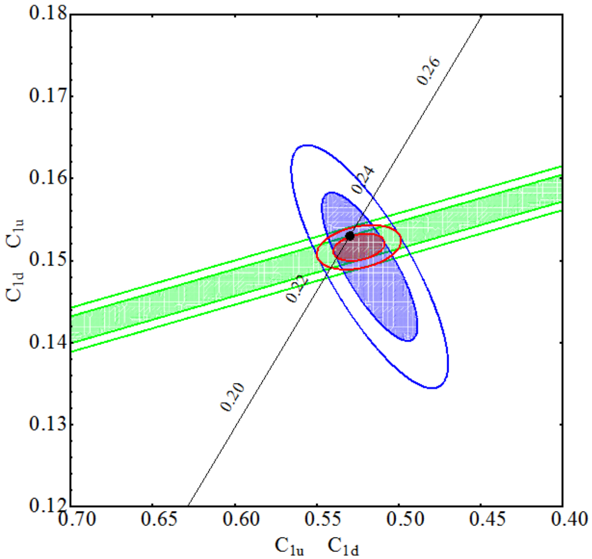




With γ-Z correction to only our Q^p_W point

$Q^p_W = 0.070 \pm 0.012$





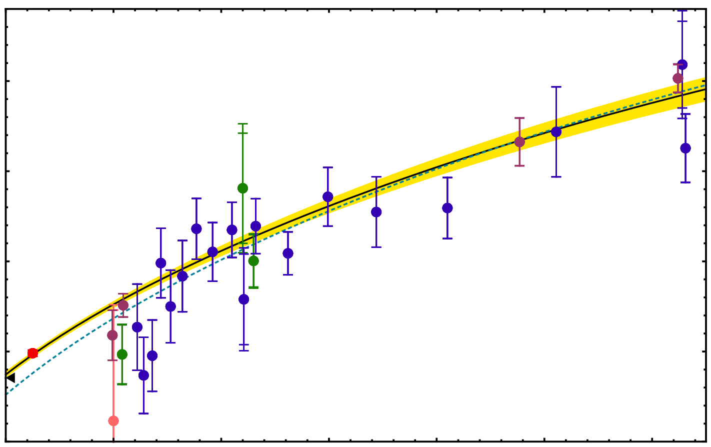
Effect of Applying γ-Z Correction to Higher Q^2 PVES Data – Simulated Full Data Set

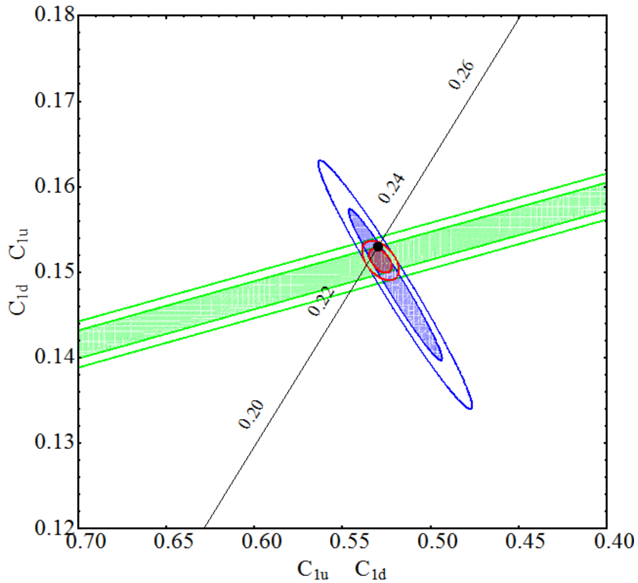
For discussion lets assume the case: $\Delta A_{ep}/A_{ep}$ = 3.4% → $\Delta Q^p_W / Q^p_W$ = 5% Measurement

γ-Z correction applied to our Pt & higher Q^2 ep data

Q^p_W (Global) = 0.0743 ± 0.0038

Q^p_W (PVES) = 0.0744 ± 0.0038

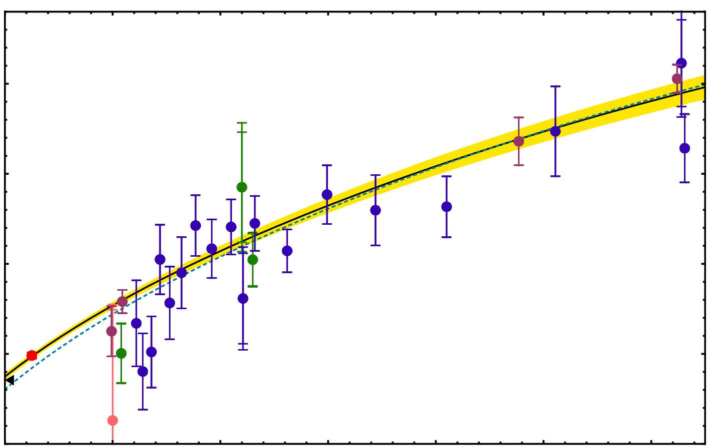


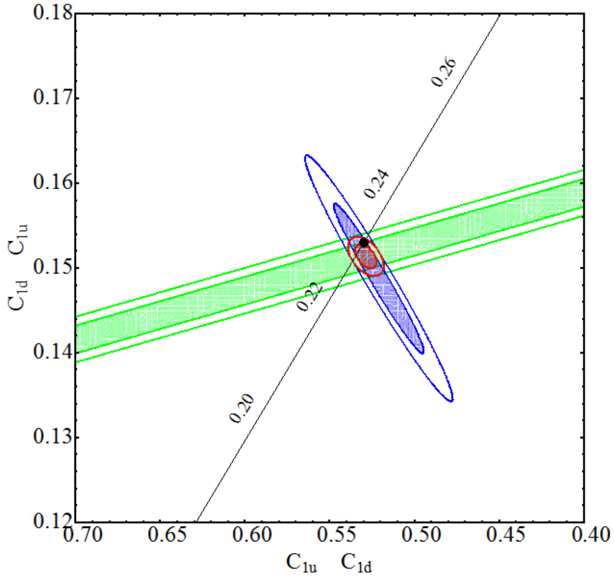


With γ-Z correction applied to only our Pt

Q^p_W (Global) = 0.0745 ± 0.0038

Q^p_W (PVES) = 0.0745 ± 0.0038





Planned & Possible Upgrades to Q^p_W Mathematica Code Extraction Procedure Prior to Un-blinding of Final Result

- Update PVES data set if any new Mainz results appear.
- Repeat studies of Chi^2 stability as function of Q^2 cut, theta cut, λ etc.
  (for final result may be better/sufficient to truncate at Q^2 of ~0.3 GeV - for example.
- Addition theoretical corrections if required for He / deuterium data for CSV or possibly use slightly larger errors. (depend on theorist interest level in the problem)
- Apply γ-Z correction to APV result – also need γ-Z correction for He / deuterium.
- Improve how ^{133}Cs APV result (and maybe other APV results) are used by code.
- Quantitative study of using different (from Kelly) or better EFF's.
- Include propagation of errors and correlations associated with EFF's into results.
  (very quick test run seems to indicate contribution a relatively small additional uncertainty). But, this needs more work!
- …..

Summary

- Measured during commissioning run:

  A_{ep} = -279 ± 35 (statistics) ± 31 (systematics) ppb
  - Smallest & most precise ep asymmetry ever measured
- First determination of Q^p_W:
  - Q^p_W = 0.063 ± 0.012 (from only 4% of all data collected)
    - SM value = 0.0710(7)
    - New PV physics reach $\Lambda/g > 1$ TeV (very consevative)
- First determination of $Q^n_W = -2(C_{1u} + 2C_{1d})$:
  - By combining our result with APV
    - Q^n_W = -0.975 ± 0.010 (SM value = -0.9890(7))
- Final results with much smaller uncertainties in 2014
  - Expected PV new physics reach of:

    $\Lambda/g \sim 2.6$ TeV (simplest and most conservative model).
  - SM test, sensitive to Z's and LQs

The Qweak Collaboration

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[13] Virginia Polytechnic Institute & State Univ
[14] Southern University at New Orleans
[15] Idaho State University
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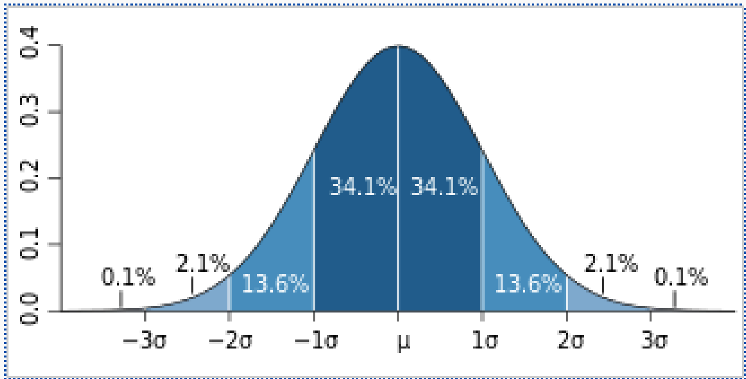
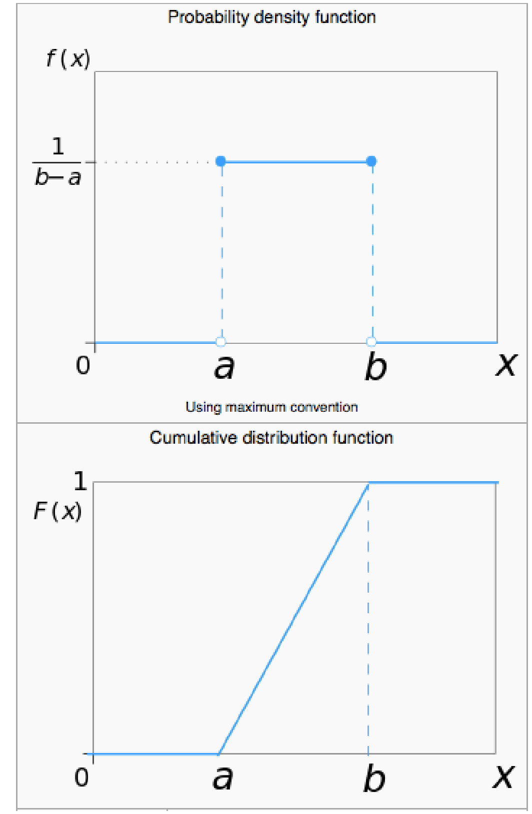
Spokespersons Project Manager Grad Students



A plot of a normal distribution (or bell-shaped curve) where each band has a width of 1 standard deviation – See also: 68–95–99.7 rule

Uniform



Notation	$\mathcal{U}(a,b)$ or $\mathrm{unif}(a,b)$
Parameters	$-\infty < a < b < \infty$
Support	$x \in [a, b]$
pdf	$\begin{cases} \frac{1}{b-a} & \text{for } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$
CDF	$\begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a,b) \\ 1 & \text{for } x \ge b \end{cases}$
Mean	$\frac{1}{2}(a+b)$
Median	$\frac{1}{2}(a+b)$
Mode	any value in $[a, b]$
Variance	$\frac{1}{12}(b-a)^2$
Skewness	0
Ex. kurtosis	$-\frac{6}{5}$
Entropy	$\ln(b-a)$
MGF	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
CF	$\frac{e^{itb}-e^{ita}}{it(b-a)}$