



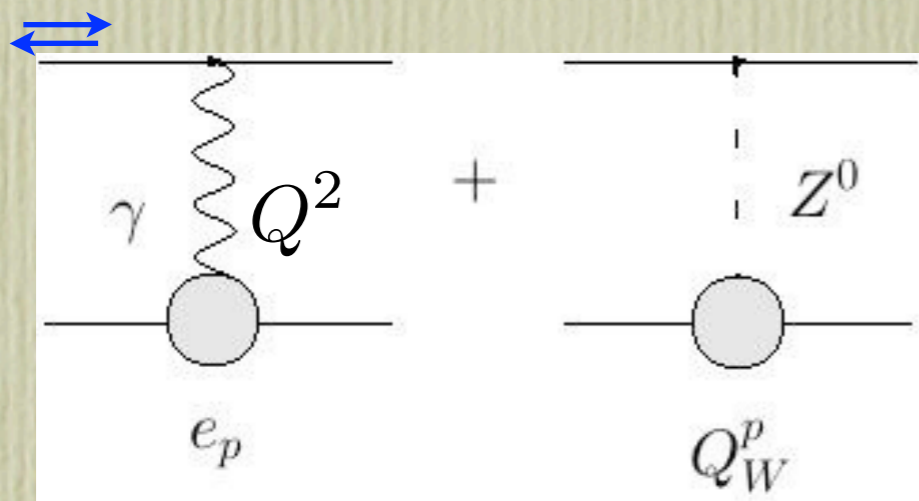
γ Z-Box from Dispersion Relations

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In collaboration with:

C.J. Horowitz, M.J. Ramsey-Musolf, X. Zhang

Weak Charge of the Proton



Elastic e-p scattering
with polarized e^- beam

$$A^{PV} = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} Q_W^p + \mathcal{O}(Q^4)$$

Effective e-q interaction

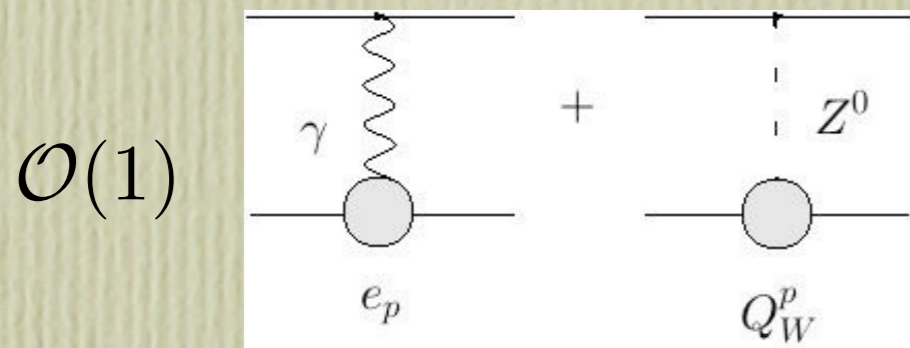
$$-\mathcal{L}^{eh} = -\frac{G_F}{\sqrt{2}} \sum_i \left[C_{1i} \bar{e} \gamma_{\mu} \gamma^5 e \bar{q}_i \gamma^{\mu} q_i + C_{2i} \bar{e} \gamma_{\mu} e \bar{q}_i \gamma^{\mu} \gamma^5 q_i \right]$$

Standard Model (tree-level)

$$Q_W^{p, tree} = -2(2C_{1u} + C_{1d}) = 1 - 4\sin^2 \theta_W \approx 0.05$$

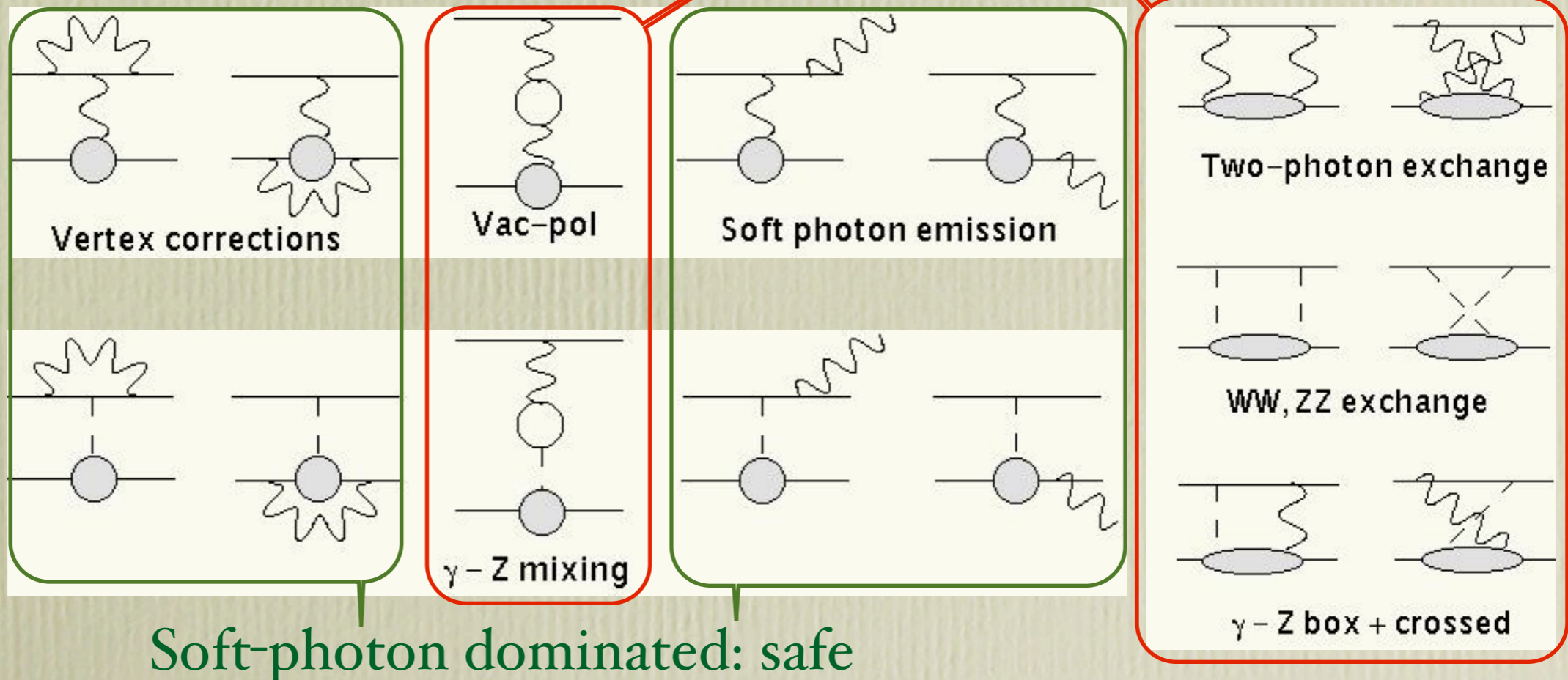
Weak Charge of the Proton: EW corrections

To match the experimental precision - include radiative corrections



Hadronic structure-dependent

$\mathcal{O}(\alpha_{em})$ $\alpha_{em} \approx 1/137$



Weak Charge of the Proton: EW corrections

Hadronic structure effects are under control

$$Q_W^p = \left[(1 + \Delta_\rho + \Delta_e)(1 - 4 \sin^2 \hat{\theta}_W + \Delta'_e) + \square_{WW} + \square_{ZZ} \right] + \square_{\gamma Z}$$

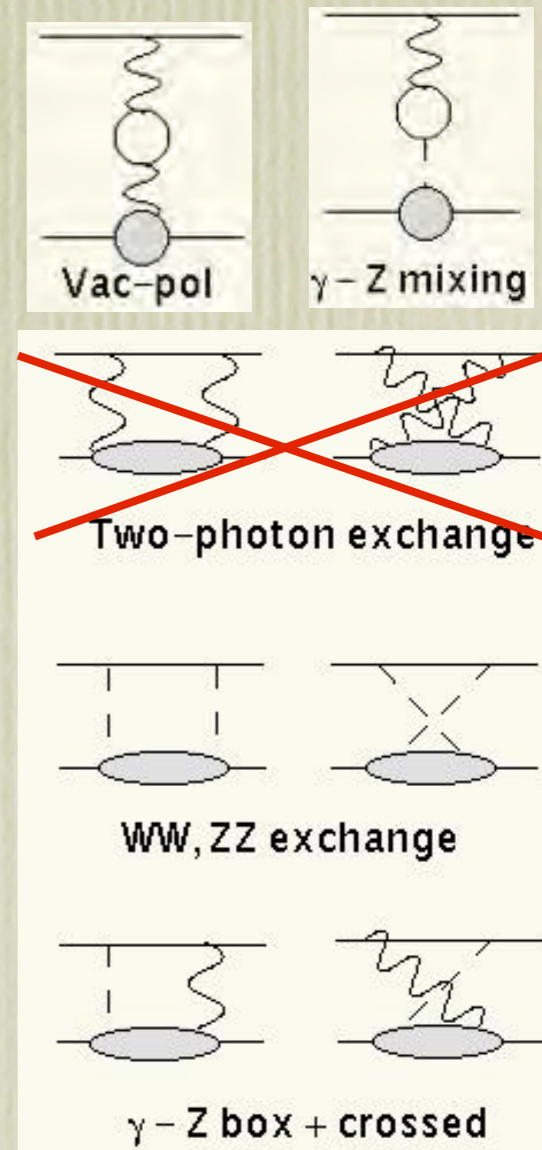
W. J. Marciano and A. Sirlin, PRD **27**, 552 (1983); **29**, 75 (1984); **31**, 213 (1985).
 M.J. Ramsey-Musolf, PRC **60**, 015501 (1999).

Vacuum polarization: reconstructed from $e^+e^- \rightarrow$ hadrons with dispersion relations

2γ -Box: kinematically suppressed

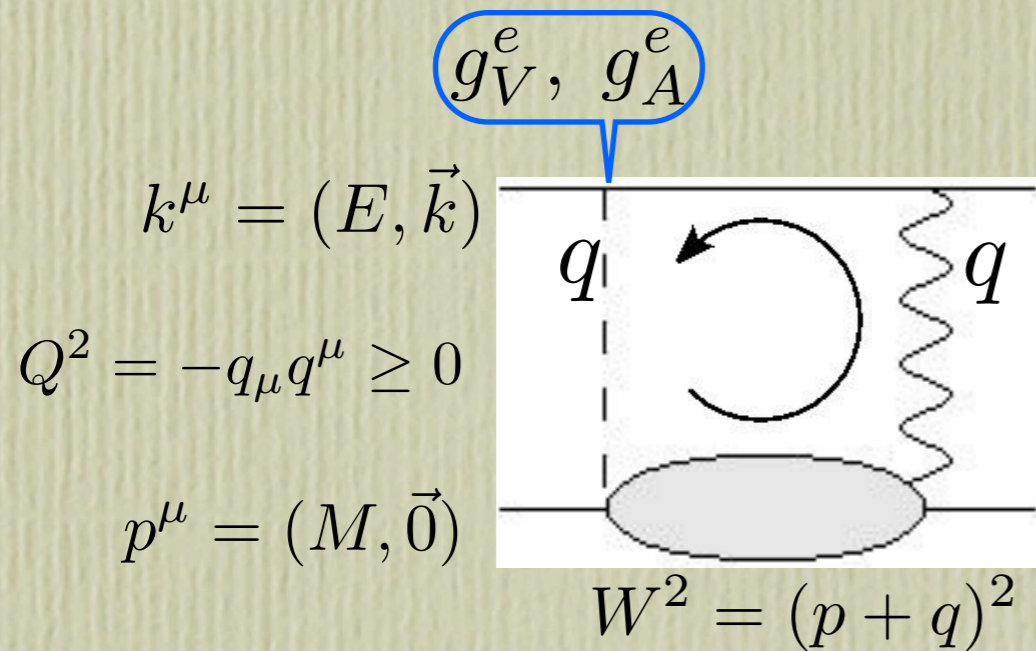
WW, ZZ -Box: perturbative- calculable reliably

γZ : for low energies (atomic PV experiments)
 cancellation between box and crossed
 - not true for ~ 1 GeV energy any more



Energy dependence of the γZ -Correction

MG & C.J. Horowitz, PRL102, 091806 (2009)



Forward dispersion relation for

$$\square_{\gamma Z} = g_V^e \square_{\gamma Z_A} + g_A^e \square_{\gamma Z_V}$$

Possess different symmetry
between box and crossed terms:

$$\text{Re} \square_{\gamma Z_A}(E) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{E' dE'}{E'^2 - E^2} \text{Im} \square_{\gamma Z_A}(E')$$

$$\text{Re} \square_{\gamma Z_V}(E) = \frac{2E}{\pi} \int_{\nu_0}^{\infty} \frac{dE'}{E'^2 - E^2} \text{Im} \square_{\gamma Z_V}(E')$$

$$\text{Re} \square_{\gamma Z_A}(0) \neq 0$$

$$\text{Re} \square_{\gamma Z_V}(0) = 0$$

APV result

Can quantify the energy dependence

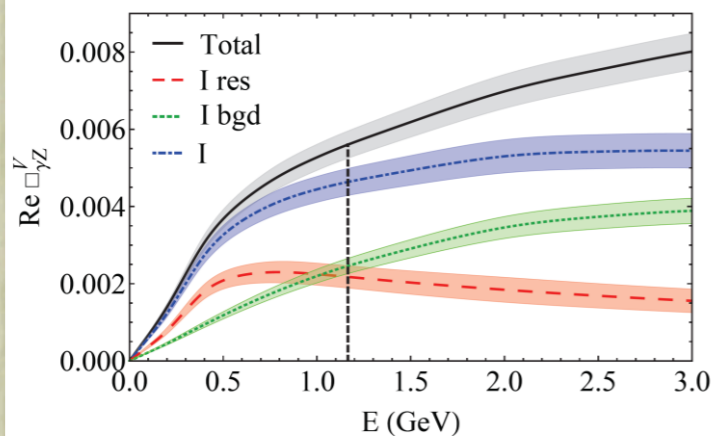
Energy dependence of the γZ -Correction

$$\text{Re} \square_{\gamma Z_V}(E) = \frac{2E}{\pi} \int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[A F_1^{\gamma Z}(W^2, Q^2) + B F_2^{\gamma Z}(W^2, Q^2) \right]$$

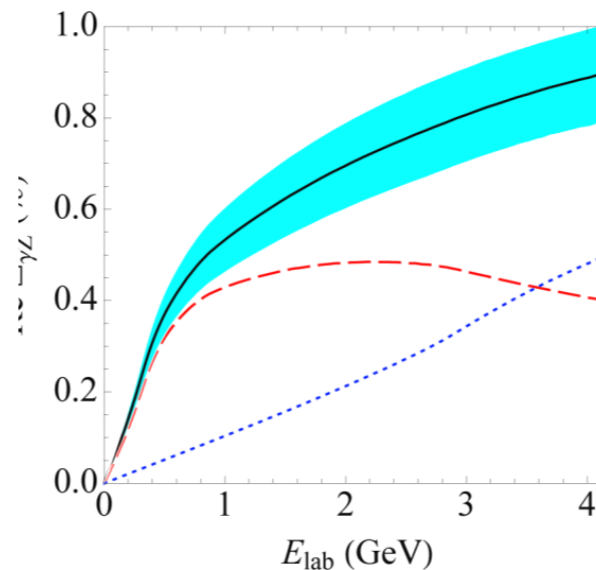
PV DIS data
- not (YET!) available

Isospin-rotate the e.-m. data
Evaluate at $E = 1.165$ GeV (QWEAK)

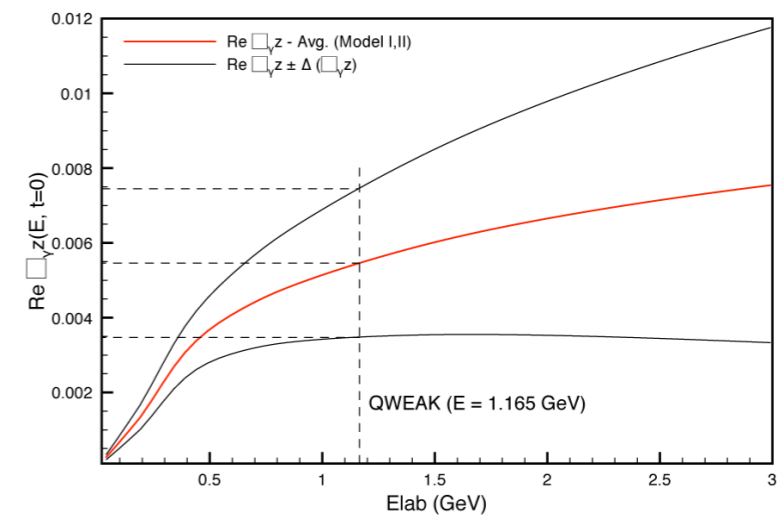
Hall *et al.*
PRD 88, 013011 (2013)



Carlson and Rislow
PRD 83, 113007 (2011)



Gorchtein *et al.*
PRC 84, 015502 (2011)

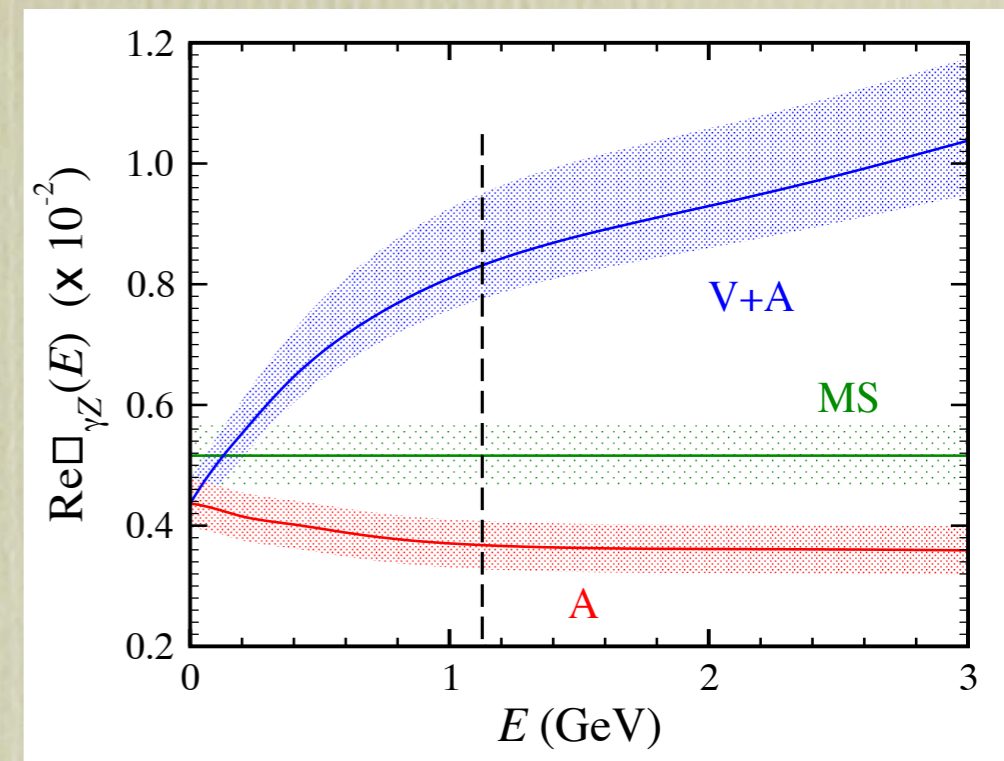


$\text{Re} \square_{\gamma Z}^V(E = 1.165 \text{ GeV})$		
$(5.6 \pm 0.36) \times 10^{-3}$	$(5.7 \pm 0.9) \times 10^{-3}$	$(5.4 \pm 2.0) \times 10^{-3}$

Energy dependence of the γZ -Correction

Evaluate the axial part through a DR
- check Marciano & Sirlin's calculation

Blunden et al., PRL 107 (2011) 081801



New SM prediction for the proton's weak charge

$$Q_W^p + \text{Re}\Omega_{\gamma Z}(E = 1.165 \text{ GeV}) = 0.0767 \pm 0.0008 \pm 0.0020_{\gamma Z}$$

To be compared to the previous prediction $Q_W^p = 0.0713 \pm 0.0008$

4σ (theory) effect was missed in the original QWEAK analysis;
Theory uncertainty needs to be further reduced

Uncertainty is dominated by the γZ_ν - asses all the sources of the uncertainty

Saturation of the dispersion integral for $\gamma Z\nu$ -box

$$\text{Re}\square_{\gamma Z\nu}(E) = \frac{2E}{\pi} \int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[AF_1^{\gamma Z}(W^2, Q^2) + BF_2^{\gamma Z}(W^2, Q^2) \right]$$

Q-Weak: $E = 1.165 \text{ GeV}$

	W < 2GeV	W < 4GeV	W < 5GeV	W < 10GeV	All W
$Q^2 < 1 \text{ GeV}^2$	62.6%	79.8%	81.2%	82.8%	83.2%
$Q^2 < 2 \text{ GeV}^2$	68.3%	85.8%	87.6%	89.9%	90.4%
$Q^2 < 3 \text{ GeV}^2$	69.4%	87.9%	90.0%	92.7%	93.3%
All Q^2	70%	91.1%	94.1%	98.6%	100%

Saturation of the dispersion integral for γZ_V -box

$$\text{Re}\Pi_{\gamma Z_V}(E) = \frac{2E}{\pi} \int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[AF_1^{\gamma Z}(W^2, Q^2) + BF_2^{\gamma Z}(W^2, Q^2) \right]$$

Mainz/MESA: $E = 0.180 \text{ GeV}$

	$W < 2\text{GeV}$	$W < 4\text{GeV}$	$W < 5\text{GeV}$	$W < 10\text{GeV}$	All W
$Q^2 < 1 \text{ GeV}^2$	75.0%	86.4%	87.4%	88.6%	88.8%
$Q^2 < 2 \text{ GeV}^2$	78.4%	90.3%	91.6%	93.2%	93.5%
$Q^2 < 3 \text{ GeV}^2$	79.1%	91.7%	93.2%	95.1%	95.5%
All Q^2	79.5%	93.9%	95.9%	99.0%	100%

Saturation of the dispersion integral for $\gamma Z\nu$ -box

For the Q-Weak kinematics

- Input at $Q^2 > 2 \text{ GeV}^2$ only constrains 10% of the $\gamma Z\nu$ -box
- Input at $Q^2 < 0.5 \text{ GeV}^2$ constrains 63% of the $\gamma Z\nu$ -box

For MESA kinematics

- Input at $Q^2 > 2 \text{ GeV}^2$ only constrains 6% of the $\gamma Z\nu$ -box
- Input at $Q^2 < 0.5 \text{ GeV}^2$ constrains 77% of the $\gamma Z\nu$ -box

Input for the dispersion integral for $\gamma Z\nu$ -box

Only indirect input available - e.-m. structure functions

We also need the rule how to obtain the γZ structure functions from the e.-m. ones

Parametrization of the inclusive data by Christy & Bosted

$$1.1 \text{ GeV} < W < 3.1 \text{ GeV}, 0 < Q^2 < 8 \text{ GeV}^2$$

M.E. Christy, P.E. Bosted, Phys.Rev. C81 (2010) 055213

This parametrization is used by all the three groups

Can be a common bias?

Sibirtsev et al., Phys.Rev. D82 (2010) 013011

$$\left(4.7_{-0.4}^{+1.1}\right) \times 10^{-3} \leftrightarrow (5.4 \pm 2.0) \times 10^{-3}$$

Identification and parameters of resonances should be (critically) assessed

Isospin rotation of e.-m. data: resonances

Based on: quantum numbers & strengths (Christy & Bosted);
relative size and sign of helicity amplitudes (PDG)

Isospin 1/2, 3/2 resonances

$$\langle X | J_{NC,V}^\mu | p \rangle = (1 - 4s^2\theta_W) \langle X | J_{em}^\mu | p \rangle - \langle X | J_{em}^\mu | n \rangle$$

$$\langle p | J_{em}^\mu | R \rangle \langle R | J_{NC,V}^\mu | p \rangle = (1 - 4s^2\theta_W) |\langle R | J_{em}^\mu | p \rangle|^2 - \langle p | J_{em}^\mu | R \rangle \langle R | J_{em}^\mu | n \rangle$$

Rescale each resonance in C&B parametrization to obtain γZ cross section

$$\xi_{Z/\gamma}^R \equiv \frac{\sigma_{T,R}^{\gamma Z,p}}{\sigma_{T,R}^{\gamma\gamma p}} = (1 - 4s^2\theta_W) - \frac{A_{R,1/2}^p A_{R,1/2}^{n*} + A_{R,3/2}^p A_{R,3/2}^{n*}}{|A_{R,1/2}^p|^2 + |A_{R,3/2}^p|^2} y_R$$

Values and uncertainties - from PDG helicity amplitudes values

	$P_{33}(1232)$	$S_{11}(1535)$	$D_{13}(1520)$	$S_{11}(1665)$	$F_{15}(1680)$	$P_{11}(1440)$	$F_{37}(1950)$
y_R	$-1.0_{+0.1}^{-0.1}$	$-0.51_{+0.35}^{-0.71}$	$-0.77_{+0.125}^{-0.125}$	$-0.28_{+0.45}^{-0.86}$	$-0.27_{+0.1}^{-0.12}$	$-0.62_{+0.19}^{-0.2}$	-1_{+1}^{-1}

Uncertainty from helicity amplitudes: An and Ap anti-correlated
Independent measurements - p and d=p+n, not p and n!

Checking input in γZ -calculation

The identification of resonances and relative strength is not the same in Christy & Bosted and PDG

PDG:

$$D_{13}(1520) : \quad A_{3/2}^p = 150(15) \text{ GeV}^{-1/2},$$
$$A_{1/2}^p = -24(9) \text{ GeV}^{-1/2}$$
$$S_{11}(1535) : \quad A_{3/2}^p = 0, \quad A_{1/2}^p = 90(30) \text{ GeV}^{-1/2}$$

Christy & Bosted:

$$D_{13}(1520) : \quad \sqrt{(A_{3/2}^p)^2 + (A_{1/2}^p)^2} \approx 23 \text{ GeV}^{-1/2}$$
$$S_{11}(1535) : \quad A_{3/2}^p = 0, \quad A_{1/2}^p \approx 170 \text{ GeV}^{-1/2},$$

Roper resonance is largely underestimated;
 $\Delta(1232)$ width 136 MeV instead of 120 MeV

Look for further input to constrain this change - GDH sum rule

Correcting input in γZ -calculation with the GDH sum rule

MG, X. Zhang, to be submitted soon

Spin-1/2 and spin-3/2 resonances can be distinguished in the helicity cross section (gr with real photons)

Data - from GDH collaboration (Mainz, Bonn)

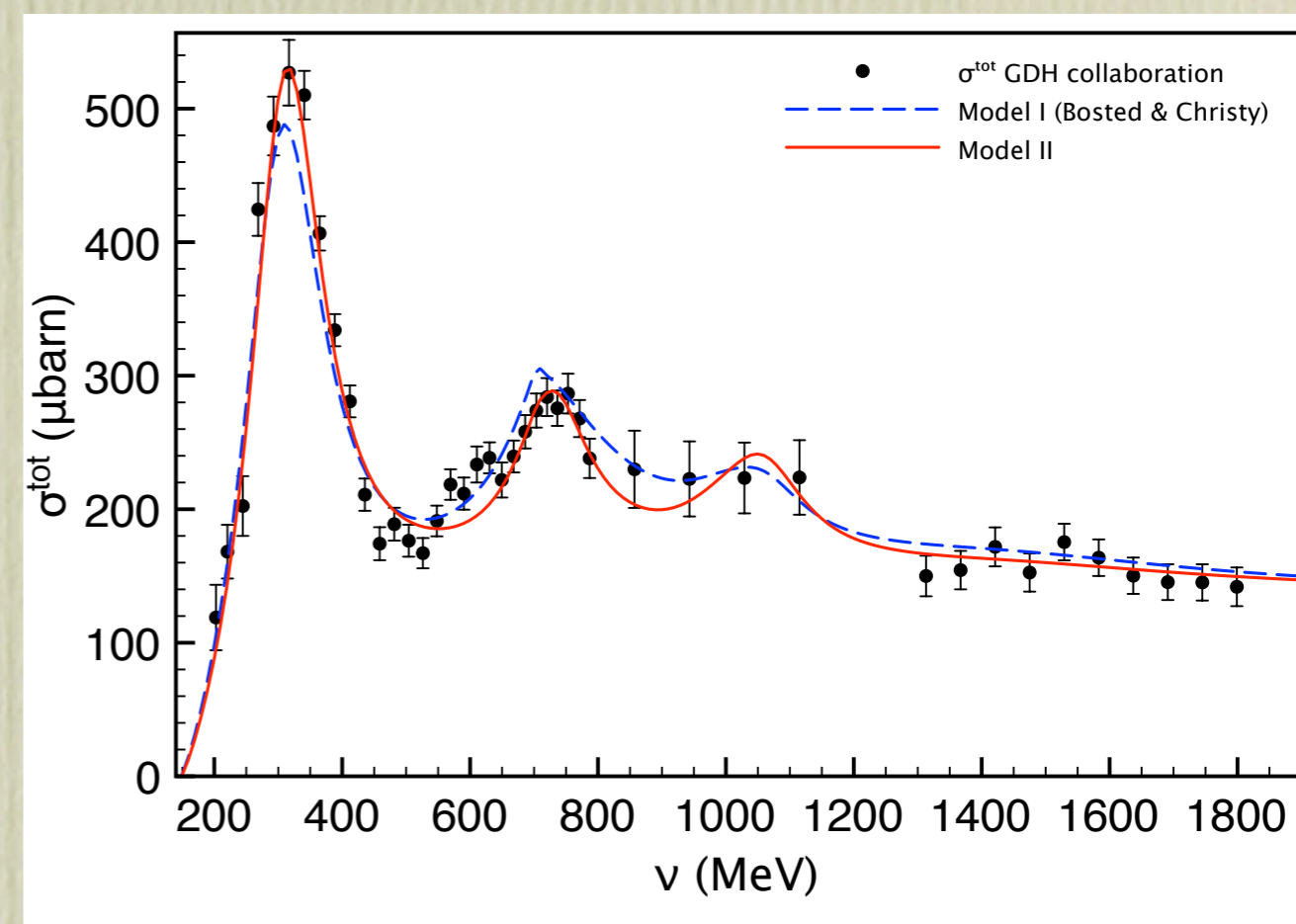
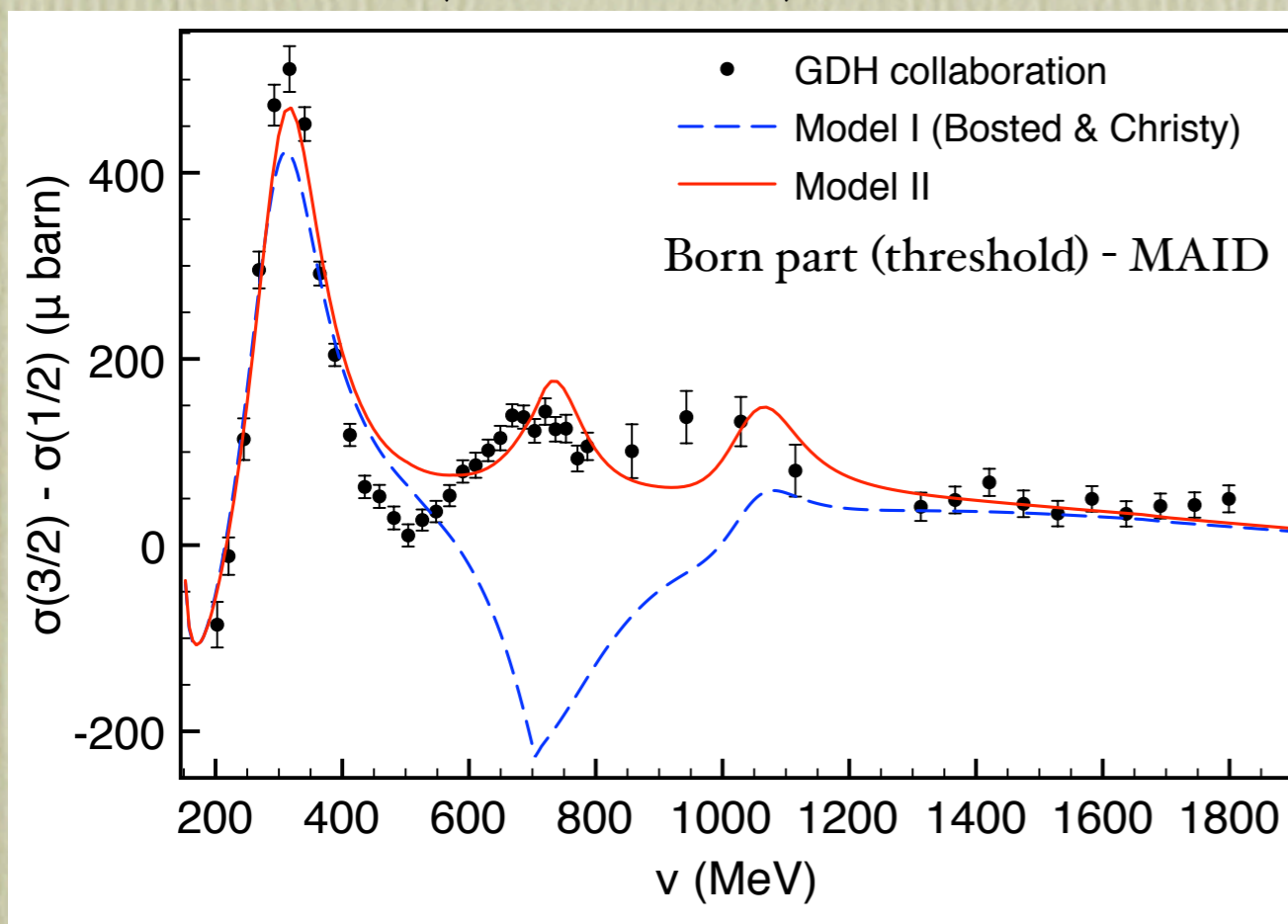
J. Ahrens et al, PRL 84(2000) 5950;

J. Ahrens et al, PRL 87(2001) 022003;

H. Dutz et al, PRL 91(2003) 192001

$$[\sigma_{3/2}^{\gamma\gamma}(\nu) - \sigma_{1/2}^{\gamma\gamma}(\nu)]$$

$$\sigma_T^{\gamma\gamma}(\nu)$$



With parameters from PDG - much better description of both observables

Not perfect: only 7 resonances, ...

Correcting input in γZ -calculation with the GDH sum rule

GDH sum rule

$$(\kappa_N^\gamma)^2 = \frac{2M^2}{\pi e^2} \int_{\nu_{thr}}^{\infty} d\nu \frac{[\sigma_{3/2}^{\gamma\gamma}(\nu) - \sigma_{1/2}^{\gamma\gamma}(\nu)]}{\nu}$$

With the “supplemented” Christy & Bosted’s fit can be evaluated

Sum rule value

$$\kappa_p^2 = 1.793^2 \approx 3.215$$

Christy & Bosted

$$\kappa_p^2 \approx 0.9$$

“PDG”

$$\kappa_p^2 \approx 3.28$$

Check the γZ -cross section

$$\kappa_N^\gamma \kappa_N^Z = \frac{2M^2}{\pi e^2} \int_{\nu_{thr}}^{\infty} d\nu \frac{[\sigma_{3/2}^{\gamma Z}(\nu) - \sigma_{1/2}^{\gamma Z}(\nu)]}{\nu}$$

Sum rule value

$$\kappa_p^\gamma \kappa_p^Z = (1 - 4s_W^2)(\kappa_p^\gamma)^2 - \kappa_p^\gamma \kappa_n^\gamma \approx 3.666$$

Christy & Bosted

$$\kappa_p^\gamma \kappa_p^Z \approx 2.247$$

“PDG”

$$\kappa_p^\gamma \kappa_p^Z \approx 3.615$$

Re-evaluate the γZ -box with the new resonance parametrization

Correcting the γZ -calculation with the resonances

($\times 10^{-3}$)	$P_{33}(1232)$	$P_{11}(1440)$	$D_{13}(1520)$	$S_{11}(1535)$	$S_{11}(1650)$	$F_{15}(1680)$	$F_{37}(1950)$
C & B	1.23 ± 0.12	0.06 ± 0.02	0.18 ± 0.03	$0.29^{+0.34}_{-0.17}$	$0.06^{+0.14}_{-0.06}$	0.04 ± 0.01	0.40 ± 0.36
PDG	1.23 ± 0.12	0.12 ± 0.03	0.31 ± 0.05	$0.06^{+0.08}_{-0.04}$	$0.06^{+0.14}_{-0.06}$	0.04 ± 0.02	0.39 ± 0.36

$$0.53^{+0.34}_{-0.17} \rightarrow 0.50^{+0.10}_{-0.07}$$

Further reduction - mainly $F_{37}(1950)$

Should be possible: it is in the right place, strength about right, proton and neutron strength is very close - quantum numbers OK

Reasonable to assume that at least 50% of F_{37} is really F_{37}

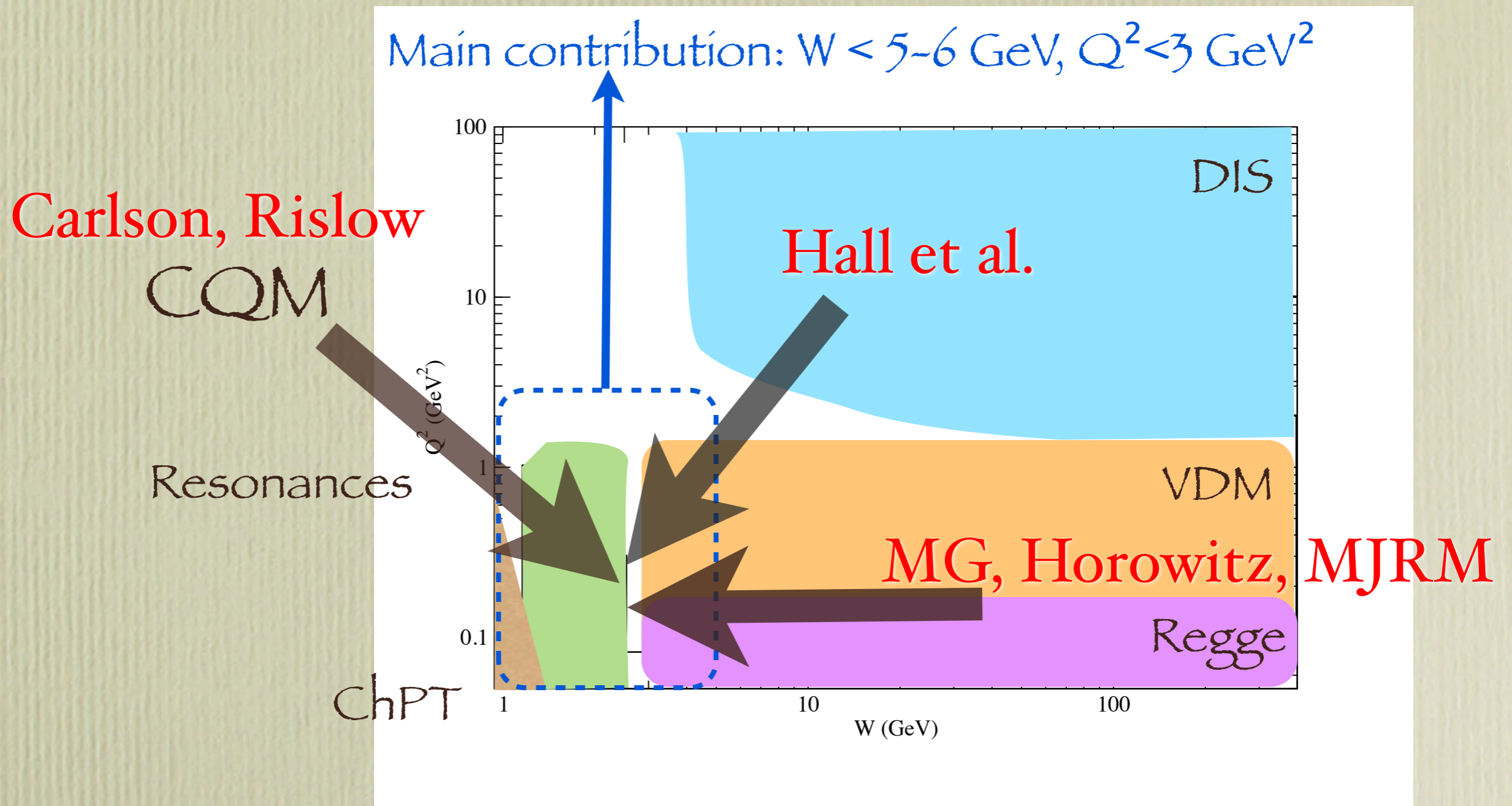
$$0.39 \pm 0.18$$

$$\sum_{res}^{OLD} = 2.24^{+0.53}_{-0.43}$$

$$\sum_{res}^{NEW} = 2.21^{+0.25}_{-0.23}$$

Uncertainty on the resonance contribution is halved.

Uncertainty of the Background



Main source of the uncertainty:

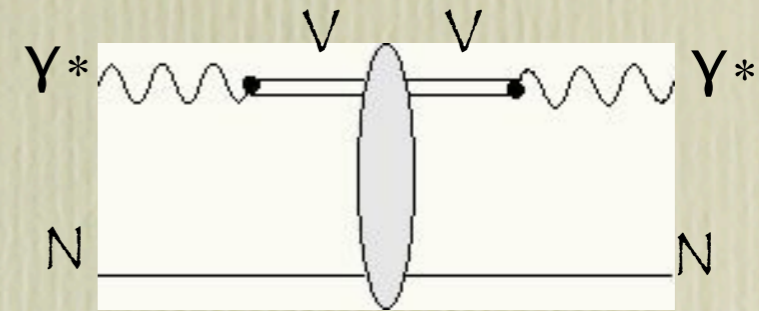
background in the resonance region and above

- how does a gamma/Z polarize vacuum in the target's proximity?

Isospin rotation of e.-m. data: background

Vector Dominance Model (VDM)

$$|\gamma^*\rangle = \sum_V C_{\gamma^*V} |V\rangle \quad V = \rho, \omega, \phi$$



$$\sigma_{tot}^{\gamma p} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2} \sigma_{Vp} \quad \text{Elastic } Vp \text{ cross section - independent of } V$$

$\frac{4\pi}{f_V^2} = 0.4545, 0.04237, 0.05435 \quad (\rho, \omega, \phi)$

VM decay constants

VDM sum rule:

$$\sigma_{tot}(\gamma p) = \sum_{V=\rho,\omega,\phi} \sqrt{16\pi \frac{4\pi\alpha}{f_V^2} \frac{d\sigma^{\gamma p \rightarrow Vp}}{dt}(t=0)}$$

Measured
experimentally

HERA: NPB' 02 $139 \pm 4 (\mu b)$ \leftrightarrow ZEUS: Z.Phys.'95,'96, PLB'96 $111 \pm 13 (\mu b)$ at $W = 70 \text{ GeV}$

Generalized VDM - continuum contribution

$$\sigma_{tot}^{\gamma p} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2} \sigma_{Vp} + \sigma_{Cp}$$

The 21% for continuum at $Q^2=0$ is the main limitation on γZ -box

Isospin rotation of e.-m. data: background

Rescale the background according to

$$\frac{\sigma^{\gamma^* p \rightarrow Z p}}{\sigma^{\gamma^* p \rightarrow \gamma^* p}} = \frac{\frac{g_V^{I=1}}{e_{I=1}} + \frac{g_V^{I=0}}{e_{I=0}} \frac{\sigma^{\gamma^* p \rightarrow \omega p}}{\sigma^{\gamma^* p \rightarrow \rho p}} + \frac{g_V^s}{e_s} \frac{\sigma^{\gamma^* p \rightarrow \phi p}}{\sigma^{\gamma^* p \rightarrow \rho p}} + \frac{X'}{\sigma^{\gamma^* p \rightarrow \rho p}}}{1 + \frac{\sigma^{\gamma^* p \rightarrow \omega p}}{\sigma^{\gamma^* p \rightarrow \rho p}} + \frac{\sigma^{\gamma^* p \rightarrow \phi p}}{\sigma^{\gamma^* p \rightarrow \rho p}} + \frac{X}{\sigma^{\gamma^* p \rightarrow \rho p}}}$$

VDM: identify X(X') with continuum

$$\frac{\sigma^{\gamma^* p \rightarrow V p}}{\sigma^{\gamma^* p \rightarrow \rho p}} = \frac{r_V m_V^4 (m_\rho^2 + Q^2)^2}{r_\rho m_\rho^4 (m_V^2 + Q^2)^2}$$

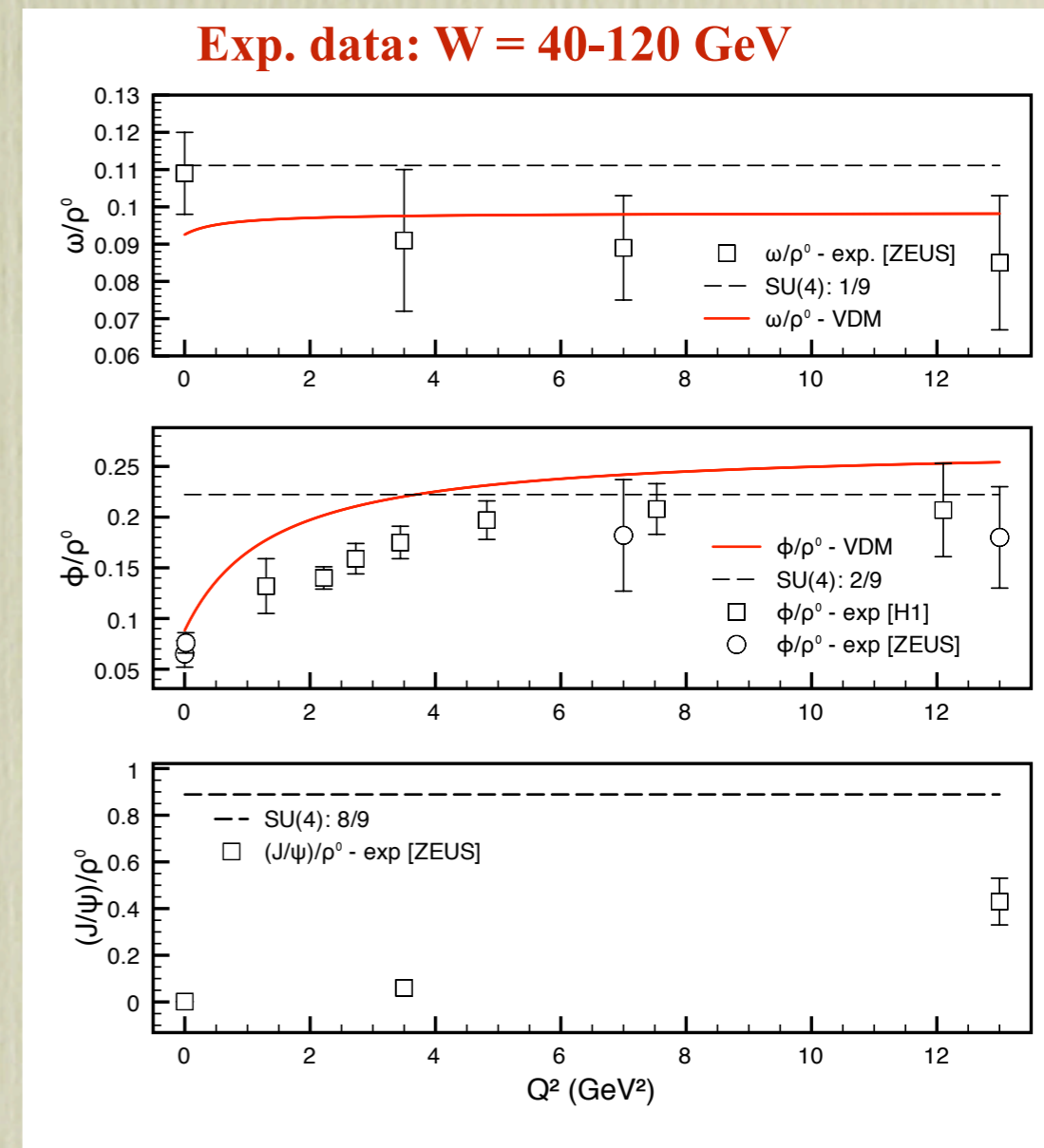
Uncertainty estimate - from data!

$$\Delta \xi_{Z/\gamma}^{V, Model A} = \left[\left(\frac{\sigma^{\gamma^* \rightarrow V}}{\sigma^{\gamma^* \rightarrow \rho}} \right)^{exp} - \left(\frac{\sigma^{\gamma^* \rightarrow V}}{\sigma^{\gamma^* \rightarrow \rho}} \right)^{Model A} \right] \sigma^{\gamma^* \rightarrow V \rightarrow Z}$$

Continuum - 100% uncertainty

Pure isovector: $r_C^{\gamma Z} / r_C^{\gamma \gamma} \approx 1$

Pure isoscalar: $r_C^{\gamma Z} / r_C^{\gamma \gamma} \approx -1$

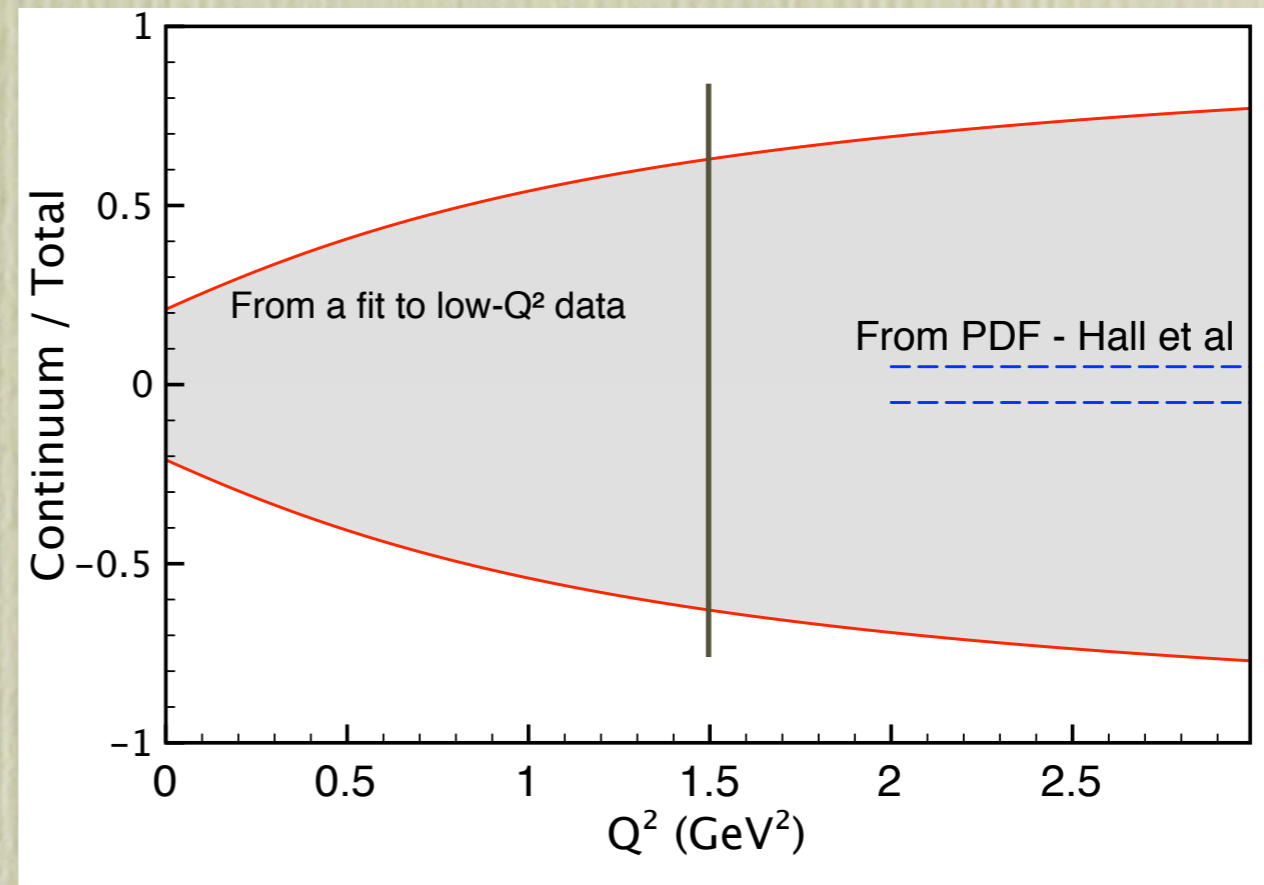
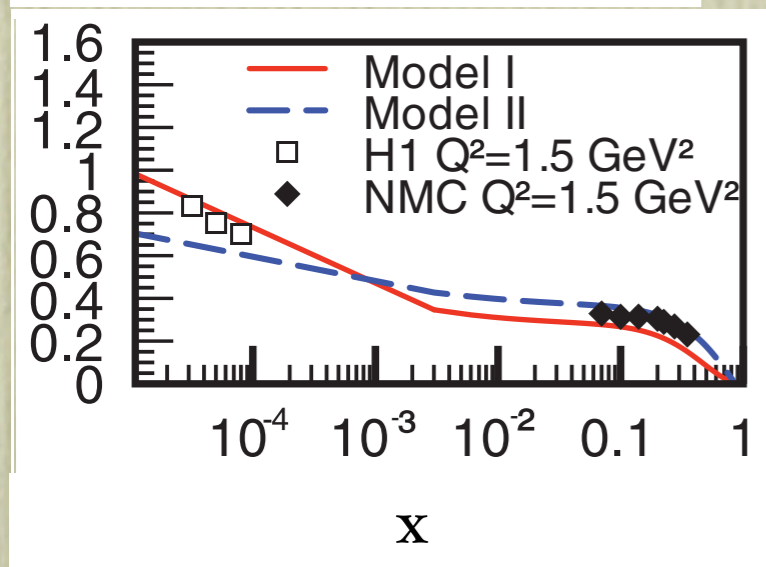
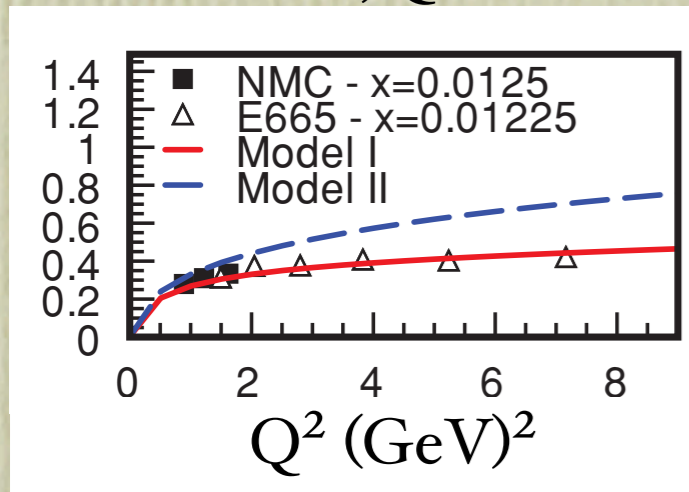


Isospin rotation of e.-m. data: background

Generalized VDM - virtual photons

$$\sigma_T^\gamma(\nu, Q^2) = \sigma_T^{Regge}(\nu) \left[\frac{0.67}{(1 + Q^2/m_\rho^2)^2} + \frac{0.061}{(1 + Q^2/m_\omega^2)^2} + \frac{0.059}{(1 + Q^2/m_\phi^2)^2} + \frac{0.21}{1 + Q^2/m_0^2} \right]$$

$F_2(x, Q^2)$



For virtual photons VDM sum rule works worse

We know less of the isospin of 1 GeV² virtual photons

How to extrapolate VDM sum rule down to JLab energies?

How to extend it to virtual photons without losing predicting power?

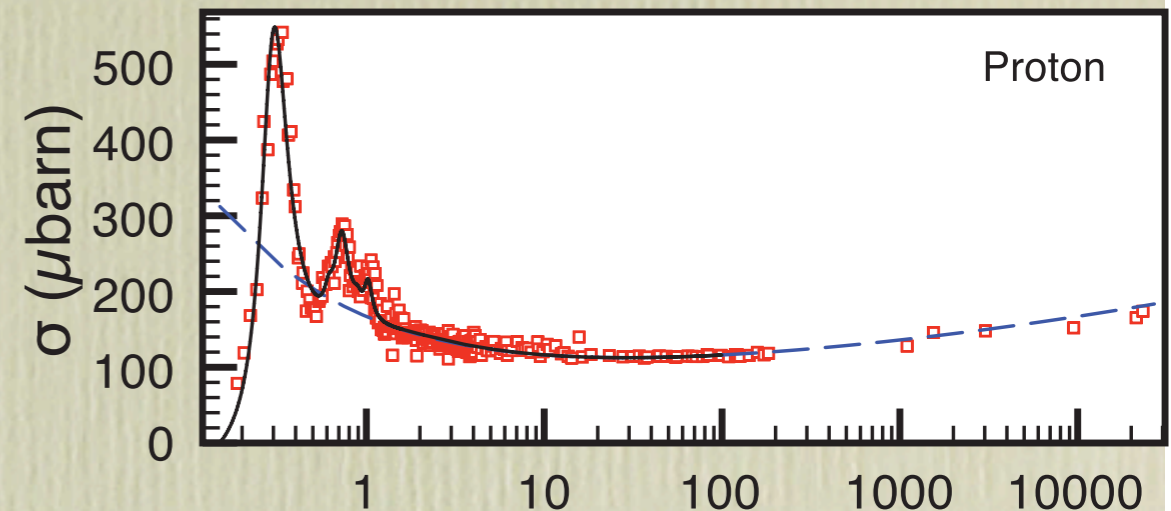
Constrain the background: Finite Energy Sum Rule

FESR: DR for Compton amplitude without Regge-behaved part

$$C_{\infty}^{\gamma\gamma} = -\frac{\alpha}{M} - \frac{1}{2\pi^2} \int_{\nu_{thr}}^N d\nu \sigma_T^{\gamma\gamma}(\nu) + \frac{\nu_0}{2\pi^2} \sum_i c_i^{\gamma\gamma} \frac{N^{\alpha_i(0)}}{\alpha_i(0)}$$

Recent extraction:

$$C_{\infty}^{\gamma\gamma} = -0.72 \pm 0.35 \mu\text{b GeV}$$



MG, T. Hobbs, A. Szczepaniak, PRC 84 (2011) 065202

Fit to real photon data only; 5 resonances

With Christy & Bosted based parametrization: new evaluation

“PDG”

$$C_{\infty}^{\gamma\gamma} = -1.12 \mu\text{b GeV}$$

Christy & Bosted

$$C_{\infty}^{\gamma\gamma} = -1.68 \mu\text{b-GeV}$$

Indicate a possibly underestimated systematical error (fit form)

New extraction:

$$C_{\infty}^{\gamma\gamma} = -0.97 \pm 0.35(\text{stat.}) \pm 0.35(\text{syst.}) \mu\text{b GeV}$$

Constrain the background: Finite Energy Sum Rule

Similar for the γZ -interference

$$C_{\infty}^{\gamma Z} = -\frac{\alpha Q_W^p}{M} - \frac{1}{2\pi^2} \int_{\nu_{thr}}^N d\nu \sigma_T^{\gamma Z}(\nu) + \frac{\nu_0}{2\pi^2} \sum_i c_i^{\gamma Z} \frac{N^{\alpha_i(0)}}{\alpha_i(0)}$$

With the parametrization of the $\gamma\gamma$ data + isospin rotation:

“PDG”

$$C_{\infty}^{\gamma Z} = 2.61 \pm 2.02(\text{back.})_{-0.78}^{+0.93}(\text{res.}) \mu\text{b GeV.}$$

Christy & Bosted

$$C_{\infty}^{\gamma Z} = 2.47 \pm 2.02(\text{back.})_{-1.33}^{+2.17}(\text{res.}) \mu\text{b GeV}$$

What is the l.h.s likely to be?

If ~ like the Thomson term:

$$\frac{C_{\infty}^{\gamma Z}}{C_{\infty}^{\gamma\gamma}} = Q_W^p(0) \approx 0.05$$

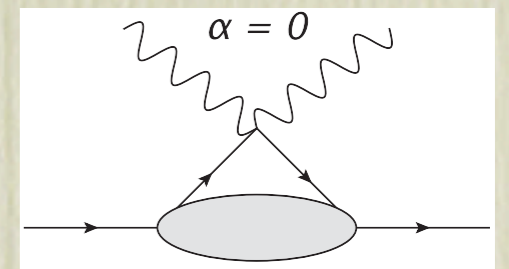
Asymptotically: effective point-like two-boson coupling

SU(6) symmetric

$$\frac{C_{\infty}^{\gamma Z}}{C_{\infty}^{\gamma\gamma}} \sim \frac{2 \sum_{q=u,d,s,c,t,b} g_V^q e_q}{\sum_{q=u,d,s,c,t,b} e_q^2} = \frac{9}{5} - 4s_W^2 \approx 0.85$$

Valence only

$$\frac{C_{\infty}^{\gamma Z}}{C_{\infty}^{\gamma\gamma}} \sim \frac{2(2g_V^u e_u + g_V^d e_d)}{2e_u^2 + e_d^2} = \frac{5}{3} - 4s_W^2 \approx 0.71$$



Only sure thing: same sign as $\gamma\gamma$, slightly smaller:

$$C_{\infty}^{\gamma Z} = -0.5 \pm 0.5 \mu\text{b GeV.}$$

Constrain the background: Finite Energy Sum Rule

“Extracted from data”

$$C_{\infty}^{\gamma Z} = -0.5 \pm 0.5 \mu\text{b GeV}$$

“PDG”

$$C_{\infty}^{\gamma Z} = 2.61 \pm 2.02(\text{back.})_{-0.78}^{+0.93}(\text{res.}) \mu\text{b GeV}$$

Continuum contribution:

It was assumed that

Equally possible
exact isovector: -1
exact isoscalar: -1

$$2.02 \pm 2.02 \mu\text{b GeV}$$

$$r_C^{\gamma Z} = r_C^{\gamma\gamma} (1 \pm 1)$$

$$r_C^{\gamma Z} = r_C^{\gamma\gamma} (0 \pm 1)$$

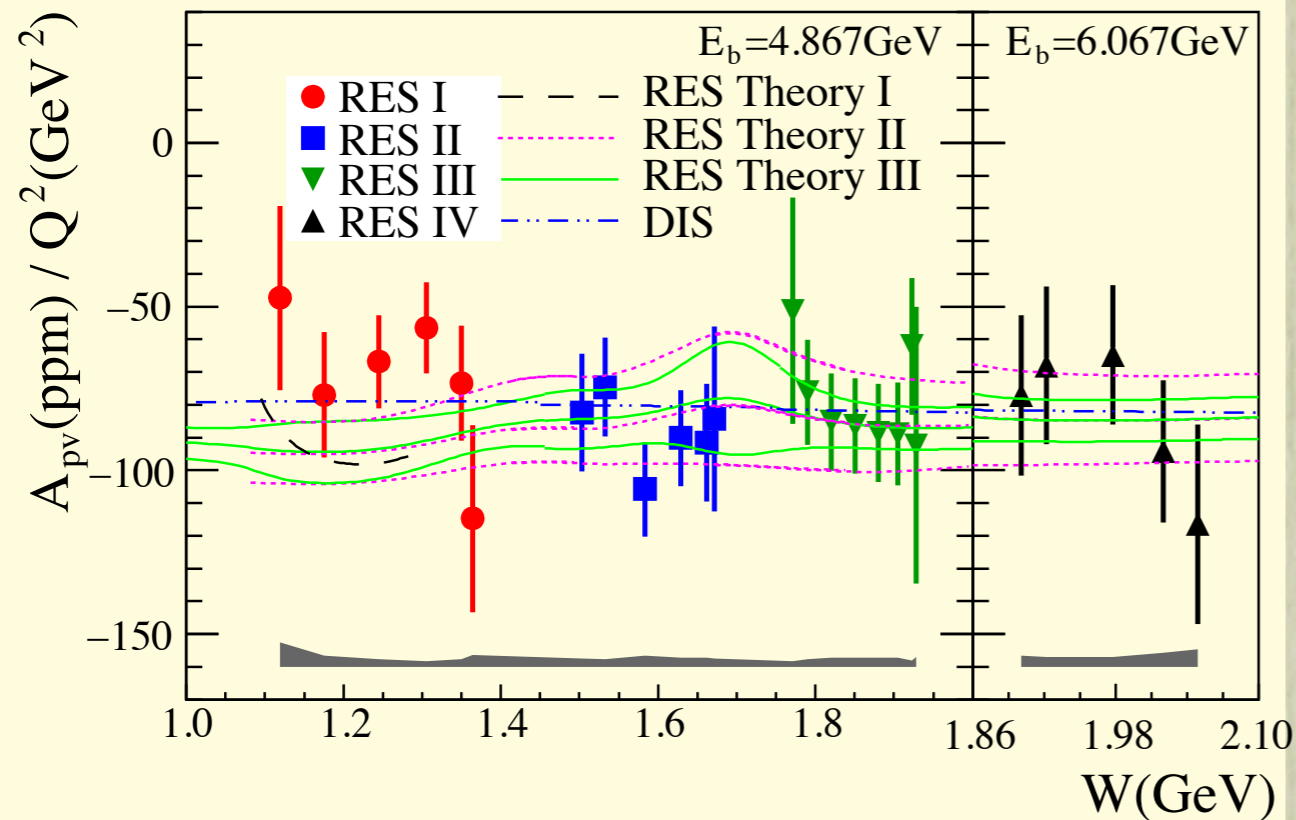
Supported by the FESR

To bring in accord, adjust the background
(resonances are fixed by c.s. and GDH)

$$(5.4 \pm 2.0) \times 10^{-3} \rightarrow (3.5 \pm 1.9 \pm 0.25) \times 10^{-3}$$

Impact of PV DIS data

FESR suggests that the continuum contribution might be largely overestimated



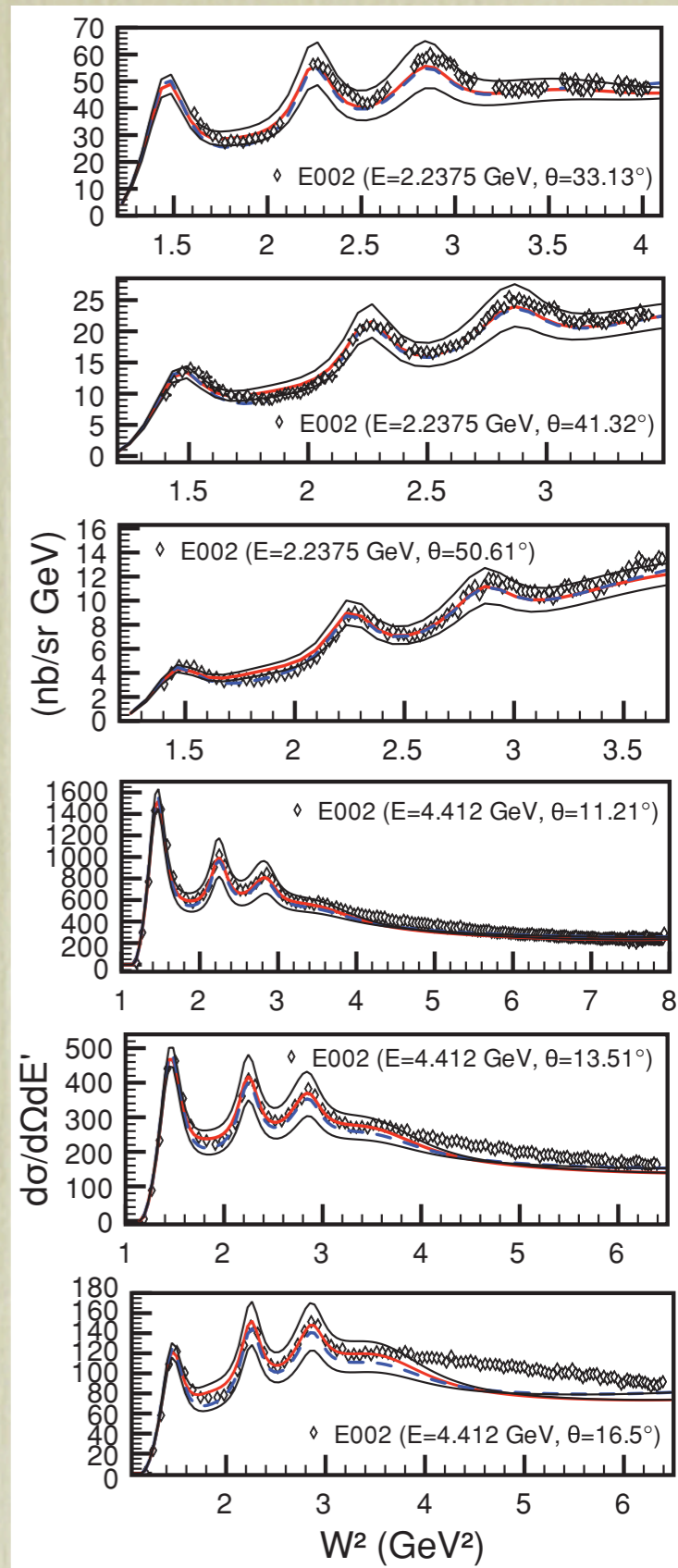
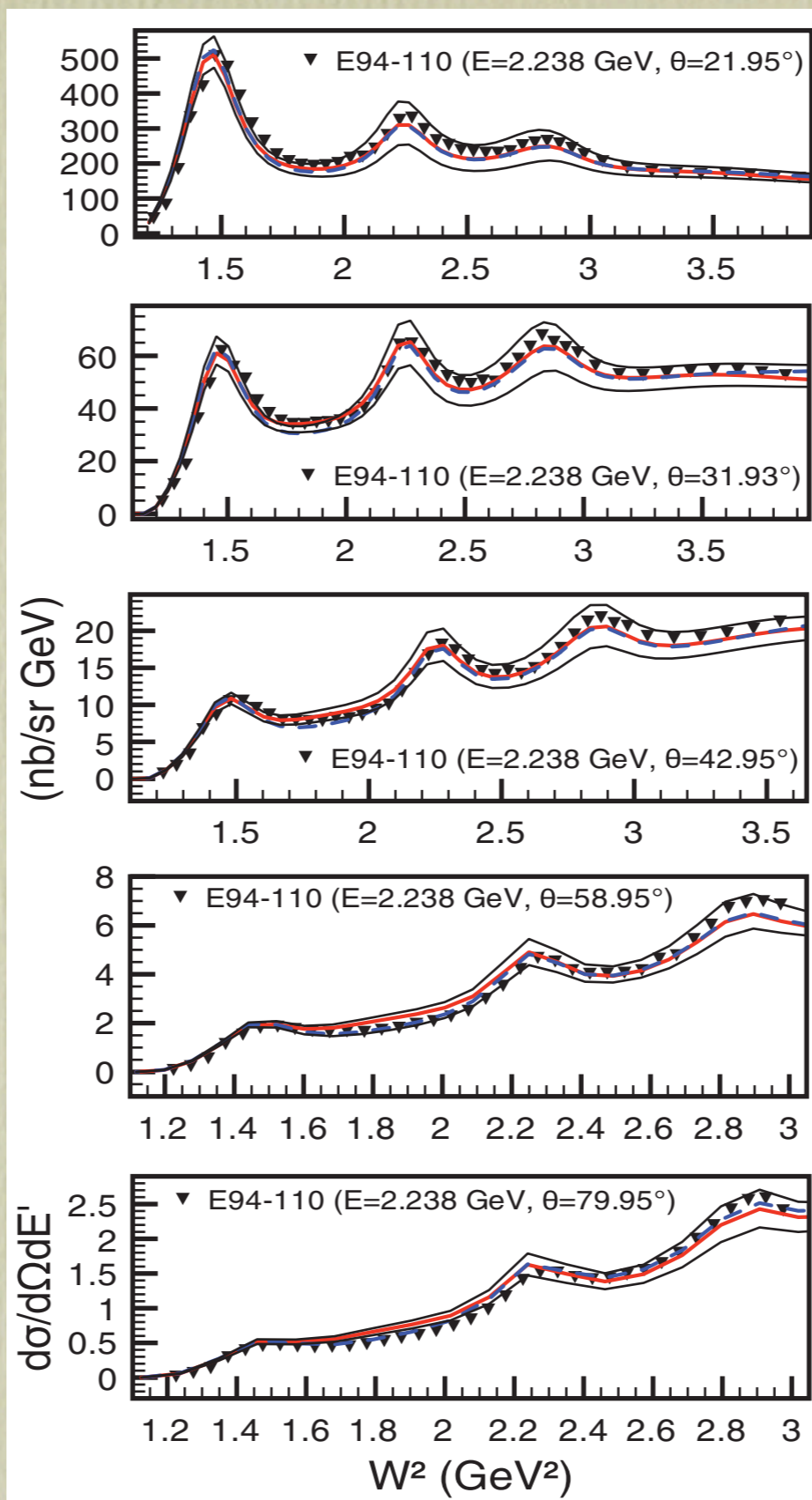
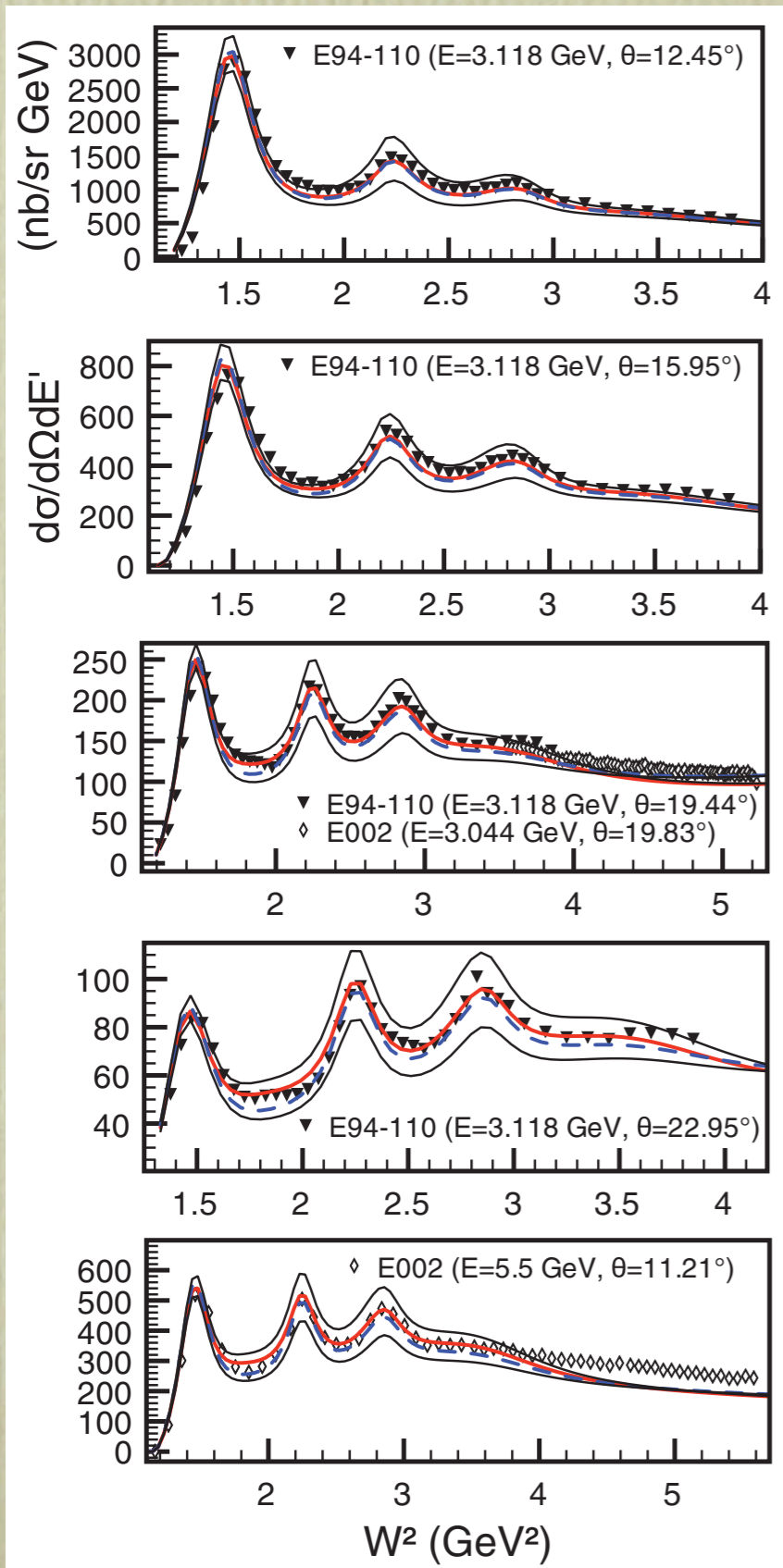
Most red points (central values) are outside of the theory band - is this uncertainty conservative?

Could this discrepancy support the FESR-driven conclusion?

New data on PV DIS structure functions coming -
PV DIS, SOLID, MOLLER

SUMMARY & OUTLOOK

- γZ box in Q-Weak kinematics: uncertainty is due to the isospin structure of the background ($\sim 66\%$) and resonance excitation of the neutron ($\sim 30\%$)
- Uncertainty estimate: combining unrelated data sets: total cross sections on proton and deuteron, helicity cross section on the proton, exp. test of the VDM sum rule
- Using GDH sum rule - half the resonance uncertainty
- FESR: continuum overestimated? (at real photon point)
- VDM sum rule at JLab energies?



SUMMARY & OUTLOOK

	W < 2GeV	W < 4GeV	W < 5GeV	W < 10GeV	All W
$Q^2 < 1$ GeV ²	62.6%	79.8%	81.2%	82.8%	83.2%
$Q^2 < 2$ GeV ²	68.3%	85.8%	87.6%	89.9%	90.4%
$Q^2 < 3$ GeV ²	69.4%	87.9%	90.0%	92.7%	93.3%
All Q^2	70%	91.1%	94.1%	98.6%	100%

