





# Extracting Q-weak from PVES

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### Proton Asymmetry

• Leading-order asymmetry (one-boson exchange)

$$\begin{split} A_{LR}(\vec{e}p) &= -\frac{G_{\mu}Q^2}{4\pi\alpha\sqrt{2}} \begin{bmatrix} \frac{g_A^e \left(\epsilon G_{Ep}^{\gamma} G_{Ep}^Z + \tau G_{Mp}^{\gamma} G_{Mp}^Z\right) + g_V^e \epsilon' G_{Mp}^{\gamma} \widetilde{G}_{Ap}}{\epsilon(G_{Ep}^{\gamma})^2 + \tau(G_{Mp}^{\gamma})^2} \end{bmatrix} \\ \tau &= \frac{Q^2}{4M^2} \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1} \\ \epsilon' &= \sqrt{(1-\epsilon^2)\tau(1+\tau)} \\ \overline{A}_{LR}^p &= \frac{A_{LR}^p}{A_0} = \frac{g_A^e \left(\epsilon G_{Ep}^{\gamma} G_{Ep}^Z + \tau G_{Mp}^{\gamma} G_{Mp}^Z\right) + g_V^e \epsilon' G_{Mp}^{\gamma} \widetilde{G}_{Ap}}{\epsilon(G_{Ep}^{\gamma})^2 + \tau(G_{Mp}^{\gamma})^2} \end{split}$$

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• Take the momentum transfer to zero for forward scattering

 $Q^2 \to 0, \quad \epsilon \to 1$ 

• Reduced asymmetry is just the weak charge (tree level for now)

 $\overline{A^p_{LR}} \to g^e_A G^Z_{Ep}$ 

• Of course there are corrections at physical kinematics: small  $Q^2$ , small  $\theta$ 

$$\begin{split} \overline{A_{RL}^{p}} &= g_{A}^{e} g_{E}^{Z(p)} \\ &+ Q^{2} \frac{1}{12M_{p}^{2}} \left[ 3g_{A}^{e} \mu_{p} \mu_{p}^{Z} + g_{A}^{e} g_{E}^{Z(p)} \left( 2M_{p}^{2} (\langle r^{2} \rangle_{Ep}^{\gamma} - \langle r^{2} \rangle_{Ep}^{Z}) - 3\mu_{p}^{2} \right) \right] \\ &+ |Q| \theta \frac{g_{v}^{e}}{2M_{p}} \mu_{p} \widetilde{G}_{A}^{p} + \mathcal{O} \left( |Q|^{3}, \theta^{2} \right) \end{split}$$

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note: expansion is just for illustration

#### Proton asymmetry

• Full expression

$$\overline{A_{LR}^p} = \frac{A_{LR}^p}{A_0} = \frac{g_A^e \left(\epsilon G_{Ep}^{\gamma} G_{Ep}^Z + \tau G_{Mp}^{\gamma} G_{Mp}^Z\right) + g_V^e \epsilon' G_{Mp}^{\gamma} \widetilde{G}_{Ap}}{\epsilon (G_{Ep}^{\gamma})^2 + \tau (G_{Mp}^{\gamma})^2}$$

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electromagnetic form factors

# The easy part

 $G_{Ep}^{\gamma}, \ G_{Mp}^{\gamma}$ • Electromagnetic form factors



Kelly parameterisation

### Proton asymmetry

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electroweak form factors

- *Z* boson couples differently to electromagnetic currents
  - Break down EM form factors into individual quark contributions

$$G_{Ep}^Z = g_V^u G_{Ep}^u + g_V^d G_{Ep}^d + g_V^s G_{Ep}^s$$

• compare usual electromagnetic

$$G_{Ep}^{\gamma} = \frac{2}{3}G_{Ep}^{u} - \frac{1}{3}G_{Ep}^{d} - \frac{1}{3}G_{Ep}^{s} \qquad \text{proton}$$
  

$$G_{En}^{\gamma} = -\frac{1}{3}G_{En}^{d} + \frac{2}{3}G_{En}^{u} - \frac{1}{3}G_{En}^{s} \qquad \text{neutron}$$

$$G_{En}^{d} = G_{Ep}^{u} - G_{E}^{\delta u}$$
$$G_{En}^{u} = G_{Ep}^{d} - G_{E}^{\delta d}$$
$$G_{En}^{s} = G_{Ep}^{s} - G_{E}^{\delta s}$$

• Charge symmetry: Protons are like neutrons

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known known "unknown"  
we will just parameterise  
our ignorance

Charge symmetry: Protons are like neutrons

 $G_{En}^{d} = G_{Ep}^{u} - G_{E}^{\delta u}$  $G_{En}^{u} = G_{Ep}^{d} - G_{E}^{\delta d}$  $G_{En}^{s} = G_{Ep}^{s} - G_{E}^{\delta s}$ 

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• Neutral current processes at the Z-pole



 $C_{1q}, C_{2q}$ 

• Absorb electron coupling; and full evolution down to low scale

 $g_A^e G_{Ep}^Z = -2(2C_{1u} + C_{1d})G_{Ep}^\gamma - 2(C_{1u} + 2C_{1d})G_{En}^\gamma + \xi_V^{(0)}G_E^s$ 

# Radiative corrections: $\gamma Z$ box

 Significant energy-dependent correction from inelastic hadronic states identified by Gorchtein & Horowitz PRL(2009)



• Forward scattering limit evaluated through dispersion relation

# Radiative corrections: $\gamma Z$ box

• Energy dependence: Hall, Melnitchouk et al. (AJM Model), PRD(2013)



• Q<sup>2</sup>-dependence: Gorchtein et al. (GHRM), PRC(2011)



# Radiative corrections: $\gamma Z$ box

- Also used for large-angle results!!
  - dispersive treatment breaks down
- For now: all data points >20 degrees
  - Use same calculation, but assign 100% uncertainty
    - Current analysis

 $\Delta Q_W^p \lesssim \pm 0.0006$ 

 $\cdot$  cf.  $\Delta Q_W^p \sim \pm 0.012$ 

# Proton asymmetry

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$$\overline{A_{LR}^p} = \frac{A_{LR}^p}{A_0} = \frac{g_A^e \left(\epsilon G_{Ep}^{\gamma} G_{Ep}^Z + \tau G_{Mp}^{\gamma} G_{Mp}^Z\right) + g_V^e \epsilon' G_{Mp}^{\gamma} \widehat{G}_{Ap}}{\epsilon (G_{Ep}^{\gamma})^2 + \tau (G_{Mp}^{\gamma})^2}$$

axial + anapole form factor

- Forward angles  $\epsilon' \sim 0$
- Significant nonperturbative radiative corrections
  - For those familiar:

 $\widetilde{G}_A^{T=1}$ : dipole form (1 GeV), normalisation *fit* to data  $\widetilde{G}_A^{T=0}$ : dipole form (1 GeV), normalisation *constrained* to theory

based upon Zhu et al. PRD(2000)

### Proton asymmetry measurements



• Note also, He-4 and Deuteron measurements also included in data ensemble
### Proton asymmetry measurements

- Even for proton alone, very hard to represent points on a single plot
  - · I've not shown any fits yet; but let's look at all points projected onto

 $\epsilon \to 1 \quad (\text{or } \theta \to 0)$ 

 $\overline{A_{LR}^{p}}^{data}(\theta = 0, Q^{2}) = \overline{A_{LR}^{p}}^{data}(\theta^{data}, Q^{2}) - \left[\overline{A_{LR}^{p}}^{fit}(\theta^{data}, Q^{2}) - \overline{A_{LR}^{p}}^{fit}(\theta = 0, Q^{2})\right]$ 

### Proton asymmetry measurements





### Proton asymmetry measurements

Forward scattering projection



$$G_E^s = \rho^s Q^2 + \rho_2^s Q^4 + \dots$$
  
 $G_M^s = \mu^s + \mu_2^s Q^2 + \dots$ 

Taylor expansion

$$G_{E}^{s} = \rho^{s}Q^{2} + \rho_{2}^{s}Q^{4} + \dots$$
$$G_{M}^{s} = \mu^{s} + \mu_{2}^{s}Q^{2} + \dots$$

"leading-order polynomial"

Taylor expansion

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"leading-order polynomial" "second-order polynomial"

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"leading-order polynomial" "second-order polynomial"

• OR I'll talk about a "dipole" form

$$G_E^s = \rho^s Q^2 \left(\frac{1}{1+Q^2/\Lambda^2}\right)^2$$
$$G_M^s = \mu^s \left(\frac{1}{1+Q^2/\Lambda^2}\right)^2$$

 Taylor expansions 1.6 leading-order fit 1.4  $\chi^2/dof$ 1.2 1.0  $0.8^{\scriptscriptstyle {ox}}_{\scriptstyle {ullet}0.0}$ 0.2 0.4 0.6 0.8 1.0  $Q^2(\text{max})$  [GeV<sup>2</sup>]











# Q-weak precision



### Q-weak determination



## Q-weak determination



## Just leading order?

- You might be nervous about using the leading Taylor expansion over such a wide range
- What about dipole-type forms?
  - Physically, strangeness would have a characteristic mass scale set by the phi meson

 $\Lambda^2 \sim 1 \, {\rm GeV}^2$ 

• But a more extreme limit would be "light-quark" mass

 $\Lambda^2 \sim 0.71 \, {\rm GeV}^2$ 

### Dipole form $\Lambda^2 \sim 0.71 \, {\rm GeV}^2$



### Dipole form $\Lambda^2 \sim 0.71 \, {\rm GeV}^2$



### Let's see what we get for the fit

• Take 1 GeV Lambda as a "central" value

### $0.71\,\mathrm{GeV}^2 < \Lambda^2 = 1\,\mathrm{GeV}^2 < \infty$

#### light-quark radii Taylor expansion

- Difference between bounds gives model-dependence uncertainty
- And let's be ambitious and use all data up to  $~Q^2 \sim 0.63 \, {
  m GeV}^2$

### "B-term" plot

• Without Q-weak



- "B-term" plot
- WITH Q-weak



## Forward rotation

Shifted data points

$$\overline{A_{LR}^{p}}^{data}(\theta = 0, Q^{2}) = \overline{A_{LR}^{p}}^{data}(\theta^{data}, Q^{2}) - \left[\overline{A_{LR}^{p}}^{fit}(\theta^{data}, Q^{2}) - \overline{A_{LR}^{p}}^{fit}(\theta = 0, Q^{2})\right]$$



### Forward rotation



### Forward rotation







#### • Remember the old status













# Dipole and Taylor

# Dipole and Taylor

 You still might be nervous about using the a constrained functional form over such a wide range
#### Dipole and Taylor

- You still might be nervous about using the a constrained functional form over such a wide range
- How about a way of resolving how sensitive we are to this parameterisation?

#### Grab local bins of the data







• Strangeness electric form factor - with Q-weak point









• Weak charges from independent fits







#### • Weak charges from independent fits



APV Q2~0.1 Q2~0.22 Q2~0.6 Q2~0.1+Qweak

Combined

• It appears that the parameterisation of the strangeness form factors over the full range is not overly ambitious



#### Current status



#### Future

- Shopping list
  - Charge symmetry violation (deuteron & Helium-4)
  - Uncertainties on EM form factors
    - already implemented MC sampling over Kelly fit parameters
    - new data(?)
  - Gamma-Z box
    - E and Q<sup>2</sup> dependence
    - (quasi-elastic) deuteron & (elastic) Helium-4