HADRON SPECTRUM COLLABORATION*

Baryon spectroscopy from lattice QCD

- Long-term goal: Solve QCD to determine the hadron mass spectrum.
- Part I. Recent progress on N, Δ, Ω excited state spectra
  - J. M. Bulava, R. G. Edwards, E. Engelson, B. Joó, H.-W. Lin, C. Morningstar, D. G. Richards and S. J. Wallace, Phys. Rev. D82, 014507 (2010)
- Part II. Spin identification for baryon states
  - R. G. Edwards, D. G. Richards and S. J. Wallace, in preparation.
- Phenomenology
- Conclusions

Matrices of correlation functions and smearing of quark fields

$$C_{ij}(t,t') = \sum_{\mathbf{xy}} \left\langle B_i(\mathbf{x},t) B_j^\dagger(\mathbf{y},t') \right\rangle$$

$$B_i(\mathbf{x},t) = C_i^{\alpha\beta\gamma} \epsilon^{abc} q_\alpha^{a f_1}(\mathbf{x},t) q_\beta^{b f_2}(\mathbf{x},t) q_\gamma^{c f_3}(\mathbf{x},t).$$

Smearing: Project to eigenvectors of Laplacian

$$q_\alpha^a(\mathbf{x},t) \longrightarrow \sum_k v_{a\mathbf{x}}^{(k)} \widetilde{q}_\alpha^{(k)}(t).$$

$$\left(-\nabla^2\right)_{\mathbf{xy}}^{ab} v_{b,\mathbf{y}}^{(k)} = \lambda_k v_{a\mathbf{x}}^{(k)}$$

$$C_{ij}(t,t') = \Phi_{i,k\ell m}^{\alpha\beta\gamma}(t) \left\langle \widetilde{q}_\alpha^{(k)}(t) \widetilde{q}_\beta^{(\ell)}(t) \widetilde{q}_\gamma^{(m)}(t) \overline{\widetilde{q}}_{\bar\alpha}^{(\bar k)}(t') \overline{\widetilde{q}}_{\bar\beta}^{(\bar\ell)}(t') \overline{\widetilde{q}}_{\bar\gamma}^{(\bar m)}(t') \right\rangle \Phi_{j,\bar k\bar\ell\bar m}^{\bar\alpha\bar\beta\bar\gamma\dagger}(t')$$

Determine energies

Calculate eigenvectors at $t^* = t_0 + 1$

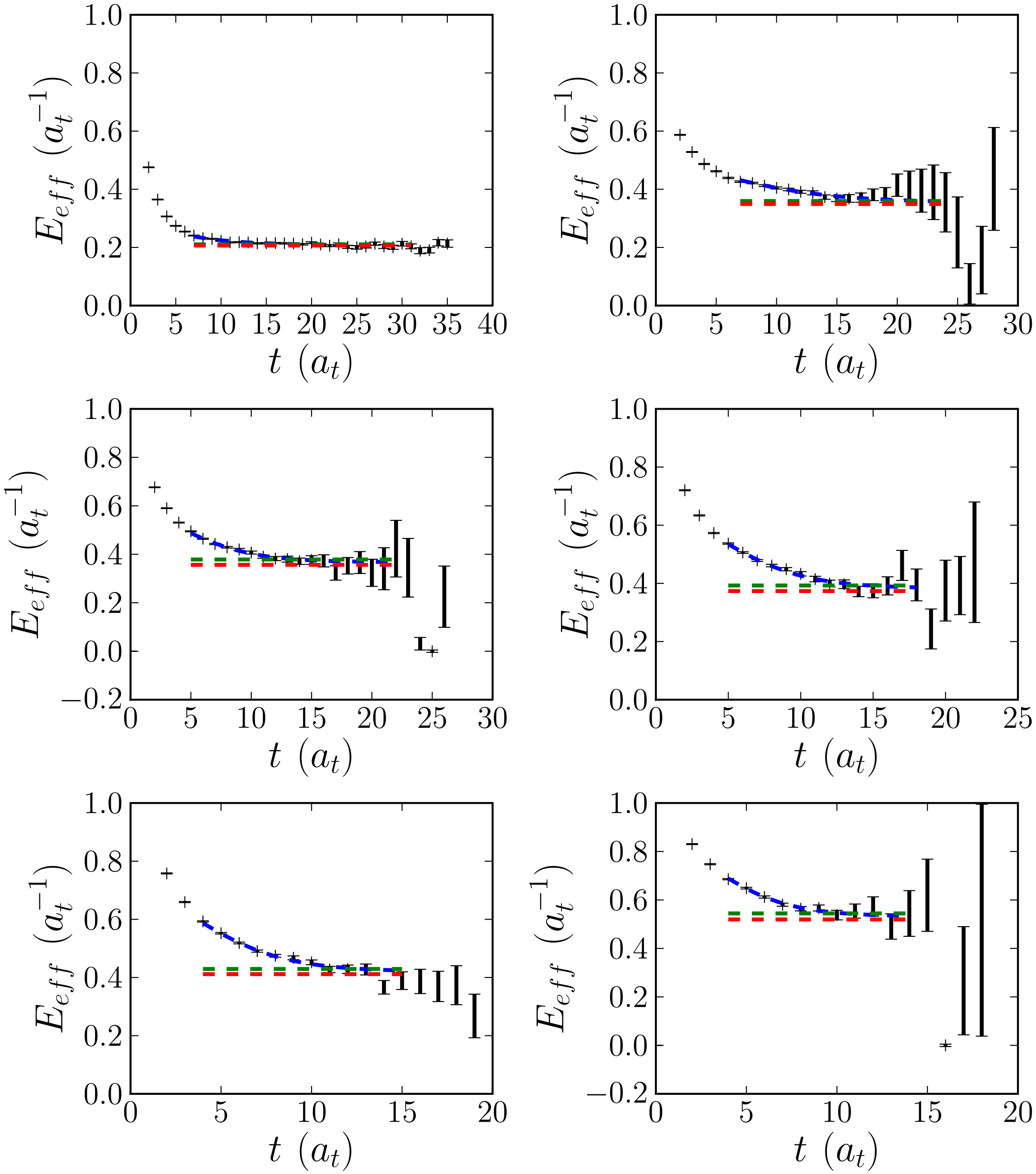
$$\overline{C}(t^*)V(t^*) = \overline{C}(t_0)V(t^*)\Lambda(t^*)$$

Rotate matrices to basis of eigenvectors, calculate diagonal elements

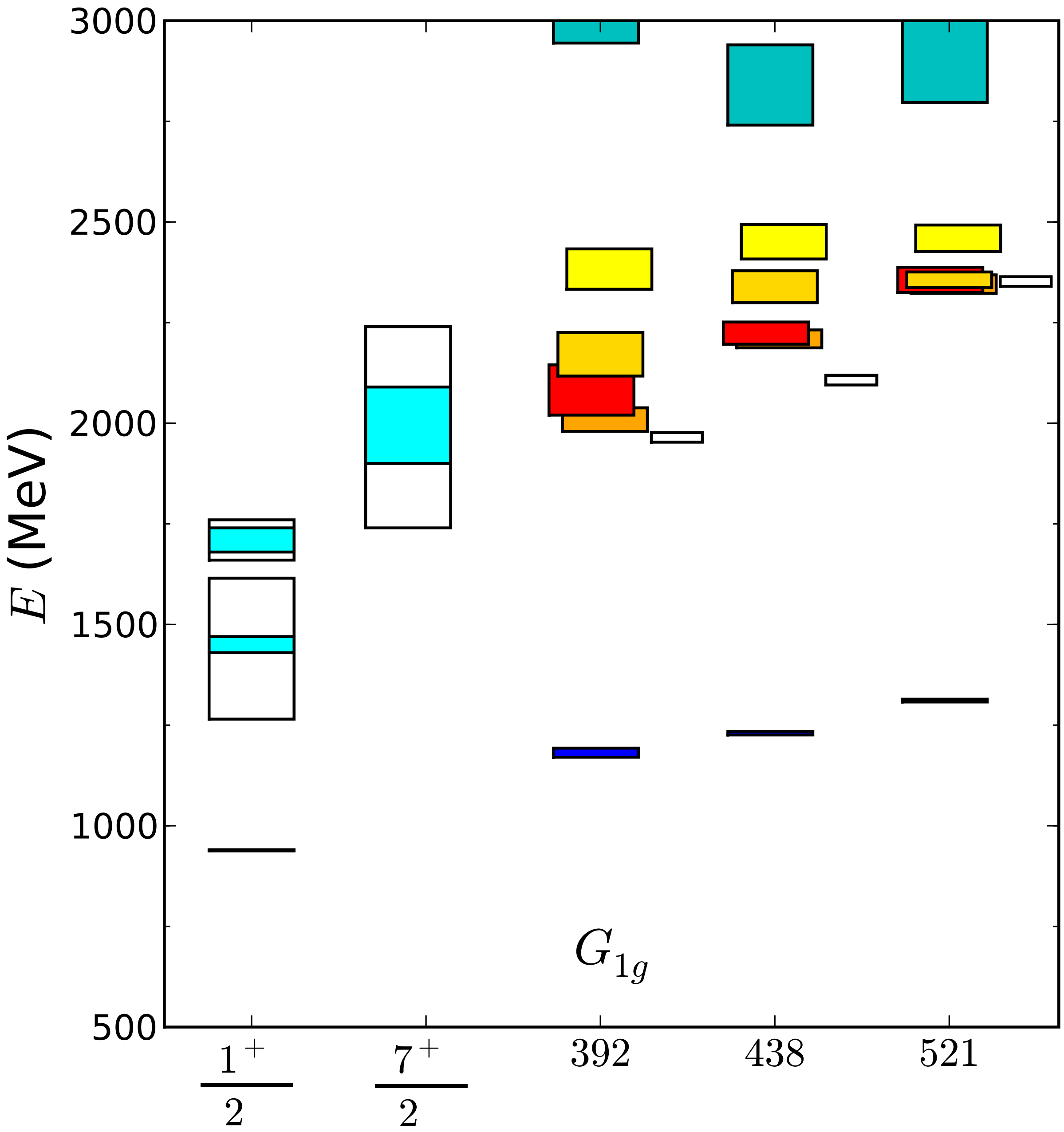
$$\widetilde{\lambda}_n(t) = \Big(V^\dagger(t^*)C(t)V(t^*)\Big)_{nn}$$

Two-exponential fits of diagonal elements

$$\lambda_{fit}(t) = (1-A)e^{-\mathrm{E}(t-t_0)} + Ae^{-E'(t-t_0)}$$



Nucleon G_{1g} effective energies: $m_\pi = 392(4)$ MeV


3000
2500
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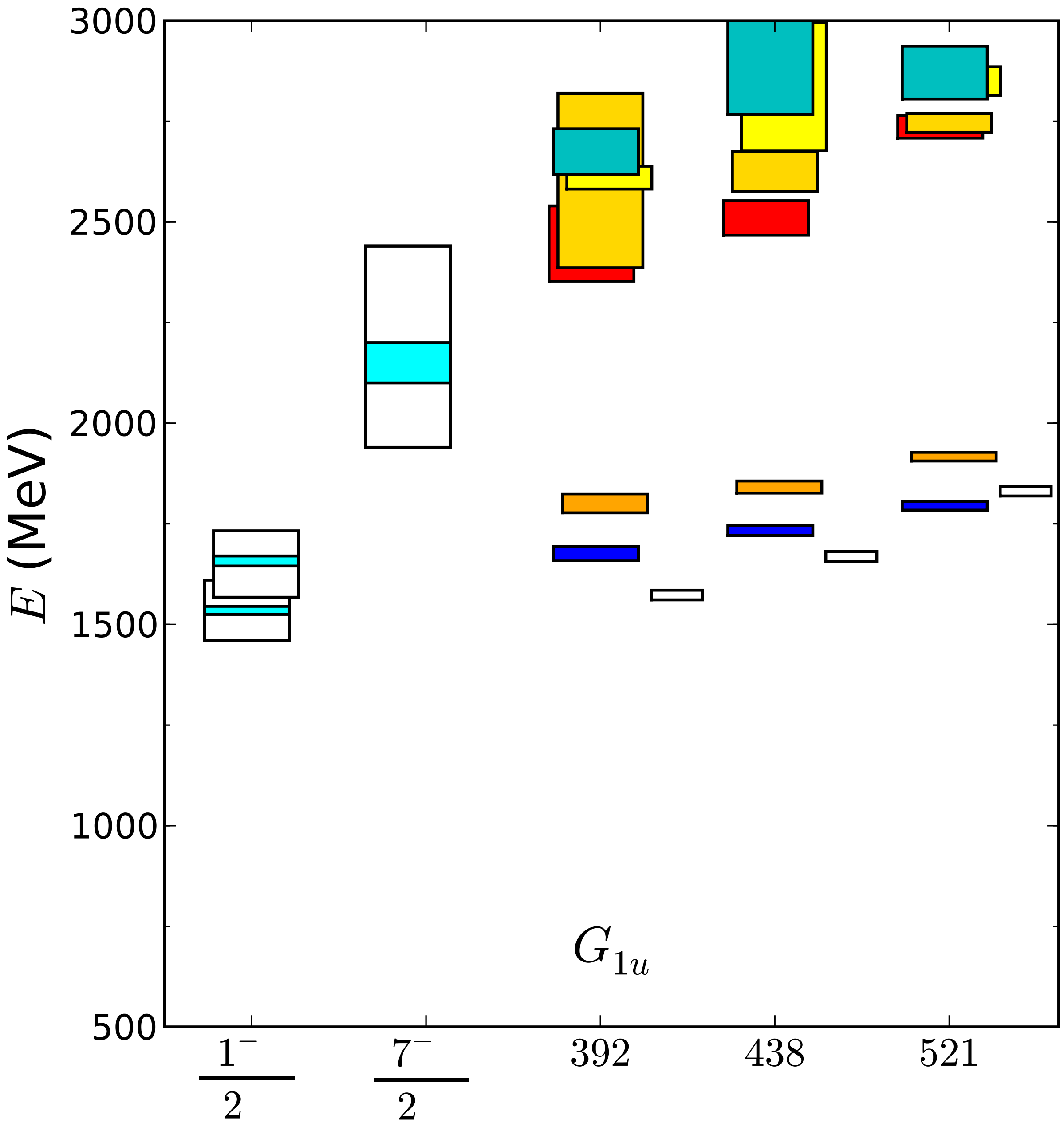
E (MeV)
G_{1g}
$\frac{1}{2}^+$
$\frac{7}{2}^+$
392
438
521



3000
2500
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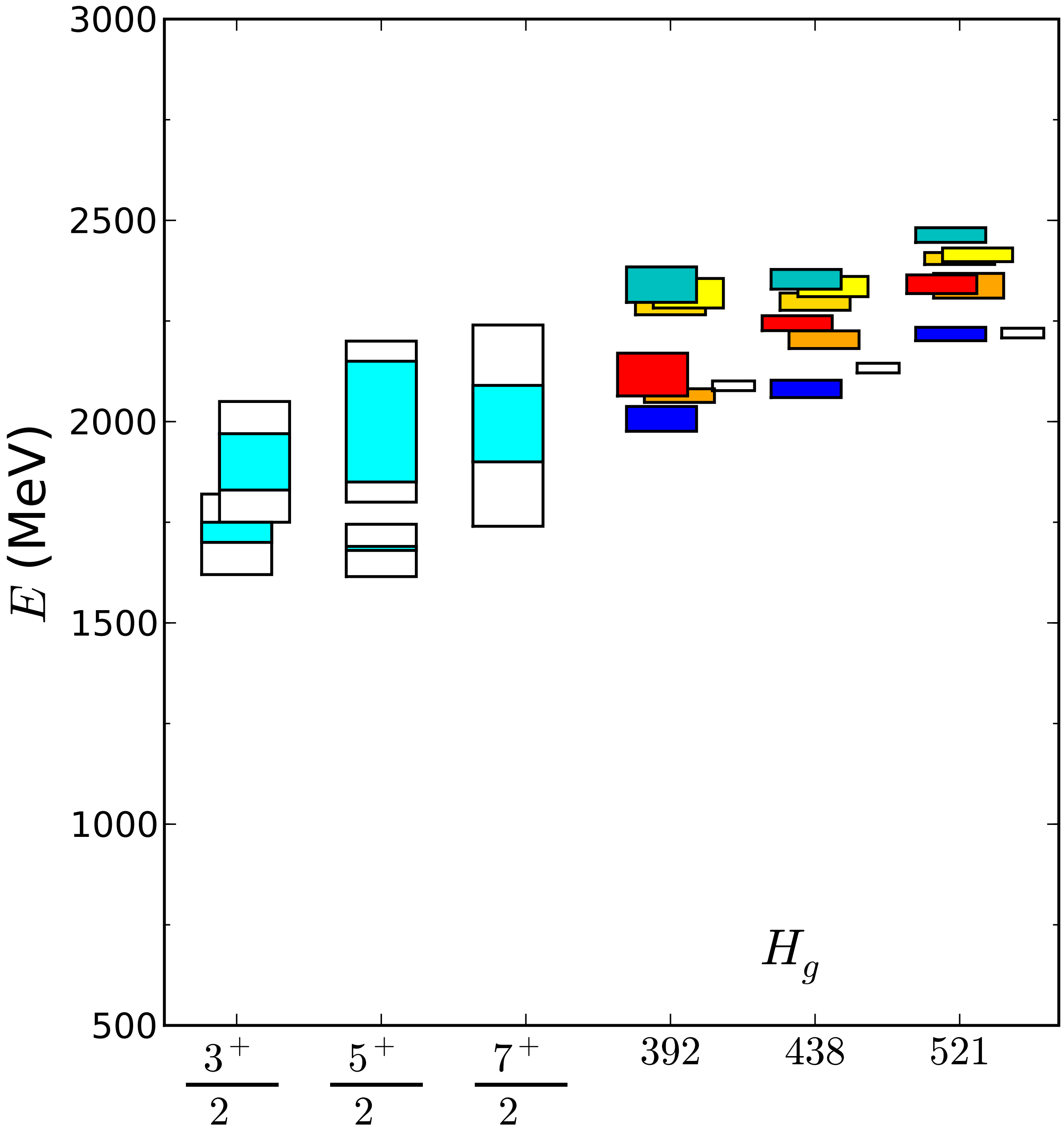
E (MeV)
G_{1u}
$\frac{1^-}{2}$
$\frac{7^-}{2}$
392
438
521


3000
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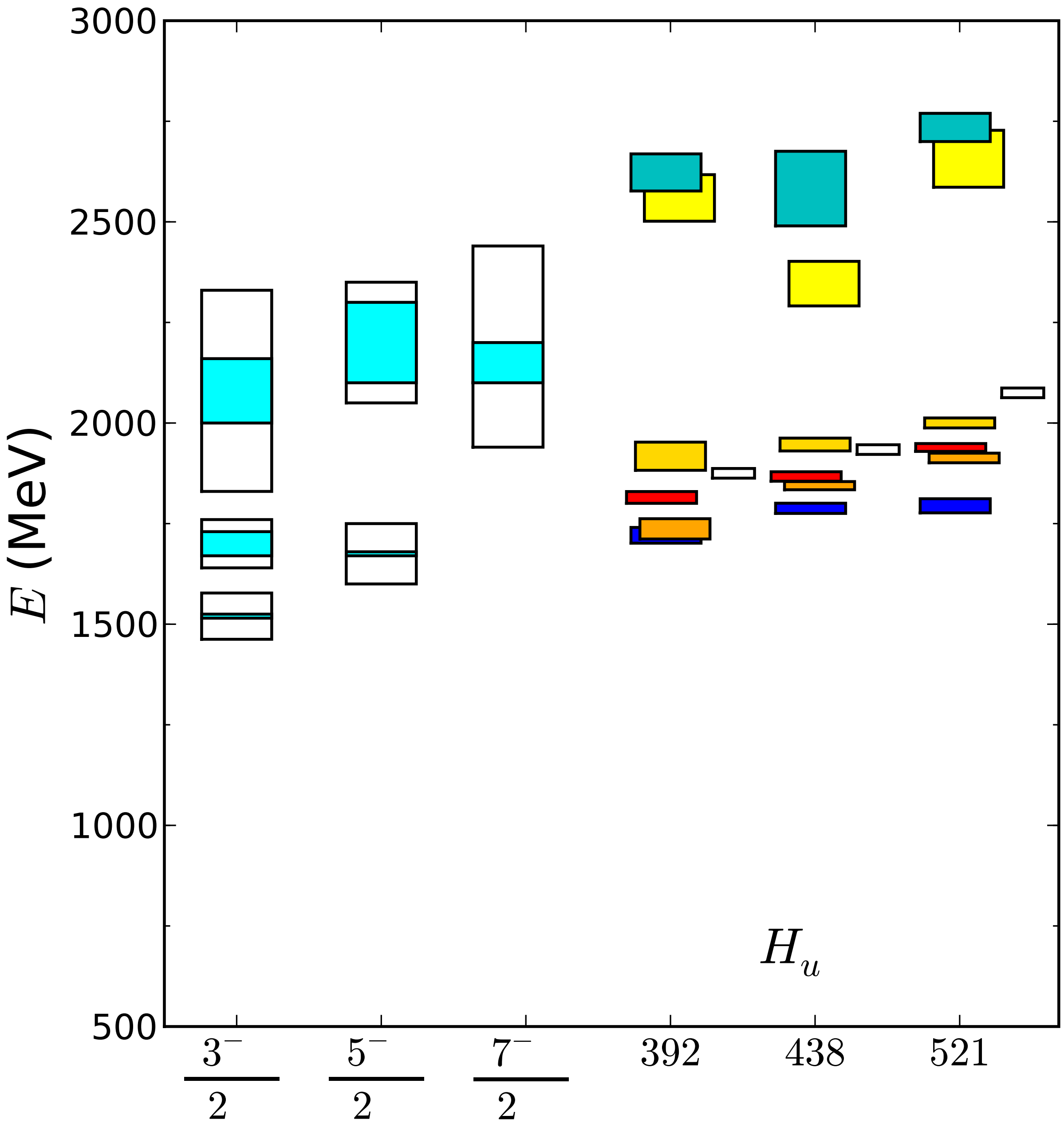
500
E (MeV)
H_g
3+/2
5+/2
7+/2
392
438
521



3000
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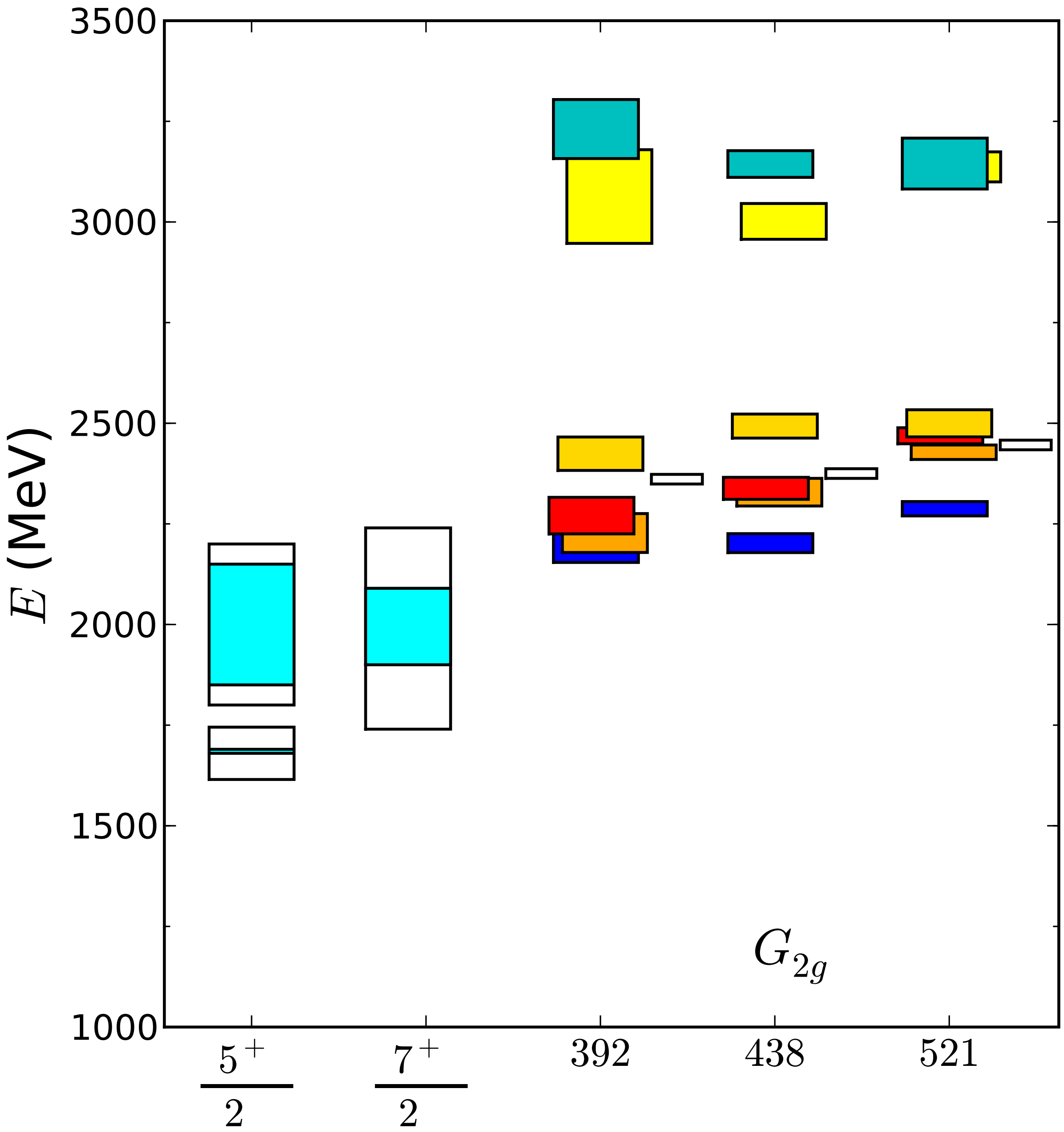
E (MeV)
H_u
$\frac{3^-}{2}$
$\frac{5^-}{2}$
$\frac{7^-}{2}$
392
438
521



3500
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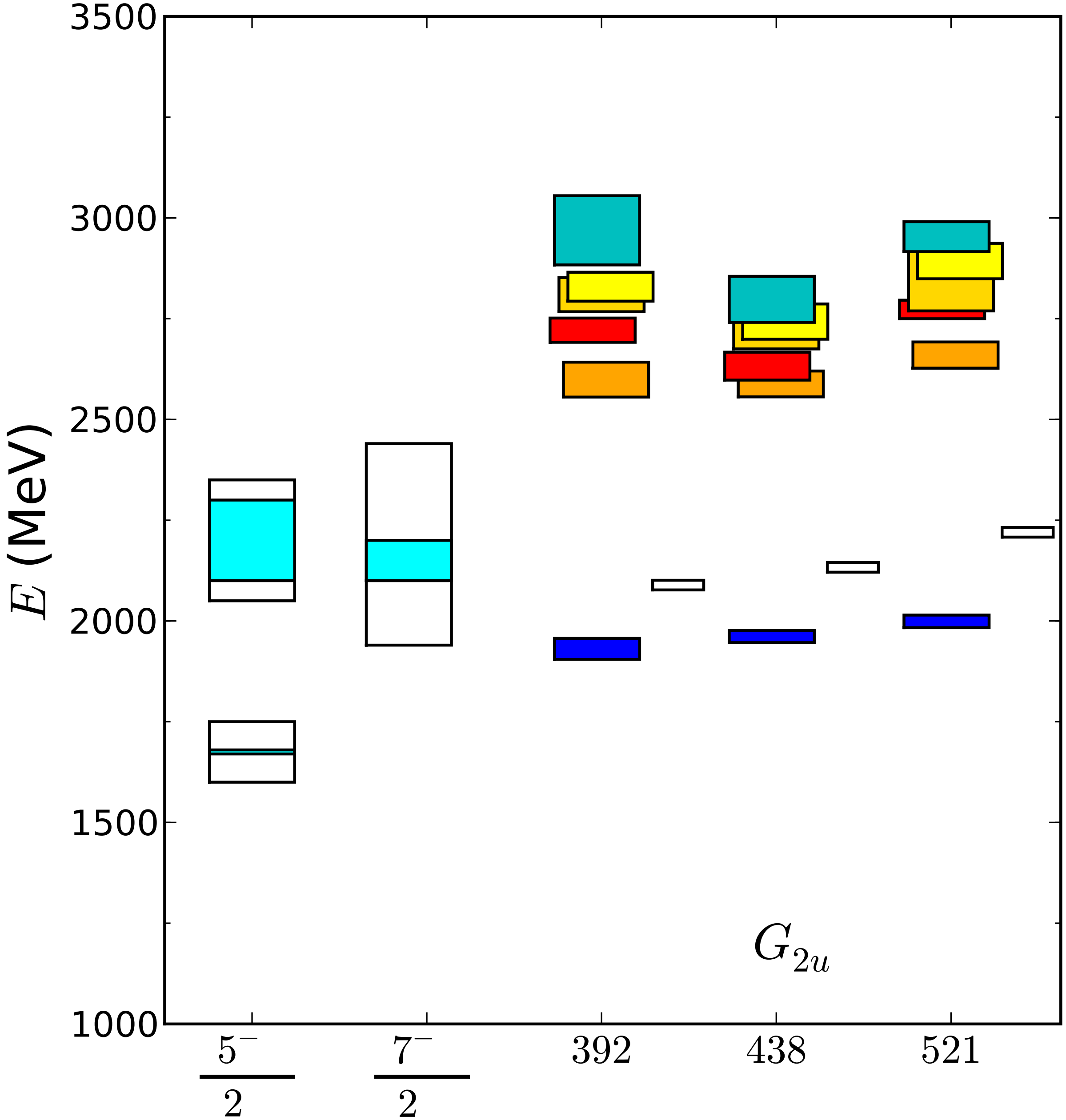
E (MeV)
G_{2g}
$\frac{5}{2}^+$
$\frac{7}{2}^+$
392
438
521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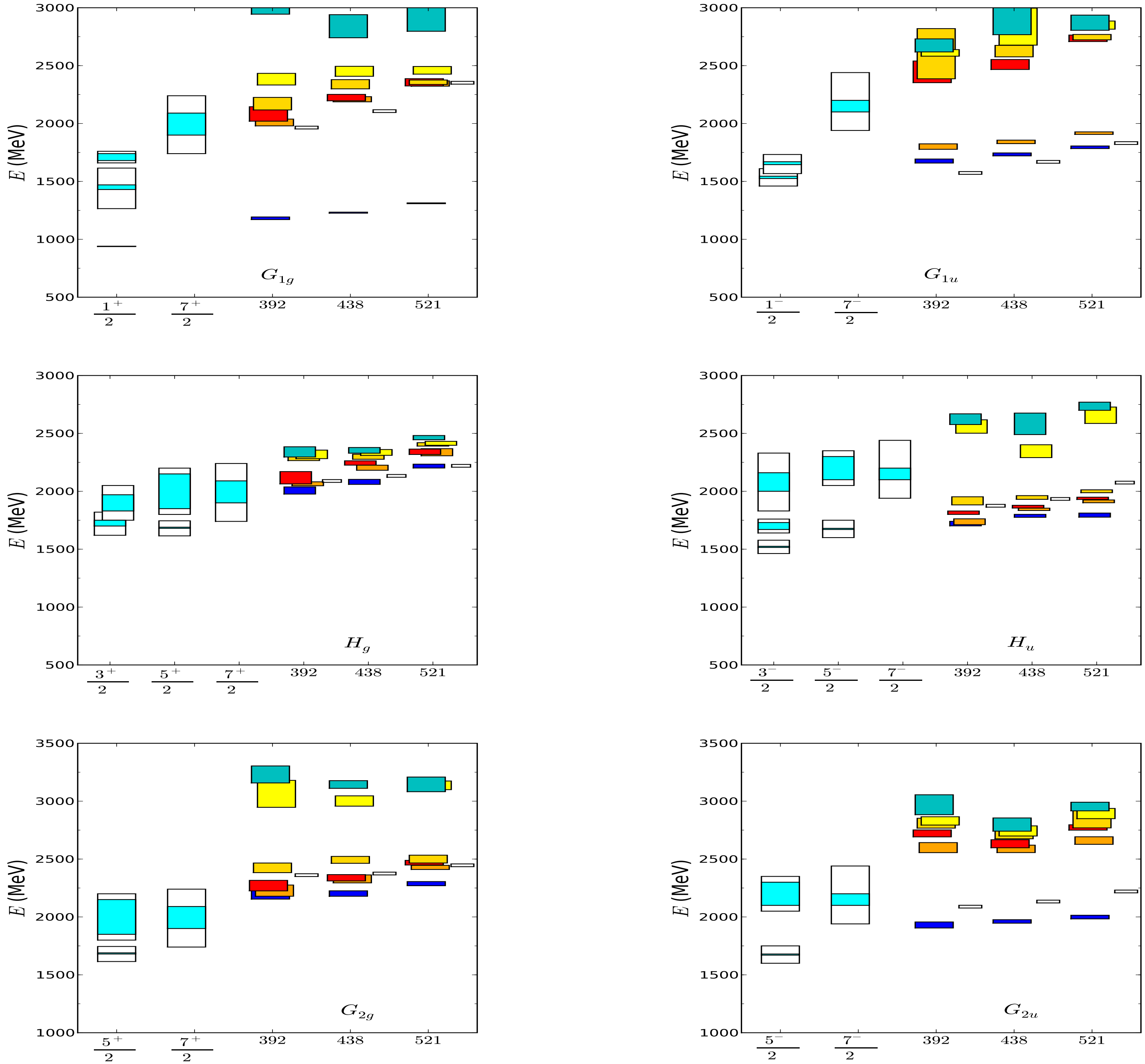
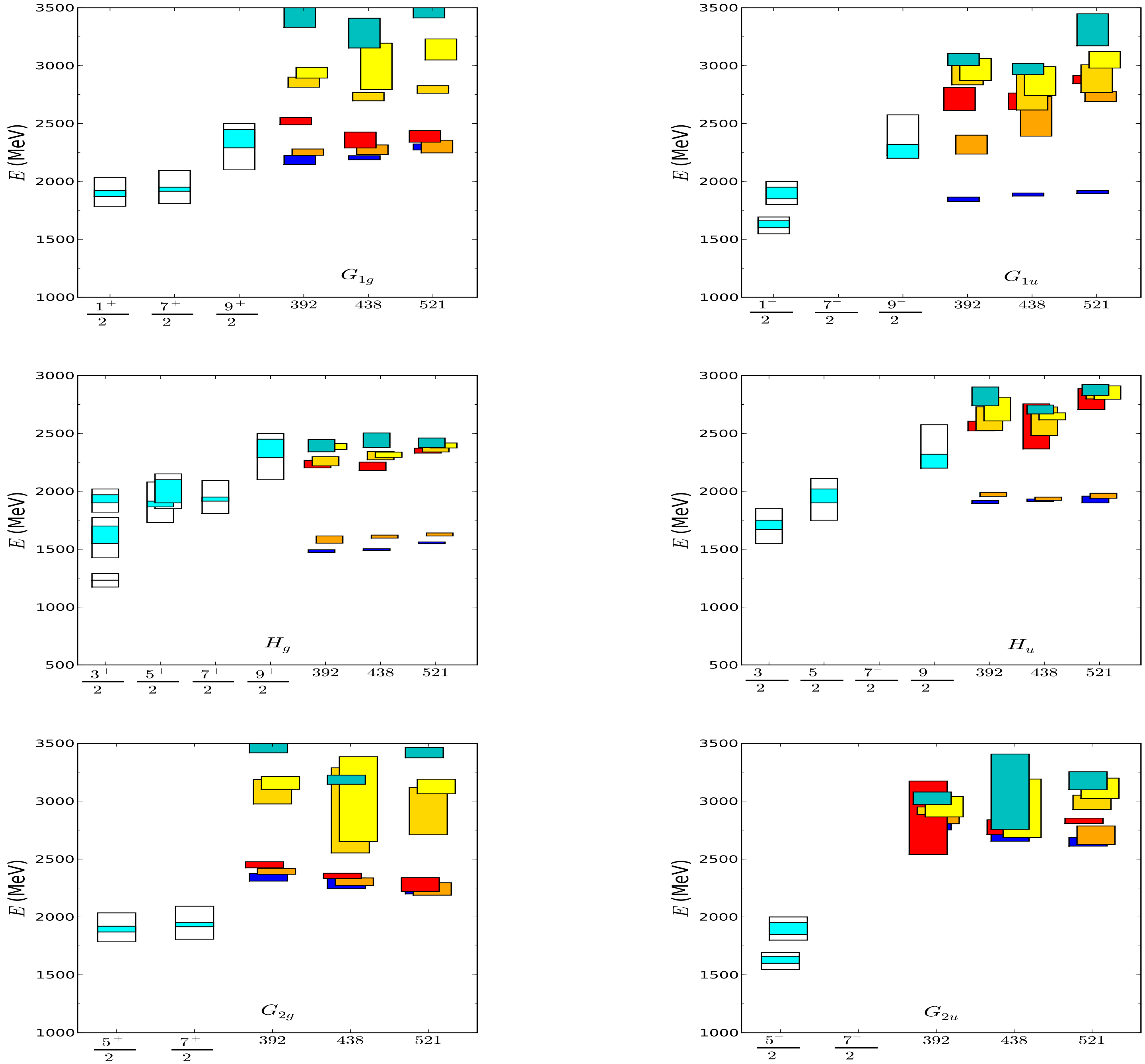
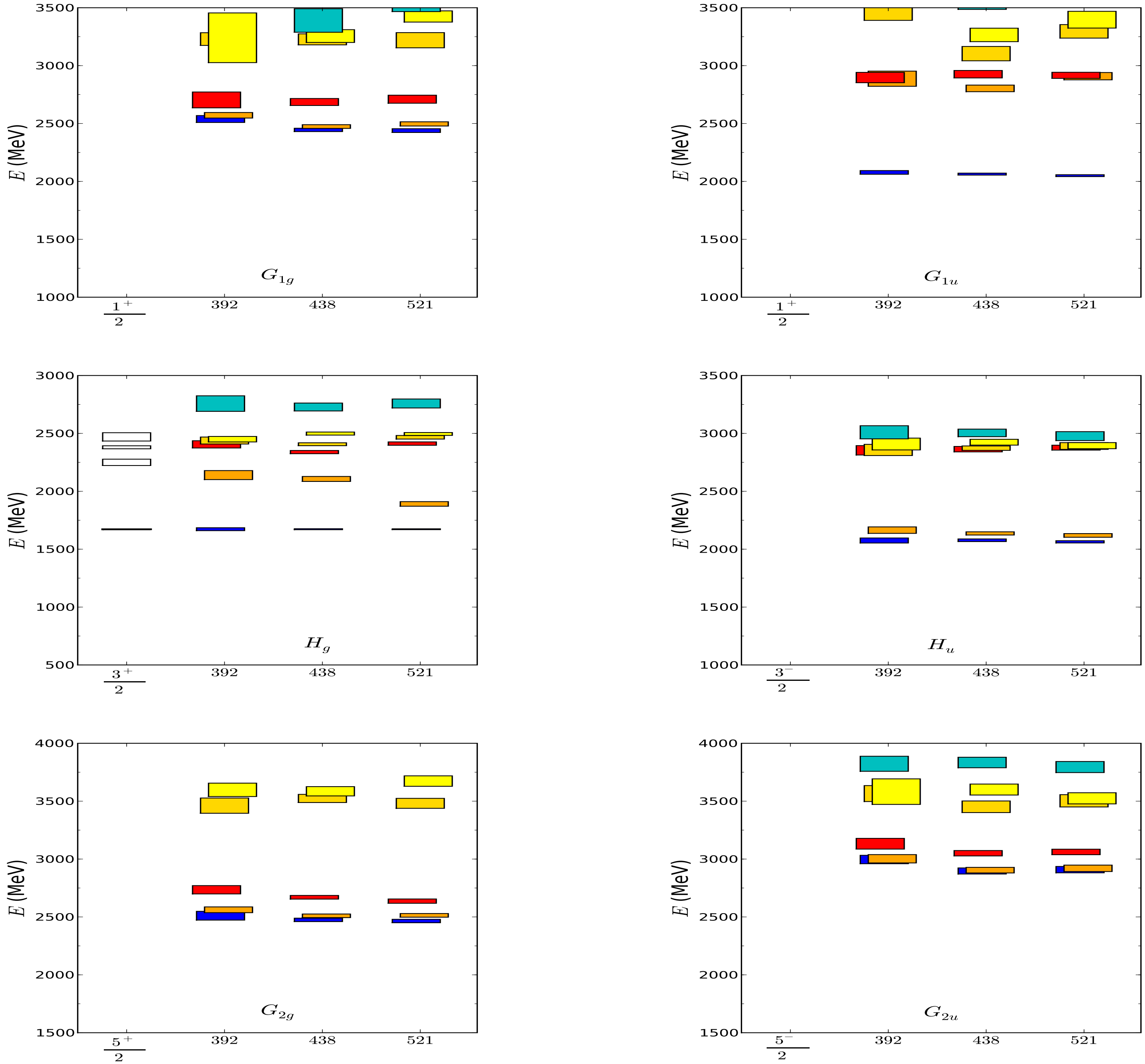


Patterns of Nucleon Spectra



Patterns of Delta Spectra



Patterns of Omega Spectra

Part II. Spin identification

- **Rotational symmetry is broken at $\mathcal{O}(\mathbf{a}^2)$ by lattice action**
- **Typical lattice spacing is 0.1 fm**
- **Typical hadron size is 1 fm**
- $\mathcal{O}(a^2) \approx \left(\frac{0.1 fm}{1.0 fm}\right)^2 \approx 0.01$
- **For hadrons, rotational symmetry should be broken weakly.**

Fresh start: Construction of operators with good J in continuum

- Mesons: Dudek, *et al.*, Phys.Rev.D80:054506,2009
- Baryons: Color singlet structure for 3 quarks, symmetric in space & spin
- $\mathbf{J} = \mathbf{L} + \mathbf{S}$ with
  - S = $\frac{1}{2}$ or $\frac{3}{2}$ from quark spins
  - L = 1 or 2 from covariant derivatives
  - J = $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ and $\frac{7}{2}$
- Lots of operators $\mathcal{O}^{[J,M]}$ with good spin in continuum limit
- Feynman, Kislinger and Ravndal formalism for quark states applied to operator construction.

Subduction to IRs of cubic group

- Why? Because lattice IRs provide orthogonal basis, not the J,M IRs
- In quantum mechanics, subduction is a change of basis $|J, M\rangle \to |\Lambda, r; J\rangle$.
- $|\Lambda, r; J\rangle = \sum_M |J, M\rangle\langle J, M|\Lambda, r; J\rangle$

  $= \sum_M |J, M\rangle\ S^{J,M}_{\Lambda,r}.$
- Subduction coefficients: $S^{J,M}_{\Lambda,r}$
- Operators: $\mathcal{O}^{[\Lambda,r;J]} = \sum_M \mathcal{O}^{[J,M]} S^{J,M}_{\Lambda,r}$
- If rotational symmetry is broken weakly,

  $\langle 0|\mathcal{O}^{[\Lambda,r;J]}(t)\mathcal{O}^{[\Lambda,r;J']\dagger}(0)|0 > \approx \delta_{J,J'}$

  is block diagonal in J.

Test 1: Block diagonality

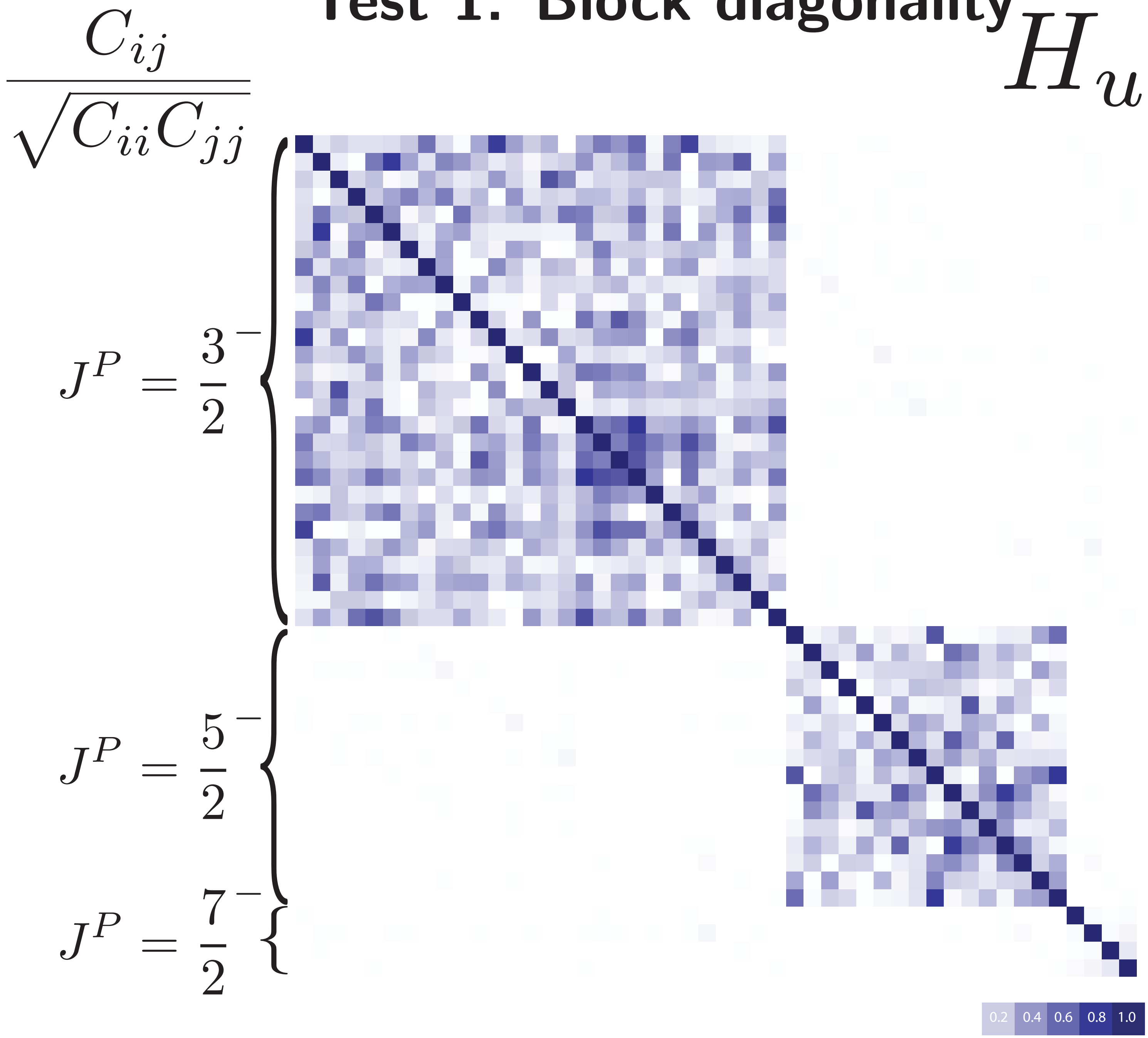
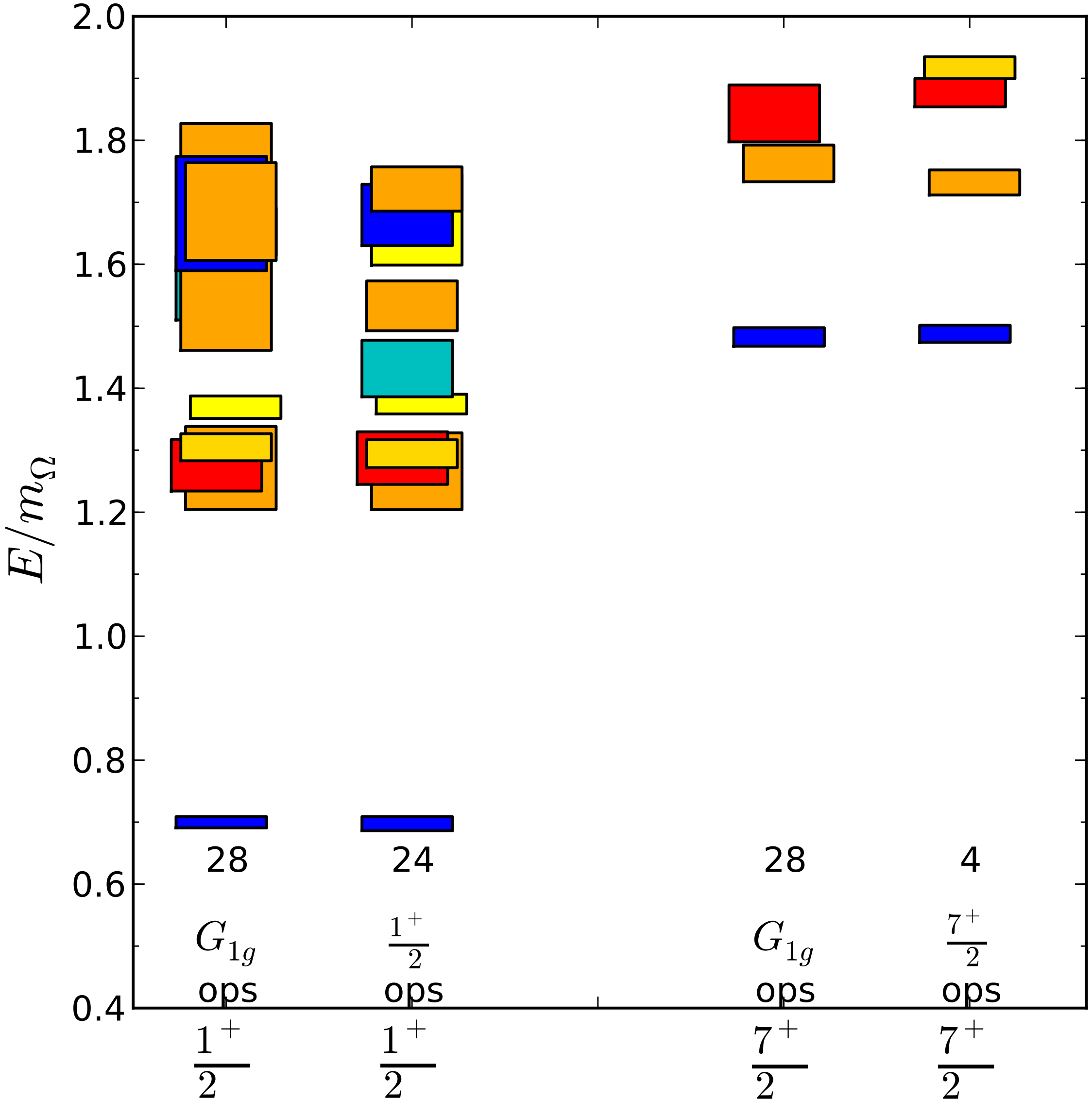
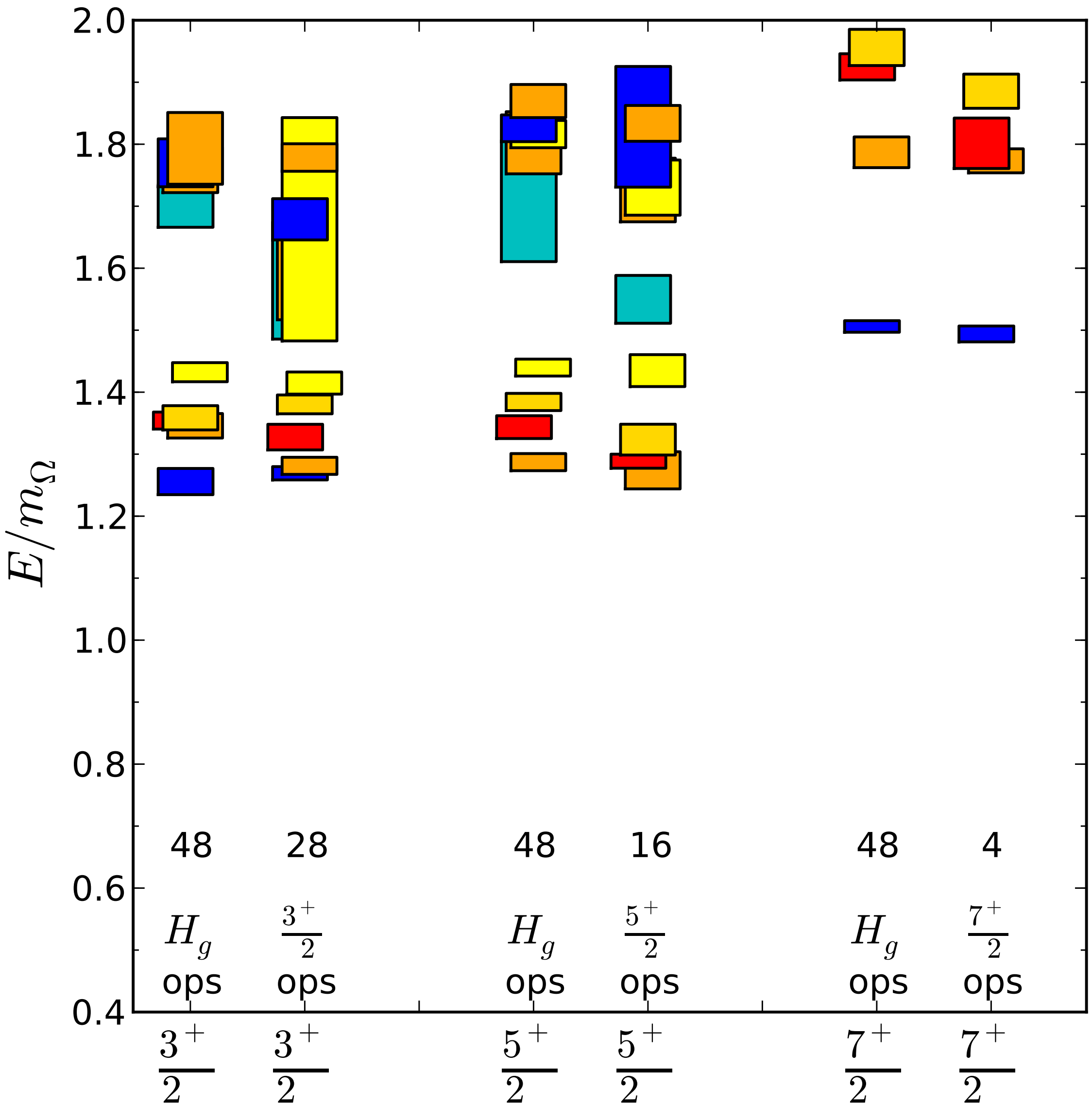


Figure 4: Magnitude of matrix elements in a matrix of correlation functions at timeslice 5.

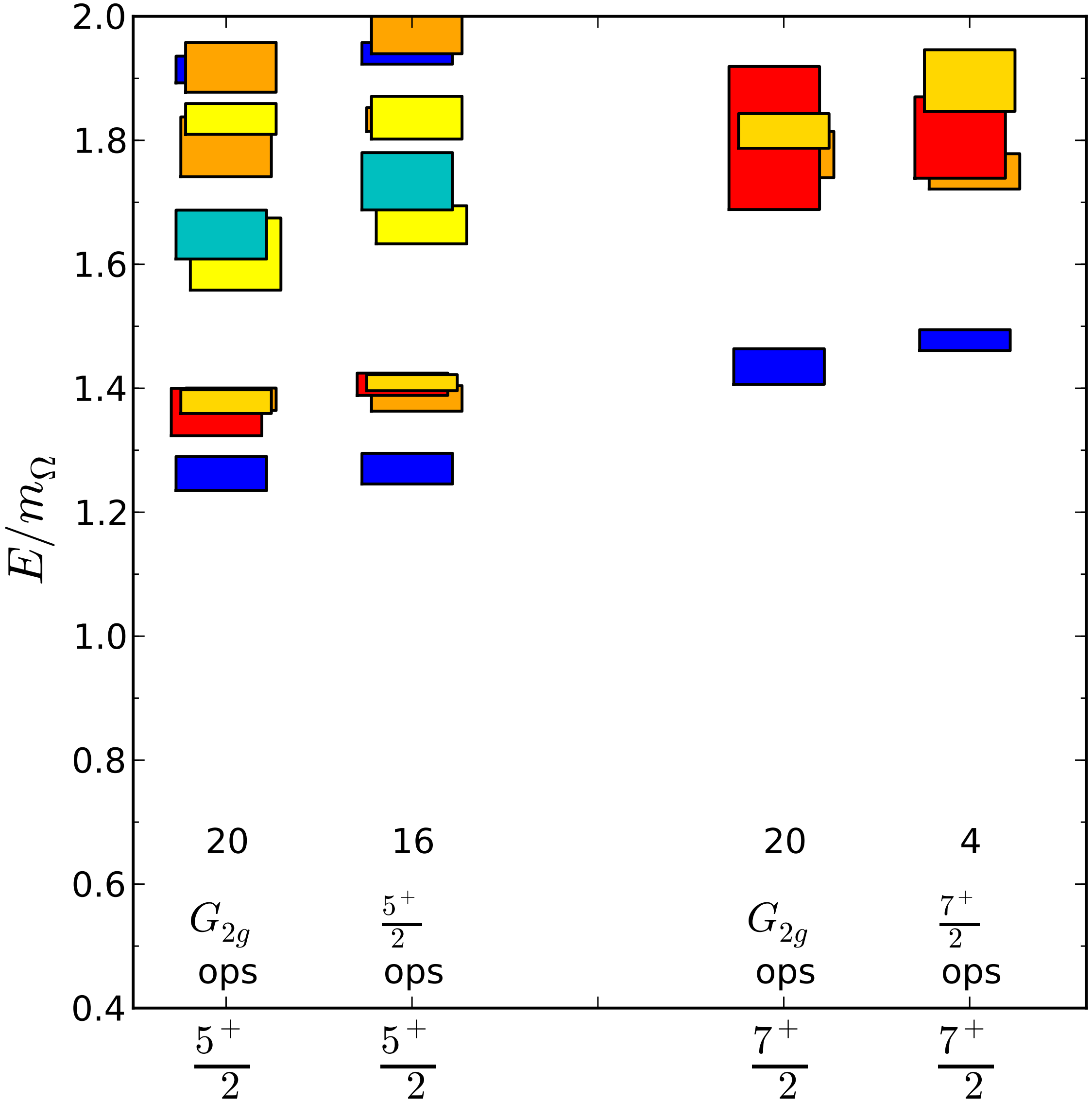
Test 2 for 28 G_{1g} energies



Test 2 for 48 H_g energies



Test 2 for 20 G_{2g} energies



How were the spins identified?

$$C_{ik}(t) = \sum_n \langle 0|\mathcal{O}_i(0)|n\rangle \mathbf{e^{-E_n t}} \langle n|\mathcal{O}_k^\dagger(0)|0\rangle$$

$$= \sum_n \quad Z_{ni}^* \quad \mathbf{e^{-E_n t}} \quad Z_{nk}$$

Spin weights

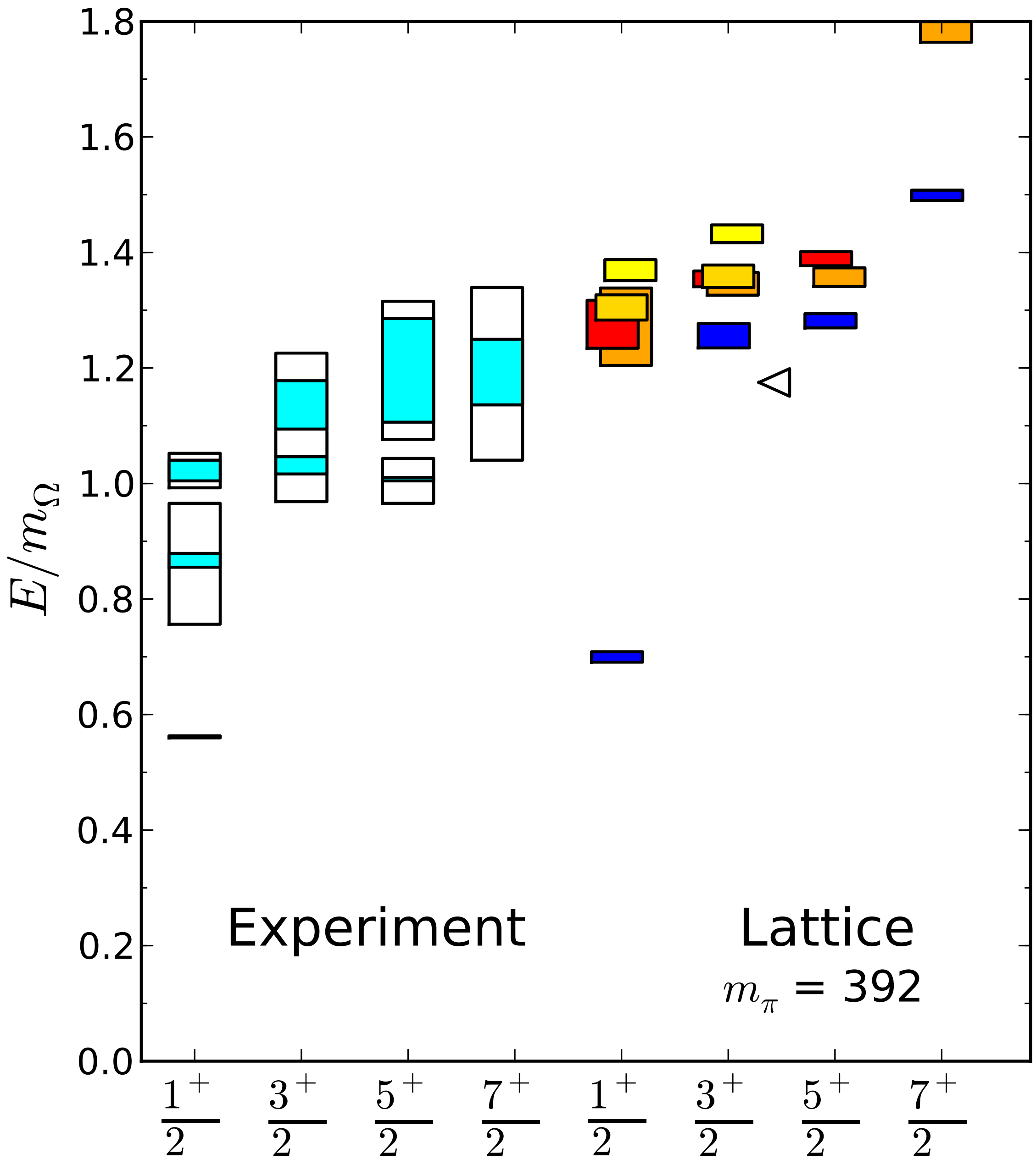
$$W_{nJ} = \frac{\sum_{k\subset J} |Z_{nk}|^2}{\sum_k |Z_{nk}|^2}$$

W_{nJ} is the relative weight for operators subduced from spin J in the creation of state $|n\rangle$:

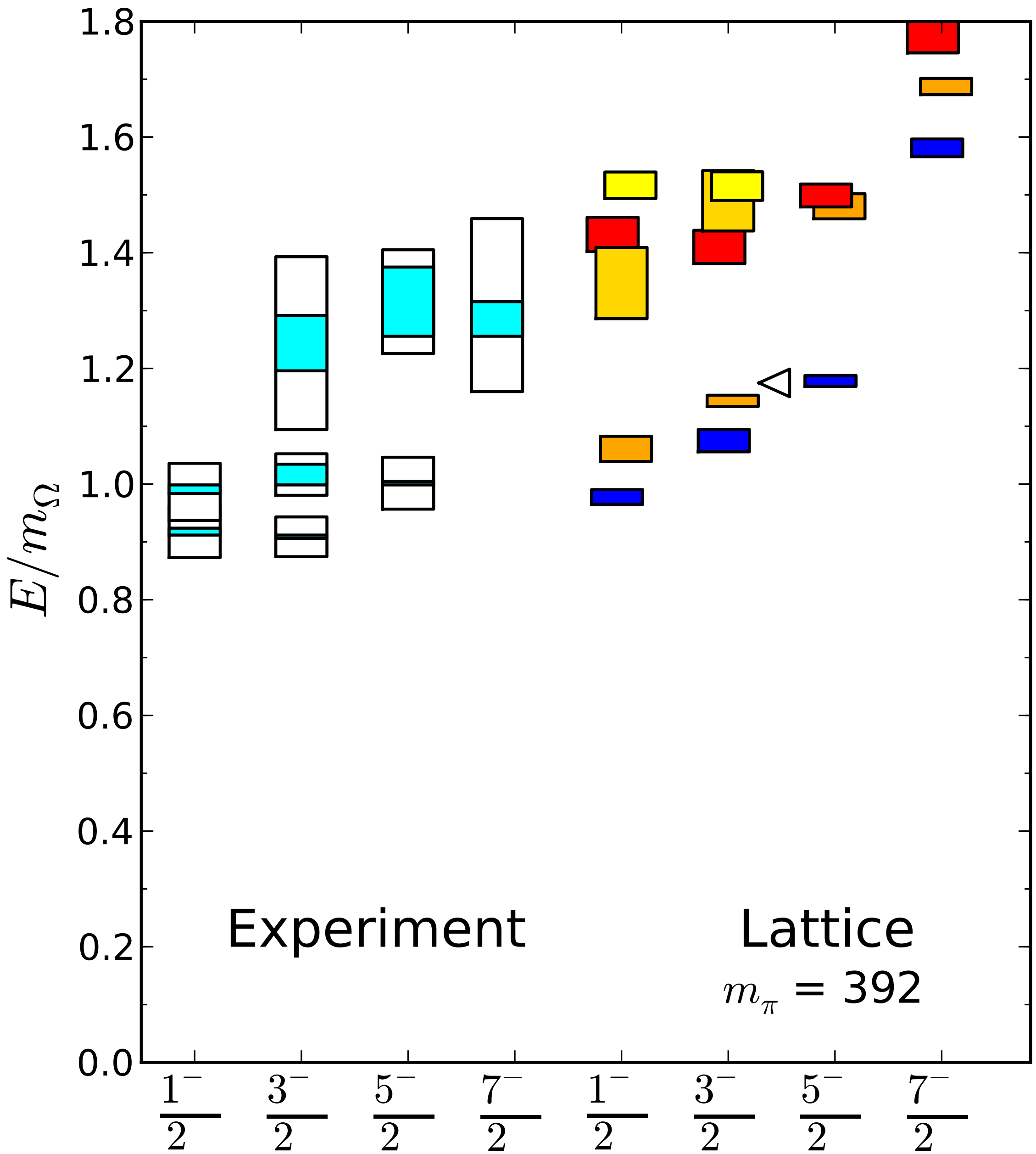
How well do weights identify the spins?

Table 1: Spin weights in % for ten G_{1g} energy levels.

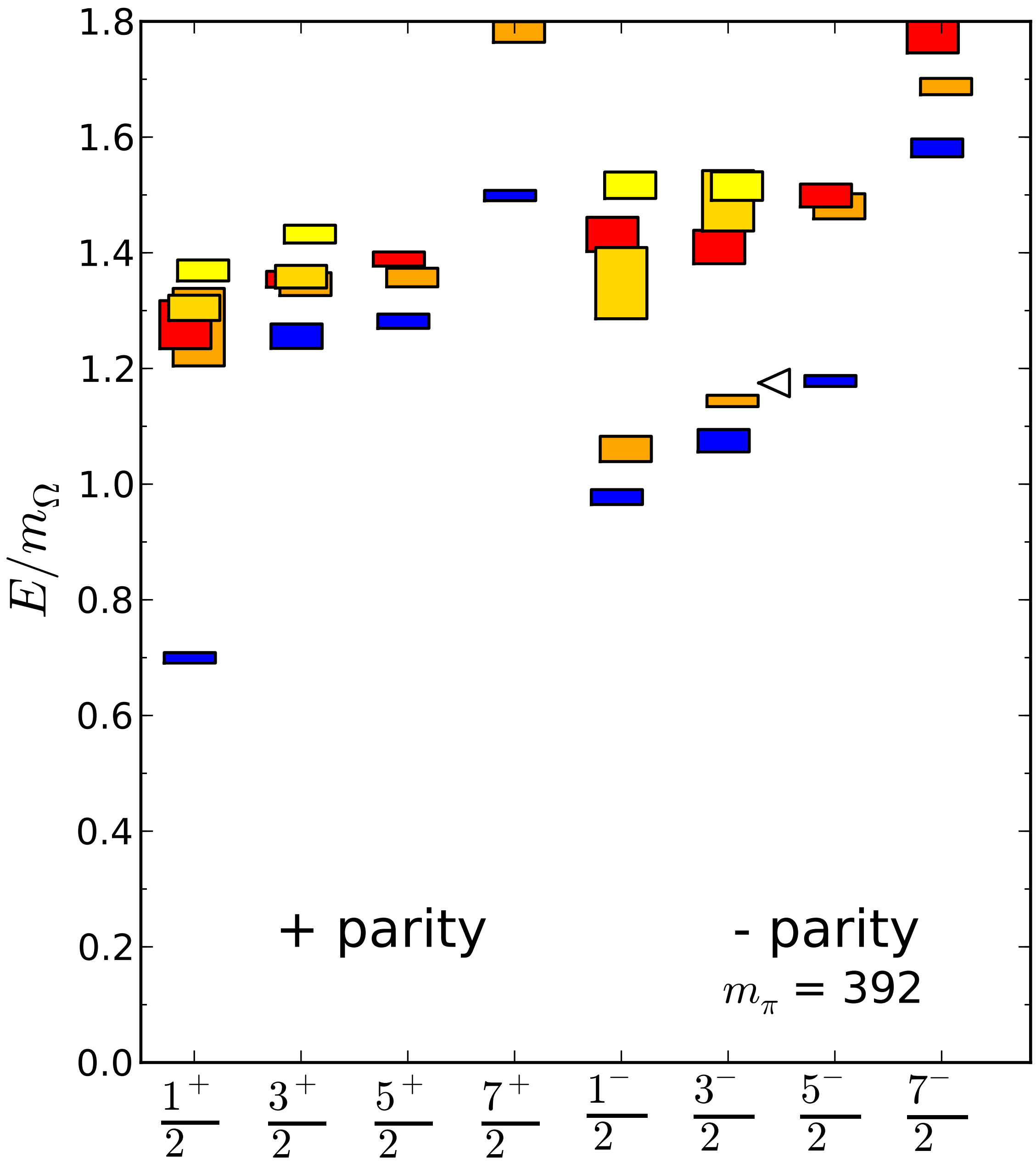
	E_n	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
G_{1g}-0	0.2081(16)	99.9	0	0	0
G_{1g}-1	0.3752(52)	99.9	0	0	0
G_{1g}-2	0.3830(66)	99.6	0	0	0.3
G_{1g}-3	0.3922(78)	99.9	0	0	0
G_{1g}-4	0.3944(71)	99.7	0	0	0.2
G_{1g}-5	0.4263(103)	99.6	0	0	0.3
G_{1g}-6	0.4398(41)	0.5	0	0	99.4
G_{1g}-7	0.5003(166)	97.9	0	0	2
G_{1g}-8	0.5020(114)	80.2	0	0	19.7
G_{1g}-9	0.5060(167)	99.7	0	0	0.2



Positive parity nucleon spectrum



Negative parity nucleon spectrum



Parity + and - nucleon spectrum

Nucleon states similar to 'quark-model' pattern

Phenomenology: Nucleon spectrum

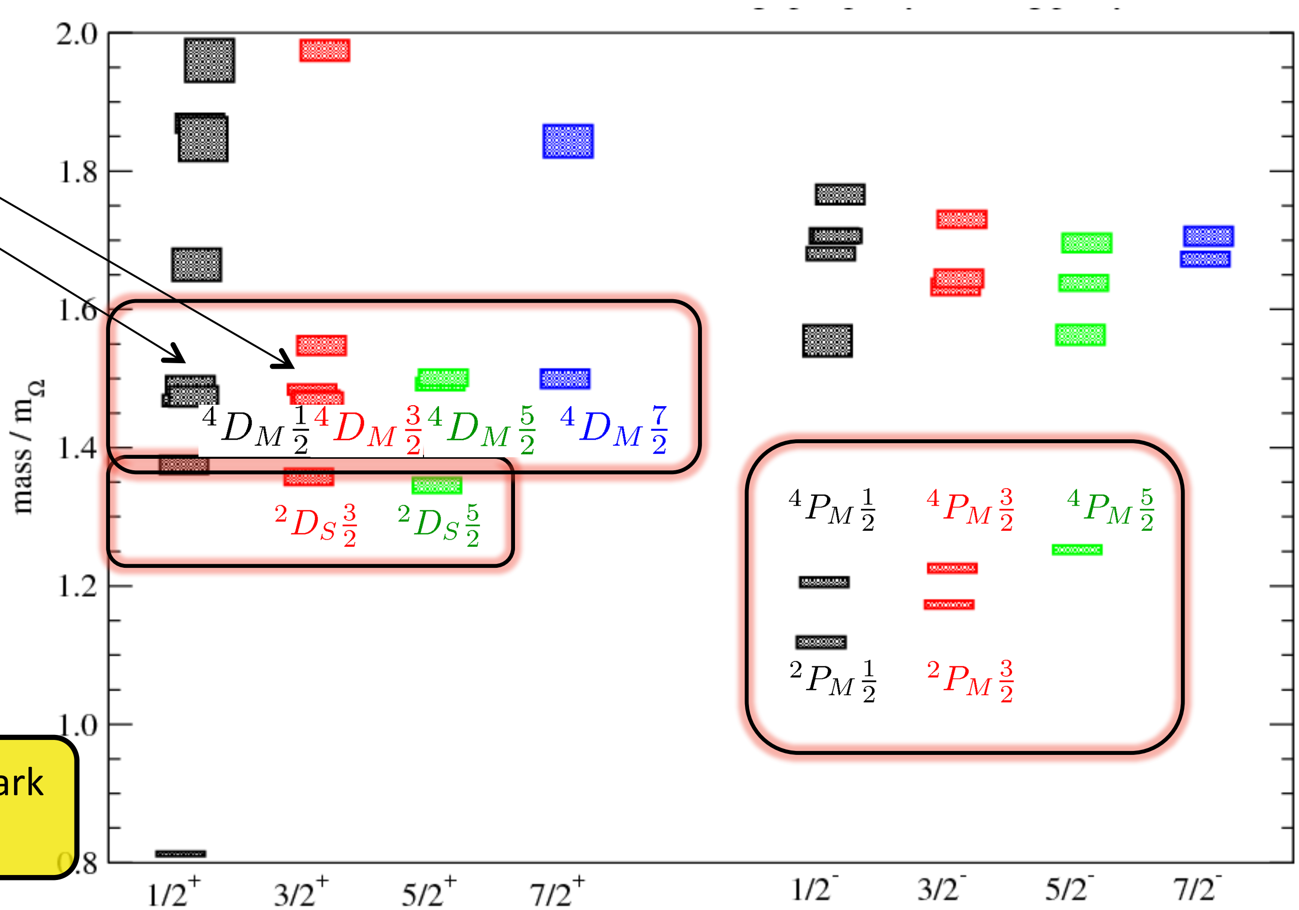
Discern structure: wave-function overlaps

$m_{1/4}$ ~ 520MeV

[20,1⁺] P-wave

[70,2⁺] D-wave

[56,2⁺] D-wave



[70,1⁻] P-wave

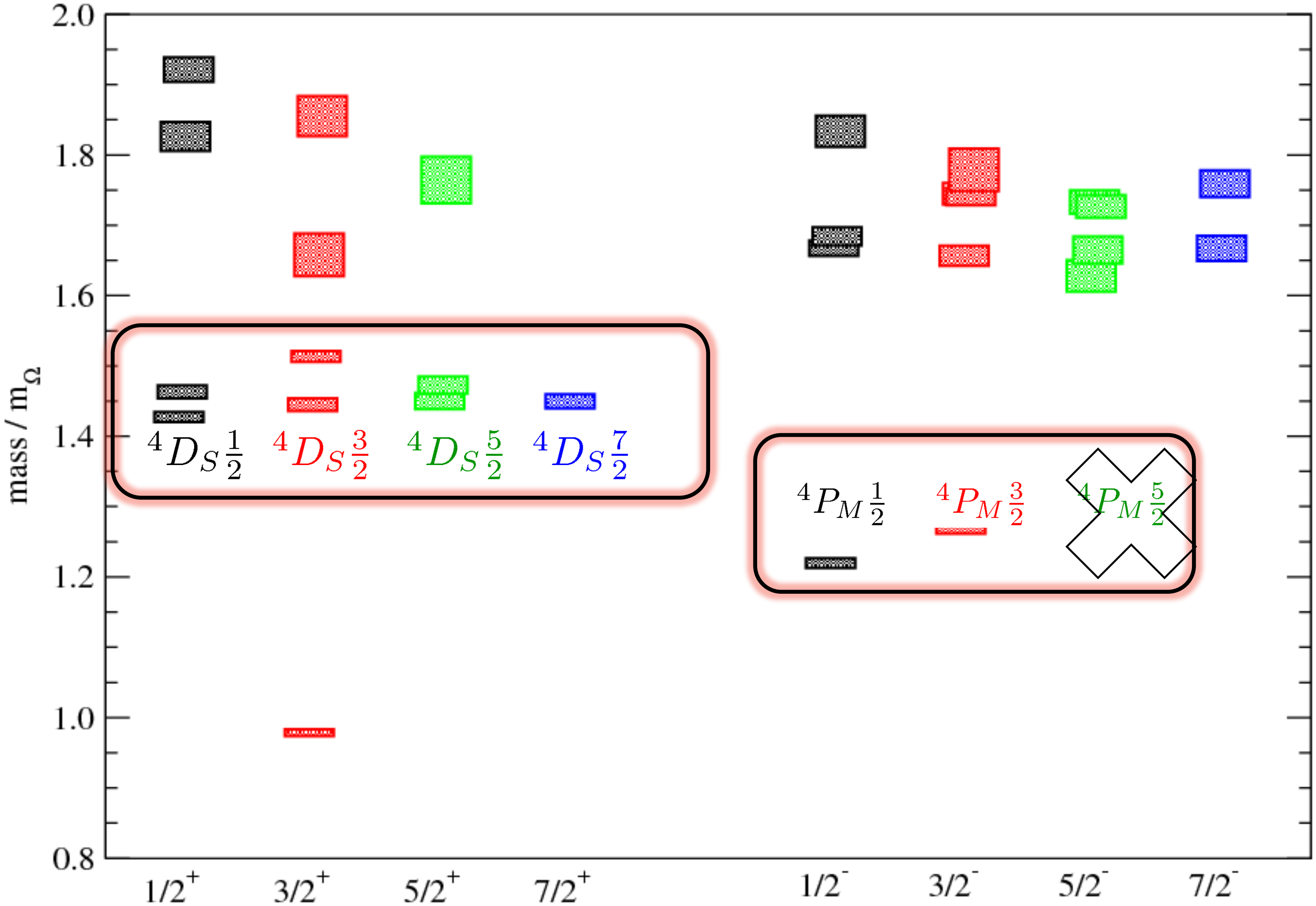
Looks like quark model?

Δ states also similar to 'quark-model' pattern

Spin identified ⊄ spectrum

Spectrum slightly higher than nucleon

[56,2⁺]
D-wave



[70,1⁻]
P-wave

Conclusions

- The patterns of lattice baryonic states are similar to the patterns of physical resonance states.
- Spin identification based on subduction of continuum J works well.
- Lots of baryonic states, but no sign of chiral restoration.

The path forward

- Multiparticle operators are needed to include scattering states (e.g, πN).
- Multiple volumes are needed for determination of phase shifts using Luscher's formalism.
- Lower m_π is needed in order to approach the physical limit.

Lattice parameters

- N_f = 2+1 QCD
  - Gauge action: Symanzik-improved
  - Fermion action: Clover-improved Wilson
- Anisotropic: a_s= 0.122 fm, a_t = 0.035 fm

ensemble	1	2	3
m_ℓ	−.0840	−.0830	−.0808
m_s	−.0743	−.0743	−.0743
Volume	$16^3 \times$ 128	$16^3 \times$ 128	$16^3 \times$ 128
$N_{\rm cfgs}$	344	570	481
$t_{\rm sources}$	4	4	4
m_π	0.0691(6)	0.0797(6)	0.0996(6)
m_K	0.0970(5)	0.1032(5)	0.1149(6)
m_Ω	0.2951(22)	0.3040(8)	0.3200(7)
m_π (MeV)	392(4)	438(3)	521(3)

Part I. N, Δ and Ω spectra

- **Many interpolating field operators in each IR of octahedral group:** Prune to $\approx$ 10
- **"Distillation" technology for smearing: Peardon, *et al.*, Phys. Rev. D80, 054506 (2009)** Use 32 eigenvectors of Laplacian
- **Matrices of correlation functions:** Diagonalize them at $t^* \approx 8$, Fix eigenvectors at t^*.
- **Diagonal correlation functions:** Fit them & extract six energies
- **Lattice spectra:** Compare patterns with experimental resonance spectra.

Limitations

- **Three-quark operators:**
  - No multiparticle operators
  - Scant evidence for scattering states

- **One (small) volume:** No extrapolations or δ's

- $m_\pi =$ **392, 438, 521 MeV :** Energies generally are high.

- **Spins: A single** $J^P = \frac{5}{2}^-$ **pattern is seen.** Patterns for higher spins are ambiguous.

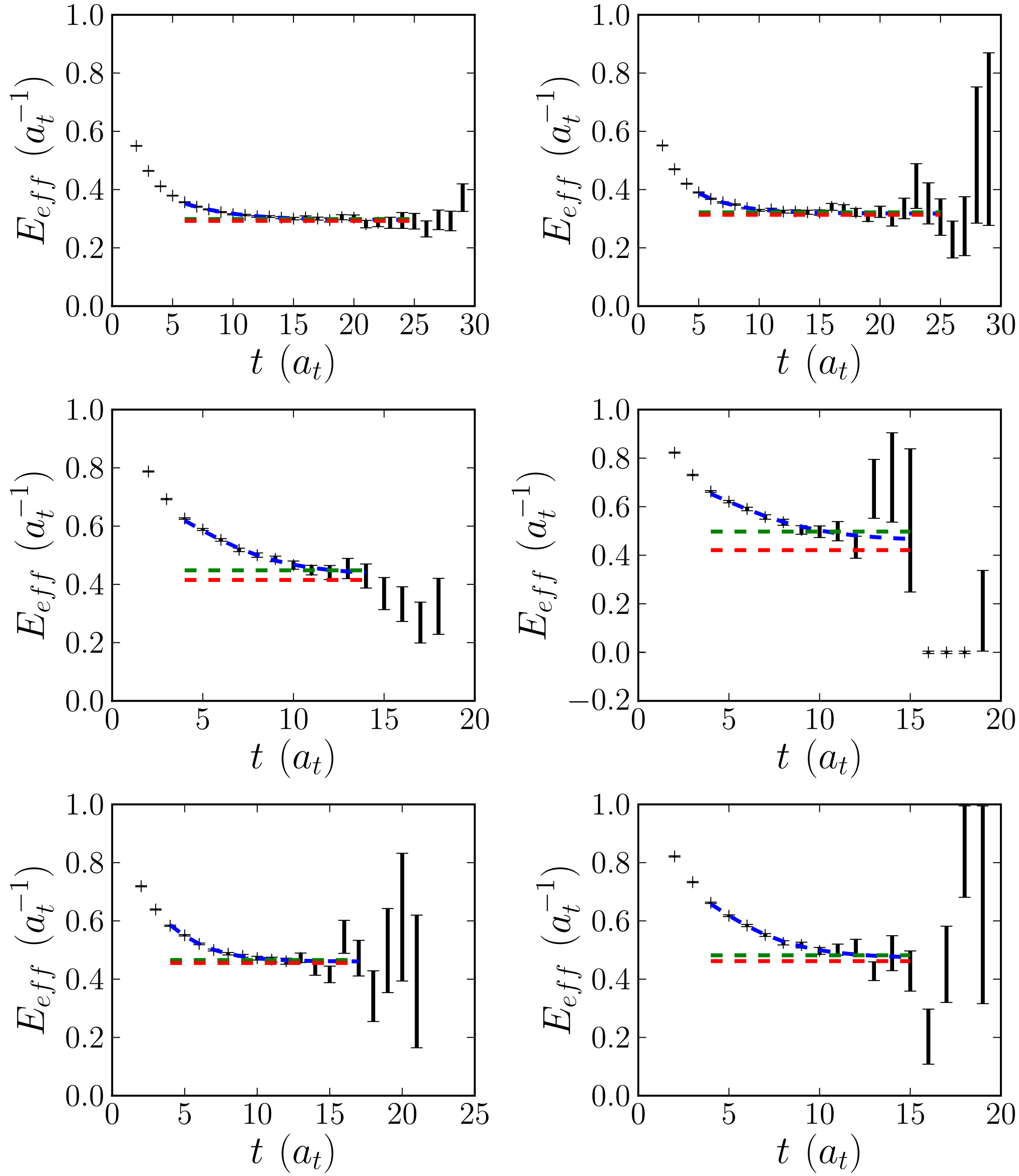
Computational Resources

- USQCD allocations
- Jefferson Laboratory clusters
- Fermi National Accelerator Lab clusters
- and the Chroma software system (Edwards *et al.*)

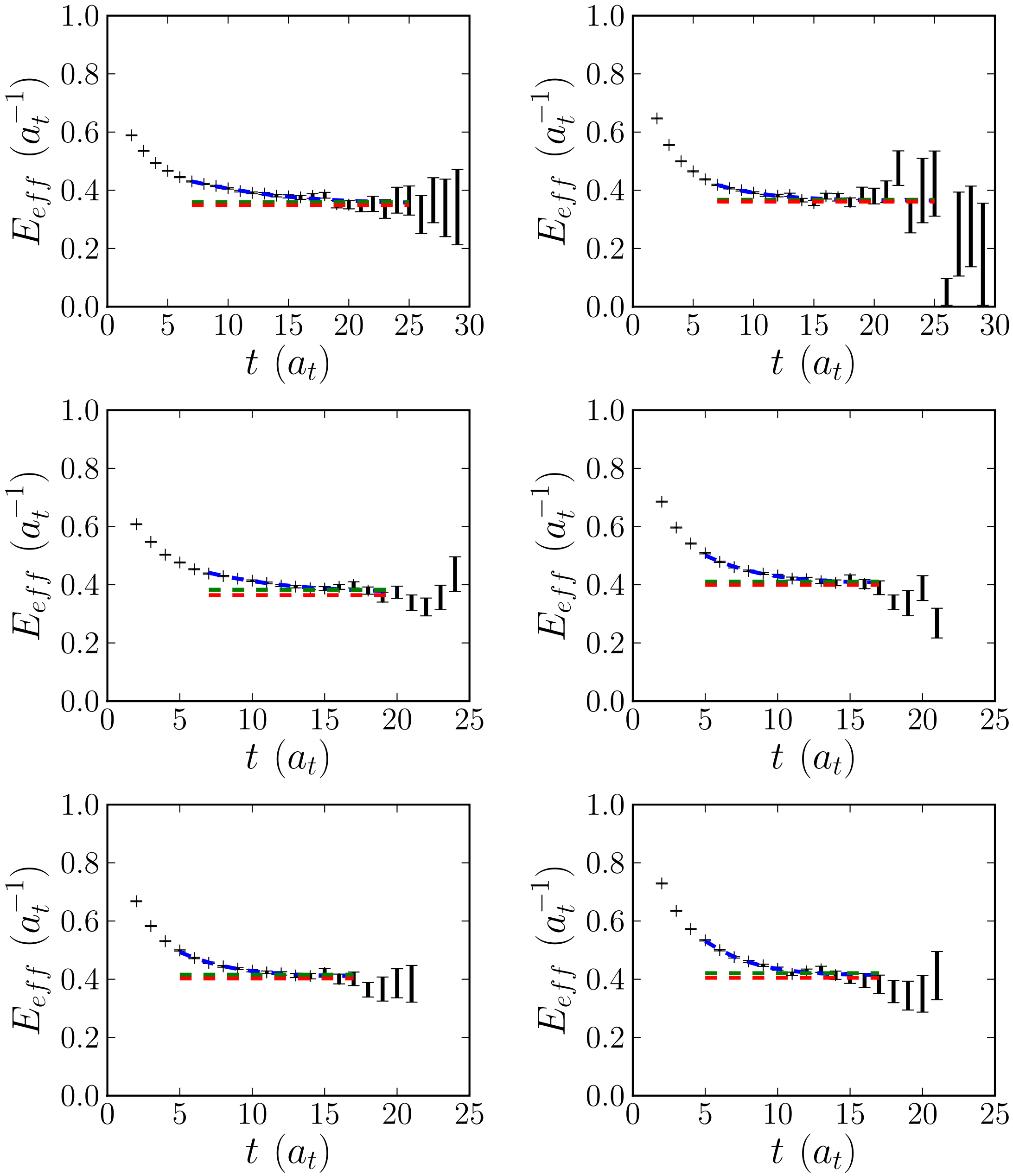
Subduction of J to $\mathcal{O}_D$

IR	Parity	Dimension	J $\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
G_{1g}	+1	2	1			1
H_g	+1	4		1	1	1
G_{2g}	+1	2			1	1
G_{1u}	-1	2	1			1
H_u	-1	4		1	1	1
G_{2u}	-1	2			1	1

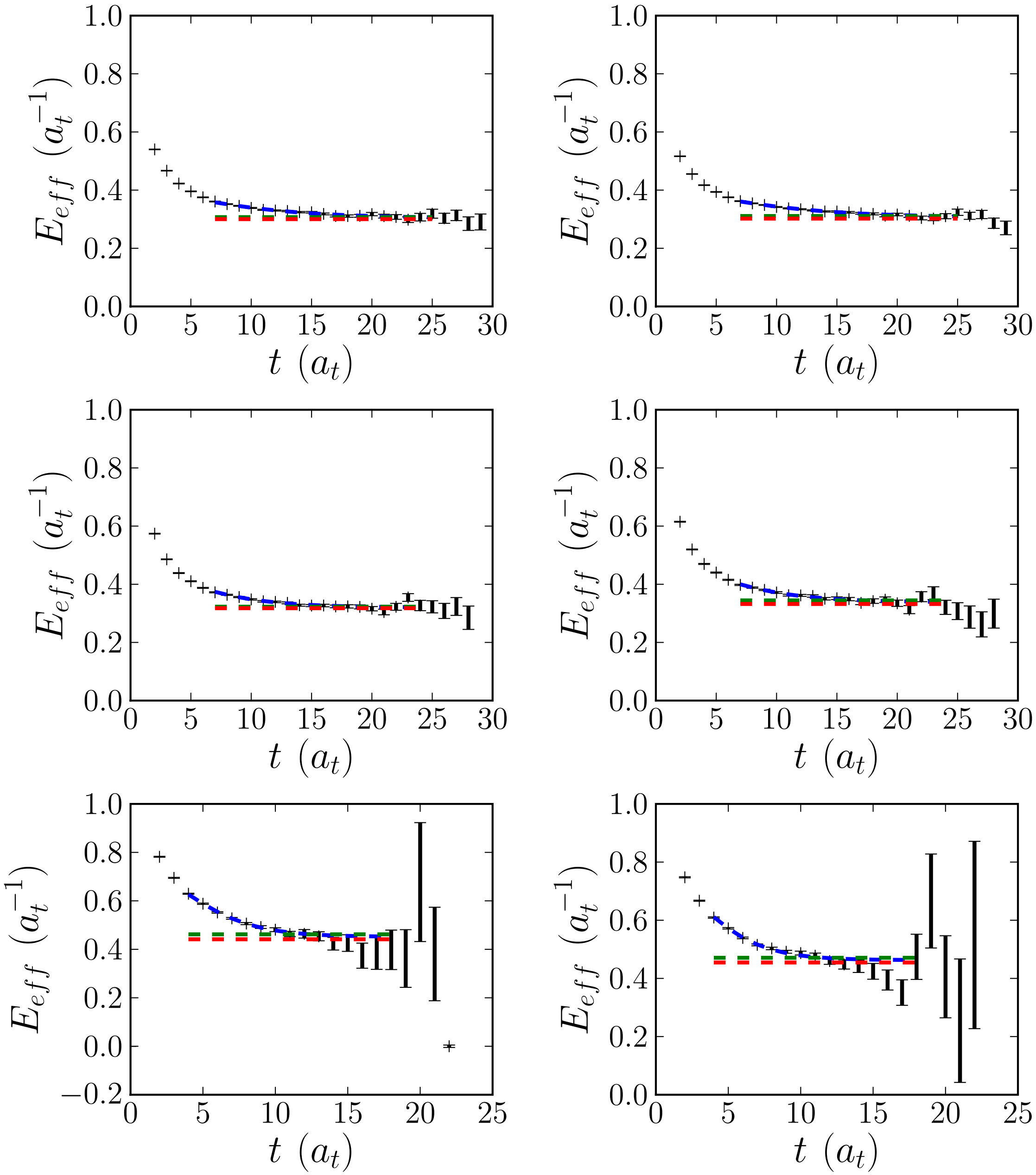
- Isolated G_1 state $\rightarrow$ Spin $\frac{1}{2}$
- isolated H state $\rightarrow$ Spin $\frac{3}{2}$
- Degenerate G_2 and H states $\rightarrow$ Spin $\frac{5}{2}$
- Degenerate G_1, H and G_2 states $\rightarrow$ Spin $\frac{7}{2}$:



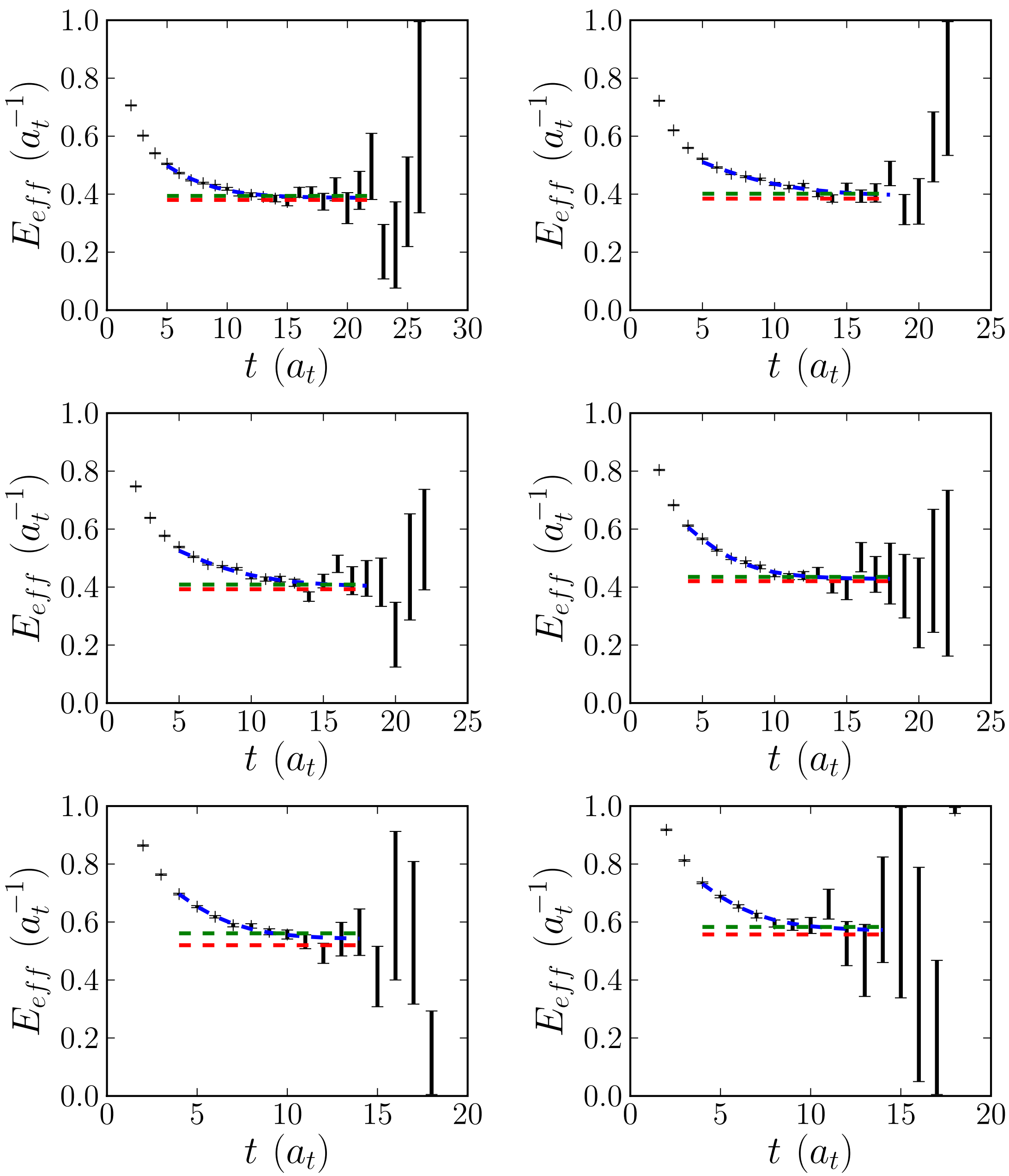
Nucleon G_{1u} effective energies: $m_\pi = 392(4)$ MeV



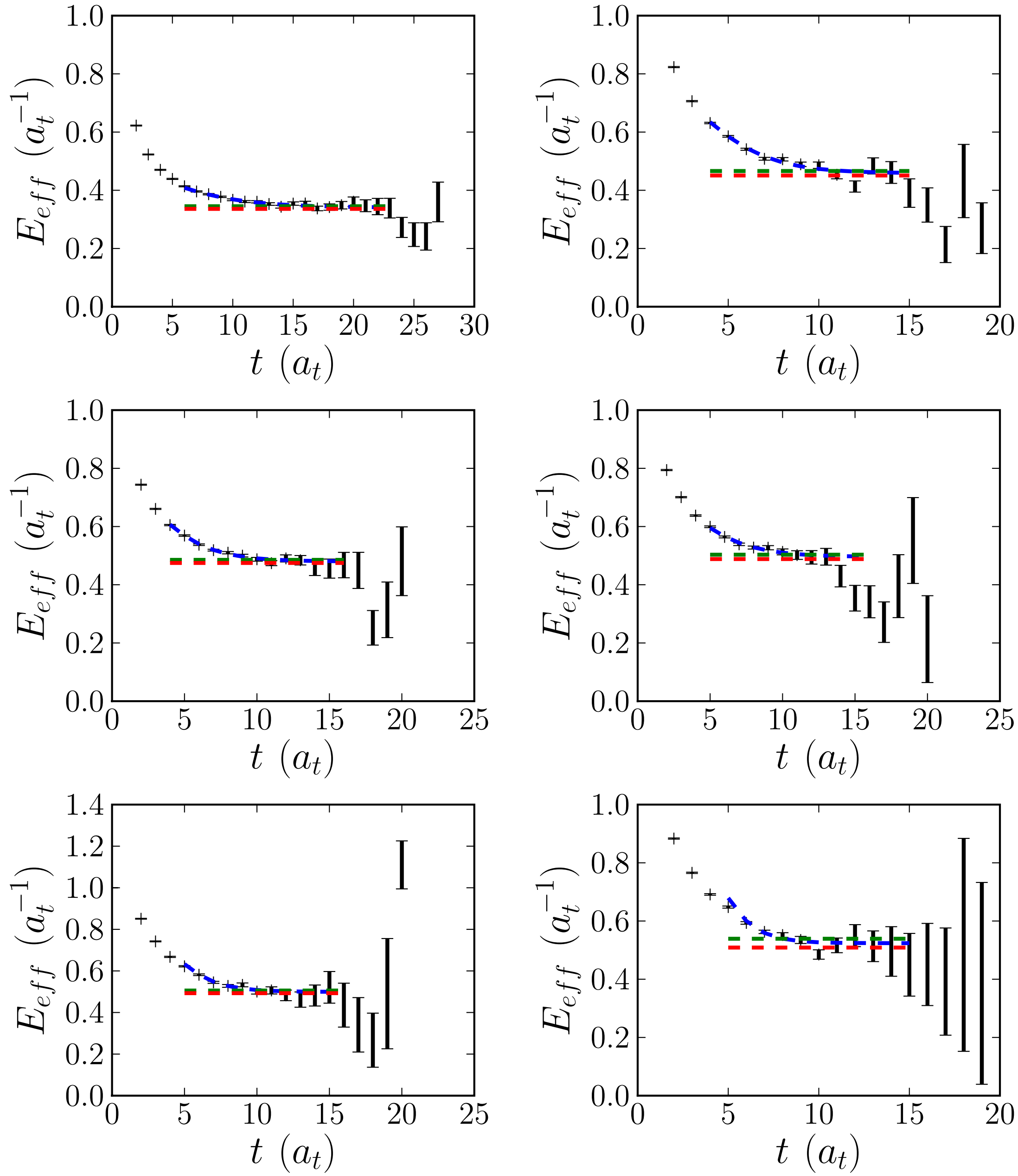
Nucleon H_g effective energies: m_π = 392(4) MeV



Nucleon H_u effective energies: m_π = 392(4) MeV



Nucleon G_{2g} effective energies: $m_\pi = 392(4)$ MeV



Nucleon G_{2u} effective energies: m_π = 392(4) MeV

Summary of Part I.

- First excited baryon spectrum based on $N_f = 2{+}1$ QCD using anisotropic lattices

- 6 lowest energy N, Δ and Ω states in each IR for $m_\pi = 392(4)$, 438(3) and 521(3) MeV.

- Patterns of lowest energies are similar to the patterns of lowest physical resonance states.

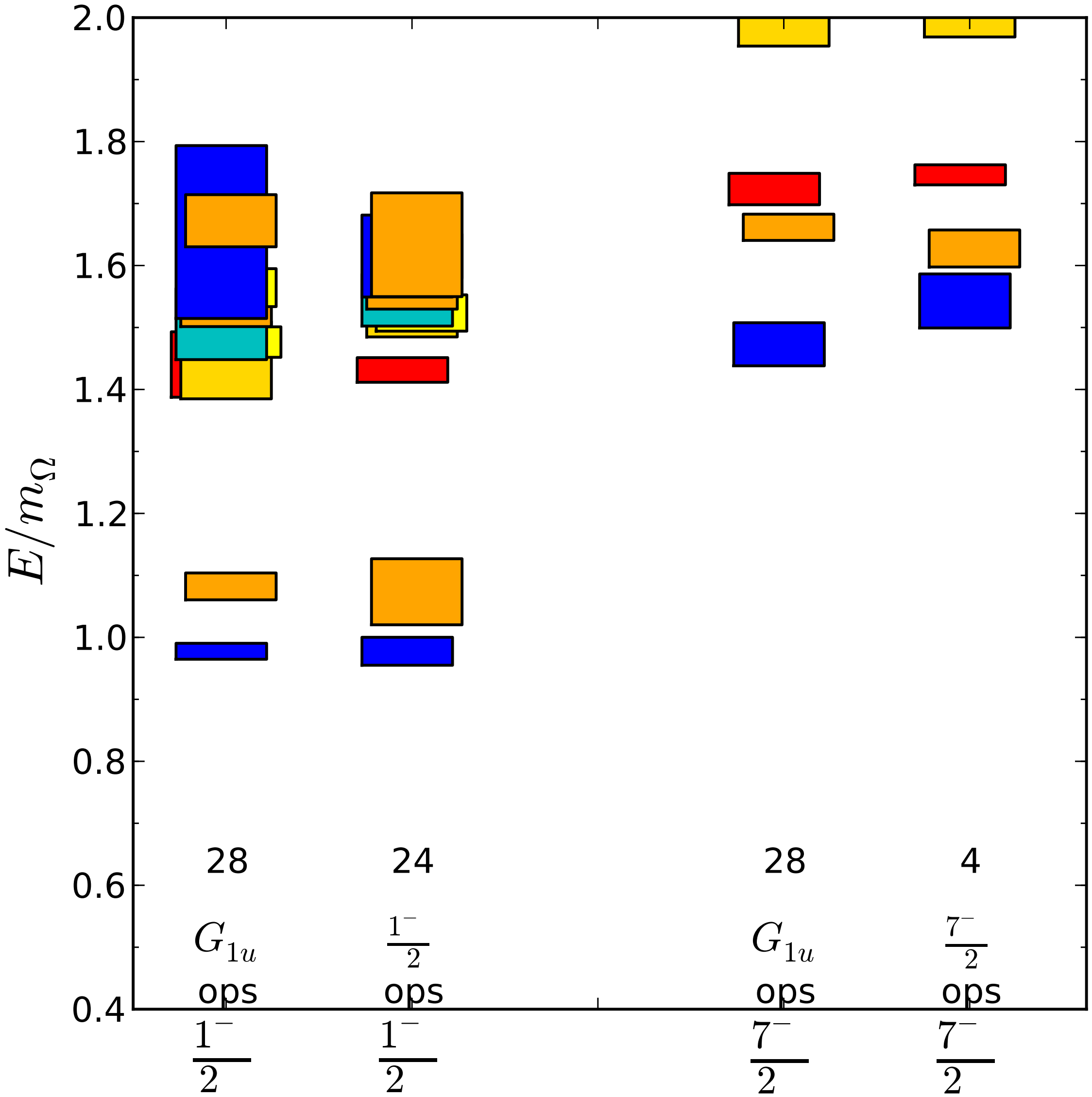
- Spin identification is very difficult. Degeneracies allow several subduction patterns to be compatible with results.

- Degenerate states in G_1, H and G_2: Could be a $J = \frac{7}{2}$ state or degenerate $J = \frac{1}{2}$ and $J = \frac{5}{2}$ states?

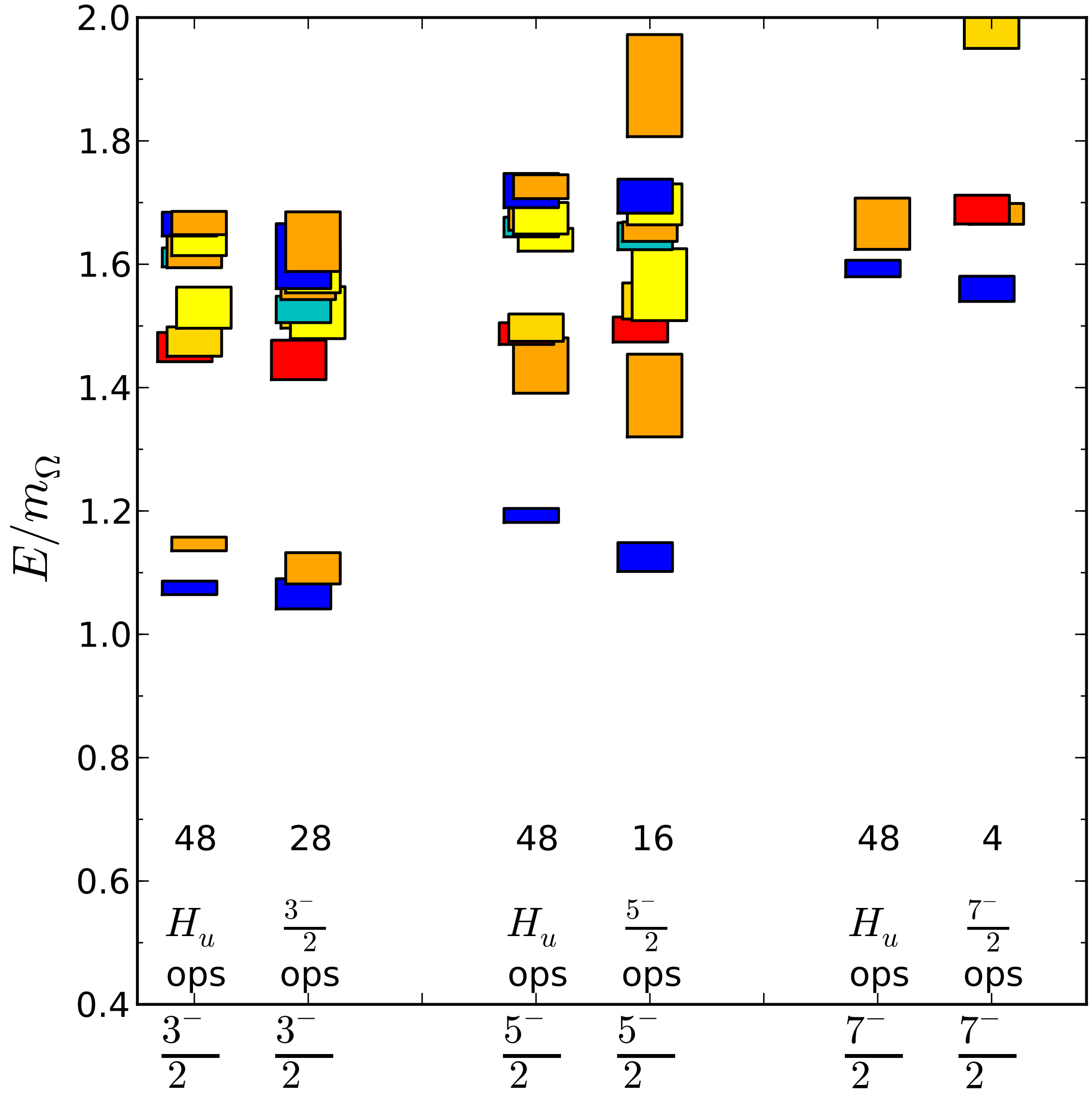
Test 2: Spectra with and without couplings between operators subduced from different J's

- Small couplings can mix different J's when states are degenerate
- Compare energies based on $C_{ij}(t)$ using all operators in an IR (include $J \neq J'$ couplings)
- and energies based on $C_{ij}(t)$ using only operators subduced from a single J (omit $J \neq J'$ couplings)
- For example, we have 28 G_{1g} operators in all. They include 24 subduced from $J = \frac{1}{2}$ and 4 subduced from $\frac{7}{2}$.
- Are the $J = \frac{1}{2}$ energies using all 28 G_{1g} operators similar to those using only the 24 operators subduced from $J = \frac{1}{2}$?

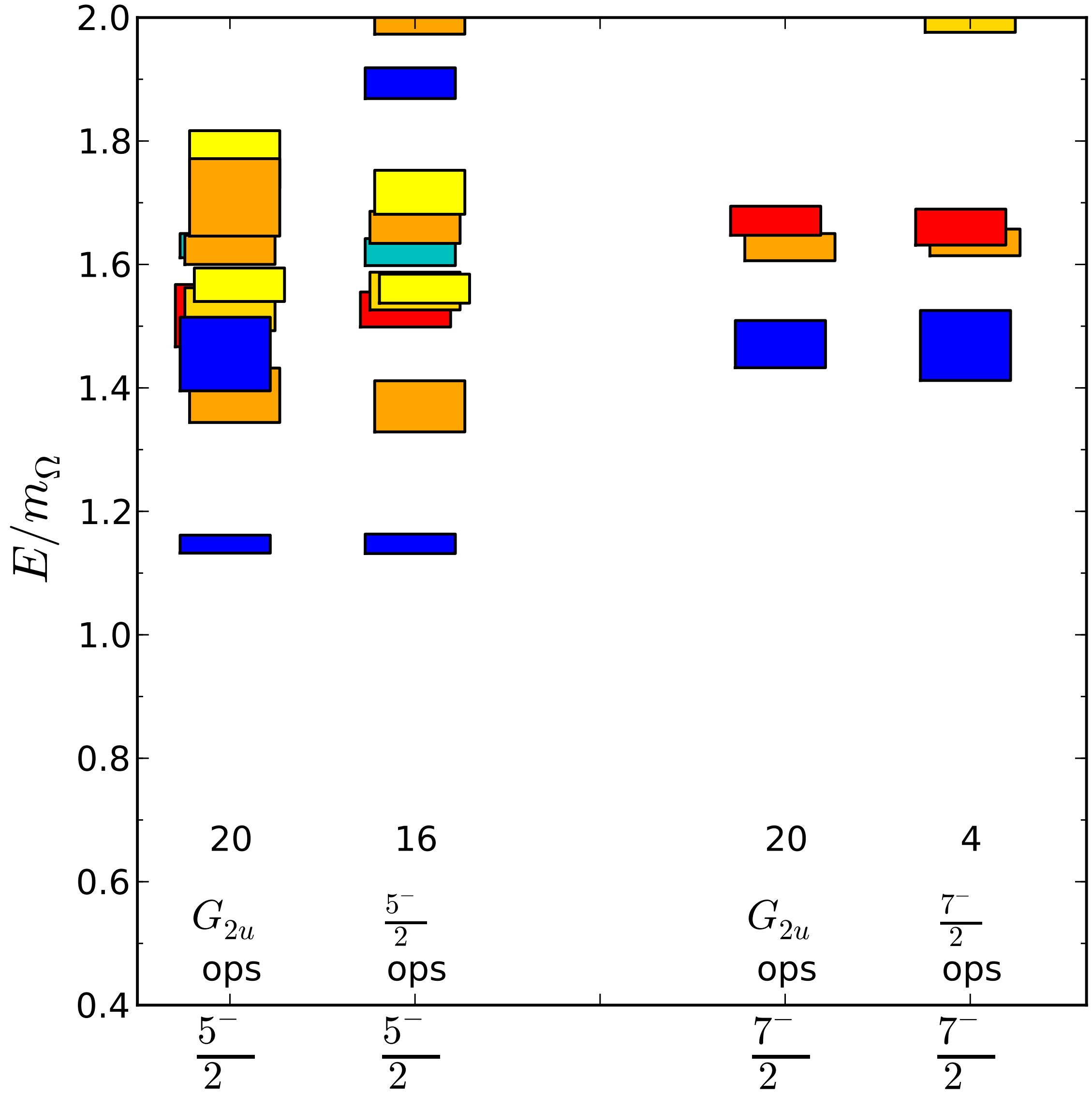
Test 2 for 28 G_{1u} energies



Test 2 for 48 H_u energies



Test 2 for 20 G_{2u} energies



How well do weights identify the spins?

Table 2: Spin weights in % for ten H_g energy levels.

	E$_n$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
H_g-0	0.3705(90)	0	98.2	1.6	0.1
H_g-1	0.3816(38)	0	5.1	94.7	0
H_g-2	0.4005(48)	0	0.8	98.9	0.1
H_g-3	0.4013(61)	0	96.4	3.4	0
H_g-4	0.4030(43)	0	99.1	0.8	0
H_g-5	0.4113(42)	0	99.3	0.4	0.1
H_g-6	0.4237(60)	0	96.1	3.6	0.1
H_g-7	0.4267(35)	0	3.2	96.4	0.2
H_g-8	0.4414(38)	0	0.6	0.3	98.9
H_g-9	0.5050(224)	0	91.3	5.9	2.7

How well does the spin identification work?

Table 3: Spin weights in % for ten G_{2g} energy levels.

	E_n	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
G_{2g}-0	0.3717(54)	0	0	99.9	0
G_{2g}-1	0.4088(50)	0	0	99.9	0
G_{2g}-2	0.4151(49)	0	0	99.6	0.3
G_{2g}-3	0.4307(58)	0	0	0.6	99.3
G_{2g}-4	0.4854(393)	0	0	99.6	0.3
G_{2g}-5	0.5095(158)	0	0	92.4	7.5
G_{2g}-6	0.5178(112)	0	0	34.5	65.4
G_{2g}-7	0.5184(87)	0	0	77.8	22.1
G_{2g}-8	0.5368(108)	0	0	86.8	13.1
G_{2g}-9	0.5480(187)	0	0	13.9	86

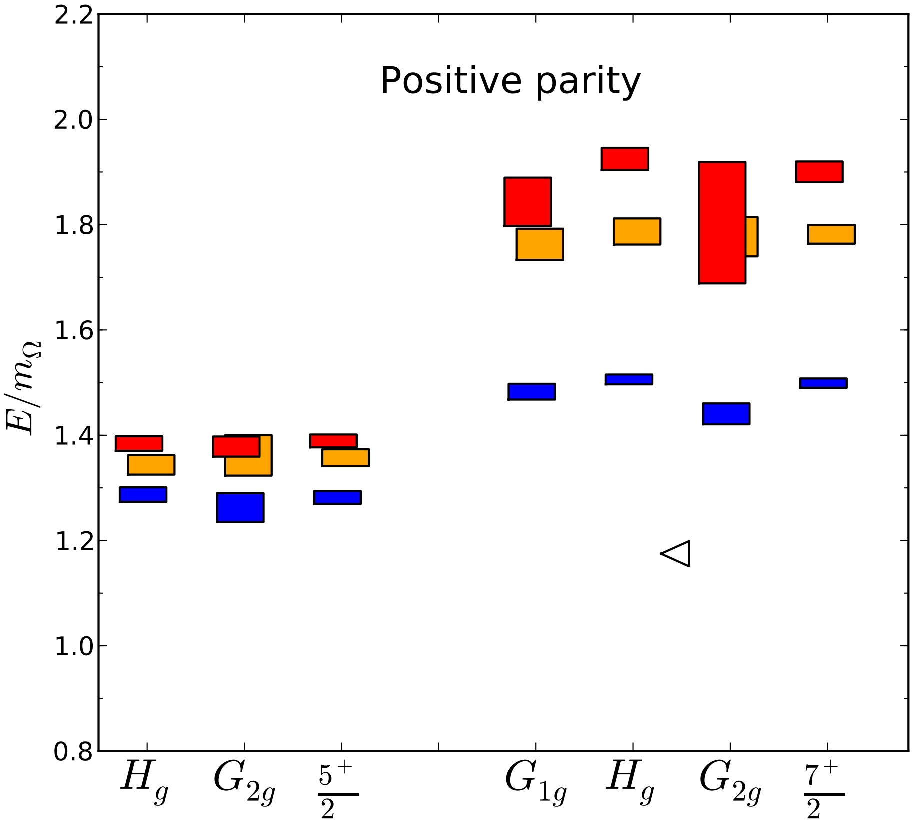
Results of spin identification analysis for baryon excited states

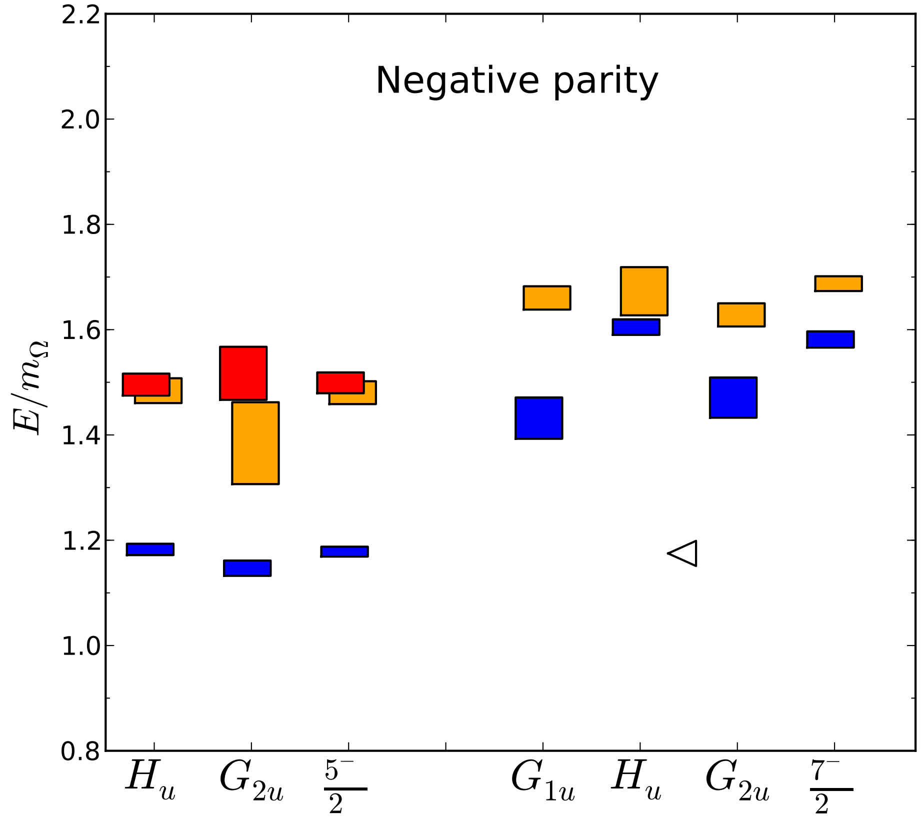
- **The spin of a baryon excited state is equal to J when the state is created predominantly by operators subduced from continuum spin J.**

- **Some baryon excited states that are nearly degenerate can have significant mixings of their J parentage.**

Spin $\frac{5}{2}$ and $\frac{7}{2}$ states based on average over M

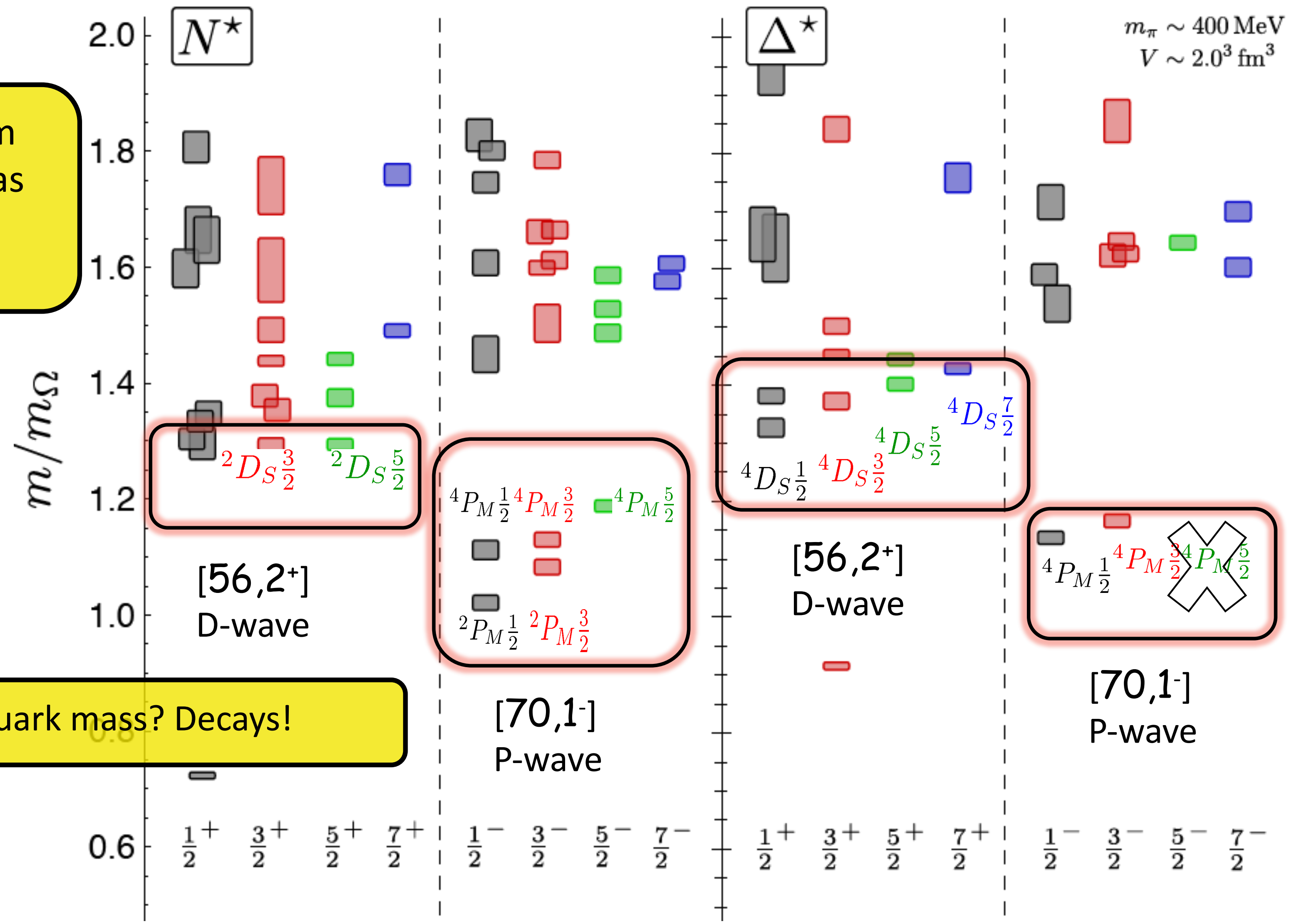
$$C^{[J]}(t) = \frac{1}{2J+1} \sum_{\Lambda,r} C^{[\Lambda,r;J]}(t)$$
$$= \frac{1}{2J+1} \sum_{M} C^{[J,M]}(t)$$





Nucleon & Delta Spectrum

Suggests spectrum at least as dense as quark model



Change at lighter quark mass? Decays!