Meson spectroscopy from lattice QCD

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Jefferson Lab - Excited states workshop,
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- **Myths:** Lattice QCD can . . .
  - . . . only compute **ground-state** properties.
  - . . . not compute **scattering** properties.
  - . . . not handle **high spin** states.
  - . . . not handle **isoscalar** mesons.

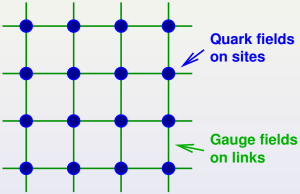
- **Where do these myths come from?**
- **New methods for making hadrons.**
- **Myth-busting results.**

Where do these myths come from?

. . . mostly from the way numerical simulations are done, particularly when including quarks.

Lattice regularisation

- Lattice provides a **non-perturbative, gauge-invariant** regulator for the QCD path integral.
- Quarks live on sites
- Gluons live on links
- a - lattice spacing
- $a \sim 0.1$ fm



- Chirally symmetric quarks are missing, but can discretise quarks by trading-off some symmetry
- In a finite volume $V = L^4$, finite number of degrees of freedom and path-integral is an ordinary (but large) integral.

High-dimension integrals estimated by Monte Carlo

Monte Carlo sampling the QCD lattice vacuum

Variance of estimators is huge unless we use importance sampling — must work with QFT in Euclidean space-time

- In a **Euclidean** metric:

$$C_\pi(t_1, t_0) = \frac{\int DUD\bar{\psi}D\psi \quad \bar{\psi}_u(t_1)\gamma_5\psi_d(t_1)\bar{\psi}_d(t_0)\gamma_5\psi_u(t_0) \quad e^{-S_G-\bar{\psi}_u M\psi_u-\bar{\psi}_d M\psi_d}}{\int DUD\bar{\psi}D\psi \quad e^{-S_G-\bar{\psi}_u M\psi_u-\bar{\psi}_d M\psi_d}}$$

- Hard to deal with Grassmann algebra

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- Quenched approximation was to ignore $\det M^2$
- $N_f = 2$ **importance sampling measure**
- Non-negative, thanks to Euclidean metric

Spectroscopy and Euclidean space-time

- Energies of colourless QCD states extracted from **two-point functions** in Euclidean time

$$C(t) = \langle \Phi(t) | \Phi^\dagger(0) \rangle$$

- Euclidean time: $\Phi(t) = e^{Ht} \Phi e^{-Ht}$ so $C(t) = \langle \Phi | e^{-Ht} | \Phi \rangle$. Insert a complete set of states then:

$$C(t) = \sum_{k=0}^{\infty} |\langle \Phi | k \rangle|^2 \; e^{-E_k t}$$

- $\lim_{t \to \infty} C(t) = Z e^{-E_0 t}$: observe large-t exponential fall-off, then **energy of ground-state** measured

Euclidean metric very useful for spectroscopy: provides a way of isolating and examining low-lying states

- **Excited-state** energies measured from matrix of correlators:

$$C_{ij}(t) = \langle \Phi_i(t) | \Phi_j^\dagger(0) \rangle$$

- Solve generalised eigenvalue problem:

$$C(t_1)\,\underline{v} = \lambda\, C(t_0)\,\underline{v}$$

for different t_0 and t_1 [Lüscher & Wolff, C. Michael]

- Then $\lim_{(t_1-t_0)\to\infty} \lambda_n = e^{-E_n(t_1-t_0)}$
- Method constructs optimal ground-state creation operator, then builds orthogonal states.

Excited states accessed if basis of creation operators is used and the matrix of correlators can be computed

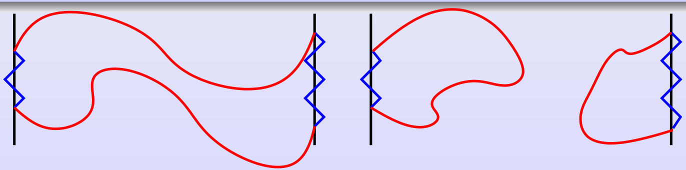
Isoscalar meson correlation functions

- Isovector mesons: Wick contraction gives

- Isoscalar – extra diagrams. Wick contraction:

$$\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle = M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{il} M^{-1}_{jk}$$

$$\langle 0 | \Phi_{I=0}(t) \Phi^{\dagger}_{I=0}(0) | 0 \rangle =$$
$$\langle 0 | \Phi_{I=1}(t) \Phi^{\dagger}_{I=1}(0) | 0 \rangle - \langle 0 | \mathrm{Tr}\, M^{-1} \Gamma U_{\mathcal{C}}(t) \mathrm{Tr}\, M^{-1} \Gamma U_{\mathcal{C}}(0) | 0 \rangle$$

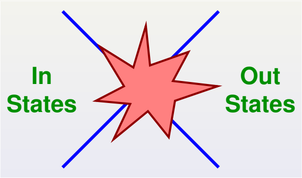


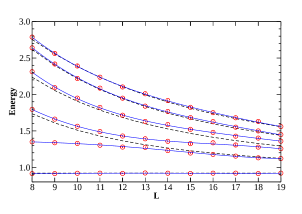
Scattering matrix elements are not directly accessible from Euclidean QFT [Maiani-Testa theorem]

- Scattering matrix elements: asymptotic $|\text{in}\rangle, |\text{out}\rangle$ states.

$\langle \text{out} \, |e^{i\hat{H}t}| \, \text{in}\rangle \rightarrow \langle \text{out} \, |e^{-\hat{H}t}| \, \text{in}\rangle$

- Euclidean space: project onto ground-state only



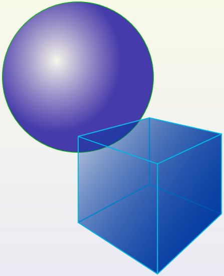


D. McManus, P. Giudice & MP

- **Lüscher's formalism:** information on elastic scattering inferred from **volume dependence** of spectrum
- Method requires precise data

Talks → Schierholz, Renner

Spin on the lattice

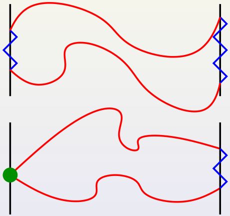


- Lattice breaks $O(3) \to O_h$
- Lattice states classified by quantum letter, $R \in \{A_1, A_2, E, T_1, T_2\}$
- Link to continuum by subducing reps of $O(3)$ into O_h
- Look for degeneracies. Problem: spin-4 has same pattern as $0 \oplus 1 \oplus 2$.

- Better spin assignment by constructing operators from lattice representation of convariant derivative.
- Start in continuum with operator of definite J, then subduce this into O_h and then replace derivatives with lattice equivalent.
- Also measure $\langle 0|\Phi^\dagger|J^{PC}\rangle$ and look for remnants of continuum symmetries.

The numerical tool-kit for quarks

- Physics focus of LQCD has been matrix elements, not spectroscopy.
- Traditionally, quark propagation computed starting with **point source**: $\eta(\underline{x}, t) = \delta_{t,0}\delta_{\underline{x},\underline{0}}$



- Solve $M\psi = \eta$, then ψ is one column of M^{-1}
- QCD lagrangian is translationally invariant
- With this trick, can make simple mesons and baryons cheaply.
- Isoscalar mesons, higher-spin states, hybrids, large operator bases not so well constructed.

The "point-to-all" propagator has restricted the focus of physics lattice QCD has addressed

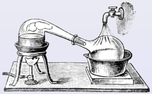
In principle:

- **Variational calculations** allow access to excited states
- **Lüscher's method** enables lattice QCD computations of elastic scattering properties.
- **Isoscalar mesons** just need extra diagrams to be computed.
- **High-spin-state** measurements just need more than local bilinears

In practice:

Problems with **manipulating quarks on the computer** has made all this very challenging and progress has been slow

A new framework
“Distillation”



“*distill*: to extract the quintessence of” [OED]

- **Smeared field:** $\tilde{\psi}$ from ψ, the “raw” quark field in the path-integral:

$$\tilde{\psi}(t) = \square[U(t)]\ \psi(t)$$

- Extract confinement-scale degrees of freedom, while preserving symmetries of quarks.
- Creation operator now acts on **smeared quark fields**:

$$\Phi_M^\dagger(t) = \bar{\tilde{\psi}}(t)\Gamma\tilde{\psi}(t)$$

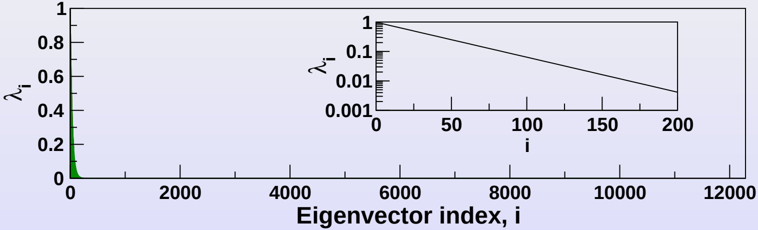
Γ is operator in $\{\underline{s}, \sigma, c\} \equiv \{\text{position,spin,colour}\}$

Smearing extracts the essential degrees of freedom needed to make mesons: overlap $\langle n|\Phi_M^\dagger|0\rangle$ is large for low-lying eigenstate $|n\rangle$

- **Gaussian** smearing:

$$\lim_{n\to\infty}\left(1+\frac{\sigma\nabla^2}{n}\right)^n = \exp(\sigma\nabla^2)$$

- This acts in the space of coloured scalar fields on a time-slice: $N_s \times N_c$



- Data from $a_s \approx 0.12\text{fm}\ 16^3$ lattice: $16^3 \times 3 = 12288$.

- Distillation: **define** smearing to be explicitly a very low-rank operator. Rank is $N_{\mathcal{D}} (\ll N_s \times N_c)$.

Distillation operator

$$\square(t) = V(t)V^{\dagger}(t)$$

with $V^a_{\underline{x},c}(t)$ a $N_{\mathcal{D}} \times (N_s \times N_c)$ matrix

- We use: $\square_{\nabla}$ the **projection operator into $\mathcal{D}_{\nabla}$, the space spanned by the lowest eigenmodes of the 3-D laplacian**
- Eigenvectors of ∇^2 not the only choice and may not be optimal
- Because V is a narrow column matrix, **quark-line diagrams factorise**: much cheaper to compute.

Distillation enables:

- Arbitrary operator insertions
- large variational basis
- isoscalar correlators
- multi-hadron states

...to be computed.

...but work to do

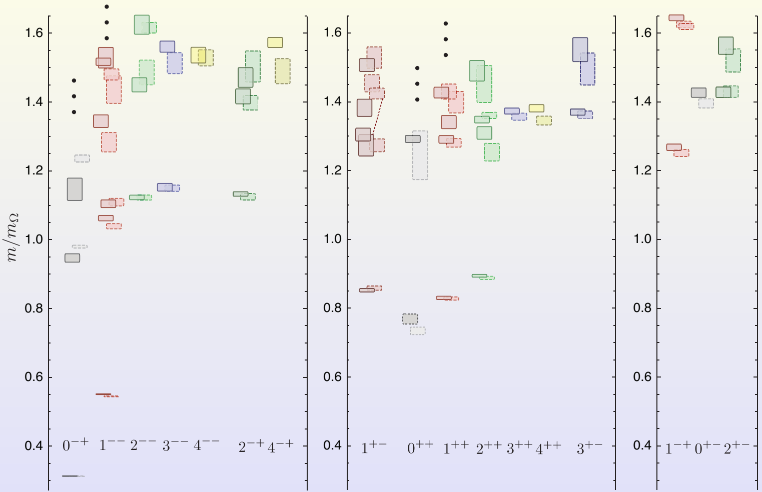
- poor cost scaling with V

Results

Caveat emptor:

- No $a \longrightarrow 0$ extrapolation
- Finite V
- unphysical m_π

Isovector meson spectroscopy

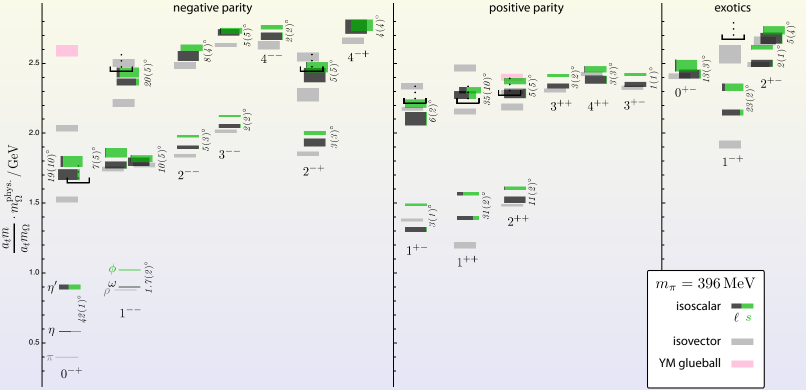


- $m_\pi = 400$ MeV
- No 2-meson operators

Should be a dense spectrum of two-meson states:

— Not seen at all

Isoscalar meson spectroscopy



- **No 0^{++} data presented**
- $m_\pi = 400$ MeV
- No glueball, 2-meson ops
- Still expensive calculations

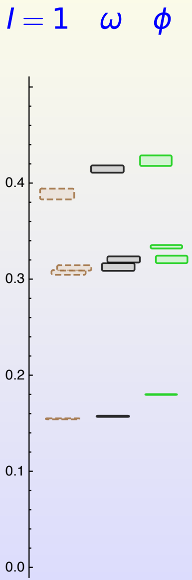
Statistical precision:

η	0.5%
η'	1.9%

Vector mesons: the ρ, ω and ϕ

- Mix light and strange quarks in correlators
- Resolve $\rho - \omega$ mass difference (QCD only) clearly:

$$m_\omega - m_\rho = 19(4)\ \text{MeV}$$



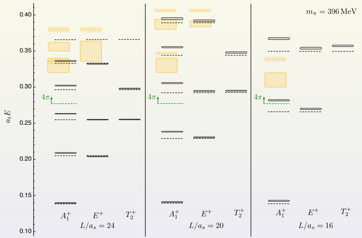
The power of GPUs

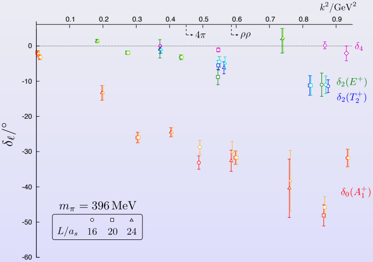
- 479 configurations, $N_{\mathcal{D}} = 64 \rightarrow 31M$ solutions of $M\psi = \eta$

- Enabled by use of **Graphics Processing Units (GPU)** and QUDA software library written by Clark and Babich .
- Linear solver integrated into `chroma` by Bálint Joó
- Sustains more than 100 GFlops per GPU for linear system solver.

GPUs offer excellent price/performance since manufactured in huge numbers

- Lüscher's method: first compute energy shifts in the finite volume
- Data for $L = 16, 20, 24a_s$
- Energy shifts well resolved





- Measure δ_0, δ_2 and small δ_4
- $I = 0, 1$ under investigation (more diagrams, including $t \to t$)

Problem: volume cost scaling

- So far - results on modest lattice sizes: $N_s = 16^3 \equiv (1.9fm)^3$. Used $N_{\mathcal{D}} = 64$

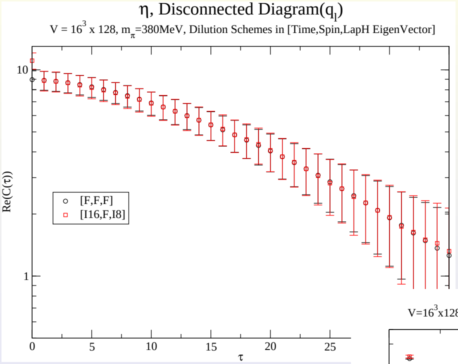
Cost vs volume

- Maintaining constant resolution of structures in the vacuum (of fixed physical size) means increasing $N_{\mathcal{D}} \propto N_s$
- Computer costs grow like N_s^2 (for inversions), N_s^3 for meson contractions, N_s^4 for baryons . . .

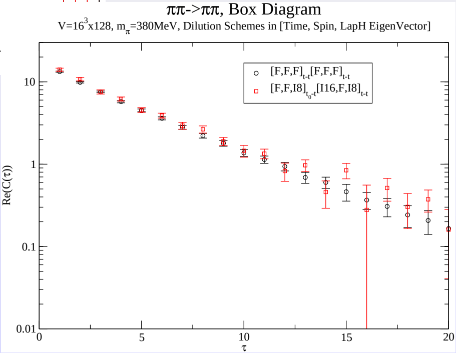
- **One Solution:** use stochastic estimator techniques (Monte Carlo within a Monte Carlo)
- This works well because distillation rank-reduces the problem substantially

NOT a problem with taking $a \to 0$.

Stochastic estimation



- Disconnected contribution to $\bar{\psi}\gamma_5\psi$
- Signal reproduced for substantial cost saving

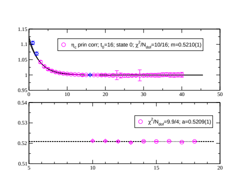


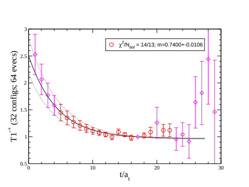
- One diagram contributing to $I = 0, 1$ $\pi\pi$ scattering

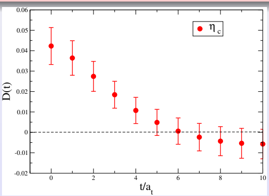
Charmonium spectroscopy

- New TCD/JLab project starting: (TCD: Liuming Liu, Graham Moir, MP, Sinéad Ryan, Pol Mainar)
- So far, tuning action and testing distillation method
- Preliminary data suggest method works for heavy quarks too

ALL DATA PRELIMINARY (and low statistics, 35 cfgs)







- Good resolution seen, as with light quark sector
- GPUs very helpful again . . .

The Myth-busters scorecard

. . . only compute properties of ground-states

BUSTED

excited-state spectroscopy ← variational techniques

. . . not compute scattering properties

BUSTED (elastic scattering)

Lüscher's method ← V-dependence of full spectrum

← two-meson states in operator basis

. . . not handle high-spin states

BUSTED

Link lattice data to continuum ← Derivative operators

. . . not handle isoscalar mesons

BUSTED

Disconnected diagrams

Distillation has enabled a lot of this progress

The fine print:

- Distillation has poor volume scaling - larger volumes still expensive
- Stochastic estimation makes the method more numerically efficient
- Crucial next step is to include two-meson operators to put in the “dark matter” missing from the spectra observed so far
- This may need reworking of the choice of distillation space

References

Distillation	Phys.Rev.D80:054506,2009.
I=1 mesons	Phys.Rev.Lett.103:262001,2009
	Phys.Rev.D82:034508,2010.
I=0 mesons	arXiv:1102.4299
$\pi\pi$ scattering	arXiv:1011.5277, arXiv:1011.6352
Stochastic	arXiv:1011.0481