Aspects of the rho-meson

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JLab

work with

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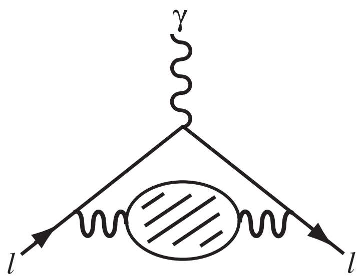
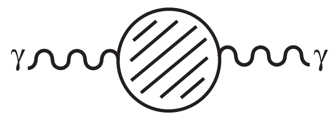
Outline

- motivation: muon $g-2$
  - role of rho-meson
  - dilepton rate of rho-meson
- rho-meson as resonance
  - π-π p-wave scattering phase
  - rho-resonance mass
  - rho-resonance width

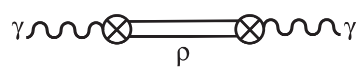
Muon anomalous magnetic moment, a_μ

- leading hadronic contribution comes from vacuum polarization





- the vector-mesons dominate the hadronic contribution to a_μ



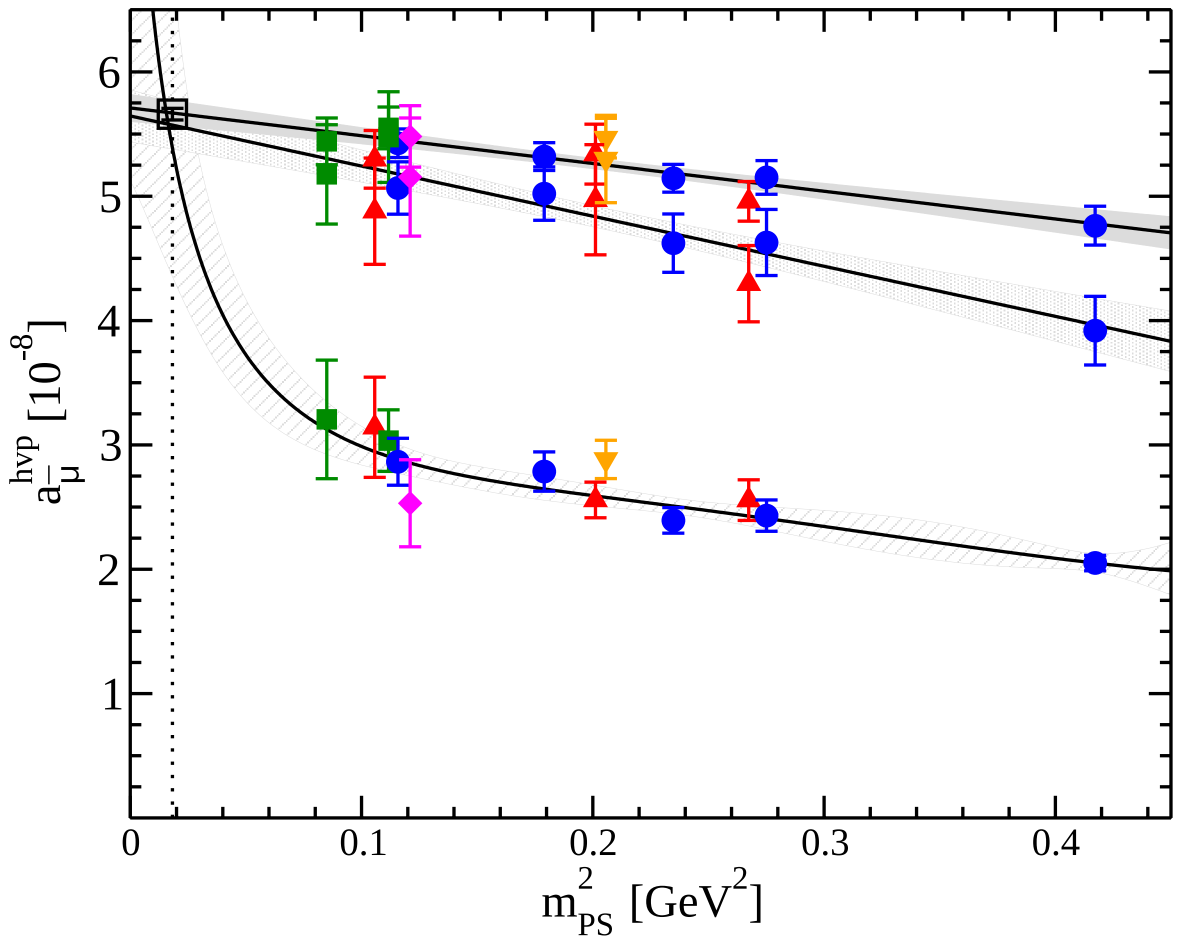
- vector meson induces strong m_{PS} dependence in a_μ

$$a_{\mu,V} \approx \frac{2}{3}\alpha^2 g_V^2 \frac{m_\mu^2}{m_V^2}$$

- insight from rho leads to new observables $a_{\overline{\mu}}$ with same physical limit

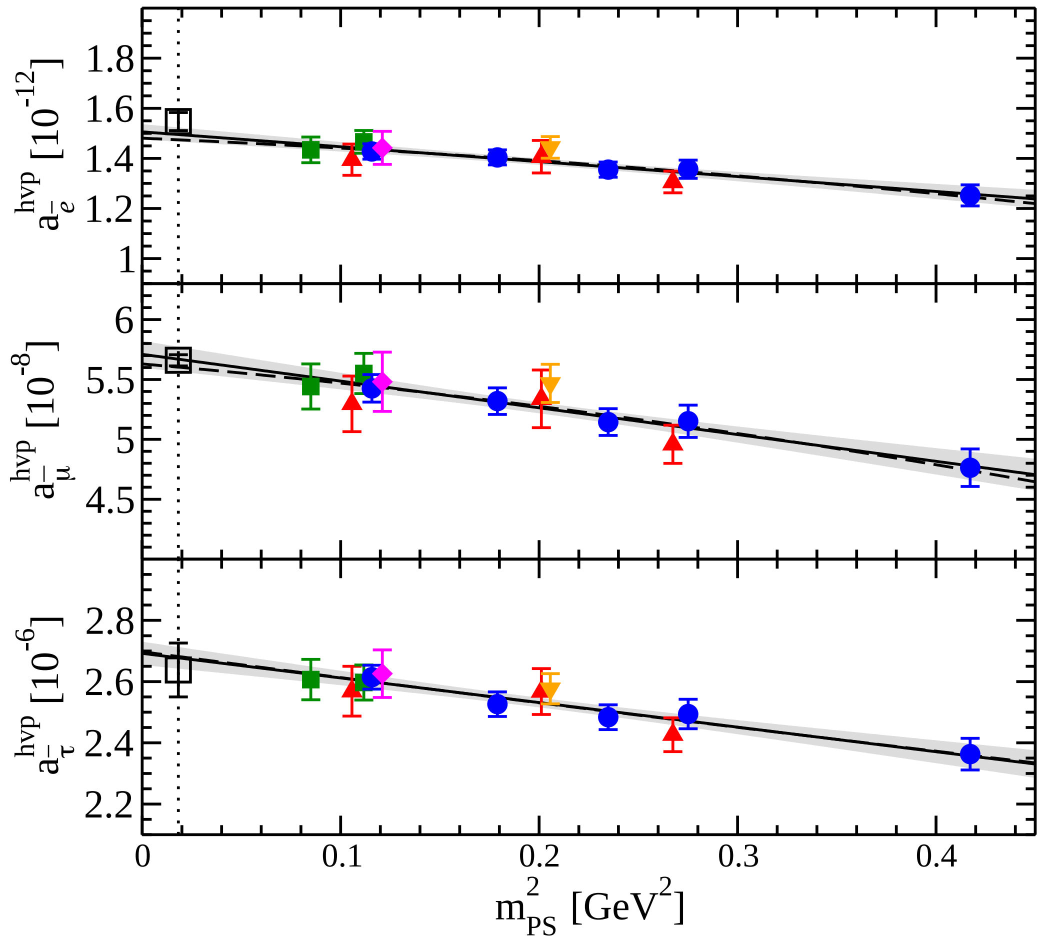
$$a_{\overline{\mu}}(m_{PS} \to m_\pi) = a_\mu(m_{PS} \to m_\pi)$$

- standard method (lower curve) has apparently strong m_{PS} dep.



- physical limit agrees for std. and two variations of an imp. method

- the same technique for all three charged leptons of standard model



- purely non-perturbative calculation with 2% or less errors

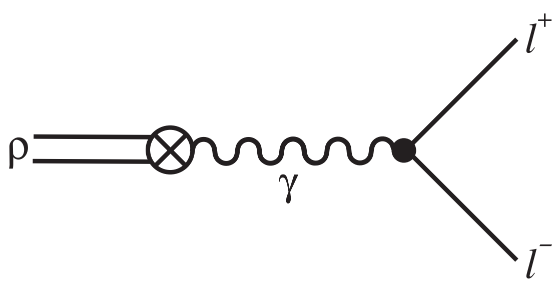
[Feng, Jansen, Petschlies, Renner]

Rho-meson electromagnetic coupling

- coupling of the rho-meson to the electromagnetic current

$$\langle \Omega | J_\mu^{em}(0) | V, p, \epsilon \rangle = \frac{m_V^2 g_V}{\sqrt{2}} \epsilon_\mu(p)$$

- this can be related to the dilepton rate $\Gamma(\rho \to l^+ l^-)$



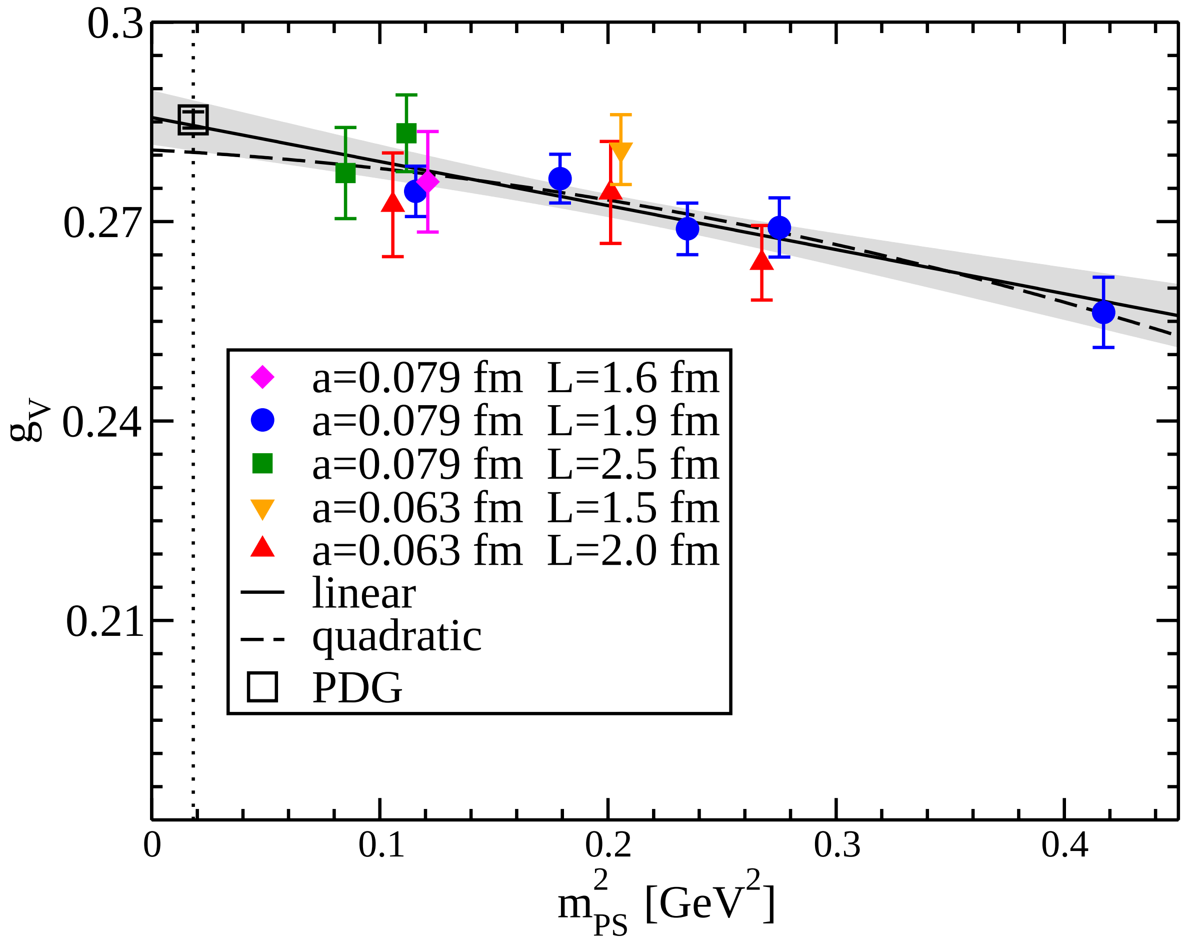
$$\Gamma(V \to l^+ l^-) \approx \frac{2\pi}{3} \alpha^2 g_V^2 m_V$$

- using PDG values, the coupling is

$$g_\rho = 0.2853\,(12)$$

Rho-meson electromagnetic coupling

- dimensionless quantities are generally expected to be cleaner



- proper resonance treatment on the lattice would be interesting

[Feng, Jansen, Petschlies, Renner]

Finite-size methods and scattering on the lattice

- π-π states at finite volume are shifted due to self-interaction

$$E_{\pi\pi} \neq \sqrt{m_\pi^2 + (2\pi/L)^2 \vec{n}_1^2} + \sqrt{m_\pi^2 + (2\pi/L)^2 \vec{n}_2^2}$$

- Lüscher relates $E_{\pi\pi}$ levels to p-wave scattering phase $\delta(E)$ for $I = 1$

$$E_{\pi\pi} = 2\sqrt{m_\pi^2 + (2\pi/L)^2 q^2} \qquad \tan\delta(E_{\pi\pi}) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(q^2)}$$

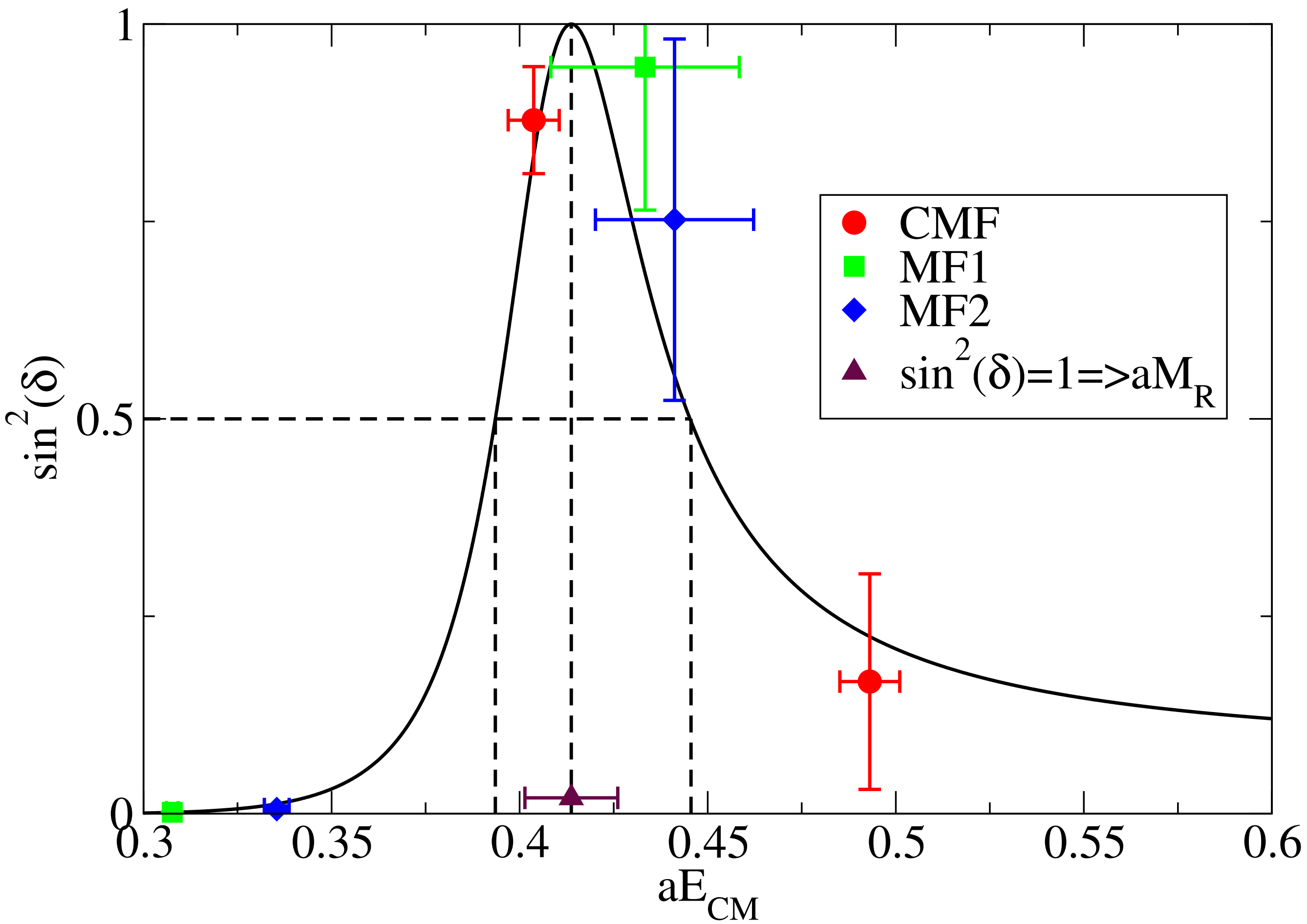
- above example is in the center-of-mass and $\mathcal{Z}_{00}(q^2)$ is a known function

$$\mathcal{Z}_{00}(q^2) = \frac{1}{\sqrt{4\pi}} \sum_{n \in Z^3} \frac{1}{|n|^2 - q^2}$$

- generalized to moving frames by Rummukainen and Gottlieb and Feng

$\mathrm{I} = 1$ π-π scattering phase

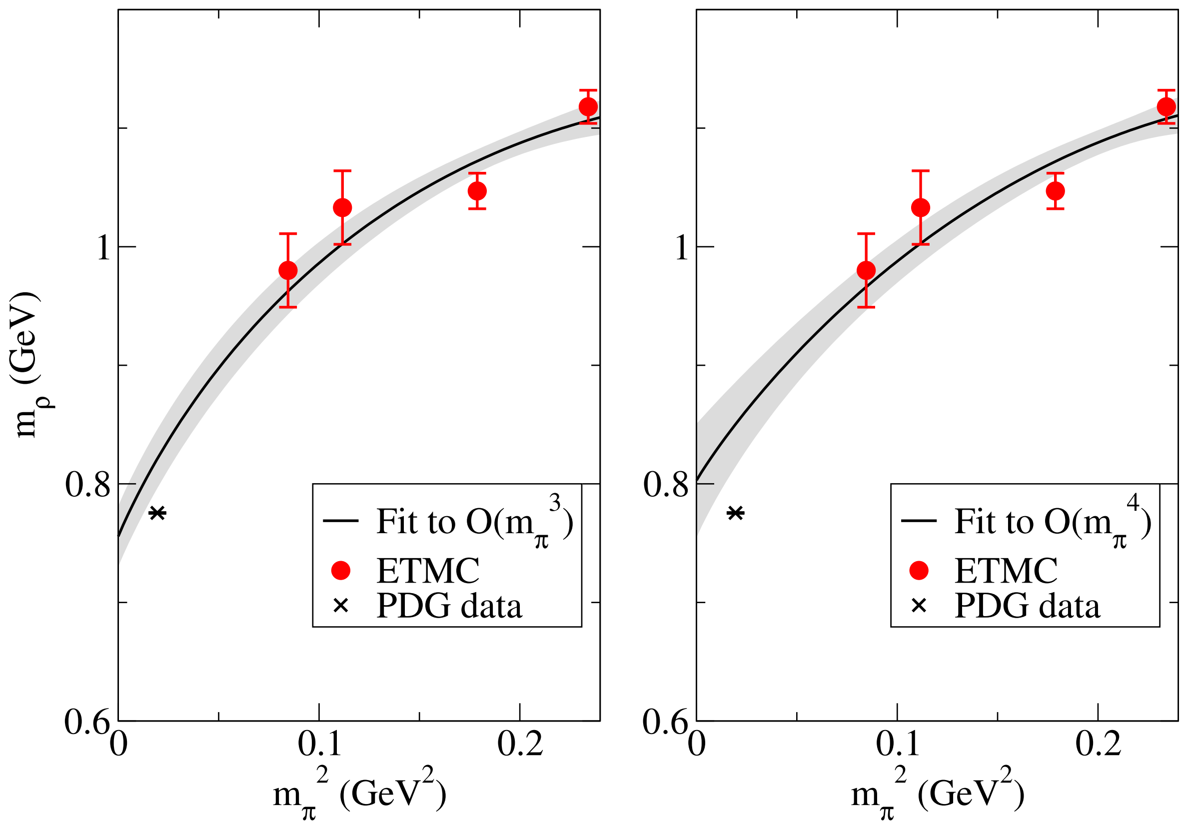
- energy dependence mapped out with three distinct frames



- fit to Breit-Wigner form to extract resonance properties

Rho-resonance mass

- fit to an effective field theory for the rho-resonance

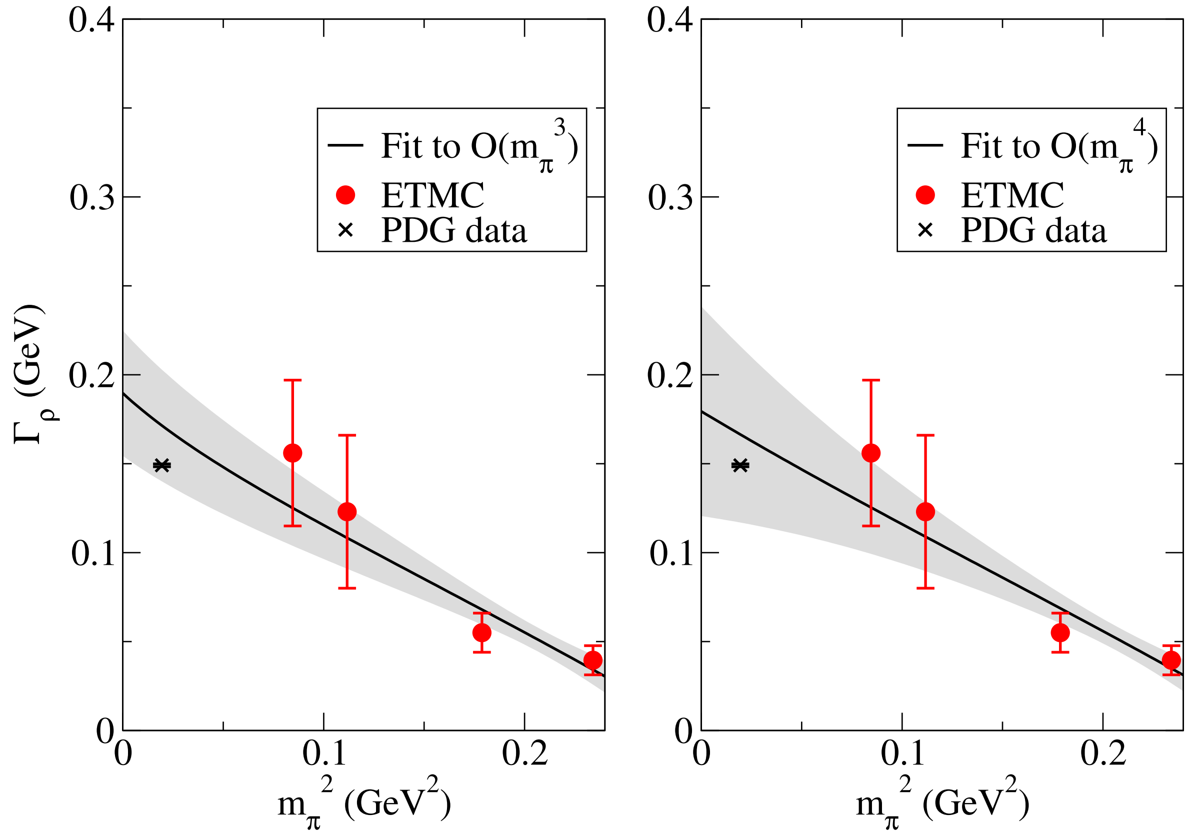


- cutoff effects, chiral dynamics can account for discrepancy

[Feng, Jansen, Renner 1011.5288]

Rho-resonance width

- again fit to an effective field theory for the rho-resonance

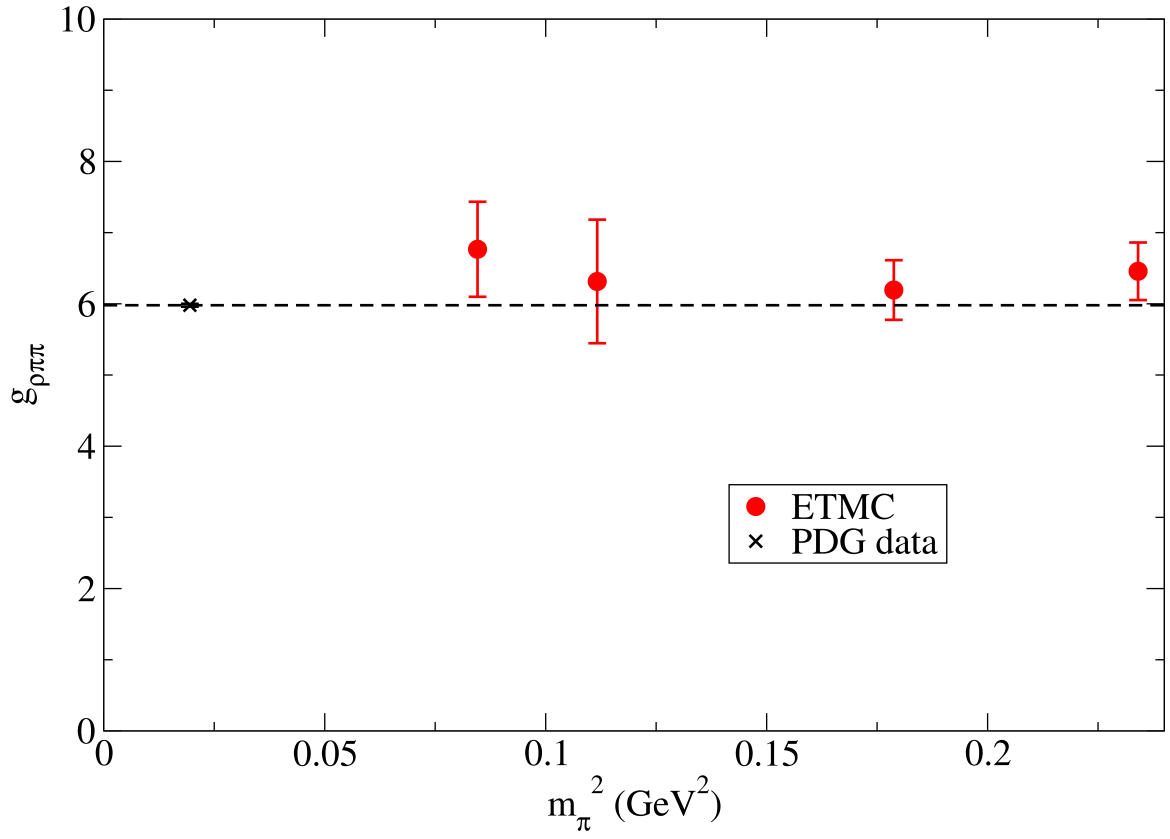


- agreement with experiment but notice larger errors, milder m_{PS} dep.

[Feng, Jansen, Renner 1011.5288]

Rho-pi-pi coupling

- $g_{\rho\pi\pi}$ is dimensionless combination of m_ρ and Γ_ρ



- apparently very flat m_{PS} dependence

[Feng, Jansen, Renner 1011.5288]

Conclusions

- understanding (and mitigating) ρ contribution important to obtaining $g_l - 2$ with 2% or less errors for all three electron, muon and tau

- electromagnetic coupling of the ρ (treated as stable) is already accurately reproduced with a nearly 1% accuracy

- finite-size methods for scattering used to calculate $I = 2$ s-wave scattering length accurate to 1%

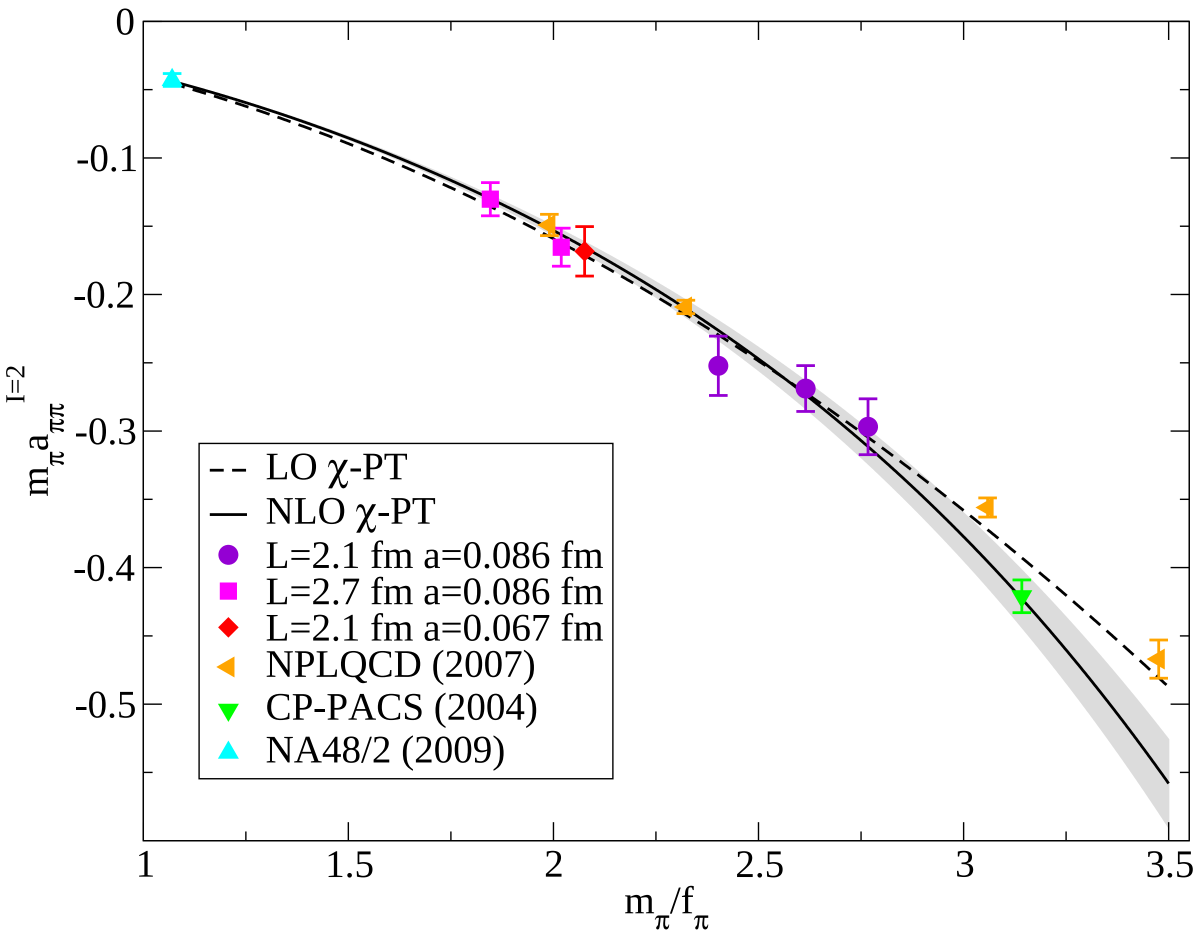
- energy dependence of $\delta_1(E)$ for $I = 1$ p-wave π-π has been calculated

- rho-resonance mass and width consistent with measured values to within systematics of our first exploratory calculation

Extra slides

I = 2 π-π scattering length

- our "warm up" exercise with finite-size methods

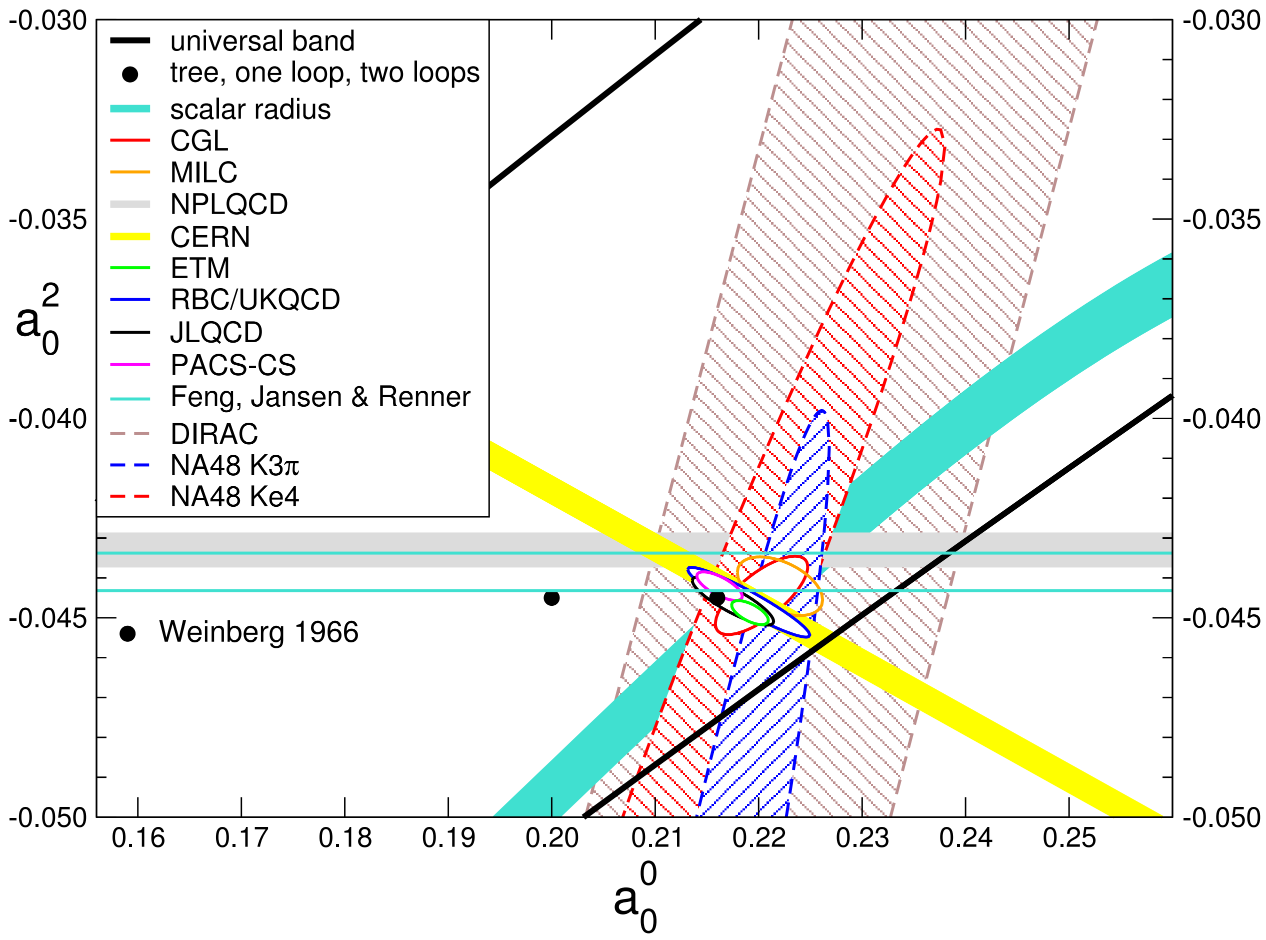


- $m_\pi a_{\pi\pi}^{I=2} = -0.0439(5)$ (ETMC) vs. $-0.0444(9)$ (NA48/2 and χPT)

[Feng, Jansen, Renner 0909.3255]

π-π scattering lengths

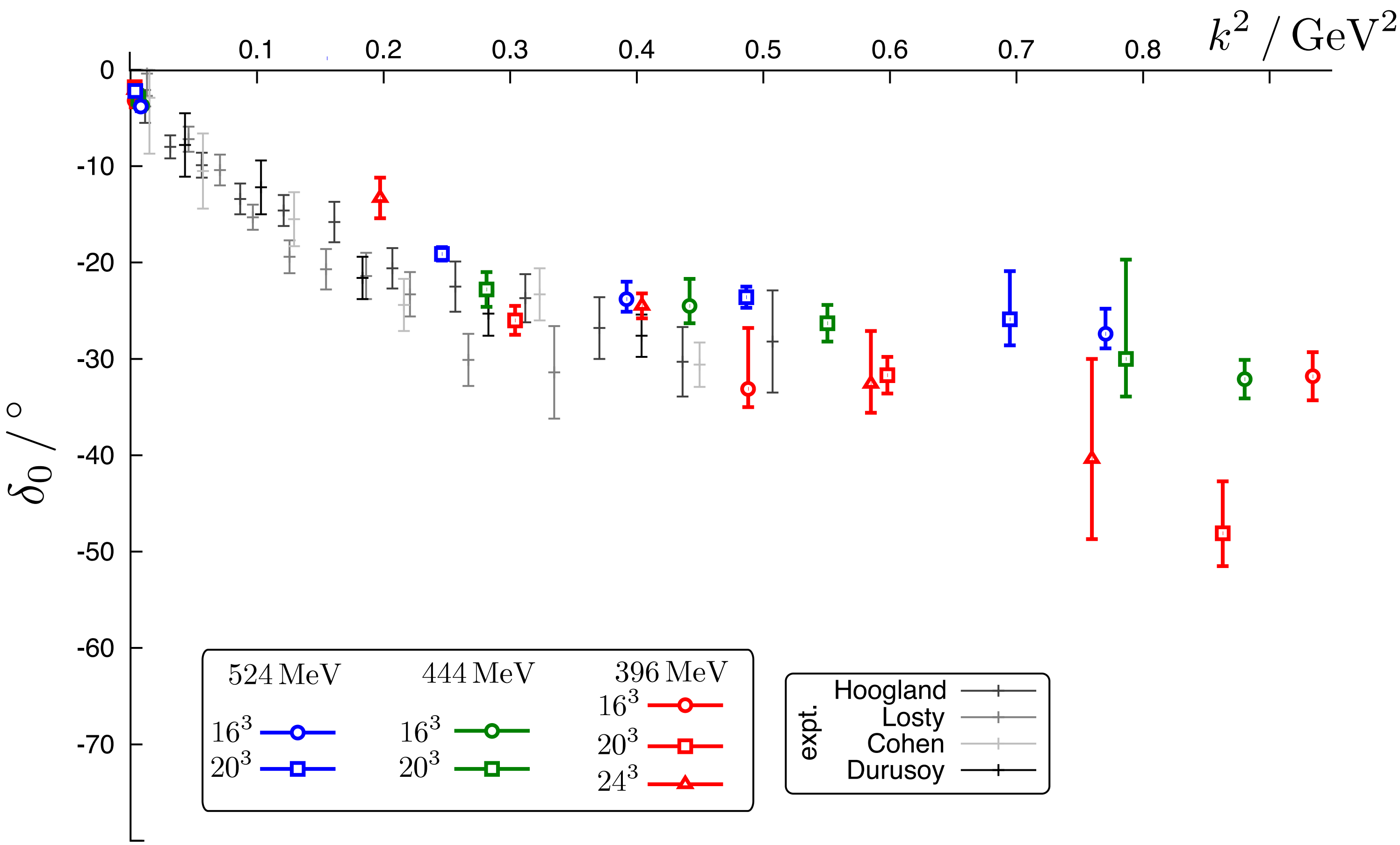
- compilation of $I = 2$ and $I = 0$ s-wave π-π scattering lengths



- lattice calc. of $I = 0$ scattering length is challenging but possible

$\mathrm{I} = 2$ π-π scattering phase

- recent extensive calculation of scattering phase



- results in scattering lengths in agreement with previous plots

[Dudek, Edwards, Peardon, Richards, Thomas 1011.6352]