Hadron Resonances and Decays from the Lattice

G. Schierholz

Deutsches Elektronen-Synchrotron DESY

With

T. Burch, M. Göckeler, S. Gutzwiller, R. Horsley, U.-G. Meißner, Y. Nakamura,

A. Nogga, H. Perlt, D. Pleiter, P.E.L. Rakow, A. Rusetsky, J. Zanotti

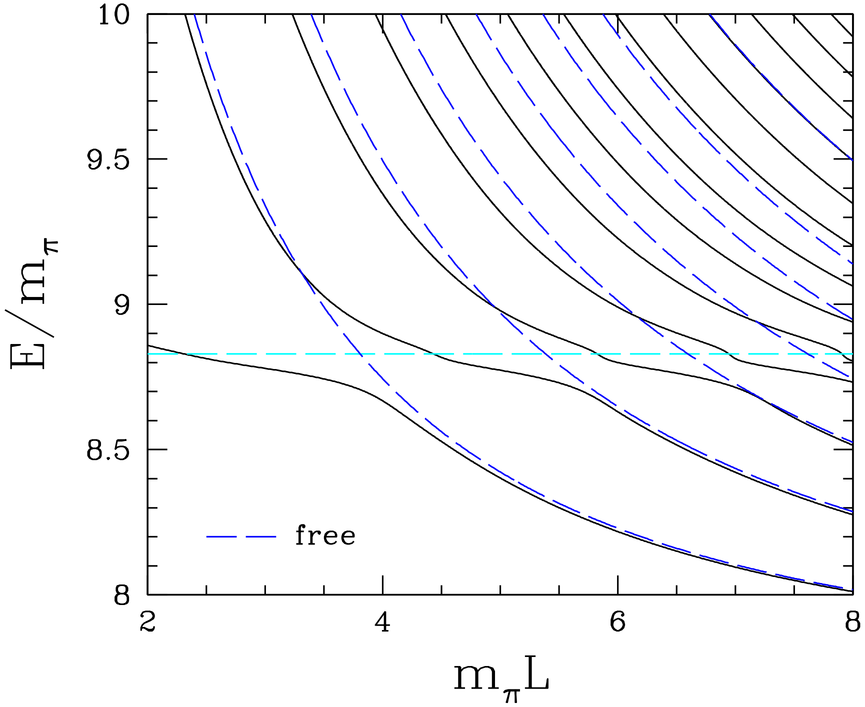
The Task

- Apart from the nucleon, hadrons of most phenomenological interest are resonances

- Resonance states cannot be identified with a single energy eigenstate of the lattice Hamiltonian

- The method of choice is to compute masses and widths from the volume dependence of the energy levels

Lüscher, Wiese



$\Delta \rightarrow N\pi$

Scalar Nonet

$\kappa^0(660)$ $\kappa^+(660)$

$a_0(980)$

$a_0^-(980)$ $\sigma(440)$ $f_0(980)$ $a_0^+(980)$

$\kappa^-(660)$ $\bar{\kappa}^0(660)$

- Does not fit a $q\bar{q}$ description, but seems to have a significant $qq\bar{q}\bar{q}$ (tetraquark ?) component

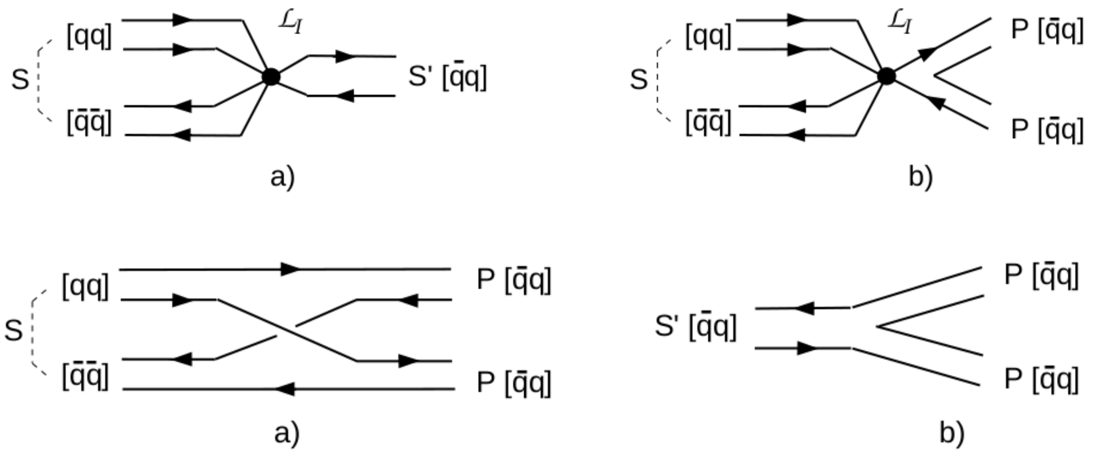
  't Hooft et al.

- Instead, a heavier nonet above 1 GeV appears to be largely $q\bar{q}$

- Scalar mesons are especially important to understand because they have the same quantum numbers as the vacuum, and therefore, they can condense into the vacuum and break global symmetries such as chiral $U(N_f) \times U(N_f)$. The details of how this symmetry breaking is implemented in Nature is one of the most profound problems in particle physics.

Decays

$$\left.\begin{array}{lll} \sigma(440) & \longrightarrow & \pi\pi \\ \kappa(660) & \longrightarrow & K\pi \\ f_0(980) & \longrightarrow & \pi\pi \\ & \longrightarrow & K\bar{K} \\ a_0(980) & \longrightarrow & \pi\eta \\ & \longrightarrow & K\bar{K} \end{array}\right\}$$

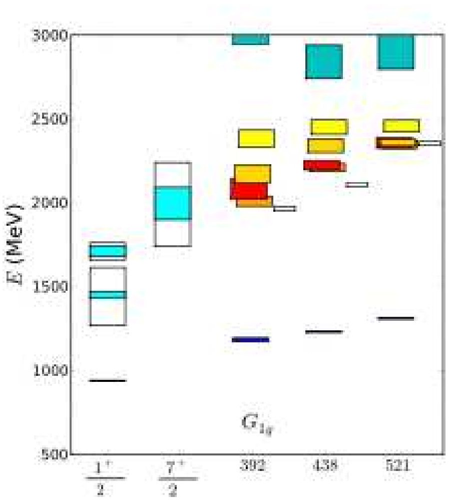


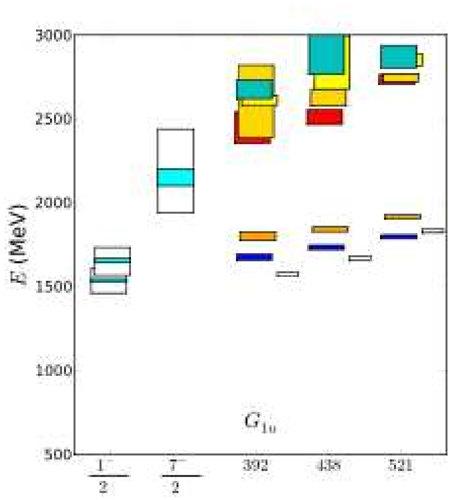
$$C = \begin{pmatrix} [q\bar{q}]\,[q\bar{q}] & [q\bar{q}]\,[qq\,\bar{q}\bar{q}] & [q\bar{q}]\,[q\bar{q}\,q\bar{q}] \\ [qq\,\bar{q}\bar{q}]\,[q\bar{q}] & [qq\,\bar{q}\bar{q}]\,[qq\,\bar{q}\bar{q}] & [qq\,\bar{q}\bar{q}]\,[q\bar{q}\,q\bar{q}] \\ [q\bar{q}\,q\bar{q}]\,[q\bar{q}] & [q\bar{q}\,q\bar{q}]\,[qq\,\bar{q}\bar{q}] & [q\bar{q}\,q\bar{q}]\,[q\bar{q}\,q\bar{q}] \end{pmatrix}$$

Mixing with glueballs can be neglected

Excited Baryons

$$\left.\begin{array}{rl} N(1440) \rightarrow & N\pi \\ & \Delta\pi \\ & N\eta \\ & \\ N^*(1535) \rightarrow & N\pi \\ & N\eta \\ \vdots & \end{array}\right\}$$

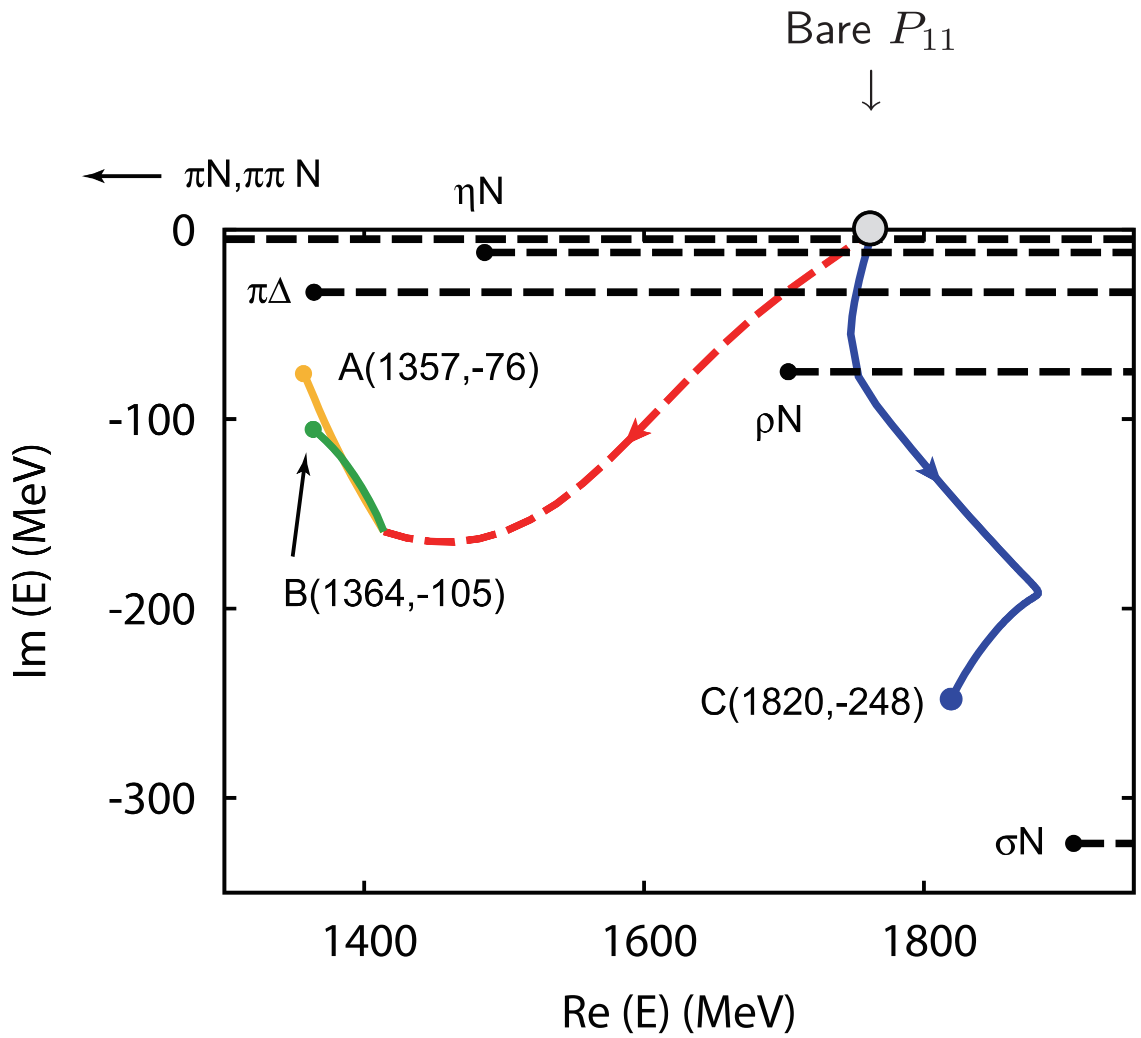




Bulava et al.

- The $N(1440)$, or Roper resonance, is a subject of extensive interest because of its surprisingly low mass
- In constituent quark models the lowest-lying negative parity state N^* occurs below the Roper, whereas in Nature the $N^*(1535)$ is almost 100 MeV above the Roper resonance
- This phenomenon has led to wide speculation on the possible exotic nature of the Roper resonance

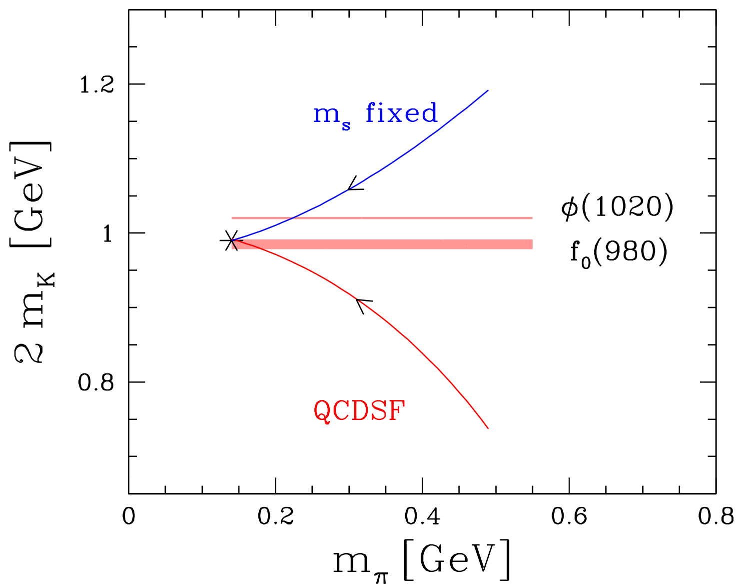
Dynamical Origin of Roper



Suzuki et al.

Lattice

- Finite volume $V = L^3 \times T$ with periodic bc: $$p = \frac{2\pi|\mathbf{n}|}{L}\,, \quad \mathbf{n} \in \mathbb{N}^3$$ $$O(4) \to H(4)$$
- Need small quark masses so that $\sigma(440)$, $\kappa(660)$, $\rho(770)$, $\Delta(1232)$, $\cdots$ resonances can decay
- Quark masses need to be tuned carefully



QCDSF: $m_u + m_d + m_s =$ fixed

This Talk

$SU(2)_F$

$$\left.\begin{array}{lcl} \rho(770) & \rightarrow & \pi\pi \\ \Delta(1232) & \rightarrow & N\pi \end{array}\right\} \quad \text{elastic}$$

$I = 1$

$I = \frac{3}{2}$

↑

Flavor symmetry essential

Benchmark Calculation

Action

$$S \quad = \quad S_G + S_F$$

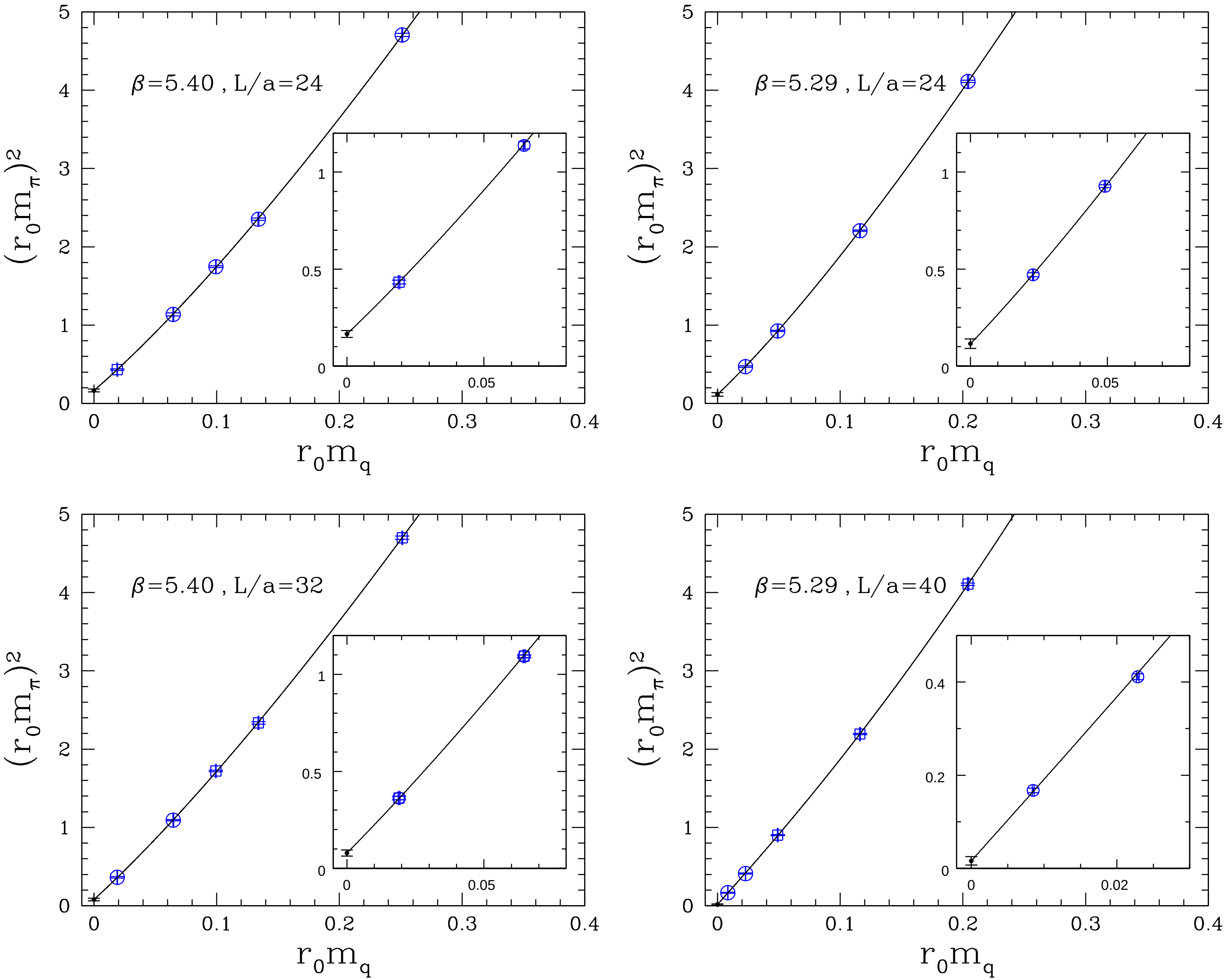
$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}(x)\right)$$

$$\begin{aligned} S_F = \sum_x \Big\{ & \bar\psi(x)\psi(x) - \kappa\,\bar\psi(x)U_\mu^\dagger(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) \\ & - \kappa\,\bar\psi(x)U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\, c_{SW}\, g\,\bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)\Big\} \end{aligned}$$

Clover Fermions

$$\boxed{N_f = 2}$$

Acceptable Volumes



δ Regime $\quad m_\pi L \ll 1\,,\ L \ll T$

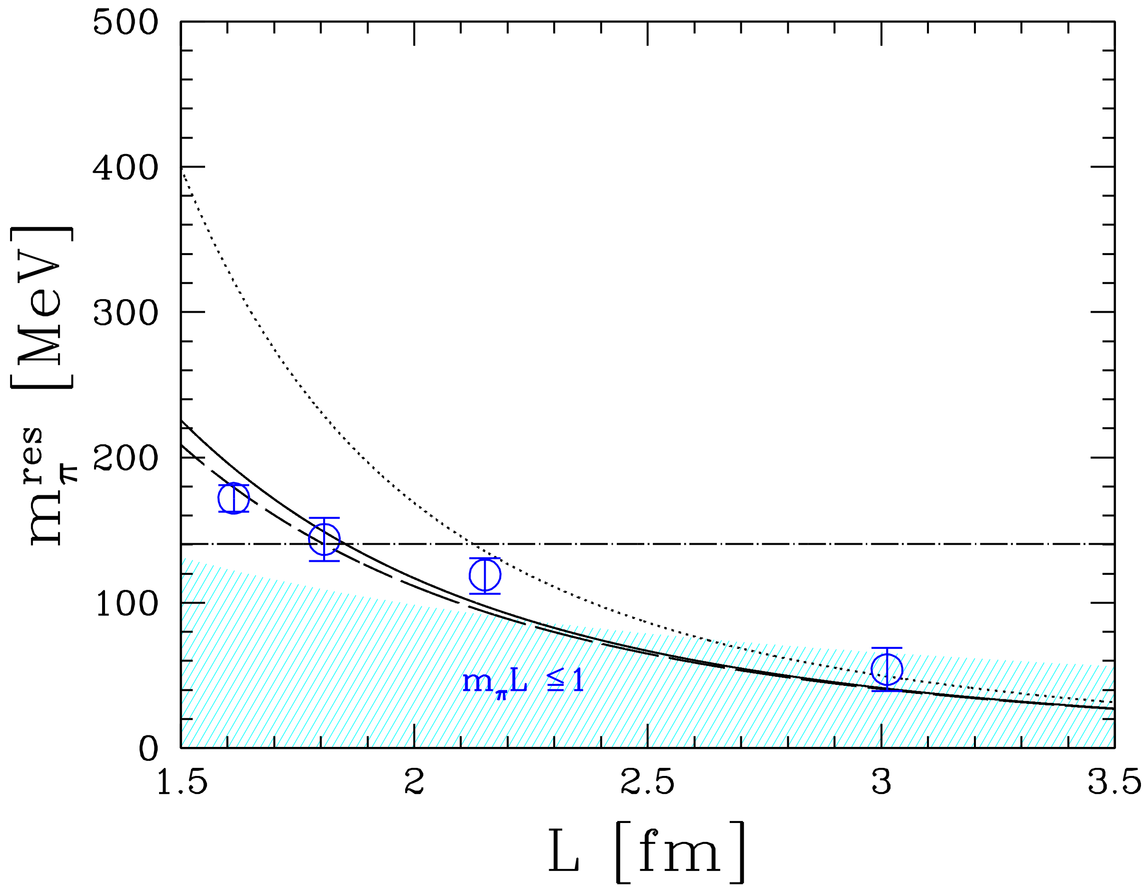
$$m_\pi^{\text{res}} = \frac{3}{2F_\pi^2 L^3 (1+\Delta)}$$

with

$$\Delta = \frac{2}{F_\pi^2 L^2}\, 0.2257849591 + \frac{1}{F_\pi^4 L^4}\left[0.088431628 - \frac{0.8375369106}{3\pi^2}\left(\frac{1}{4}\ln{(\Lambda_1 L)}^2 + \ln{(\Lambda_2 L)}^2\right)\right]$$

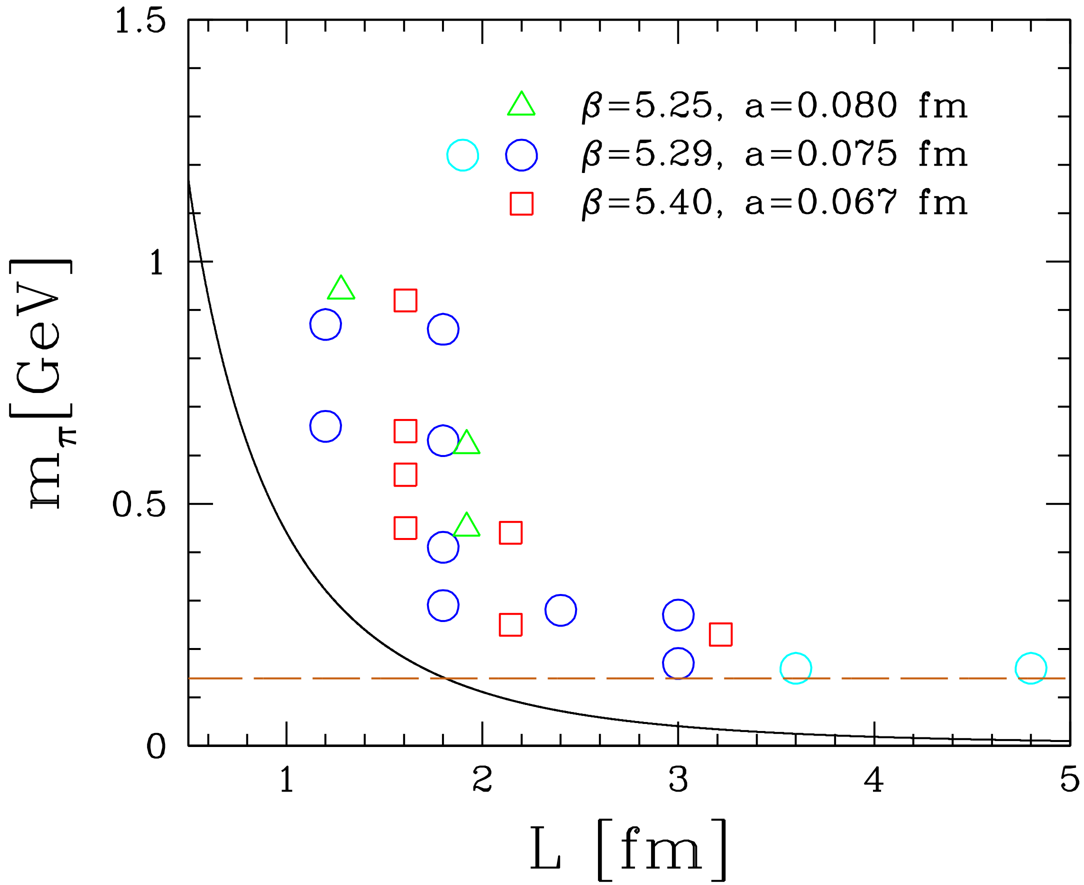
Leutwyler, Niedermayer & Hasenfratz

Residual Mass



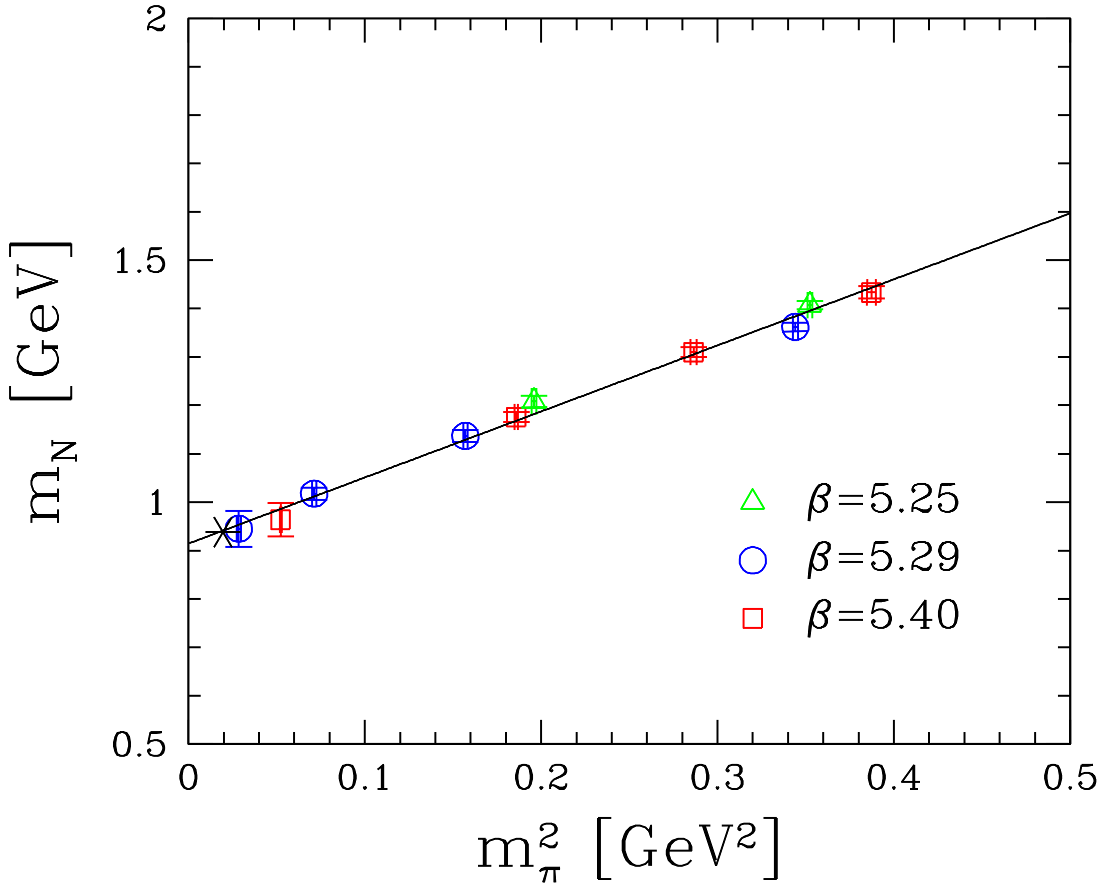
$$F_0 = F_\pi|_{m\pi=0} = 78^{+14}_{-10}\,\text{MeV}$$

Landscape



β=5.25, a=0.080 fm
β=5.29, a=0.075 fm
β=5.40, a=0.067 fm
1.5
1
0.5
0
1
2
3
4
5
m_π[GeV]
L [fm]

Scale



$$r^2 \left.\frac{\partial V(r)}{\partial r}\right|_{r=r_0} = 1.65 \qquad r_0 = 0.47(1)\ \text{fm}$$

Resonances on the Lattice

Continuum

$$T = K + K\ G\ T$$

$$t_l =\ k_l + k_l\ ip\ t_l$$

$$\det\left[\tan\delta_l - p\,k_l\right] = 0$$

Euclidean lattice

$$T^{\rm lat} = K + K\ \ G^{\rm lat}\,T^{\rm lat}$$

$$t_l^{\rm lat} =\ k_l + k_l\ \underbrace{pM}_{H(4)}\ \ t_l^{\rm lat}$$

$\uparrow$ pole on-shell

$$\det\left[\tan\delta_l\,M - 1\right] = 0$$

$$M = M(W, L)$$

Energy Levels

Correlation matrix

$$C_{ij}(t) = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_k \langle 0|\mathcal{O}_i|k\rangle \langle k|\mathcal{O}_j^\dagger|0\rangle \, \mathrm{e}^{-tW_k} \, , \quad \mathcal{O} = q\Gamma\bar{q} \, , \; q\Gamma\bar{q}\, q\Gamma\bar{q} \, , \cdots$$

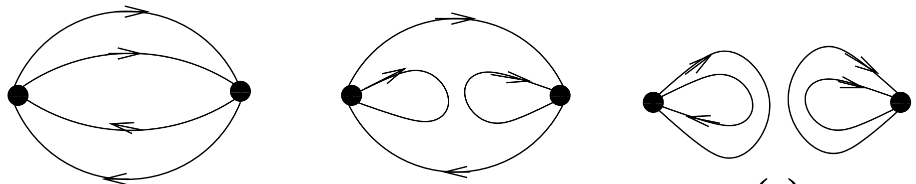
Energies W_k are obtained from generalized eigenvalue equation

$$C_{ij}(t)\, v_j = \lambda(t, t_0)\, C(t_0)_{ij}\, v_j$$

with eigenvalues

$$\lambda_k(t, t_0) = \mathrm{e}^{-(t-t_0)W_k} \Big(1 + O(\mathrm{e}^{-(t-t_0)\Delta W_k})\Big)$$

Lüscher & Wolff



(a) (b) (c)

Rho

$$W = \sqrt{\mathbf{P}^2 + E^2} \qquad \mathbf{P} = \frac{2\pi}{L}\mathbf{d}\,, \quad \mathbf{d} \in \mathbb{Z}^3$$

$$E = 2\sqrt{k^2 + m_\pi^2}$$

Consider most general case

$$\mathbf{d} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{d} = d\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{d} = \frac{d}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{d} = \frac{d}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

↑ Lüscher

↑ Gottlieb & Rummukainen

For general partial waves l

$$M^{\mathbf{d}}_{lm,l'm'}(q) = \gamma^{-1}\frac{(-1)^l}{\pi^{3/2}} \sum_{j=|l-l'|}^{l+l'} \sum_{s=-j}^{j} \frac{i^j}{q^{j+1}} \mathcal{Z}^{\mathbf{d}}_{js}(1;q^2)^* C_{lm,js,l'm'} \qquad q = \frac{kL}{2\pi}, \quad \gamma = \frac{W}{E}$$

Phase

$$\det\left[\tan\delta_{lI}\, M - 1\right] = 0$$

Notation

$$w_{lm} = \gamma^{-1}\frac{\pi^{-3/2}}{\sqrt{2l+1}q^{l+1}}\mathcal{Z}^{\mathbf{d}}_{lm}(1;q^2)^*$$

$$\mathcal{Z}^{\mathbf{d}}_{lm}(s;q^2) = \sum_{\mathbf{r}\in P_{\mathbf{d}}} \frac{\mathcal{Y}_{lm}(\mathbf{r})}{(\mathbf{r}^2-q^2)^s} \quad s \to 1$$

$$P_{\mathbf{d}} = \left\{ \mathbf{r} \middle| \mathbf{r} = \left(\frac{1}{\gamma}-1\right)\frac{\mathbf{n}\mathbf{d}}{d^2}\mathbf{d} + \frac{1}{2\gamma}\mathbf{d} + \mathbf{n}, \mathbf{n} \in \mathbb{Z}^3 \right\}$$

$\mathbf{d}/\lvert\mathbf{d}\rvert$	M
$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} w_{00} & 0 & 0 \\ 0 & w_{00} & 0 \\ 0 & 0 & w_{00} \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} w_{00} - w_{20} & 0 & 0 \\ 0 & w_{00} + 2w_{20} & 0 \\ 0 & 0 & w_{00} - w_{20} \end{pmatrix}$
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} w_{00} - w_{20} & 0 & \sqrt{6}w_{22} \\ 0 & w_{00} + 2w_{20} & 0 \\ -\sqrt{6}w_{22} & 0 & w_{00} - w_{20} \end{pmatrix}$
$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} w_{00} & \sqrt{6}w_{22}e^{-i\pi/4} & \sqrt{6}w_{22} \\ -\sqrt{6}w_{22}e^{i\pi/4} & w_{00} & -\sqrt{6}w_{22}e^{-i\pi/4} \\ -\sqrt{6}w_{22} & \sqrt{6}w_{22}e^{i\pi/4} & w_{00} \end{pmatrix}$

$\mathbf{d}/\lvert\mathbf{d}\rvert$	$H(4)$	Spin	$\cot\delta_{11}$
$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	A_1^+		w_{00}
$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	E^-	$\parallel$	$w_{00} + 2w_{20}$
	A_2^-	$\perp$	$w_{00} - w_{20}$
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	B_2^+	$\parallel$	$w_{00} - w_{20} + i\sqrt{6}w_{22}$
	B_1^+	$\perp$	$w_{00} - w_{20} - i\sqrt{6}w_{22}$
	E^+	$\perp$	$w_{00} + 2w_{20}$
$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	B_1^-	$\parallel$	$w_{00} + 2i\sqrt{6}w_{22}$
	B_2^-	$\perp$	$w_{00} - i\sqrt{6}w_{22}$

Hypothetical Energy Levels

Effective range formula

$$\frac{k^3}{E} \cot\, \delta_{11}(k) = \frac{24\pi}{g^2_{\rho\pi\pi}} \left(k_\rho^2 - k^2\right) \qquad E = 2\sqrt{k^2 + m_\pi^2}\,, \quad k_\rho = \frac{1}{2}\sqrt{m_\rho^2 - 4m_\pi^2}$$

$$\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k_\rho^3}{m_\rho^2}$$

Experimentally: $\Gamma_\rho = 146$ MeV $\Longrightarrow$ $g_{\rho\pi\pi} = 5.9$

Free case

$$k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n} \in \mathbb{N}^3$$

$$\frac{E}{m_\pi} = 2\sqrt{1 + \frac{(2\pi\vec{n})^2}{(m_\pi L)^2}}$$

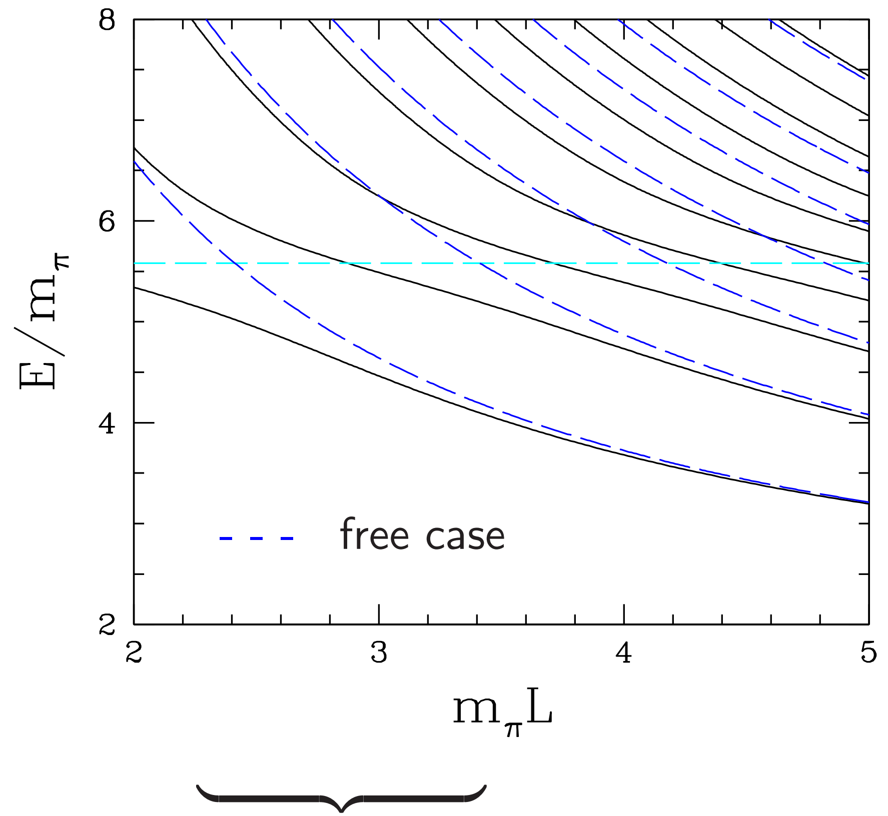
Interacting case $\boxed{\vec{P} = 0}$

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}\right\} \bmod \pi\,, \quad q = \frac{kL}{2\pi}$$

Lüscher

$\vec{P} = 0$

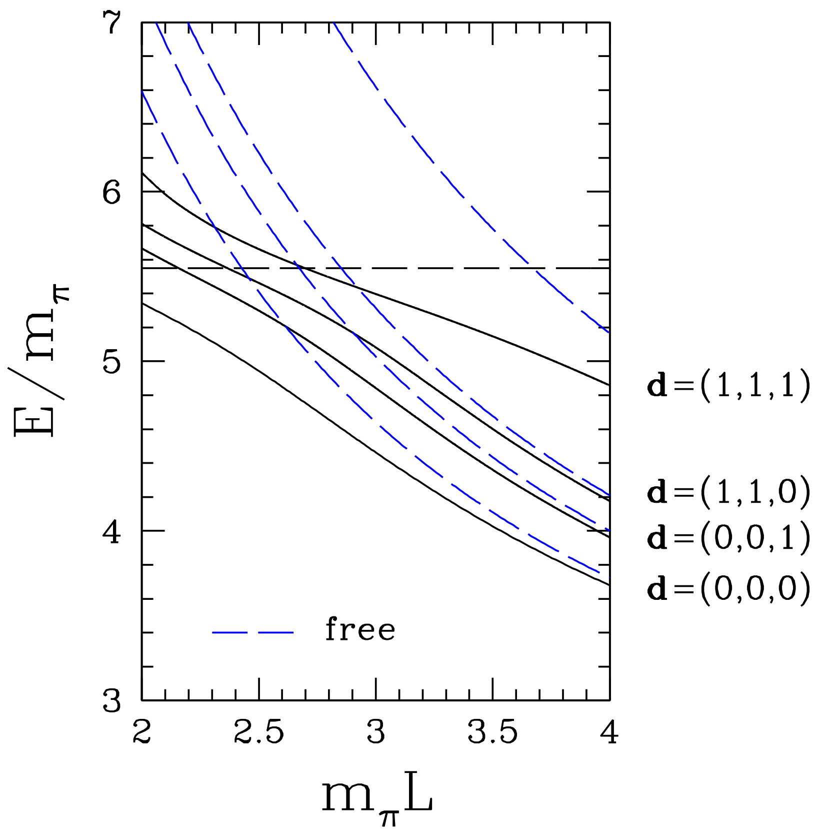
Physical m_π, m_ρ and Γ_ρ



Useful region

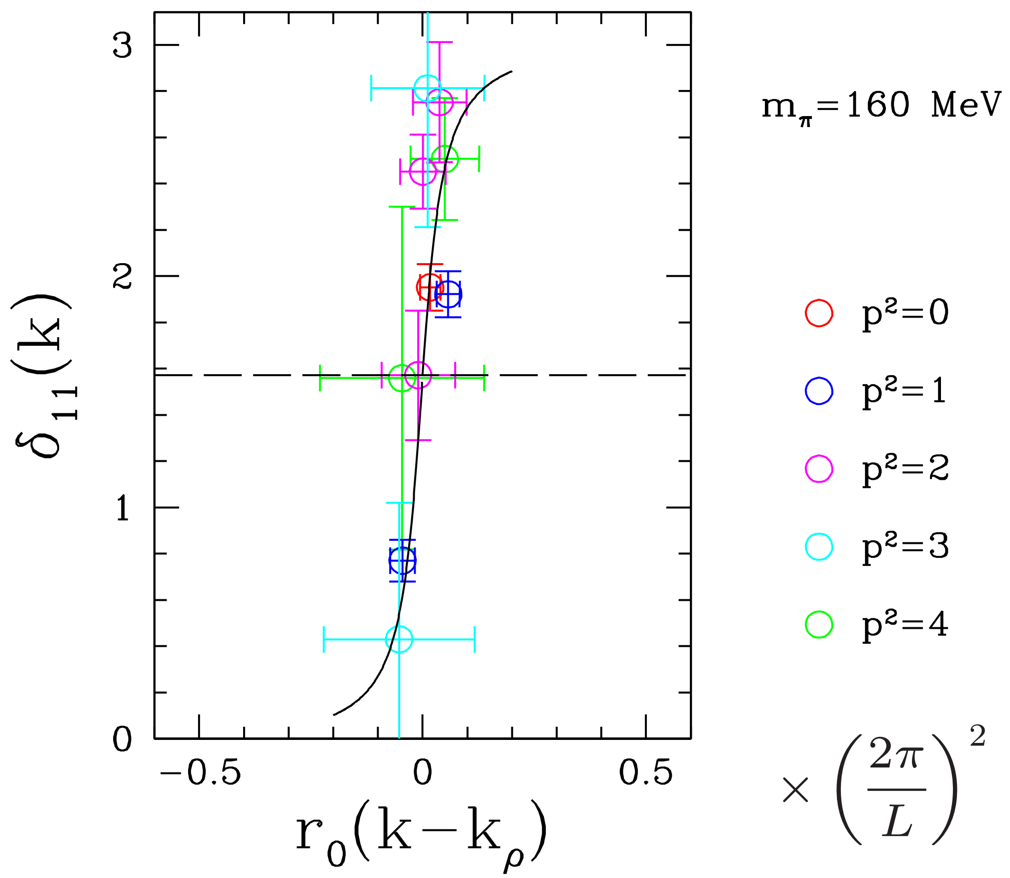
$\vec{P} \neq 0$

Physical m_π, m_ρ and Γ_ρ

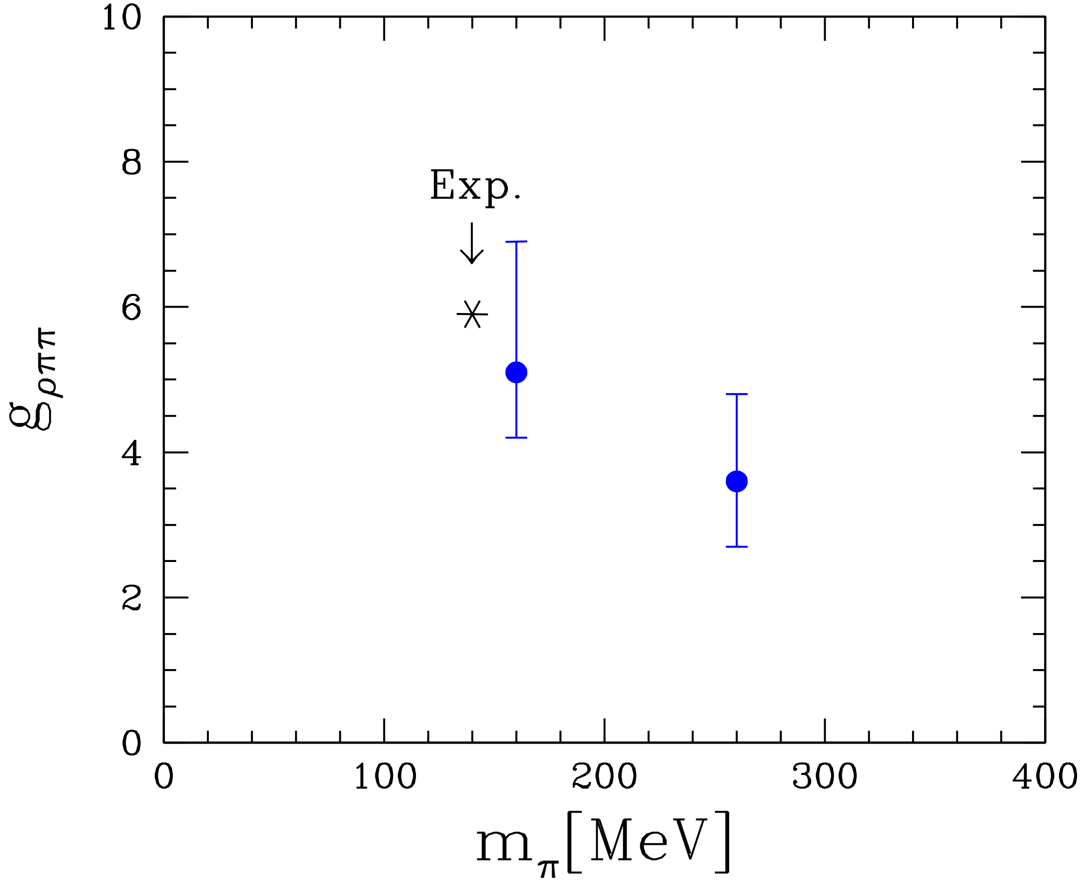


Phases

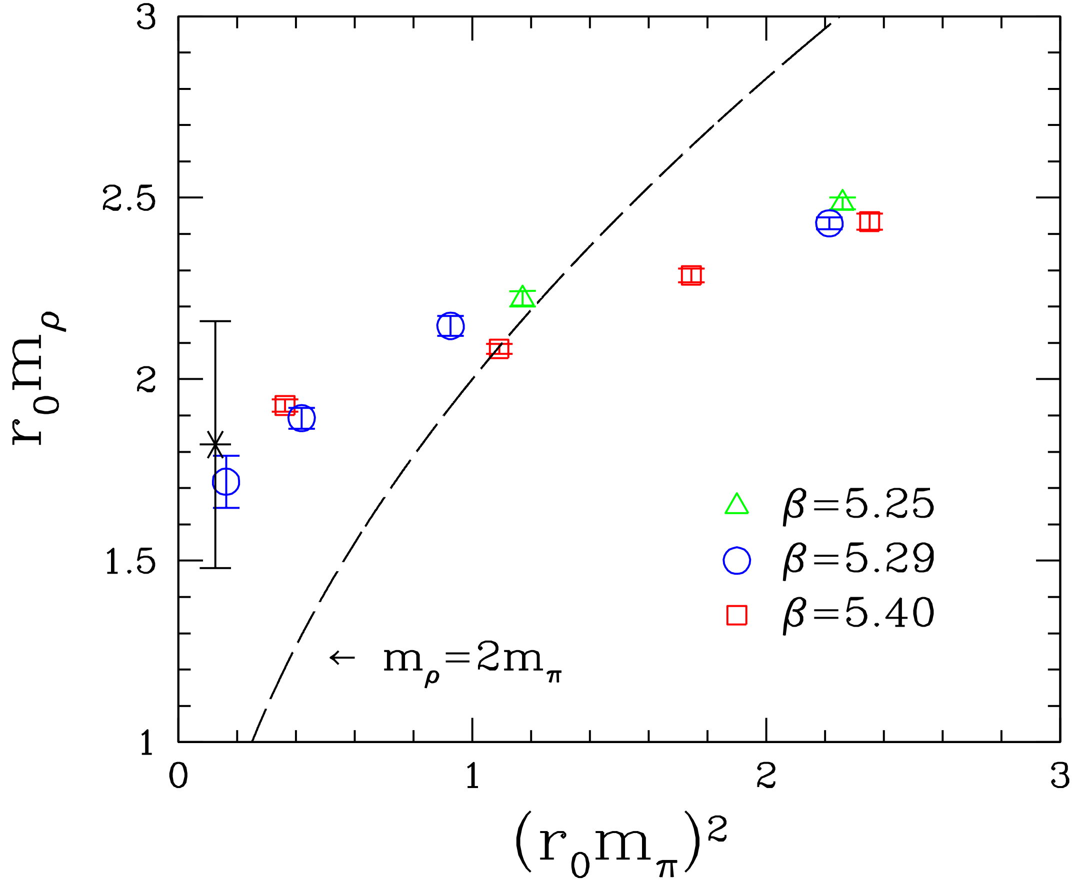
Preliminary



Width



Mass



3
2.5
2
1.5
1
0
1
2
3
$r_0 m_\rho$
$(r_0 m_\pi)^2$
$\leftarrow$ $m_\rho = 2m_\pi$
β=5.25
β=5.29
β=5.40

Delta

Effective range formula

$$\frac{k^3}{E} \cot \delta_{3/2\,1}(k) = \frac{24\pi}{g^2_{\Delta N\pi}} \left(m^2_\Delta - E^2\right)$$

Here

$$E = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_N^2}\,, \quad m_\Delta = \sqrt{k_\Delta^2 + m_\pi^2} + \sqrt{k_\Delta^2 + m_N^2}$$

$$\Gamma_\Delta = \frac{g^2_{\Delta N\pi}}{6\pi} \frac{k_\Delta^3}{m_\Delta^2}$$

Experimentally: $\Gamma_\Delta = 118\,\text{MeV} \quad \Longrightarrow \quad \frac{g^2_{\Delta N\pi}}{4\pi} = 14.6$

Free case

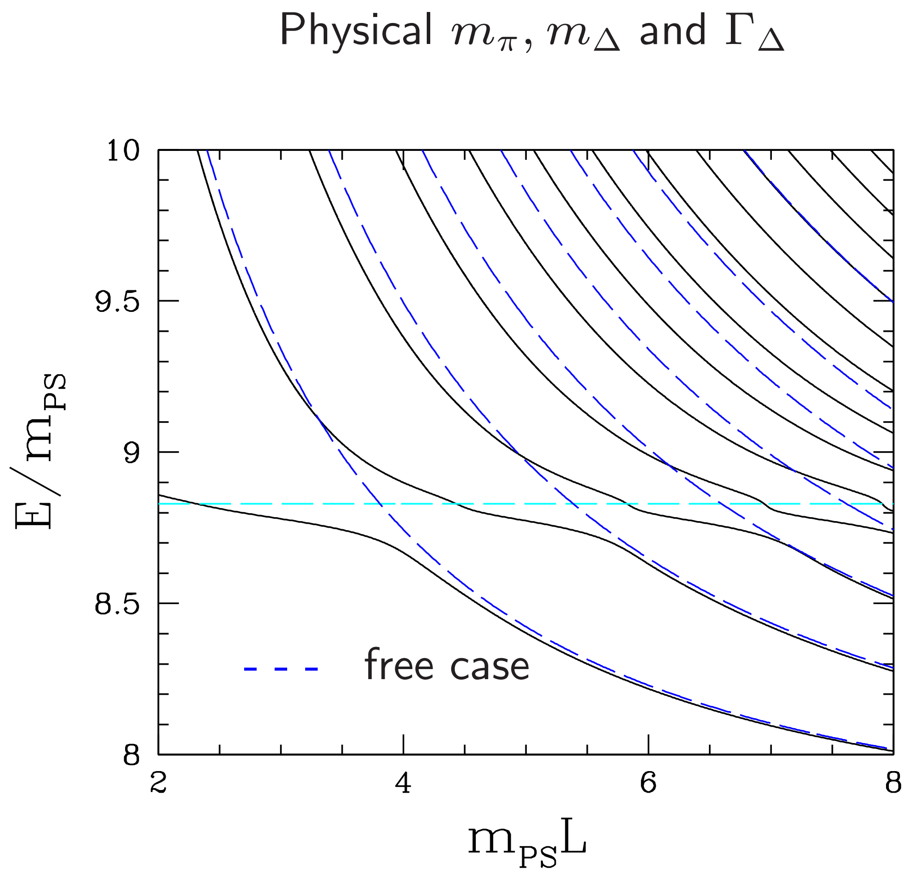
Interacting case $\boxed{\vec{P} = 0}$

$$k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n} \in \mathbb{N}^3$$

$$\delta_{3/2\,1}(k) = \arctan\left\{\frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}\right\} \bmod \pi\,, \quad q = \frac{kL}{2\pi}$$

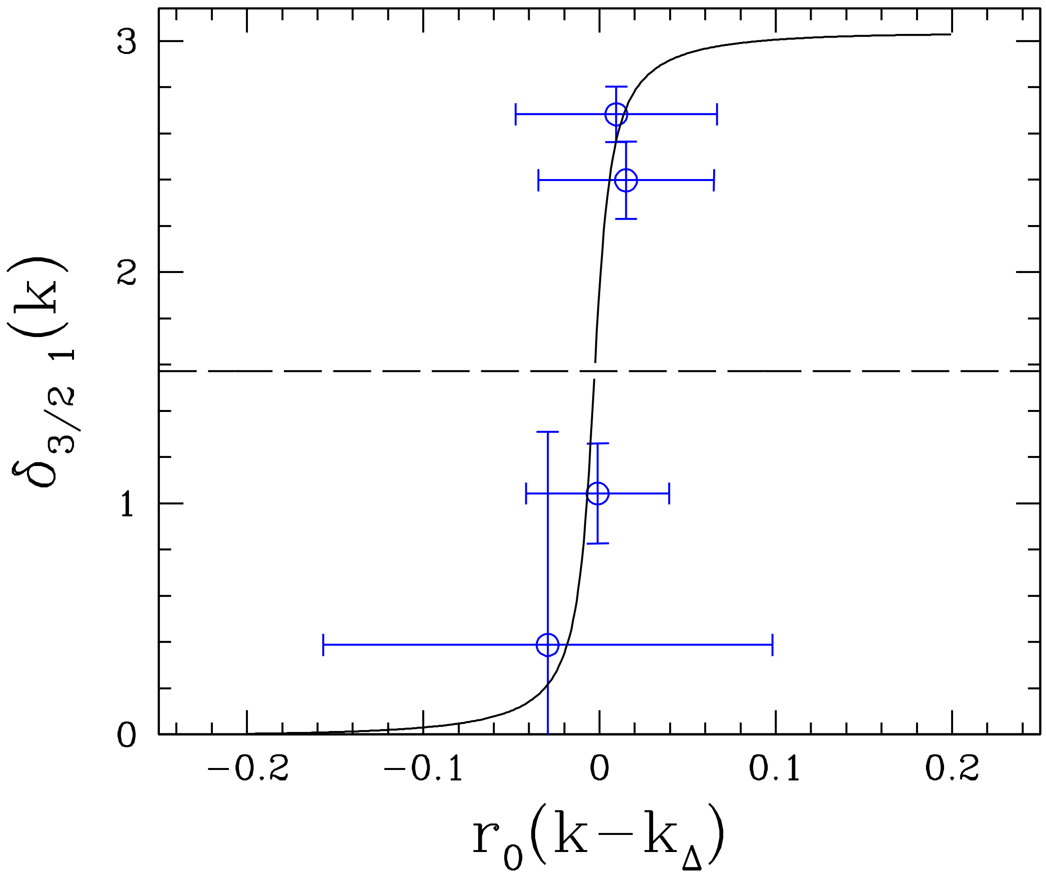
Bernard et al.

Hypothetical Energy Levels



Phase

Very preliminary



Summary

- Simulations at the physical pion mass with Wilson-type fermions progressing
  - Improvement of algorithms
  - Increase of computing power
  - QPACE

- Benchmark calculation of ρ resonance parameters successful

  Precision of the calculation largely question of statistics

  Ideal volume: $m_\pi L = 2-4$

  $\approx 3-6$ fm

- Calculation of Δ resonance parameters will follow shortly

  Lowest energy level E sufficient

- Work on scalar resonances in progress. For $N(1440)$ and $N^*(1535)$ matching with energy levels computed from HBChPT in finite volume might be advantageous

  Meißner et al.